Thermodynamics Lecture Notes: Chapter 2

Michaela Hirschmann

Contents

2 Temperature and the zeroth law of Thermodynamics

2.1 Temperature

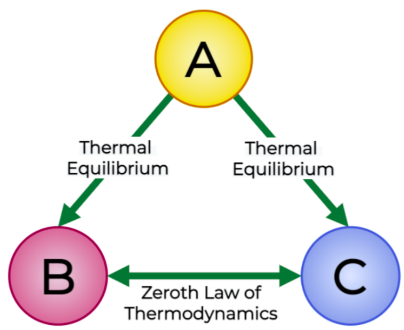
Temperature: a measure of thermal energy

- Celsius: water freezing point = 0° C , boiling point = 100° C , equally divided by 100
- Fahrenheit: water freezing point = 32° F, boiling point = 212° F , divided by 180
- Kelvin: = 0° K = -273.15° C = -459.67° F, all thermal motion ceases in the classical description of thermodynamics

When two objects with different temperatures are put into thermal contact, they will then converge to the same temperature, meaning they will be in thermal equilibrium, i.e. there will be no net transfer of thermal energy anymore (Note that the exact definition of thermal energy will be given in Chapter 6).

2.2 Zeroth law of Thermodynamics

If two systems are in thermal equilibrium with a third system, then they are in thermal equilibrium with each other.



2.3 Applications: Thermal Expansion

Linear expansion:

The length of a solid generally increase with increasing temperature. The change in length Δl is, to a good approximation, directly proportional to the change in temperature ΔT, as long as ΔT is not too large. Also, Δl is proportional to the original length l_0,

$$\Delta l = \alpha l_0 \Delta T \tag{1}$$

To obtain equality in the above equation, we introduce the parameter α, which is called the **coefficient of linear expansion** (Values for α of different materials are shown in Table 1.).

Material	Aluminium	Bras	Iron	Lead	Quartz
$\alpha(C^{\circ})^{-1}$	25×10^{-6}	19×10^{-6}	12×10^{-6}	29×10^{-6}	0.4×10^{-6}

Table 1: Numerical value of the coefficient of linear expansion

Let's consider a system (such as iron rails) at an initial state of temperature T_0 and the length l_0, reaching a final state of temperature T and the length l. The change in temperature and length as well as the final length can be expressed as:

$$\Delta l = l - l_0, \text{ and } \Delta T = T - T_0, \text{ i.e. } l = \Delta l + l_0 = \alpha l_0 \Delta T + l_0 = l_0(1 + \alpha \Delta T) \quad (2)$$

An iron rail of 200 m long at 20° C, shrinks at −30° C by:

$$\Delta l = 12 \times 10^{-6} (^{\circ}C)^{-1} (-30 - 20)^{\circ}C \times 200m = -1.2 \times 10^{-1} \text{ m} \quad (3)$$

and expands at 40° C by:

$$\Delta l = 12 \times 10^{-6} (^{\circ}C)^{-1} (40 - 20)^{\circ}C \times 200m = 0.48 \times 10^{-1} \text{ m} \quad (4)$$

Thus, the total change of the length from 40° C to −30° C is:

$$0.48 \times 10^{-1} m + 1.2 \times 10^{-1} m = 1.68 \times 10^{-1} \text{ m} \quad (5)$$

Hence, you can open a tight jar of lid by pouring hot water as the diameter of a metal ring gets larger.

Volume Expansion:

The volume of solid or liquid changes with a change of temperature as:

$$\Delta V = \beta V_0 \Delta T, \quad (6)$$

where V_0 is the initial volume. The parameter β is called the **coefficient of volume expansion** (Example values for different materials shown in Table. 2).

Material	Aluminium	Bras	Iron	Lead	Quartz	Mercury	Ethyl Alcohol
$\beta(C^{\circ})^{-1}$	75×10^{-6}	56×10^{-6}	35×10^{-6}	87×10^{-6}	1×10^{-6}	180×10^{-6}	1100×10^{-6}

Table 2: Numerical value of the coefficient of volume expansion

The value of β for liquids is much larger than for solids, which is why mercury and alcohol are used in thermometers.

