General Physics II at EPFL

2018-2019 SS: Lecture; Wed 17:15-19:00 and Thu 8:15-9:00, Special session; 9:15-10:00

Exercise; Thu 10:15-12:00

Special Relativity (1st week)

Galilean-Newtonian Relativity

Inertial Frames: frames where Newton's first law is valid (frame is where observer sits)

- Object at rest remains at rest
- Object moving with a constant velocity keep moving with a constant velocity

Figure 1

Case I: Object and observer are in the train and at rest

Velocity of the object seen by the observer: $\vec{u} = (0,\ 0,\ 0)$

Case II: Object in the train moving with a constant velocity $\vec{U} = (0,\ 0,\ 10\ \text{m/sec})$

Velocity of the object seen by the observer outside of the train: $\vec{u}' = (0,\ 0,\ 10\ \text{m/sec}) = \vec{u} + \vec{U}$

Case III: Train is on the Earth moving with a constant velocity $\vec{V} = (0,\ 0,\ 250\ \text{m/sec})$

Velocity of the train $\vec{U}' = (0,\ 0,\ 260\ \text{m/sec}) = \vec{U} + \vec{V}$

Velocity of the object seen by the observer outside of the earth:

$$\vec{u}'' = \vec{u}' + \vec{V} = \vec{u} + \vec{U} + \vec{V} = (0,\ 0,\ 260\ \text{m/sec})$$

Assumed: **distance and time interval unchanged for Case I → Case II → Case III**

For all three cases, Newton's first low is valid

Any frames moving with constant velocities respect another inertial frame are also inertial frames.

There is no absolute inertial frame = Galilean-Newtonian relativity

NB: The velocities of an object are different from one frame to another frame, but the accelerations are the same.

Case I: Object accelerated from $\vec{u}_1 = (0,\ 0,\ 0)$ to $\vec{u}_2 = (0,\ 0,\ 2\ \text{m/sec})$ in $\Delta t = 1\ \text{sec}$. Acceleration is $\vec{a} = (\vec{u}_2 - \vec{u}_1)/\Delta t = (0,\ 0,\ 2\ \text{m/sec}^2)$

Then move to Case II

Object accelerated from $\vec{u}_1' = \vec{u}_1 + \vec{U} = (0,\ 0,\ 10\ \text{m/sec})$ to $\vec{u}_2' = \vec{u}_2 + \vec{U} = (0,\ 0,\ 12\ \text{m/sec})$ in $\Delta t = 1\ \text{sec}$. Acceleration is $\vec{a}' = (\vec{u}_2' - \vec{u}_1')/\Delta t = (0,\ 0,\ 2\ \text{m/sec}^2) = \vec{a}$.

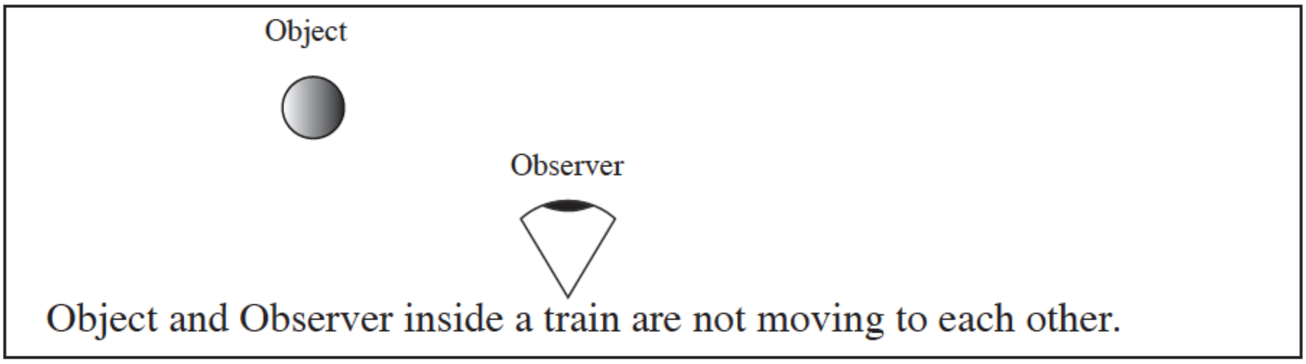
Acceleration is unchanged.

Intuitively, mass and force would be unchanged between different inertial frames. Since the acceleration is also unchanged, $\vec{F} = m\vec{a}$ is valid in all the inertial frames.

Move to more general definition of the Galilean-Newtonian relativity, i.e. in all the inertial frames, laws of physics are unchanged, three Newton's laws and the laws of gravity.

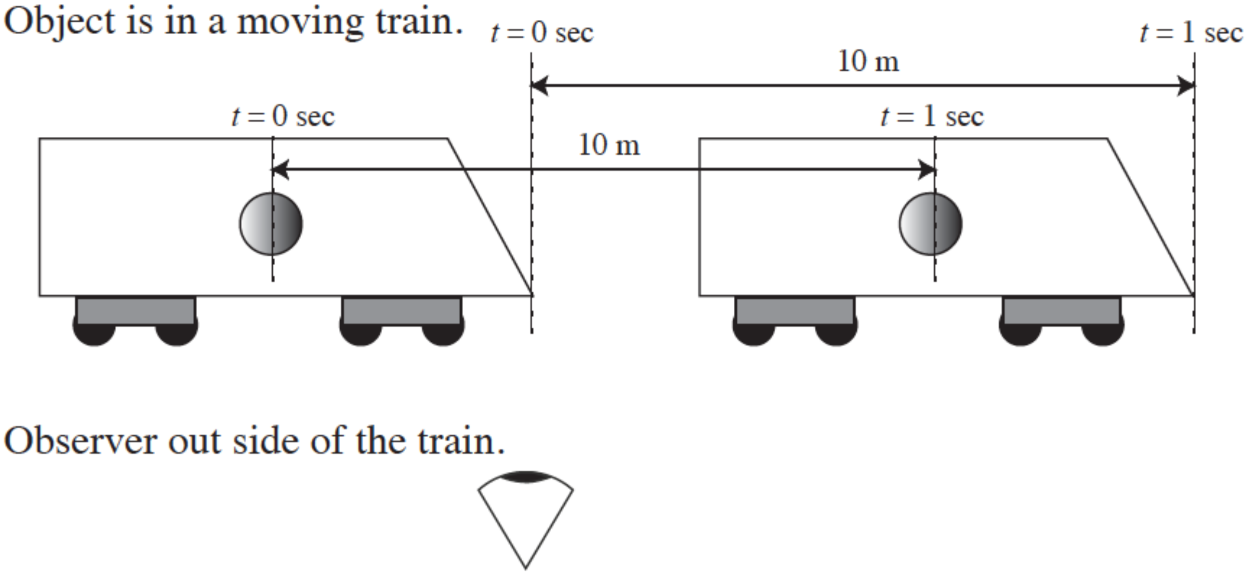
Case I': Object at $\vec{r}_0 = (0,\ 0,\ 0)$ with a velocity $\vec{u}_0 = (0,\ 0,\ 0)$, at $t = 0$, starts to fall with gravitation with a constant acceleration toward the z-direction, i.e. $\vec{a} = (0,\ 0,\ 9\ \text{m/sec}^2)$.

