Relativity 1

Your intuition about space and time is likely wrong

Question for you

Suppose you are standing on a train going 100 km/h. You throw a ball either in the direction of the train or against the direction of the train at 20 km/h. What speeds will an observer standing on the ground see the ball moving?

Take-away

This way of adding or subtracting velocities, which feels intuitive, is called '**Galilean-Newtonian relativity**'.

Question for you or DeepSeek

If a train is not shaking, not accelerating nor decelerating, and you do not look out of the window, would you feel that you are on a train moving?

Take-away

As long as you are going at a constant velocity, you should not feel that you are moving at all. You are in an inertial frame of reference. You will not spill your tea in a train even if it is going fast with respect to the ground (unless there are bumps or turns).

The key idea of special relativity is that all the laws of physics are the same in any inertial frame of reference, that is, a frame that is not accelerating, even if it is moving with respect to other frames. In particular, the speed of light is the same. This innocent looking statement causes big problems for Galilean-Newtonian relativity.

The problem is illustrated when you think about simultaneous and non-simultaneous events.

Question for you or DeepSeek

1) Imagine you stand between two loud speakers, one on your left and one on your right, each a distance l from you. The one on your left makes a sound at time 0, the one on your right makes a sound at time 0 also. At what time tA and tB will you hear the sounds? Consider the speed of sound to be c.

Take-away

The answer is that you will hear the sounds at the same time, so simultaneously.

Question for you or DeepSeek

2) Now suppose you are yourself moving with speed v to right. When will you hear these two sounds? Derive the answer.

Take-away

The answer is that you will hear the sounds at different times, so not simultaneously.

So, being on a train is different from being on the ground. In this case, this makes sense because the air, which is the medium for sound is stationary in the frame where the sound is made. You are moving. But that is a problem for light, which does not have a medium. Do you see the light arrive at the same time or at different times?

Translating time-space coordinates for events in different inertial frames

The easy (intuitive) case – Galilean-Newtonian transformation

Question for you or DeepSeek

[I need to illustrate this on the board.]

Suppose there are two observers S and S’, where S’ is moving in the positive x-direction with respect to S. The coordinates for time and space for S are t, x, y, z, and for S’ they are t’, x’, y’, z’. At t=0, both S and S’ are in the same location and synchronize their clocks. So, at t=t’=0, x=x’=y=y’=z=z’=0. Some time later an event happens at t’, x’, y’, z’ for S’. According to your usual intuition (Galilean-Newtonian relativity), when and where does the event occur for S?

Question for you or DeepSeek

Put this transformation in vector and matrix form, first for an observer S’ that moves in the positive x-direction with velocity vx, then for an observer S’ that moves in any direction in space with velocity vx, vy, vz.

Question for you or DeepSeek

Now write the vector-matrix form of this Galilean-Newtonian transformation if S’ was going in the opposite direction in space. Then, show that transforming the time-space coordinates for an event for an observer going in one direction and then an observer going in the opposite direction, you end up with coordinates that do not change at all.

Question for you or DeepSeek

Suppose an object moves in inertial frame S’ with velocity ux’ in the x’ direction. The inertial frame S’ moves with velocity v in the x-direction with respect to S. What is the velocity of the object ux in frame S? Use the Galilean-Newtonian transformations you just derived.

Take-away

You learned how to transform space-time coordinates between different inertial frames for Galilean-Newtonian relativity. You also learned how velocities simply add in Galilean-Newtonian relativity. This is a problem if one of the ‘objects’ goes at the speed of light, because it the other frame, the object would go with a velocity different from the speed of light.

Fixing the problem: Lorentz transformation

Question to you or DeepSeek

Let us try to find a way to change the Galilean-Newtonian transformations so that an object moving at the speed of light in frame S will also be moving at the speed of light in frame S', which is moving at velocity v with respect to S. Multiply the transformation for x' to x by a factor gamma. By symmetry, gamma needs to also multiply the transformation for x to x'. By assuming the object travels at the same speed "c" in both frames, solve for t' in terms of t and t' in terms of t. By plugging one equation into the other, get an expression for gamma.

Question to you or DeepSeek

Write these transformations again in vector-matrix form. Show how to transform from one frame to the other and back in matrix form. Show that performing both transformations gives you the identity transformation.

Take-away

By multiplying the x-transformation by gamma and ensuring that an object going at the speed of light (which must be light itself as we will see later) has the same velocity in all frames, you derived the Lorentz transformations, which are the correct way to transform between space-time coordinates between different frames.

Question for DeepSeek

Compute the velocity ux of an object in the S frame if it is going with velocity ux' in the S' frame.

Take-away

Velocities cannot simply be added. Look at the denominator. It makes sure you can never go faster than the speed of light.