General Physics II at EPFL

(2018-2019 SS, Wed 17:15-19:00 and Thu 8:15-10:00, Exercise Thu 10:15-12:00)

Thermodynamic (1st week)

Matter

Three phases of matter
gas, liquid and solid
for water: vapour - boiling point - water - freezing point or melting point - ice

From macroscopic point of view:
solid: maintains a fixed shape and a fixed size even if a large force is applied
liquid: does not maintain a fixed shape (takes a shape of the container), but not compressible
gas: has neither fixed shape nor volume, expands and fills the container

From microscopic point of view:
Solid: The force between the particles (atoms or molecules) is strong enough that they cannot move free and can only vibrate.
Liquid: Atoms have many nearest neighbours in contact, yet no long-range order is present.
Gas: In a gas, the molecules have enough kinetic energy so that the effect of intermolecular forces is small (or zero for an ideal gas), and the typical distance between neighboring molecules is much greater than the molecular size.

Molecule - Atom - Nucleus (protons and neutrons) & electrons
Atomic number (Z) and Mass number (A)
Z = number of proton, A = number of nucleons (protons plus neutrons)
Unified atomic mass unit (u): mass of ^{12}C = 12 u (1 u = 1.6605×10^{-27} kg)
^{12}C: 6 protons and 6 neutrons, i.e. one proton or neutron ≈ 1 u

Typical size of atom = 10^{-10} m

Cupper (solid) density = $8.9 \times 10^3\ \mathrm{kg/m^3}$
mass of one Cu atom = 62.930 u = $62.930 \times 1.661 \times 10^{-27}$ kg = 1.045×10^{-25} kg
The number of Cu atoms per 1 m^3 = density of Cu / weight of Cu atom

$$= \frac{8.9 \times 10^3\ \mathrm{kg/m^3}}{1.045 \times 10^{-25}\ \mathrm{kg/atom}} = 8.5 \times 10^{28}$$

The number of atom along the one dimension: n
The total number of atoms in a cube: $N = n^3$
For 1 m × 1 m × 1 m

$$n = \sqrt[3]{N} = \sqrt[3]{8.5 \times 10^{28}} = 4.4 \times 10^9$$

The distance between the atoms = distance / number of atoms

$$\frac{1\ \mathrm{m}}{4.4 \times 10^9\ \mathrm{atoms}} = 2.3 \times 10^{-10}\ \mathrm{m}$$

Indicating that the distance between the atoms is close to the size of the atom.

Oxygen gas density at 0° C with the normal pressure = $1.429\ \text{kg}/\text{m}^3$
Weight of one O atom = 15.999 u = $15.999 \times 1.661 \times 10^{-27}$ kg = 2.657×10^{-26} kg
The number of O atoms per $1\ \text{m}^3$ = density of O / weight of O atom

$$= \frac{1.429\ \text{kg}/\text{m}^3}{2.657 \times 10^{-26}\ \text{kg}/\text{atom}} = 5.4 \times 10^{25}$$

The number of atom along the one dimension: n
The total number of atoms in a cube: $N = n^3$
For $1\ \text{m} \times 1\ \text{m} \times 1\ \text{m}$

$$n = \sqrt[3]{N} = \sqrt[3]{5.4 \times 10^{25}} = 3.8 \times 10^8$$

The distance between the atoms = distance / number of atoms

$$\frac{1\ \text{m}}{3.8 \times 10^8\ \text{atoms}} = 2.6 \times 10^{-9}\ \text{m}$$

Distance between the atoms is an order of magnitude larger than the size of the atom.

Temperature

Temperature: a measure of thermal energy

Celsius: water freezing point = 0° C, boiling point = 100° C, equally divided by 100

Fahrenheit: water freezing point = 32° F, boiling point = 212° F, divided by 180

Kelvin: 0° K = -273.15° C = -459.67° F, all thermal motion ceases in the classical description of thermodynamics

When two objects with different temperatures are put to thermal contact, they will then become the same temperature: they are in thermal equilibrium, i.e. there is no net transfer of thermal energy.

Zero-th law of thermodynamics
If two systems are in thermal equilibrium with a third system, then they are in thermal equilibrium with each other.