Case II': Object is in the train moving with a constant velocity



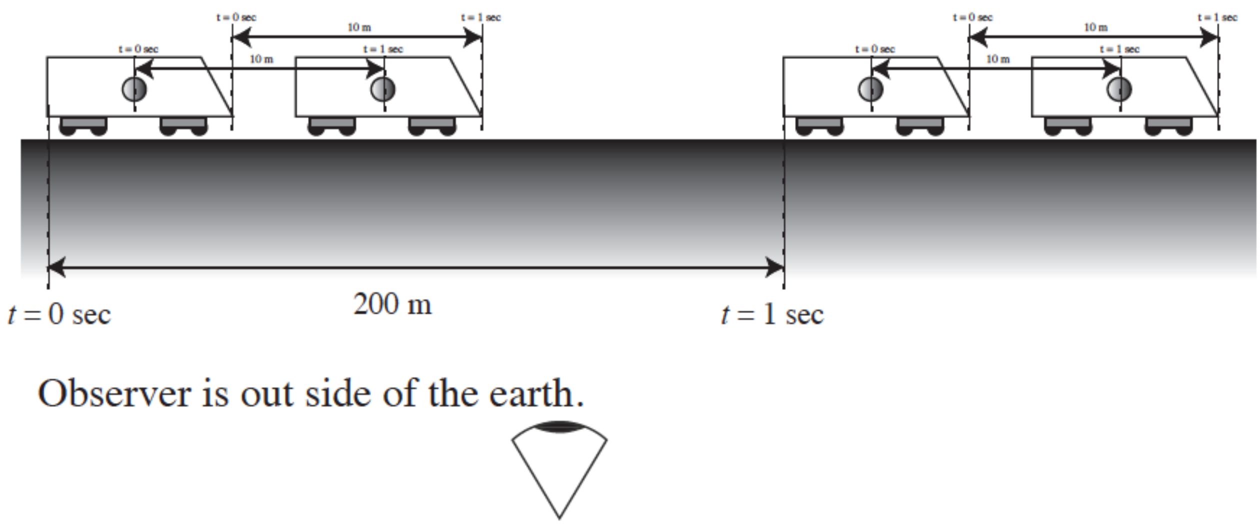
Object and Observer inside a train are not moving to each other.

Object is in a moving train.



Observer out side of the train.

Object is in a moving train on the moving earth.



Observer is out side of the earth.

Postulation of the Special Relativity Theory

1. The laws of physics have the same form in all inertial frames.
2. Light propagates through the vacuum with a definite speed c independent of the speed of the source or observer: → Information cannot propagate faster than c.

Simultaneity

Event: something that happens at a particular place at a particular time:
Two events $E_1(\vec{r}_1, t_1)$, $E_2(\vec{r}_2, t_2)$ are simultaneous if $t_1 = t_2$.
This can be easily examined if the two events occur at the same place ($\vec{r}_1 = \vec{r}_2$). If not, how can we know whether the two events are simultaneous?

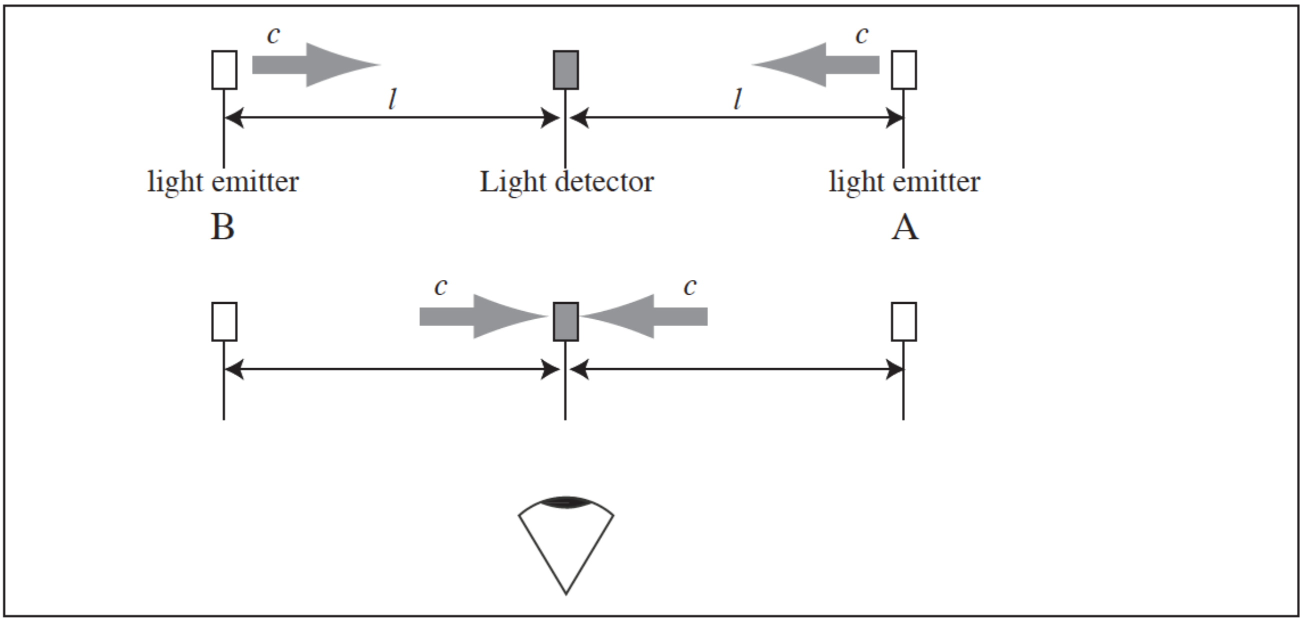
1) Light was emitted from the emitter A and B at t_0^{A} and t_0^{B} respectively and a light detection system is in the half way between A to B with a distance l from the each. An observer is standing at next to the detection system. The arrival times of the light from A and B are given by t^{A} and t^{B}, respectively:

$$t^{\mathrm{A}} - t_0^{\mathrm{A}} = \frac{l}{c},\ t^{\mathrm{B}} - t_0^{\mathrm{B}} = \frac{l}{c}$$

It follows

$$t^{\mathrm{A}} - t_0^{\mathrm{A}} = t^{\mathrm{B}} - t_0^{\mathrm{B}} \Rightarrow t_0^{\mathrm{A}} - t_0^{\mathrm{B}} = t^{\mathrm{A}} - t^{\mathrm{B}}$$

Thus if the two lights arrives at the same time, i.e. $t^{\mathrm{A}} = t^{\mathrm{B}}$, $t_0^{\mathrm{A}} = t_0^{\mathrm{B}}$ and two events happened simultaneously.



2) The whole set-up is in the train moving a constant speed and an observer is outside of the train watching the train and set-up moving. The light was emitted from A and B simultaneously at $\tilde{t}_0^{\mathrm{A}} = \tilde{t}_0^{\mathrm{B}} = 0$: by denoting $\tilde{t}^{\mathrm{A}}$ and $\tilde{t}^{\mathrm{B}}$ as arrival times of A and B seen by the outside observer, we have

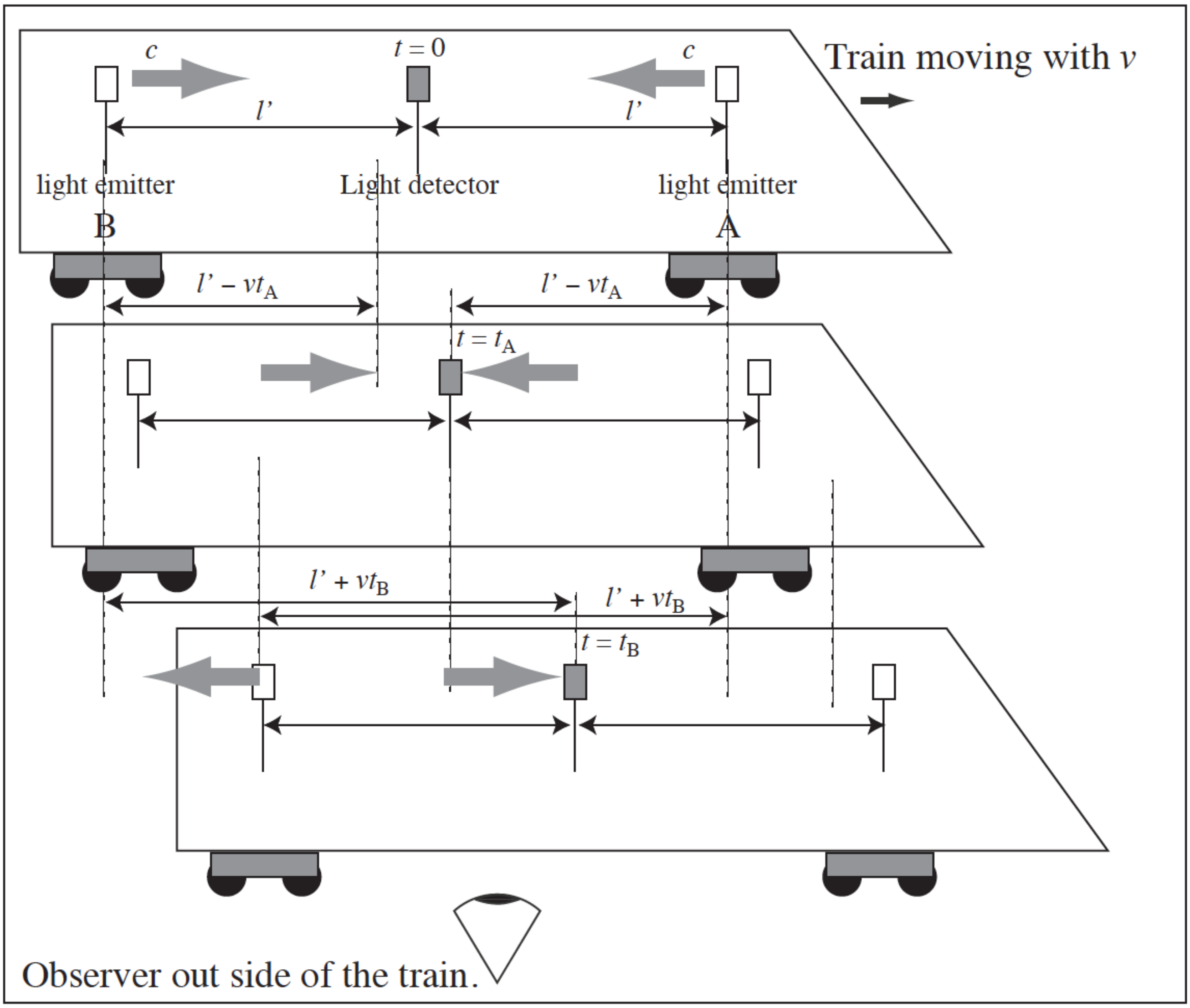
$$\tilde{t}^{\mathrm{A}} = \frac{l' - v\tilde{t}^{\mathrm{A}}}{c},\ \tilde{t}^{\mathrm{B}} = \frac{l' + v\tilde{t}^{\mathrm{B}}}{c}$$

