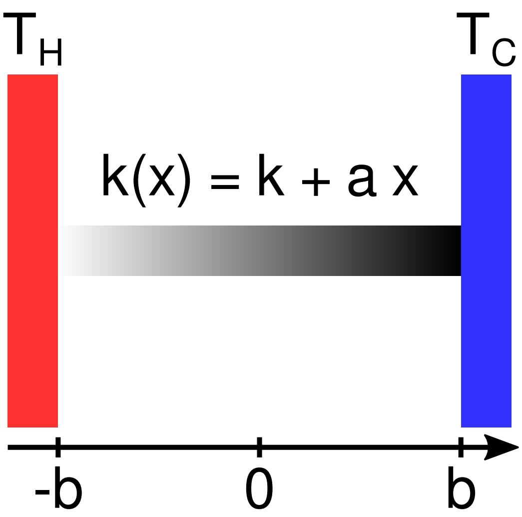
General instructions:

- You are given five sheets of A3 paper. The sheet with your name will be the envelop to put the other sheets inside of and return to us. Use the envelop sheet as scrap paper. However, cross all calculations on this envelop sheet out before you return the exam. Use each other sheet for a different problem in this exam.
- You must return these two pages of questions along with your exam inside the envelop.
- Always work with variables until the end of a calculation. If the question asks for it, plug in for a numerical answer at the very last step.
- Circle your final answers to each question.

Problem 1: Heat conduction **25 points total**



A hot object at temperature T_H is on the left at $x = -b$. A cold object at temperature T_C is on the right at $x = b$. The two objects are connected by a rod of length $2b$ and cross-sectional area A. Unlike the cases solved in class and in the exercises, the thermal conductivity of the rod is not uniform. It is given by $k(x) = k + ax$, where x goes from $-b$ to b. Furthermore, $k(x)$ is always positive, so $k > ab$.

a) The hot and cold objects on the left and right, respectively, stay at their respective temperatures forever despite the rod connecting them. What are their heat capacities? 5 points

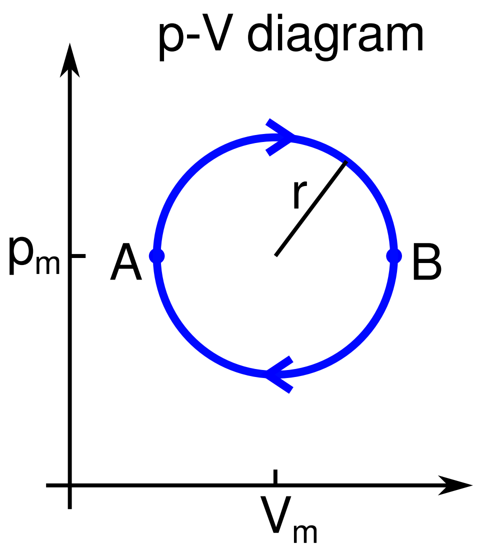
b) Solve for the heat flux $\dot{Q}$ from the hot to the cold object using $\dot{Q} = -k(x)A\frac{dT(x)}{dx}$. 5 points

c) Perform a Taylor expansion of the expression for the heat flux $\dot{Q}$ you found in part b) in the parameter $\frac{ab}{k} \ll 1$ and keep the expression to lowest order in a. 5 points

d) What is the heat flux if the heat conductivity $k(x)$ is just equal to k? 5 points

e) Compare your results in c) and d). Does varying the heat conductivity in space – maintaining the same average heat conductivity along the rod – change the heat flux? If so, does the heat flux become smaller or larger if the heat conductivity is inhomogeneous? Explain your reasoning briefly based on your results in c) and d). 5 points

Problem 2: Heat engine / refrigerator **25 points total**



The p-V diagram above illustrates the changes that n moles of an ideal gas undergo. The gas can be atomic or molecular, so do not assume either. The path in the p-V diagram constitutes a circle of radius r centered at p_m and V_m, so for the path from A to B, p is given by $p = p_m + \sqrt{r^2 - (V - V_m)^2}$. (You are not suggested to use this formula, it is only given to you if you feel you need it.)

Make sure you get all of the signs right. $+$: energy added to the system, -: energy taken out of the system.

a) Calculate the work $W_{A \to B}$ performed on the system in the transition from A to B. 5 points

b) Calculate the heat $Q_{A \to B}$ taken in by the system on the path from A to B. 5 points

Hints: Start with $dE(T) = dQ - pdV$. Use $dE(T) = nC_V dT$. Then, relate dT to infinitesimal changes in dp and dV for an ideal gas. (You have done something roughly similar when we discussed adiabatic transformations in the lectures.) Finally, integrate to obtain $Q_{A \to B}$.