Thermal Expansion

Linear expansion:

The length of solid generally increase with increasing temperature.
The change in length Δl is, to a good approximation, directly proportional to the change in temperature ΔT, as long as ΔT is not too large. Also, Δl is proportional to the original length l_0,

$$\Delta l = \alpha l_0 \Delta T$$

The parameter α is called the **coefficient of linear expansion**.

Material	Aluminium	Bras	Iron	Lead	Quartz
$\alpha\left(\text{C}^\circ\right)^{-1}$	25×10^{-6}	19×10^{-6}	12×10^{-6}	29×10^{-6}	0.4×10^{-6}

Initial state: temperature T_0 and the length l_0.
Final state: temperature T and the length l,

$$\Delta l = l - l_0,\ \text{and}\ \Delta T = T - T_0,\ \text{i.e.}\ l = \Delta l + l_0 = \alpha l_0 \Delta T + l_0 = l_0(1 + \alpha \Delta T)$$

An iron rail of 200 m long at 20° C, shrink at -30° C by

$$\Delta l = 12 \times 10^{-6}(-30 - 20) \times 200 = -1.2 \times 10^{-1}\ \text{m}$$

and expand at 40° C by

$$\Delta l = 12 \times 10^{-6}(40 - 20) \times 200 = 0.48 \times 10^{-1}\ \text{m}$$

The total change of the length from -30° C to 40° C is thus

$$0.48 \times 10^{-1} + 1.2 \times 10^{-1} = 1.68 \times 10^{-1}\ \text{m}.$$

Opening a tight jar lid by pouring hot water.
The diameter of a metal ring gets larger.

Volume Expansion

The volume of solid or liquid changes with a change of temperature as

$$\Delta V = \beta V_0 \Delta T$$

where V_0 is the initial volume. The parameter β is called the **coefficient of volume expansion.**

Material	Aluminium	Bras	Iron	Lead	Quartz	Mercury	Ethyl alcohol
$\beta\left(\text{C}^\circ\right)^{-1}$	75×10^{-6}	56×10^{-6}	35×10^{-6}	87×10^{-6}	1×10^{-6}	180×10^{-6}	1100×10^{-6}

The value of β for liquid is much larger than solid:→ mercury and alcohol thermometer

The volume can be given as the product of the three sides, i.e.

$$V = \prod_{i=1}^{3}(1 + \alpha\Delta T) l_0^i = (1 + \alpha\Delta T)^3 l_0^1 l_0^2 l_0^3 = (1 + \alpha\Delta T)^3 V_0$$

For $|\alpha\Delta T| << 1$, we have

$$(1 + \alpha\Delta T)^3 = 1 + 3\alpha\Delta T + 3(\alpha\Delta T)^2 + \cdots \approx 1 + 3\alpha\Delta T,$$

thus

$$V \approx (1 + 3\alpha\Delta T)V_0,$$

or

$$\Delta V \approx 3\alpha\Delta T V_0,$$

i.e.

$$\beta \approx 3\alpha.$$

Indeed this is the case for the solids given in the table.

A steel container of $1\ \text{m} \times 1\ \text{m} \times 1\ \text{m}$ with an open top is filled with Mercury at 20° C. If the temperature increases to 40° C, how much mercury will overflow the container? The volume change of the container can be given by, i) same as the solid steel block, or ii) from the length expansion of the three sides. Since $3\alpha \approx \beta$, the two will lead to a similar conclusion. We use here i) and the volume increase of the container is given by,

$$\Delta V_{\text{cont}} = \beta\Delta T V_0 = 35 \times 10^{-6} \times (40 - 20) \times 1 = 0.0007\ \text{m}^3.$$

Equally for the mercury, we obtain

$$\Delta V_{\text{merc}} = \beta\Delta T V_0 = 180 \times 10^{-6} \times (40 - 20) \times 1 = 0.0036\ \text{m}^3.$$

Therefore, the amount of mercury overflows is,

$$\Delta V_{\text{merc}} - \Delta V_{\text{cont}} = 0.0036 - 0.0007 = 0.0029\ \text{m}^3.$$