Note that the length between the light emitters and the detector might be different between the two observers. It follows that

$$\tilde{t}^{\mathrm{A}} = \frac{l'}{c+v},\ \tilde{t}^{\mathrm{B}} = \frac{l'}{c-v}$$

thus $\tilde{t}^{\mathrm{A}} \neq \tilde{t}^{\mathrm{B}}$, while the observer on the train sees $t^{\mathrm{A}} = t^{\mathrm{B}}$.

If it were a ball with a velocity, u, the observer outside of the train sees the velocity of the ball moving from A to the detector becoming $u-v$, and for the ball from B, $u+v$. In this case, $t^{\mathrm{A}} = t^{\mathrm{B}} = l'/u$ (however not l/u).

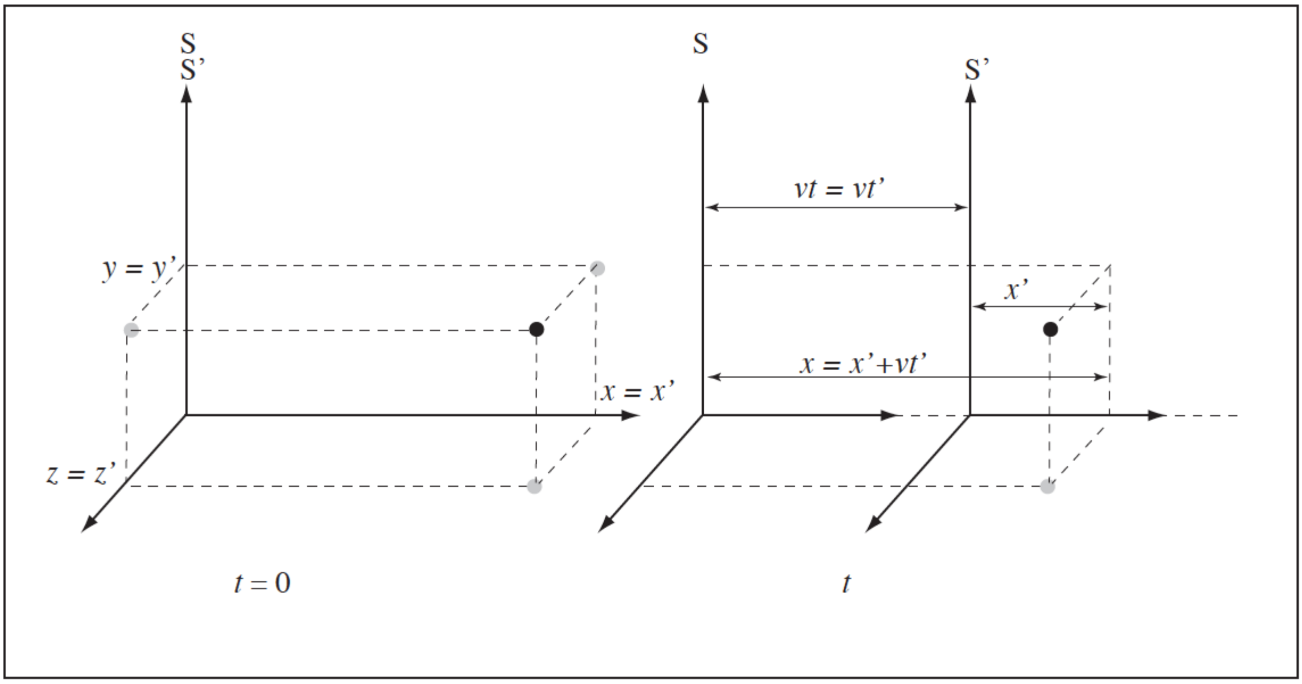


Galilean and Lorentz transformation

A frame is defined by a coordinate system: e.g. a Cartesian coordinate system characterised by a set of three coordinate axes, x, y, and z, which are orthogonal to each other, describing our three dimensional space. Every inertial frame has its own coordinate systems and space points can be specified in different inertial frames.

Let us consider two inertial frames, S with its coordinate system (x, y, z) and S' with (x', y', z'). At $t = 0$, the two systems overlap each other, and S' moves with a constant velocity, v, in a positive x direction. One considers that S' corresponds to an observer in the train watching a object, while S corresponds to an observer outside of the train watching both observer and object moving together with the train with a constant velocity, v.

Galilean transformation



An event is defined in S' by the time and space coordinate (t', x', y', z'), The same event is defined in S as (t, x, y, z) where

$$t = t'$$

$$x = x' + vt' = x' + vt$$

$$y = y'$$

$$z = z'$$

called Galilean transformation. In linear algebra, we have

$$\begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ v & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix}.$$

In a more general form:

$$\begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ v_x & 1 & 0 & 0 \\ v_y & 0 & 1 & 0 \\ v_z & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix}$$

or more simply

$$\begin{pmatrix} t \\ \vec{r} \end{pmatrix} = \begin{pmatrix} 1 & \vec{0}^{\mathrm{T}} \\ \vec{v} & I \end{pmatrix} \begin{pmatrix} t' \\ \vec{r}' \end{pmatrix}, \text{ where } \vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \vec{r}' = \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}, \vec{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}, \vec{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \text{ and } I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Similarly for a transformation from S to S':

$$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -v & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$$

