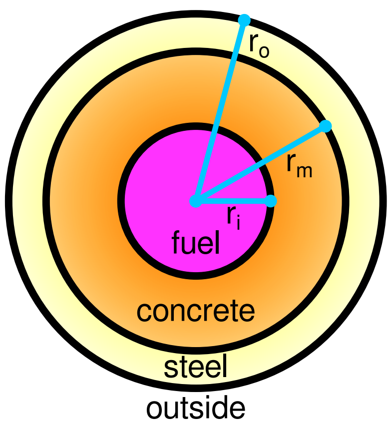
Phys-106 - Thermodynamics - Prof. Sahand Rahi
Final Exam - 07.08.2020

General instructions:

- Always work with variables until the end of a calculation. If the question asks for it, plug in for a numerical answer at the very last step.
- Circle your final answers to each question.

Problem 1: Heat conduction and thermal expansion **25 points total**



Radius	r_i	0.5 m
Radius	r_m	1 m
Radius	r_o	1.25 m
Temperature	T_{fuel}	1200 K
Temperature	T_{outside}	300 K
Thermal conductivity	k_{concrete}	1 W/(m K)
Thermal conductivity	k_{steel}	10 W/(m K)
Coefficient of linear expansion	α_{concrete}	$13 \cdot 10^{-6}$ K^{-1}
Coefficient of linear expansion	α_{steel}	$12 \cdot 10^{-6}$ K^{-1}
Young's modulus	E_{concrete}	$20 \cdot 10^{9}$ N/m^2
Young's modulus	E_{steel}	$200 \cdot 10^{9}$ N/m^2
Tensile strength	σ_{concrete}	$2 \cdot 10^{6}$ N/m^2
Tensile strength	σ_{steel}	$800 \cdot 10^{6}$ N/m^2

A sphere of spent nuclear fuel is enclosed in a sphere of concrete which is enclosed in steel. All dimensions, parameters, and coefficients you may or may not need are given in the table above.

a) Calculate the temperature $T(r)$ as a function of the distance to the center (r) in the concrete and steel layers. Draw $T(r)$ from $r = r_i$ to $r = r_o$. 5 points

b) What is the temperature at the concrete-steel boundary? Give a numerical answer. Approximate $4\pi \approx 10$ and expand any $\frac{1}{1+x}$ terms, where $x \ll 1$ is small, to first order in x. 5 points

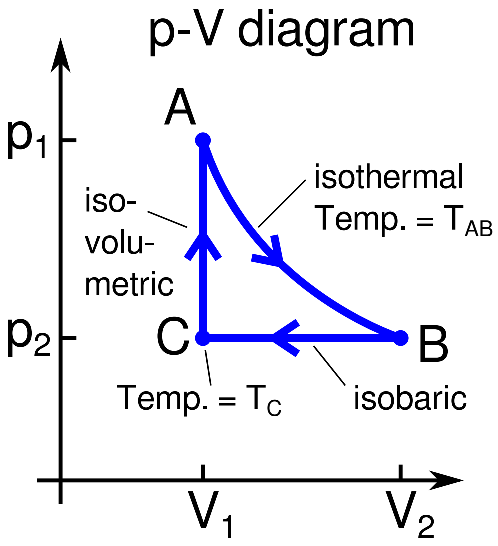
c) How much heat is lost by the nuclear fuel per unit time? Give a numerical answer using the same approximations as in (b). 5 points

The next questions do not depend on your answers to a)-c):

d) Suppose now that the whole ball is enclosed in a perfect insulator. What is the temperature profile in the concrete and steel layers? 5 points

e) Suppose the concrete and steel layers started at temperature T_{outside}. Then, the fuel was inserted into the core and the system was sealed. Then, the whole system was enclosed in a perfect insulator. Will the thermal expansions of the materials cause the steel to burst? Supply numerical values and a yes/no answer. 5 points

Problem 2: Heat engine / refrigerator **25 points total**



The p-V diagram above illustrates the changes that n moles of an atomic gas go through.

a) Calculate for the the isothermal transformation A→B: 5 points

- the work done on the system,
- the heat taken up by the system, and
- the change in internal energy.
- Indicate for each of these three quantities whether it is bigger or smaller than zero.
- Replace p and V by temperatures T_{AB} or T_{c} in your formulas wherever possible.