Precise formula for the length expansion

Let us denote l_0 to be the initial length of a bar. According to the formula of the linear expansion, the length of the bar after increasing the temperature by ΔT, l, is given by

$$l = \Delta l + l_0 = \alpha l_0 \Delta T + l_0 = l_0(1+\alpha\Delta T)$$

where a is the coefficient of linear expansion. We now consider this in two steps: Step-1 increase the temperature by $\Delta T/2$, Step-2 increase the temperature by another $\Delta T/2$. After the Step-1, the length of the bar is given by

$$l_{\text{Step-1}} = l_0\left(1+\alpha\Delta T/2\right)$$

and after Step-2

$$l_{\text{Step-2}} = l_{\text{Step-1}}\left(1+\alpha\Delta T/2\right) = l_0\left(1+\alpha\Delta T/2\right)^2$$

If we consider n-steps to reach the temperature difference of ΔT by increasing the temperature by $\Delta T/n$ for the each step, the length of the bar after n-steps is given by

$$l_{\text{Step-n}} = l_0\left(1+\alpha\Delta T/n\right)^n$$

A continuous transition is then obtained by increasing the number of steps to the infinity, i.e.

$$l_{\text{contineous}} = \lim_{n\to\infty} l_0\left(1+\alpha\Delta T/n\right)^n = l_0 e^{\alpha\Delta T}$$

which is the **exact expression** for the length expansion.

By noting

$$e^{\alpha\Delta T} = 1 + \alpha\Delta T + \frac{1}{2!}\left(\alpha\Delta T\right)^2 + \frac{1}{3!}\left(\alpha\Delta T\right)^3 + \frac{1}{4!}\left(\alpha\Delta T\right)^4 + \cdots$$

we can conclude that the formula of the linear expansion is valid only if $\alpha|\Delta T| << 1$.

Anomalous behaviour of water

Normally, the volume of material increases with temperature, i.e. the density decreases as a function of temperature. Also the solid phase of a material has a higher density than that at the liquid phase. The water has a very different behaviour. The density of ice at the normal pressure decreases slightly with increasing temperature, i.e.

at -180° C, 0.9340 g/cm^3, and at 0° C, 0.9167 g/cm^3,

which is similar behaviour to the usual material. When it melts at 0° C, the liquid water has a density of 0.9998 g/cm^3, higher than that of the ice at the same temperature. This is why the ice floats in the liquid water. The density of water increases (volume decreases) with increasing temperature, and it reaches at its maximum at 4° C, 1.00 g/cm^3. Further increases in the temperature decreases the density (increases the volume) as usual material.

→Important for the earth life form.

Thermal Stress

If the two ends of an iron beam if rigidly fixed, changes of temperature induces stress due to the thermal expansion or contraction: thermal stress,

i) the iron beam try to expand by $\Delta l = \alpha \Delta T l_0$.

ii) External force, F, is required to push back the base to stay with the same length.

The reduction of the length due to the external force is given by

$$\Delta l' = \frac{1}{E}\frac{F}{A} l_0 .$$

where E is Young's module for material and A is the transversal area of the beam. Since this should compensate the thermal expansion, $\Delta l = \Delta l'$,

$$\alpha \Delta T l_0 = \frac{1}{E}\frac{F}{A} l_0$$

giving the thermal stress to be

$$\frac{F}{A} = \alpha E \Delta T .$$

Let us consider a concrete structure, like a beam of concrete supporting the motor way. The Young's module for a concrete is $20 \times 10^9 \ \mathrm{N/m^2}$, and the coefficient of linear expansion, $12 \times 10^{-6} \ \mathrm{C^{\circ -1}}$. For a 40 degrees temperature difference, the stress is given by

$$\frac{F}{A} = \alpha E \Delta T = 12 \times 10^{-6} \mathrm{C^{\circ -1}} 20 \times 10^9 \mathrm{Nm^{-2}} 40 \mathrm{C^{\circ}} = 9.6 \times 10^6 \mathrm{Nm^{-2}}$$

Noting the tensile strength, $2 \times 10^6 \ \mathrm{N/m^2}$, and compressive strength, $2 \times 10^7 \ \mathrm{N/m^2}$, for the concrete, it appears to be not safe without having a gap between the beams and fixation.