Note that

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ v & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ -v & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ v-v & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

i.e. two transformation matrices are inverse to each other. In general

$$\begin{pmatrix} t' \\ \vec{r}' \end{pmatrix} = \begin{pmatrix} 1 & \vec{0}^{\mathrm{T}} \\ \vec{v} & I \end{pmatrix}^{-1} \begin{pmatrix} t \\ \vec{r} \end{pmatrix} = \begin{pmatrix} t' \\ \vec{r}' \end{pmatrix} = \begin{pmatrix} 1 & \vec{0}^{\mathrm{T}} \\ -\vec{v} & I \end{pmatrix} \begin{pmatrix} t \\ \vec{r} \end{pmatrix}$$

For a case where P corresponds to the space-time position of a particle, the velocity in the frame S' is given by

$$\begin{pmatrix} u'_x \\ u'_y \\ u'_z \end{pmatrix} = \begin{pmatrix} dx'/dt' \\ dy'/dt' \\ dz'/dt' \end{pmatrix}$$

Transformation to the frame S is then given by

$$u_x = \frac{dx}{dt} = \frac{d(x' + vt')}{dt'} = \frac{dx'}{dt'} + \frac{dv}{dt'}t' + v = u'_x + v, \; u_y = u'_y, \; u_z = u'_z$$

since v is constant. Therefore, we have

$$\begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} = \begin{pmatrix} u'_x + v \\ u'_y \\ u'_z \end{pmatrix}$$

and in general

$$\vec{u} = \vec{u}' + \vec{v}$$

This transformation is non relativistic, i.e. if $u = c$, $c = c + v$, which is contradiction to the special relativity principle.

Lorentz transformation

In order to obtain the relativistic transformation, let us assume that the coordinate transformation is changed as

$$\left.\begin{array}{r} x = x' + vt' \\ y = y' \\ z = z' \end{array}\right\} \Rightarrow \left\{\begin{array}{l} x = \gamma(x' + vt') \\ y = y' \\ z = z' \end{array}\right.$$

where γ is still to be determined. In the non-relativistic limit, γ=1. From the relativity principle, transformation from S to S' should be given by replacing v to $-v$

$$x' = \gamma(x - vt)$$

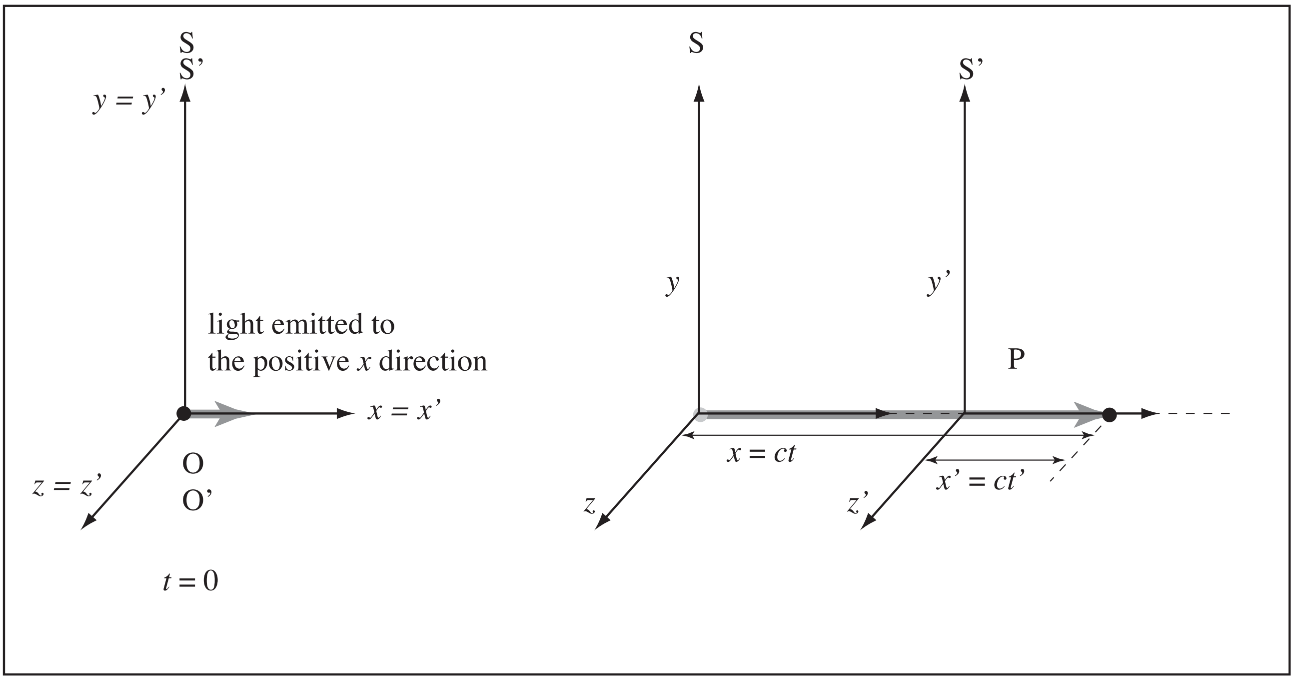
$$y' = y$$

$$z' = z$$

which is a general transformation rule.