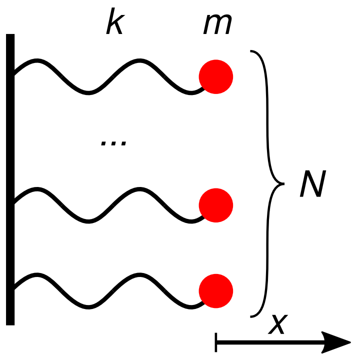
c) Calculate the change in internal energy $E_{A \to B}$ of the system on the path from A to B. 5 points

d) Calculate the entropy change $S_{A \to B}$ of the system on the path from A to B. You can use a short-cut to get the right answer instead of integrating along the semi-circle but for full credit, you must integrate along the semi-circle. 5 points

e) How many external heat baths does the transformation from A to B involve? 2.5 points

f) Is the system a heat engine, a refrigerator, or a heat pump? 2.5 points

Problem 3: Statistical physics **25 points total**



N particles of mass m are attached to a wall by springs with spring constant k. The particles can only move in the direction of the spring and have potential energy $\frac{1}{2}kx^2$, where x is the deviation from the equilibrium length of the spring.

a) If the system is at temperature T, what is the internal energy of the whole system? 5 points

b) What is the heat capacity of the system? 5 points

c) Suppose each spring breaks if stretched beyond $x_\text{b} = \sqrt{\frac{50k_BT}{k}}$. What is the fraction of springs that will break? Use the numerical values at the bottom of this page to supply a numerical answer. 5 points

d) Now, forget about any springs breaking. The N springs-and-masses are put inside a box together with $2N$ particles of a gas of particles of mass M. The gas is initially at 0 K. The N springs are all stretched to a distance x_max from equilibrium and released. What is the final temperature inside the box if the particles on the springs and the particles in the gas interact with one another and equilibrate? 10 points

The following quantities may be helpful:

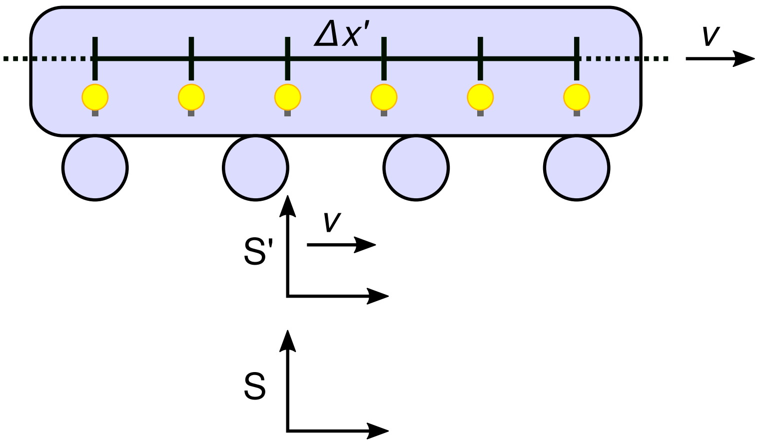
$\frac{1}{\sqrt{\pi}}\int_1^\infty e^{-t^2}dt = 8\cdot 10^{-2}$ $\quad\frac{1}{\sqrt{\pi}}\int_2^\infty e^{-t^2}dt = 2\cdot 10^{-3}$ $\quad\frac{1}{\sqrt{\pi}}\int_3^\infty e^{-t^2}dt = 1\cdot 10^{-5}$

$\frac{1}{\sqrt{\pi}}\int_4^\infty e^{-t^2}dt = 8\cdot 10^{-9}$ $\quad\frac{1}{\sqrt{\pi}}\int_5^\infty e^{-t^2}dt = 8\cdot 10^{-13}$ $\quad\frac{1}{\sqrt{\pi}}\int_6^\infty e^{-t^2}dt = 1\cdot 10^{-17}$

$\frac{1}{\sqrt{\pi}}\int_1^\infty t^2e^{-t^2}dt = 1\cdot 10^{-1}$ $\quad\frac{1}{\sqrt{\pi}}\int_2^\infty t^2e^{-t^2}dt = 1\cdot 10^{-2}$ $\quad\frac{1}{\sqrt{\pi}}\int_3^\infty t^2e^{-t^2}dt = 1\cdot 10^{-4}$

$\frac{1}{\sqrt{\pi}}\int_4^\infty t^2e^{-t^2}dt = 1\cdot 10^{-7}$ $\quad\frac{1}{\sqrt{\pi}}\int_5^\infty t^2e^{-t^2}dt = 2\cdot 10^{-11}$ $\quad\frac{1}{\sqrt{\pi}}\int_6^\infty t^2e^{-t^2}dt = 4\cdot 10^{-16}$

Problem 4: Special relativity **25 points total**



A very long train moving with velocity v to the right with respect to another frame S has light bulbs placed at a distance $\Delta x'$ from one another, so at positions $x'_i = i\Delta x'$ where i is an integer. Let us call the frame of reference of the train S'.

a) Suppose two neighboring light bulbs flash at the same time $t' = 0$ in frame S'. There are observers all along the train in frame S standing right next to the train and looking directly at the train. What is the time delay between the flashes recorded by two observers standing right next to the two bulbs in frame S? 5 points

b) Suppose all the bulbs flash with a period of $\Delta t'$ in the S' frame, that is, at $t' = n\Delta t'$ where n is an integer. How far apart must the bulbs be on the train in the S' frame, that is, what does $\Delta x'$ have to be, for one observer in frame S to see each bulb, which passes her, flash? 5 points

c) Now suppose the distance between the light bulbs is as you found them in b). What is the period between flashes that the observer in frame S sees?

d) Does your answer in c) agree with time dilation of the period $\Delta t'$ as the proper time? 5 points

e) Suppose instead of light bulbs separated by a distance $\Delta x'$, there are continuous lighting strips emitting light with frequency f' from the sides of the train. Use your results to derive the frequency of the light an observer standing right next to the train in S and looking directly at the train sees. 5 points