The volume can be given as the product of the three sides, i.e.

$$V = \prod_{i=1}^{3} (1 + \alpha \Delta T) l_0^i = (1 + \alpha \Delta T)^3 l_0^1 l_0^2 l_0^3 = (1 + \alpha \Delta T)^3 V_0. \quad (7)$$

For $|\alpha \Delta T| \ll 1$, we have:

$$(1 + \alpha \Delta T)^3 = 1 + 3\alpha \Delta T + 3(\alpha \Delta T)^2 + ... \approx 1 + 3\alpha \Delta T, \quad (8)$$

thus

$$V \approx (1 + 3\alpha\Delta T)V_0, \tag{9}$$

or,

$$\Delta V = 3\alpha\Delta T V_0, \tag{10}$$

i.e. $\beta = 3\alpha$. Indeed this is the case for the solids as shown in Tables 1 and 2.

Example:
A steel container of 1 m ×1 m ×1 m with an open top is filled with Mercury at 20° C. If the temperature increases to 40° C, how much mercury will overflow the container?

The volume change of the container can be given by, i) same as the solid steel block, or ii) from the length expansion of the three sides. Since $3\alpha \approx \beta$, the two will lead to a similar conclusion. We use here i) and the volume increase of the container is given by,

$$\Delta V_{cont} = \beta\Delta T V_0 = 35 \times 10^{-6}(^\circ C)^{-1} \times (40 - 20)^\circ C \times 1\text{m}^3 = 0.0007\text{m}^3. \tag{11}$$

Equally for the mercury, we obtain:

$$\Delta V_{merc} = \beta\Delta T V_0 = 180 \times 10^{-6}(^\circ C)^{-1} \times (40 - 20)^\circ C \times 1\text{m}^3 = 0.0036\text{m}^3. \tag{12}$$

Therefore, the amount of mercury overflows is,

$$\Delta V_{merc} - \Delta V_{cont} = 0.0036 - 0.0007 = 0.0029\text{m}^3 \tag{13}$$

Precise formula for the length expansion

Let us denote l_0 to be the initial length of a bar. According to the formula of the linear expansion, the length of the bar after increasing the temperature by ΔT, l, is given by:

$$l = \Delta l + l_0 = \alpha l_0 \Delta T + l_0 = l_0(1 + \alpha\Delta T) \tag{14}$$

where α is the coefficient of linear expansion. We now consider an expansion in two steps:

- Step-1 increase the temperature by $\frac{\Delta T}{2}$,
- Step-2 increase the temperature by another $\frac{\Delta T}{2}$.

After the Step-1, the length of the bar is given by:

$$l_{step-1} = l_0 \left(1 + \alpha\frac{\Delta T}{2}\right) \tag{15}$$

and after Step-2:

$$l_{step-2} = l_{step-1}\left(1 + \alpha\frac{\Delta T}{2}\right) = l_0\left(1 + \alpha\frac{\Delta T}{2}\right)^2. \tag{16}$$

If we consider n-steps to reach the temperature difference of ΔT by increasing the temperature by $\frac{\Delta T}{n}$ for the each step, the length of the bar after n-steps is given by:

$$l_{step-n} = l_0\left(1 + \alpha\frac{\Delta T}{n}\right)^n. \tag{17}$$

A continuous transition is then obtained by increasing the number of steps to infinity, i.e.

$$l_{contineous} = \lim_{n\to\infty} l_0 \left(1 + \alpha \frac{\Delta T}{n}\right)^n = l_0 e^{\alpha \Delta T}. \tag{18}$$

which is the **exact expression** for the length expansion.

By noting:

$$e^{\alpha\Delta T} = 1 + \alpha\Delta T + \frac{1}{2!}(\alpha\Delta T)^2 + \frac{1}{3!}(\alpha\Delta T)^3 + \frac{1}{4!}(\alpha\Delta T)^4 + ... \tag{19}$$

we can conclude that the formula of the linear expansion is valid only if $\alpha|\Delta T| \ll 1$.