For the determination of γ, we consider a special case where light is emitted along the x (x') direction at $t = t' = 0$, from the origin of S, which identical to the origin of S'. In the S' frame, an event where the light arrives at t' to the Point P is given by an event (t', ct', 0, 0). In the S frame, this event is given by (t, ct, 0, 0) and the transformation from S' is given by

$$x = ct = \gamma(x' + vt') = \gamma(ct' + vt') = \gamma(c + v)t'$$



From the inverse transformation, S to S' is given by

$$x' = ct' = \gamma(x - vt) = \gamma(ct - vt) = \gamma(c - v)t$$

It follows that

$$t' = \frac{\gamma(c - v)t}{c}$$

thus

$$ct = \gamma(c + v)t' = \frac{\gamma^2(c + v)(c - v)t}{c} = \gamma^2 \frac{(c^2 - v^2)t}{c}$$

$$\gamma^2 = \frac{c^2}{(c^2 - v^2)} = \frac{1}{1 - (v/c)^2}$$

$$\gamma = \frac{1}{\sqrt{1 - (v/c)^2}}$$

The γ determined here is valid for a general case where the transformation is given by

$$\begin{cases} x = \gamma(x' + vt') \\ y = y' \\ z = z' \end{cases} \quad \text{and} \quad \begin{cases} x' = \gamma(x - vt) \\ y' = y \\ z' = z \end{cases}$$

For the transformation of t' to t **for the general case**, using $x' = \gamma(x - vt)$, it follows that

$$t = \frac{x}{v} - \frac{x'}{v\gamma}$$

and using $x = \gamma(x' + vt')$

$$t = \frac{\gamma}{v}(x' + vt') - \frac{x'}{v\gamma} = \gamma\left[t' + \left(1 - \frac{1}{\gamma^2}\right)\frac{x'}{v}\right] = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

In summary, we obtain **the following general transformation law** for x, y, z and t

$$\begin{cases} x = \gamma\left(x' + vt'\right) \\ y = y' \\ z = z' \\ t = \gamma\left(t' + \beta\dfrac{x'}{c}\right) \end{cases}$$

where

$$\beta = \frac{v}{c},\ \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

And in a matrix form

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix}$$

This is called Lorentz transformation. The inverse transformation is given by

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

Note that

$$\begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \gamma^2\left(1-\beta^2\right) & -\gamma^2\beta + \gamma^2\beta & 0 & 0 \\ \gamma^2\beta - \gamma^2\beta & \gamma^2\left(1-\beta^2\right) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

NB: for more general transformation is given by

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta_x & \gamma\beta_y & \gamma\beta_z \\ \gamma\beta_x & 1 + (\gamma-1)\dfrac{\beta_x{}^2}{\beta^2} & (\gamma-1)\dfrac{\beta_x\beta_y}{\beta^2} & (\gamma-1)\dfrac{\beta_x\beta_z}{\beta^2} \\ \gamma\beta_y & (\gamma-1)\dfrac{\beta_x\beta_y}{\beta^2} & 1 + (\gamma-1)\dfrac{\beta_y{}^2}{\beta^2} & (\gamma-1)\dfrac{\beta_y\beta_z}{\beta^2} \\ \gamma\beta_z & (\gamma-1)\dfrac{\beta_x\beta_z}{\beta^2} & (\gamma-1)\dfrac{\beta_y\beta_z}{\beta^2} & 1 + (\gamma-1)\dfrac{\beta_z{}^2}{\beta^2} \end{pmatrix} \begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix}$$

where

$$\beta_x = \frac{v_x}{c},\ \beta_y = \frac{v_y}{c},\ \beta_z = \frac{v_z}{c},\ \beta^2 = \beta_x{}^2 + \beta_y{}^2 + \beta_y{}^2$$

Four-Dimensional Space Time

In Galilean-Newtonian relativity, the time interval and the distance in space remain unchanged between the different inertial frames. However, the Lorentz transformation shows that they are no longer invariant. It also indicates that the space and time are closely related.

Relativistic Addition of Velocities

The velocity transformation in Special Relativity can be derived from the coordinate transformation,

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix}$$

e.g.

$$u_x = \frac{dx}{dt} = \frac{d}{dt}\gamma(x' + vt')$$

By noting

$$\frac{d}{dt} = \frac{d}{dt'}\frac{dt'}{dt}$$

and

$$\frac{dt}{dt'}\frac{dt'}{dt} = 1 \Rightarrow \frac{dt'}{dt} = \frac{1}{dt/dt'}$$

it follows that

$$u_x = \frac{1}{dt/dt'}\gamma\left(\frac{dx'}{dt'} + v\right) = \frac{1}{dt/dt'}\gamma(u'_x + v).$$

Using

$$\frac{dt}{dt'} = \frac{d}{dt'}\gamma\left(t' + \frac{vx'}{c^2}\right) = \gamma\left(1 + \frac{v}{c^2}u'_x\right)$$

it follows that

$$u_x = \frac{u'_x + v}{1 + vu'_x/c^2}$$

In a similar manner,

$$u_y = \frac{dy}{dt} = \frac{1}{dt/dt'}\frac{dy'}{dt'} = \frac{u'_y}{\gamma(1 + vu'_x/c^2)} = \frac{u'_y\sqrt{1 - (v/c)^2}}{1 + vu'_x/c^2}$$

$$u_z = \frac{u'_z\sqrt{1 - (v/c)^2}}{1 + vu'_x/c^2}$$

Unlike for the case of Galilean transformation, y and z components of the velocity are affected although the relative velocity between the two frames is along the x component.

When it is for the light, i.e. $\vec{u}' = (c, 0, 0)$,

$$u_x = \frac{c + v}{1 + v/c} = c\frac{c + v}{c + v} = c$$

$$u_y = u_y = 0$$

i.e. the speed of the light remains unchanged in different inertial frames.