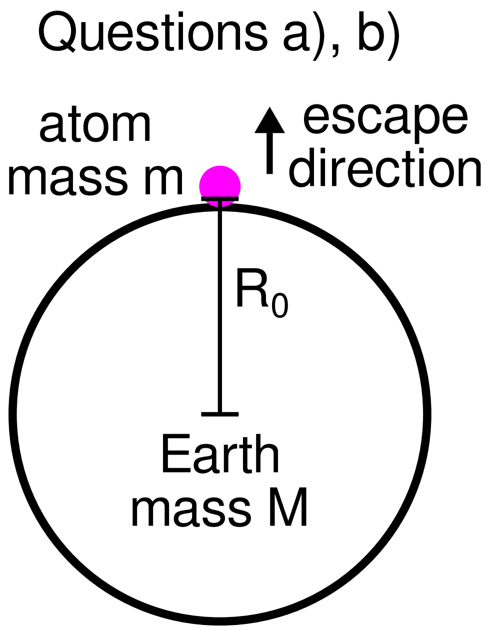
b) Perform the same calculations as in a) for the isobaric transformation B→C. 5 points

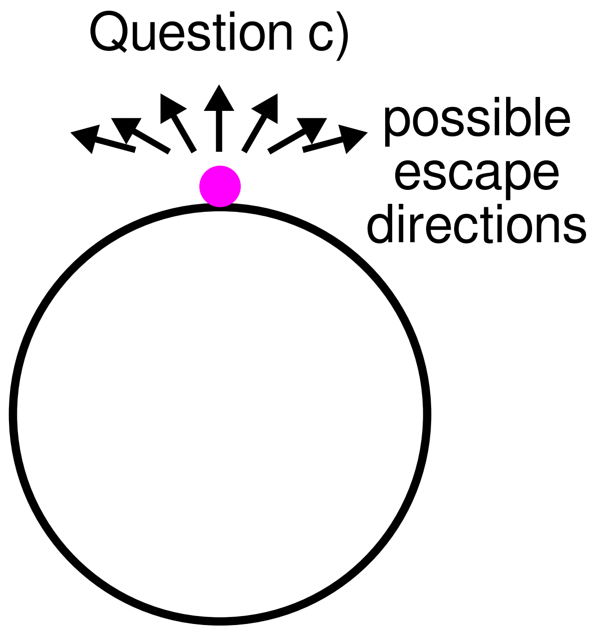
c) Perform the same calculations as in a) and b) for the isovolumetric transformation C→A. 5 points

d) Is this system a heat engine or a refrigerator? What is its efficiency? 5 points

e) Calculate the changes in entropy of the system along each of the paths A→B, B→C, and C→A, and indicate whether these changes are bigger or smaller than zero. 5 points

Make sure you get all of the signs right. +: energy added to the system, -: energy taken out of the system.





The earth's gravitational pull prevents most air molecules from escaping into space. However, the pull is not infinitely strong. In this problem, you will calculate how likely it is that air molecules do escape.

a) The potential energy of an object on the surface of the earth is $\frac{-mMG}{R_0}$, where m is the mass of the object, M is the mass of the earth, G is the gravitational constant, and R_0 is the radius of the earth. Suppose a particle can only escape if it is sufficiently fast in the direction away from the center of the earth as indicated in the figure above on the left. What condition must the kinetic energy of an air molecule satisfy to escape from the surface of the earth into space? 10 points

b) How probable is it for a molecule at temperature T to satisfy this condition and to escape from earth? Supply a numerical answer. Use the numerical values below. 10 points

c) Suppose that as long as the particle has enough kinetic energy in any direction away from the earth, even if it is not entirely perpendicular to the surface of the earth as in a), it will escape. See figure on the right. What is the probability that it will escape? Provide a numerical answer. 5 points