Anomalous behaviour of water

Normally, the volume of material increases with temperature, i.e. the density decreases as a function of increasing temperature. Also the solid phase of a material has a higher density than that at the liquid phase. The water has a very different behaviour. The density of ice at the normal pressure decreases slightly with increasing temperature, i.e. at $-180°$ C, 0.9340 g/cm^3, and at $0°$ C, 0.9167 g/cm^3, which is similar behaviour to the usual material. When it melts at $0°$ C, the liquid water has a density of 0.9998 g.cm^3, higher than that of the ice at the same temperature. This is why the ice floats in the liquid water. The density of water increases (volume decreases) with increasing temperature, and it reaches at its maximum at $4°$ C, 1.00 g/cm^3. A further increase in the temperature decreases the density (increases the volume) as for usual materials. → This is very important for life on earth.

2.4 Applications: Thermal Stress

If the two ends of an iron beam are rigidly fixed, changes of temperature induce stress due to the thermal expansion or contraction, referred to as thermal stress:

1. The iron beam tries to expand by $\Delta l = \alpha \Delta T l_0$.
2. An external force, F, is required to push back the beam to stay with the same length.

The reduction of the length due to an external force is given by:

$$\Delta l' = \frac{1}{E}\frac{F}{A} l_0, \tag{20}$$

where E is Young's module (specific for a material) and A is the transversal area of the beam. Since this should compensate the thermal expansion, $\Delta l' = \Delta l$,

$$\alpha \Delta T l_0 = \frac{1}{E}\frac{F}{A} l_0, \tag{21}$$

giving the thermal stress (pressure, as force per area) to be:

$$\frac{F}{A} = \alpha E \Delta T. \tag{22}$$

Example:
Let us consider a concrete structure, like a beam of concrete supporting the motor way. The Young's module for concrete is 2×10^{10} N/m^2, and the coefficient of linear expansion, $12 \times 10^{-6} (^\circ \text{ C})^{-1}$. For a 40 degrees temperature difference, the stress is given by:

$$\frac{F}{A} = \alpha E \Delta T = 12 \times 10^{-6} (^\circ C)^{-1} \times 2 \times 10^{10} \times 40^\circ C = 9.6 \times 10^6 N m^{-2} \tag{23}$$

Noting the tensile strength, 2×10^6 N/m^2 (maximum pulling pressure before breaking), and the compressive strength, 2×10^7 N/m^2, for the concrete (maximum compression pressure before breaking), it should not break for a 40° C temperature increase, but it would be safer to have a gap between the beam and the fixation.

2.5 Special exercise: Breaking of Copper due to cooling

Let's consider we heat a piece of copper pipe to 300°C and fasten it tightly at the ends so that it can't contract upon cooling. The tensile strength of copper is 230 MN/m^2, and the Yung's modulus E for Copper is 110 GN/m^2. At what temperature will the pipe break as it cools down?

Solution: *The length change Δl that would occur upon cooling by ΔT, if the pipe were not clamped, is compensated by an equal elongation due to the thermal stress F/A. This is linked to the length change Δl via the Yung's modulus $E = (F/A)/(\Delta l/l)$.*

First, calculate change of length of copper pipe if it wasn't fixed:

$$\Delta l_1 = \alpha l \Delta T$$

Due to a tensile stress F/A, the pipe would be elongated by Δl_2:

$$\Delta l_2 = l \frac{F/A}{E}$$

The length change Δl_1 caused by cooling compensates for the length change Δl_2caused by the tensile stress, so that their sum equals zero. Thus, we can calculate ΔT:

$$\Delta l_1 + \Delta l_2 = 0 \;\rightarrow\; \alpha l \Delta T + l \frac{F/A}{E} = 0$$

Thus:

$$\Delta T = -\frac{F/A}{\alpha E} = -\frac{230 \cdot 10^6 N/m^2}{(17 \cdot 10^6 1/K) \cdot (110 \cdot 10^9 N/m^2)} = -123K = -123^\circ C$$

Add this value to the initial temperature. This results in the final temperature, at which the copper pipe, clamped at the ends, will break due to cooling:

$$T_f = T_1 + \Delta T = 300^\circ C - 123^\circ C = 177^\circ C$$

Comment: The hot water pipes in houses are therefore never rigidly clamped, as they are heated during soldering. Moreover, temperature differences as high as those assumed here do not occur: On the one hand, the flowing water cannot be colder than 0 °C, and on the other hand, 60 °C is not usually exceeded with hot water. In addition, modern heating systems work with even lower flow temperatures. Therefore, the effect described here is not to be feared with proper installation and normal operation.