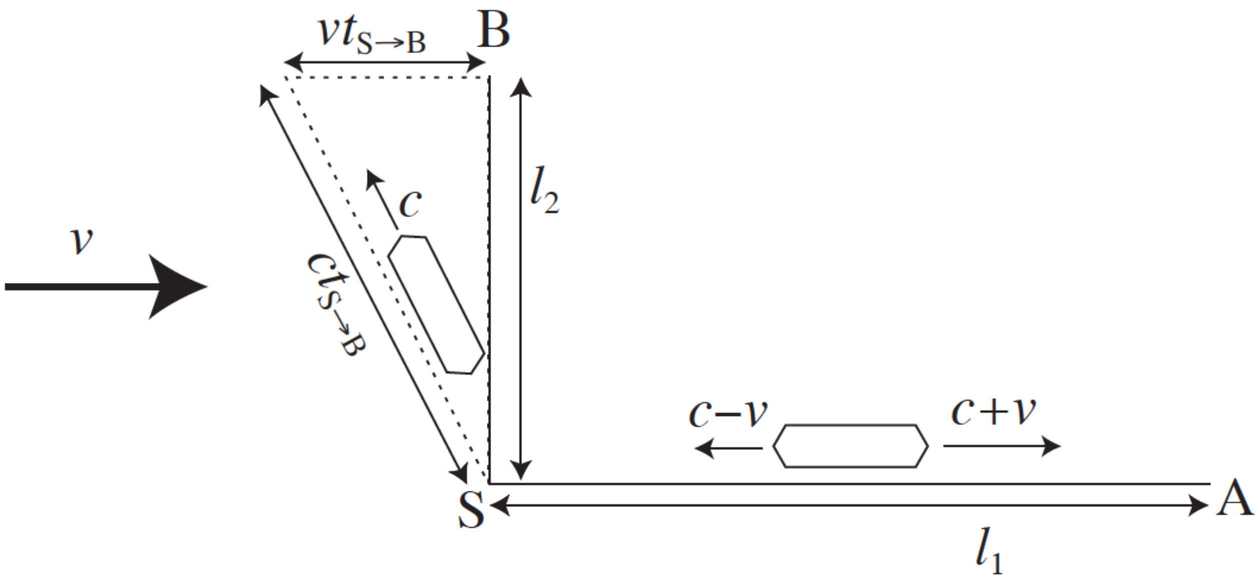
Michelson-Morley Experiment (Special session)

Maxwell's equation & theory of electromagnetism: → light is high frequency electromagnetic waves.
Speed of light in vacuum: c = 299,792,458 m / second

If the light is wave, usually we need a medium where the wave propagates.
How the light can propagate in vacuum? ⇒ an idea: vacuum is filled with Ether.
How is c defined? (Recall that the velocity is frame dependent.)

The c is measured on the earth and the earth is moving with a constant speed respect to Ether.

⇒ It should depend on the direction of the light propagation.



An example of a boat travels from the starting point S to and back with a velocity, c, to a destination A (distance S-A, l_1) that is along the direction of the stream, and B perpendicular to the direction of the stream (distance S-B, l_2). The velocity of the stream is v.

$$t_{S\to A} = \frac{l_1}{c+v},\ t_{A\to S} = \frac{l_1}{c-v}$$

$$t_{S\leftrightarrow A} = t_{S\to A} + t_{A\to S} = \frac{l_1(c-v)+l_1(c+v)}{(c+v)(c-v)} = 2\frac{l_1}{c}\frac{1}{1-(v/c)^2}$$

and

$$(ct_{S\to B})^2 = (vt_{S\to B})^2 + l_2^2,\ t_{S\to B}^2 = \frac{l_2^2}{c^2-v^2}$$

$$t_{S\to B} = \frac{l_2}{c}\frac{1}{\sqrt{1-(v/c)^2}} = t_{B\to S}$$

$$t_{S\leftrightarrow B} = 2\frac{l_2}{c}\frac{1}{\sqrt{1-(v/c)^2}}$$

The difference in the arrival times is

$$\Delta t = t_{S\leftrightarrow A} - t_{S\leftrightarrow B} = 2\frac{l_1}{c}\frac{1}{1-(v/c)^2} - 2\frac{l_2}{c}\frac{1}{\sqrt{1-(v/c)^2}}$$

We rotate the system by 90 degrees, i.e. B along the direction of stream and A perpendicular to the direction of the stream. The difference in the arrival times is given by

$$\Delta\tilde{t} = \tilde{t}_{S\leftrightarrow A} - \tilde{t}_{S\leftrightarrow B} = 2\frac{l_1}{c}\frac{1}{\sqrt{1-(v/c)^2}} - 2\frac{l_2}{c}\frac{1}{1-(v/c)^2}$$

The difference of the two is then derived to be

$$\Delta t - \Delta\tilde{t} = \left(2\frac{l_1}{c}\frac{1}{1-(v/c)^2} - 2\frac{l_2}{c}\frac{1}{\sqrt{1-(v/c)^2}}\right) - \left(2\frac{l_1}{c}\frac{1}{\sqrt{1-(v/c)^2}} - 2\frac{l_2}{c}\frac{1}{1-(v/c)^2}\right)$$

$$= 2\frac{l_1}{c}\left(\frac{1}{1-(v/c)^2} - \frac{1}{\sqrt{1-(v/c)^2}}\right) + 2\frac{l_2}{c}\left(\frac{1}{1-(v/c)^2} - \frac{1}{\sqrt{1-(v/c)^2}}\right)$$

$$= \frac{2(l_1+l_2)}{c}\left(\frac{1}{1-(v/c)^2} - \frac{1}{\sqrt{1-(v/c)^2}}\right)$$