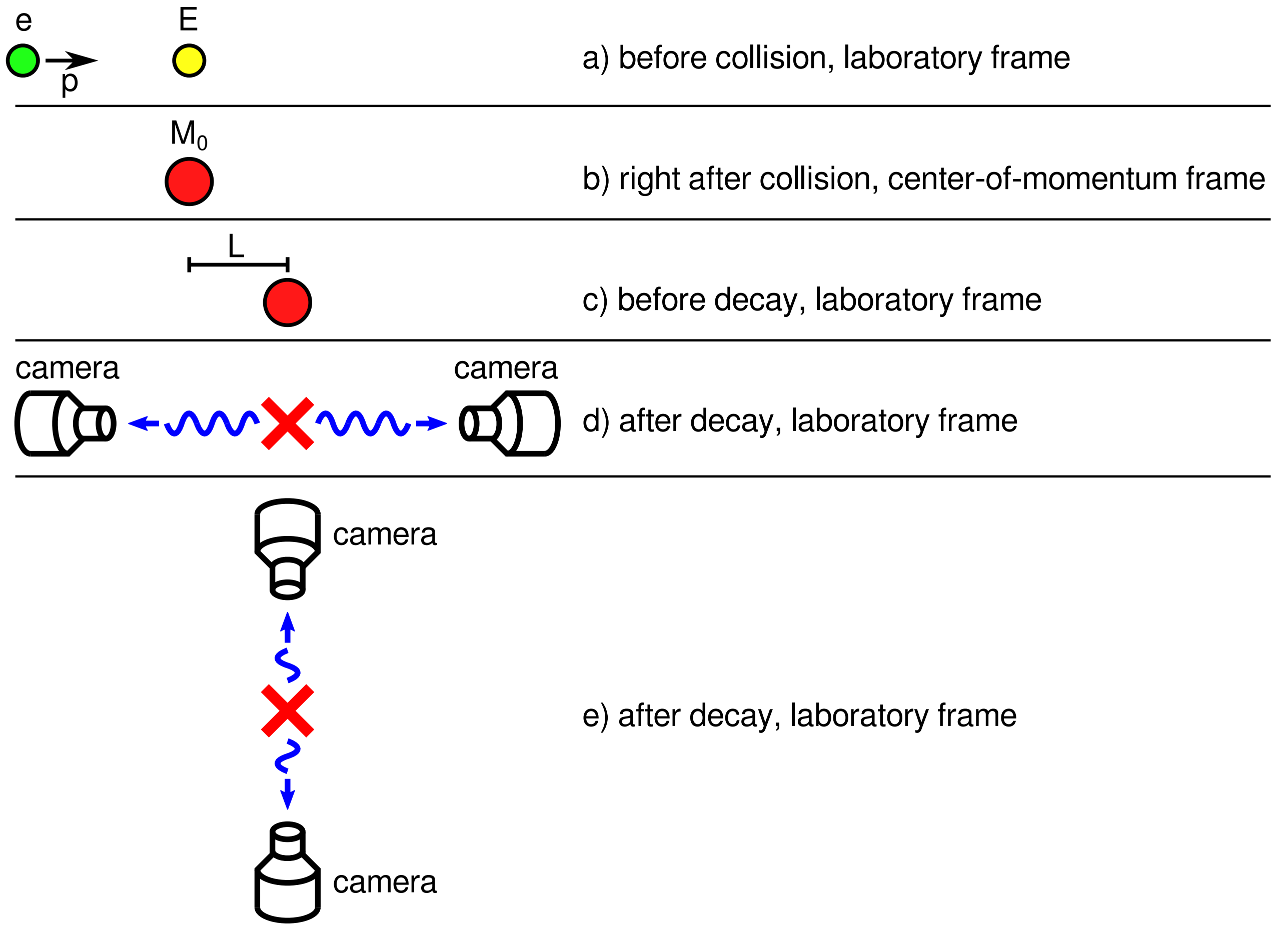
The following quantities may be helpful:

$\frac{mMG}{R_0 k_{\mathrm{B}} T} \approx 25$

$\frac{1}{\sqrt{\pi}} \int_1^\infty e^{-t^2} dt = 8 \cdot 10^{-2}$ $\frac{1}{\sqrt{\pi}} \int_2^\infty e^{-t^2} dt = 2 \cdot 10^{-3}$ $\frac{1}{\sqrt{\pi}} \int_3^\infty e^{-t^2} dt = 1 \cdot 10^{-5}$
$\frac{1}{\sqrt{\pi}} \int_4^\infty e^{-t^2} dt = 8 \cdot 10^{-9}$ $\frac{1}{\sqrt{\pi}} \int_5^\infty e^{-t^2} dt = 8 \cdot 10^{-13}$ $\frac{1}{\sqrt{\pi}} \int_6^\infty e^{-t^2} dt = 1 \cdot 10^{-17}$

$\frac{1}{\sqrt{\pi}} \int_1^\infty t^2 e^{-t^2} dt = 1 \cdot 10^{-1}$ $\frac{1}{\sqrt{\pi}} \int_2^\infty t^2 e^{-t^2} dt = 1 \cdot 10^{-2}$ $\frac{1}{\sqrt{\pi}} \int_3^\infty t^2 e^{-t^2} dt = 1 \cdot 10^{-4}$
$\frac{1}{\sqrt{\pi}} \int_4^\infty t^2 e^{-t^2} dt = 1 \cdot 10^{-7}$ $\frac{1}{\sqrt{\pi}} \int_5^\infty t^2 e^{-t^2} dt = 2 \cdot 10^{-11}$ $\frac{1}{\sqrt{\pi}} \int_6^\infty t^2 e^{-t^2} dt = 4 \cdot 10^{-16}$

Problem 4: Special relativity **25 points total**



A particle (green circle in the illustration) with total relativistic energy e is moving with relativistic momentum p toward another particle (yellow) at rest in the laboratory frame with total relativistic energy E.

a) How fast is the center-of-momentum frame moving? 5 points

b) In the collision, one new particle (red) is formed. What is its rest mass M_0? 5 points

c) In its own frame of reference, the new particle (red) decays after time T. How far did it travel (L) in the laboratory frame? 5 points

d) The new particle (red) decays and emits light of frequency f in all directions. At what frequencies will that light appear to a camera in the direction of the flight of the particle and to a camera behind the direction of the flight of the particle? What is the difference in time when the cameras will detect that light? 5 points

e) If the cameras were to the sides recording light that is emitted at a 90° angle to the path of flight, at what frequency would they record the emitted light: the same, higher, or lower than f? 5 points