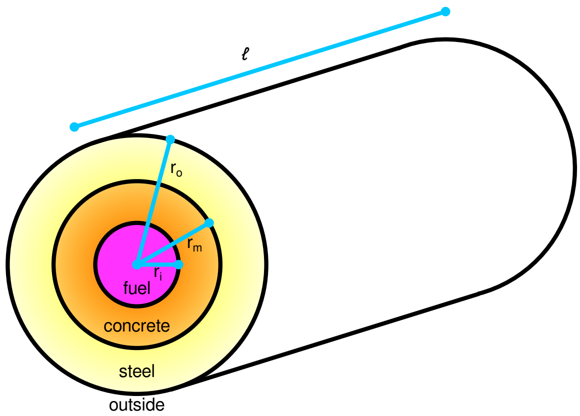
General instructions:

- Always work with variables until the end of a calculation. If the question asks for it, plug in for a numerical answer at the very last step.
- Circle your final answers to each question.

Problem 1: Heat conduction and thermal expansion **25 points total**



Radius	r_i	e^{-1} m = 0.368... m (e is Euler's number)
Radius	r_m	1 m
Radius	r_o	e m = 2.718... m (e is Euler's number)
Length	l	10 m
Temperature	T_{fuel}	1200 K
Temperature	T_{outside}	300 K
Thermal conductivity	k_{concrete}	1 W/(m K)
Thermal conductivity	k_{steel}	10 W/(m K)
Coefficient of linear expansion	α_{concrete}	$13 \cdot 10^{-6}$ K^{-1}
Coefficient of linear expansion	α_{steel}	$12 \cdot 10^{-6}$ K^{-1}
Young's modulus	E_{concrete}	$20 \cdot 10^{9}$ N/m^2
Young's modulus	E_{steel}	$200 \cdot 10^{9}$ N/m^2
Tensile strength	σ_{concrete}	$2 \cdot 10^{6}$ N/m^2
Tensile strength	σ_{steel}	$800 \cdot 10^{6}$ N/m^2

A cylinder of nuclear fuel, which is always at temperature T_i, is enclosed in concrete and steel. All dimensions, parameters, and coefficients you may or may not need are given in the table above.

a) Calculate the temperature profile $T(r)$ as a function of the distance to the center r in the concrete and steel layers. Draw $T(r)$ from $r = r_i$ to $r = r_o$. 5 points

b) What is the temperature at the concrete-steel boundary? Give a numerical answer. Approximate $2\pi \approx 6$ and expand any $\frac{1}{1+x}$ terms, where $x \ll 1$ is small, to first order in x. 5 points

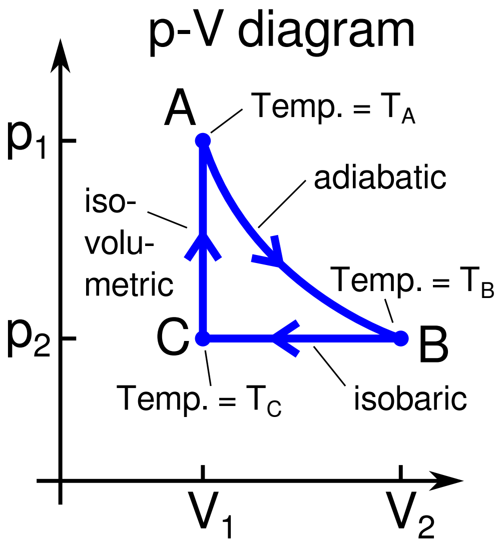
c) How much heat is lost by the nuclear fuel per unit time per unit length? Give a numerical answer using the same approximations as in (b). 5 points

The next questions do not depend on your answers to a)-c):

d) Suppose now that the whole cylinder is enclosed in a perfect insulator. What is the temperature profile $T(r)$ in the concrete and steel layers? 5 points

e) Suppose the concrete and steel layers started at temperature T_{outside}. Then, the fuel was inserted into the core and the system was sealed. Then, the whole system (fuel, concrete, and steel) was enclosed in a perfect insulator. Will the thermal expansions of the materials cause the steel to burst? Supply numerical values and a yes/no answer. 5 points

Problem 2: Heat engine / refrigerator **25 points total**



The p-V diagram above illustrates the changes that n moles of an ideal atomic gas go through.

a) Calculate for the adiabatic transformation A→B: 5 points

- the work done on the system,
- the heat taken up by the system, and
- the change in internal energy.
- Indicate for each of these three quantities whether it is bigger or smaller than zero.
- Replace p and V by temperatures T_A, T_B, and T_C everywhere.

b) Perform the same calculations as in a) for the isobaric transformation B→C. 5 points

