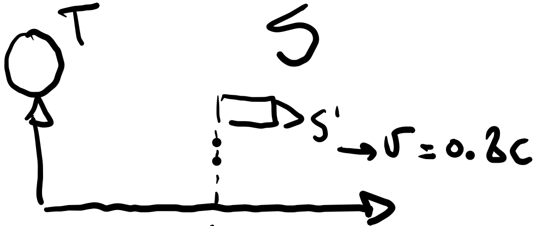
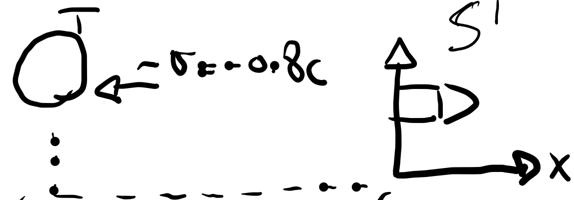
NOTE: $t_0 = t_0' = 0$ (horologes synchronisés au départ de la fusée)



(x_A, t_A)
$x_A = ?$, $t_A = ?$

$\boxed{Qa}$

$t_A' = 10s$
$x_A = ?$
$t_A = ?$



$(x_{T,A}', t_A')$

(x_A', t_A')
$x_A' = 0m$, $t_A' = 10s$

$$t_A = \gamma\left(t_A' + \frac{v \cancel{x_A'}}{c^2}\right) = \gamma t_A' \qquad (x_A' = 0\,m)$$

i) $x_A = \gamma(\cancel{x_A'} + v t_A') = \gamma v t_A'$ // (Lorentz) ($x_A' = 0$, $t_A' = 10s$)

ii) $x_A = v \cdot t_A = v \cdot \gamma t_A'$ // ($t_A = \gamma t_A'$)

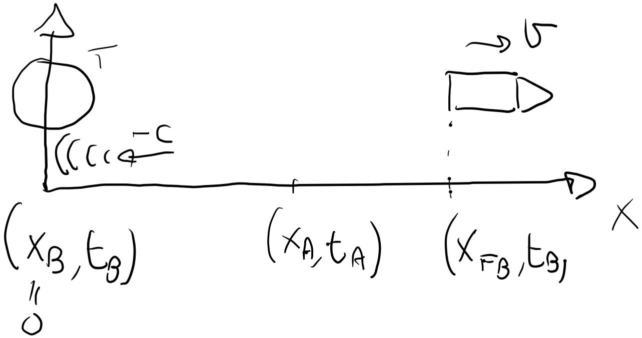
$x_T' = -v t_A'$

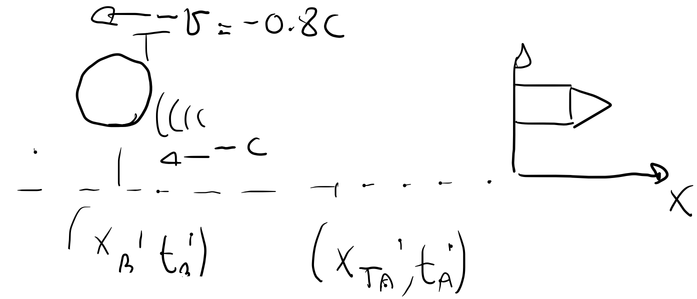
$|x_T'| = L' = v t_A'$

$x_A = L_0 = \gamma L' = \gamma v t_A'$ //

S | $\boxed{Q.B}$ | S'

$t_B = ?$





$$t_B = t_A + \frac{x_A}{c} = \gamma t_A' + \gamma \frac{v t_A'}{c} =$$

$$= \gamma t_A' \left(1 + \frac{v}{c}\right) //$$

$$x_B' = -c\underbrace{(t_B' - t_A')}_{\text{Signal}}$$

$$\underbrace{-v t_B'}_{\text{Terre}}$$

$$\Downarrow$$

$$-v t_B' = -c\,(t_B' - t_A') \Rightarrow t_B' = \frac{t_A'}{\left(1 - \frac{v}{c}\right)}$$

$$\Downarrow$$

$$x_B' = -v t_B' = \frac{-v t_A'}{\left(1 - \frac{v}{c}\right)}$$

(Lorentz)

$$t_B = \gamma\left(t_B' + v\frac{x_B'}{c^2}\right) = \gamma\left[\frac{t_A'}{\left(1 + \frac{v}{c}\right)} + \frac{v}{c^2}\left(\frac{-v t_A'}{\left(1 - \frac{v}{c}\right)}\right)\right]$$

$$= \gamma t_A' \frac{\left(1 - \frac{v^2}{c^2}\right)}{\left(1 - \frac{v}{c}\right)} = \gamma t_A' \left(1 + \frac{v}{c}\right) //$$