For $(v/c)^2 << 1$

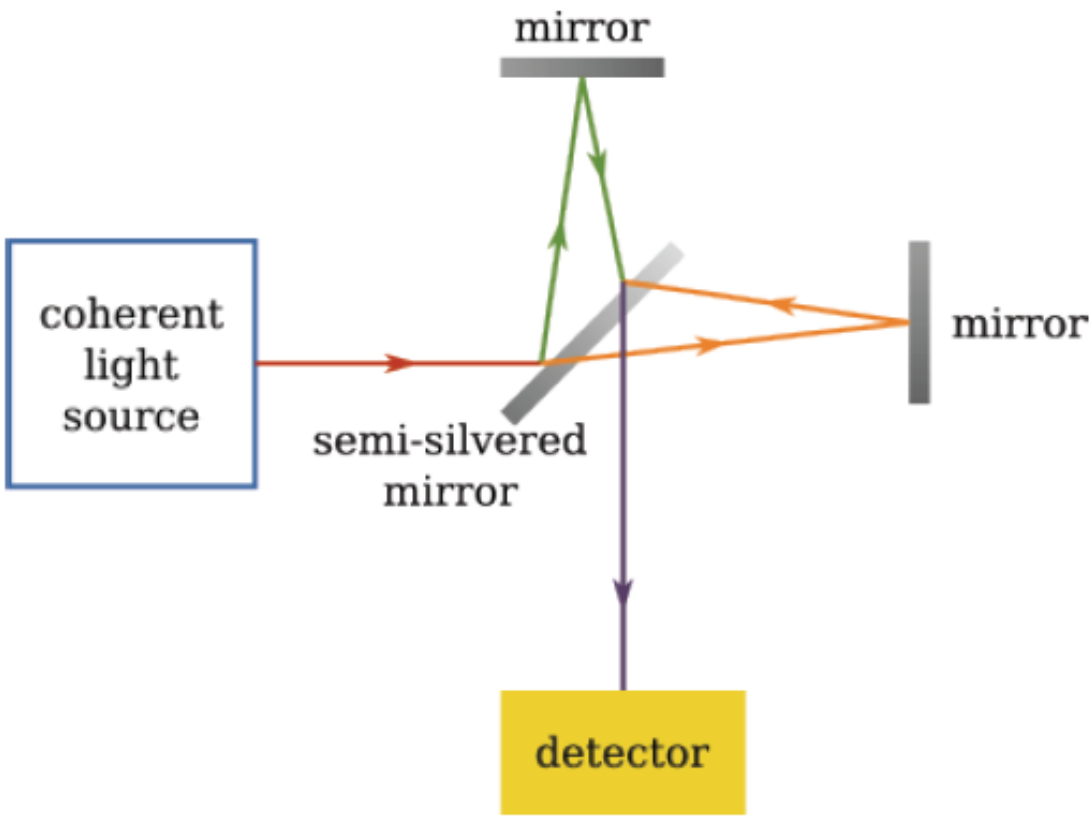
$$\frac{1}{1-(v/c)^2} = 1 + \left(\frac{v}{c}\right)^2 + O\left[\left(\frac{v}{c}\right)^{2n}, n>1\right], \quad \frac{1}{\sqrt{1-(v/c)^2}} = 1 + \frac{1}{2}\left(\frac{v}{c}\right)^2 + O\left[\left(\frac{v}{c}\right)^{2n}, n>1\right]$$

using the Taylor expansion. It follows that

$$\Delta t - \Delta\tilde{t} \approx (l_1+l_2)\frac{v^2}{c^3}$$

i.e. there should be difference if $v \neq 0$.
Experiment by Michelson and Morley did not show any difference.
The speed of light is identical in the vacuum for all inertial frames.

Schematic view of the Michelson-Morley Experiment:



Mathematical note: Approximation using the Taylor expansion

Taylor series of a function $F(x)$ around $x = a$ is given by

$$F(x) = F(a) + \frac{1}{1!}\frac{dF(a)}{dx}(x-a) + \frac{1}{2!}\frac{d^2F(a)}{dx^2}(x-a)^2 + \cdots = F(a) + \sum_{n=1}^{\infty}\frac{1}{n!}\frac{d^nF(a)}{dx^n}(x-a)^n$$

Let us consider the case $a = 0$, i.e.

$$F(x) = F(0) + \frac{1}{1!}\frac{dF(0)}{dx}x + \frac{1}{2!}\frac{d^2F(0)}{dx^2}x^2 + \cdots = F(0) + \sum_{n=1}^{\infty}\frac{1}{n!}\frac{d^nF(0)}{dx^n}x^n$$

If $|x|$ is much smaller than 1, i.e. $1 >> |x| >> |x|^2 >> |x|^3 >> \cdots$, and if $|F(0)| \approx |d^nF(0)/dx^n|$, the terms with higher order in x becomes negligible.

As an example, we consider $F(x) = 1/(1-x)$. By noting

$$F(x) = (1-x)^{-1},\ \frac{dF(x)}{dx} = (1-x)^{-2},\ \frac{d^2F(x)}{dx^2} = 2(1-x)^{-3},\ \frac{d^3F(x)}{dx^3} = 6(1-x)^{-4}, \cdots$$

the Taylor series is then given by

$$\frac{1}{1-x} = \left.\frac{1}{1-x}\right]_{x=0} + \left.\frac{1}{(1-x)^2}\right]_{x=0} x + \left.\frac{1}{(1-x)^3}\right]_{x=0} x^2 + \left.\frac{1}{(1-x)^4}\right]_{x=0} x^3 + \cdots$$
$$= 1 + x + x^2 + x^3 \cdots$$

For small x, say $x = 0.001$, we have

$$\frac{1}{1-x} = 1.001001001\cdots \text{ and } 1 + x = 1.001, \text{ i.e. } \frac{1}{1-x} \approx 1 + x$$

and we can have a good approximation by neglecting terms that are proportional to x^2 or with higher powers in x.

Another example for $F(x) = \sqrt{1-x}$, we have

$$F(x) = (1-x)^{1/2},\ \frac{dF(x)}{dx} = -\frac{1}{2}(1-x)^{-1/2},\ \frac{d^2F(x)}{dx^2} = -\frac{1}{4}(1-x)^{-3/2},\ \frac{d^3F(x)}{dx^3} = -\frac{3}{8}(1-x)^{-5/2}, \cdots$$

thus

$$\sqrt{1-x} = \left.\sqrt{1-x}\right]_{x=0} - \frac{1}{2}\left.\frac{1}{\sqrt{1-x}}\right]_{x=0} x - \frac{1}{8}\left.\frac{1}{\left(\sqrt{1-x}\right)^3}\right]_{x=0} x^2 - \frac{1}{16}\left.\frac{1}{\left(\sqrt{1-x}\right)^5}\right]_{x=0} x^3 + \cdots$$
$$= 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 \cdots$$

and for $x = 0.001$,

$$\sqrt{1-x} = 0.9994999\cdots \text{ and } 1 - \frac{1}{2}x = 0.9995$$

i.e. neglecting terms proportional to x^2 or higher is a good approximation.