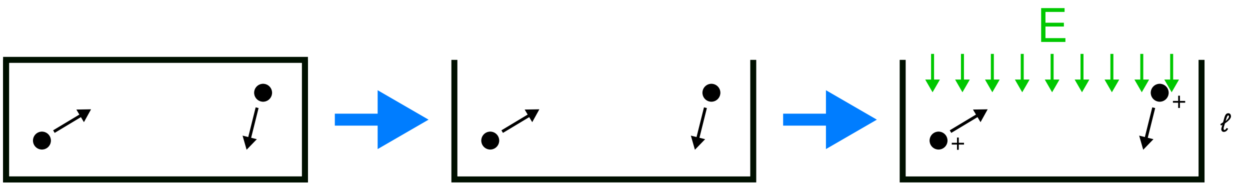
c) Perform the same calculations as in a) and b) for the isovolumetric transformation C→A. 5 points

d) Is this system a heat engine or a refrigerator? What is its efficiency? 5 points

e) Calculate the changes in entropy of the system along each of the paths A→B, B→C, and C→A, and indicate whether these changes are bigger or smaller than zero. 5 points

Make sure you get all of the signs right. +: energy added to the system, -: energy taken out of the system.

Problem 3: Statistical physics **25 points total**



A box that is very large in the x-y direction (so you can neglect the side walls) and of height l contains a gas of molecules of mass m at temperature T.

a) The lid of the box is opened up. For the molecules that are moving toward the opening of the box, what is their average velocity in the direction of the opening? 10 points

b) If you leave the lid open for a long time, what fraction of molecules will escape? 5 points

c) Suppose an electric field of strength E is turned on immediately after the lid is removed. The particles have charge q, so the force acting on them is $F = qE$. What is the probability of a particle that was at the bottom of the box and going at least partially toward the opening of the box right when the lid was removed to make it out of the box? Plug into your final formula: $qE = 10$ eV/m, $l = 0.01$ m, $k_\mathrm{B}T = 0.025$ eV, where eV is a unit of energy called 'electronvolt'. 10 points

The following quantities may be helpful:

$\frac{1}{\sqrt{\pi}} \int_1^\infty e^{-t^2} dt = 8 \cdot 10^{-2}$ $\frac{1}{\sqrt{\pi}} \int_2^\infty e^{-t^2} dt = 2 \cdot 10^{-3}$ $\frac{1}{\sqrt{\pi}} \int_3^\infty e^{-t^2} dt = 1 \cdot 10^{-5}$

$\frac{1}{\sqrt{\pi}} \int_4^\infty e^{-t^2} dt = 8 \cdot 10^{-9}$ $\frac{1}{\sqrt{\pi}} \int_5^\infty e^{-t^2} dt = 8 \cdot 10^{-13}$ $\frac{1}{\sqrt{\pi}} \int_6^\infty e^{-t^2} dt = 1 \cdot 10^{-17}$

$\frac{1}{\sqrt{\pi}} \int_1^\infty t^2 e^{-t^2} dt = 1 \cdot 10^{-1}$ $\frac{1}{\sqrt{\pi}} \int_2^\infty t^2 e^{-t^2} dt = 1 \cdot 10^{-2}$ $\frac{1}{\sqrt{\pi}} \int_3^\infty t^2 e^{-t^2} dt = 1 \cdot 10^{-4}$

$\frac{1}{\sqrt{\pi}} \int_4^\infty t^2 e^{-t^2} dt = 1 \cdot 10^{-7}$ $\frac{1}{\sqrt{\pi}} \int_5^\infty t^2 e^{-t^2} dt = 2 \cdot 10^{-11}$ $\frac{1}{\sqrt{\pi}} \int_6^\infty t^2 e^{-t^2} dt = 4 \cdot 10^{-16}$

Problem 4: Special relativity **25 points total**

Consider a particle A with rest mass M to be at rest. It decays into two particles B and C with identical rest masses m.

a) What are the magnitudes of the momenta $|\vec{p}_B|$ and $|\vec{p}_C|$? 5 points

b) What is the angle between the flight path of the two final particles? 5 points

c) If particle A had momentum $\vec{P}$, one of the final particles can be at rest. Suppose it is particle B that will be at rest. Calculate $P = |\vec{P}|$ such that particle B is at rest. 10 points

The next question does not depend on your answers to the previous questions.

d) Suppose, as in c), that particle A is moving with momentum $\vec{P}$ so that B will be at rest and C is moving. Now, suppose that A, B, and C are emitting radiation with frequencies f_A, f_B, and f_C, respectively, measured in frames where the particles are each at rest. What frequencies f'_A, f'_B, and f'_C will an observer see if A was directly flying toward the observer? 5 points