S $\boxed{Q_c}$ S'

$t_B' = ?$

$x_B' = ?$

(Lorentz inverse)

$$t_B' = \gamma\Big[\underset{\downarrow \, Q_B}{t_B} - \frac{v \overset{0}{\overset{\parallel}{x_B}}}{c^2}\Big] = \gamma^2 t_A'\left(1+\frac{v}{c}\right)$$

$$= \frac{t_A'\cancel{\left(1+\frac{v}{c}\right)}}{\left(1-\frac{v}{c}\right)\cancel{\left(1+\frac{v}{c}\right)}} = \frac{t_A'}{\left(1-\frac{v}{c}\right)} \;//$$

(Lorentz inverse)

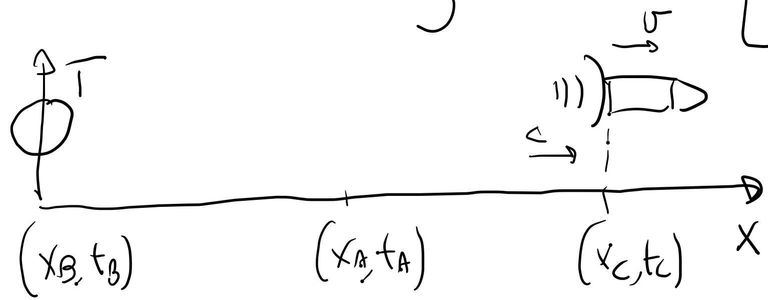
$$x_B' = \gamma(\underset{\overset{\parallel}{0}}{x_B} - v t_B) = -\gamma v \underset{\downarrow \, Q_B}{t_B} = -\gamma^2 v\, t_A'\left(1+\frac{v}{c}\right)$$

$$= -\frac{v t_A'}{1-\frac{v^2}{c^2}}\left(1+\frac{v}{c}\right) = -\frac{v t_A'}{\left(1-\frac{v}{c}\right)} \;//$$

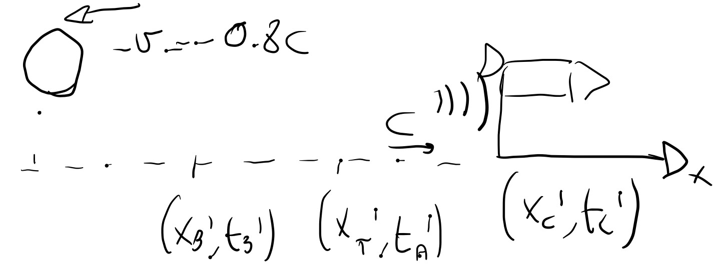
$$t_B'\Big|_{Q_B} = \frac{t_A'}{\left(1-\frac{v}{c}\right)} \;//$$

$$x_B'\Big|_{Q_B} = -\frac{v t_A'}{\left(1-\frac{v}{c}\right)} \;//$$

S $\boxed{Q_D}$ S'



$t_c' = ?$



$$t_c = t_B + \frac{\overset{v t_c}{x_c}}{c} = t_B + \frac{v}{c} t_c$$

$$\Downarrow$$

$$t_c = \frac{t_B}{1-\frac{v}{c}}$$

Lorentz inverse

$$t_c' = \gamma\left(t_c - \frac{v x_c}{c^2}\right) = \gamma\left(t_c - \frac{v^2}{c^2} t_c\right)$$

$$= \gamma\underbrace{\left(1-\frac{v^2}{c^2}\right)}_{=\frac{1}{\gamma^2}} t_c = \frac{t_c}{\gamma} = \frac{\overset{Q_B}{t_B}}{\gamma\left(1-\frac{v}{c}\right)} =$$

$$= \frac{\cancel{\gamma}\, t_A'\left(1+\frac{v}{c}\right)}{\cancel{\gamma}\left(1-\frac{v}{c}\right)} = t_A'\left(1+\frac{v}{c}\right) / \left(1-\frac{v}{c}\right) \quad //$$

$$t_c' = t_B' + \underbrace{\frac{(\overset{0}{x_c'} - x_B')}{c}}_{\text{propigation signal}} = \overset{Q_B}{t_B'} - \frac{\overset{Q_B}{x_B'}}{c}$$

$$= \frac{t_A'}{\left(1-\frac{v}{c}\right)} + \frac{v t_A'}{c\left(1-\frac{v}{c}\right)} = \frac{t_A'}{\left(1-\frac{v}{c}\right)}\left(1+\frac{v}{c}\right) \quad //$$