General Physics II: Tutorial Material

Lecture 14 (Chapter 11 – Heat transfer and Review)

1) Black body radiation: A black body is an object at equilibrium with the radiation it emits. This radiation is characterized by the fact that the internal energy density depends only on the temperature at thermal equilibrium. The internal energy of this radiation is given by:

$$E_{int}(S,V) = \frac{3}{4}\left(\frac{3c}{16\sigma}\right)^{1/3} S^{4/3}V^{-1/3} \tag{1}$$

where sigma is the Stefan Boltzmann constant (see lecture).

a) Determine the free energy $F(T,V)$ of this radiation.

b) Show that the internal energy $U(S,V)$ of the radiation can be obtained by performing an inverse Legendre transformation on the free energy $F(T,V)$ (i.e. $E_{int}(S,V) = F + ST$).

c) Find expressions for $P(T,V)$ and $P(S,V)$ for the radiation pressure.

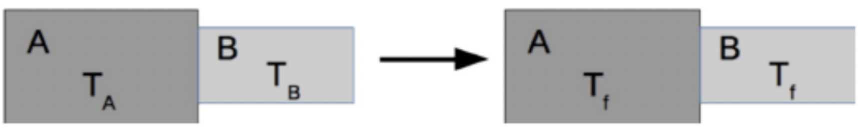
2) Exercise taken from a previous exam:
We consider a system of two solid bodies, A and B, with their masses, m_A and m_B, and specific heats per mass, c_A and c_B, respectively. And their initial absolute temperatures are T_A and T_B. Now the two bodies are put together under thermal contact while the system is thermally isolated from the environment.

a) Using the 1^{st} law of thermodynamics, calculate the temperature T_f, when the system has reached thermal equilibrium.

b) Show that T_f is always in between T_A and T_B.

c) Calculate the entropy changes for A and B, ΔS_A and ΔS_B, between the initial and the final thermal equilibrium states.

d) From ΔS_A and ΔS_B obtained above, show that the total entropy change, $\Delta S_A + \Delta S_B$, is always ≥ 0.



3) Exercise taken from a last year's exam: Atkinson Cycle
James Atkinson was a British engineer who designed several combustion engines. The Atkinson cycle is a modification of the Otto cycle intended to improve its efficiency. The trade-off in achieving higher efficiency is a decrease in the work performed per cycle. The idealized Atkinson cycle consists of the following reversible processes

- $1 \to 2$: adiabatic compression
- $2 \to 3$: isochoric heating
- $3 \to 4$: isobaric expansion
- $4 \to 5$: adiabatic expansion
- $5 \to 6$: isochoric cooling
- $6 \to 1$: isobaric cooling

Assume that the cycle is operated on an ideal gas. The following physical quantities that characterize the cycle are assumed to be known: volumes V_1, V_2, and V_6, pressure P_3 and P_5, temperature T_5 and the number of moles n of the gas. Analyze this cycle by answering the following questions:

a) Draw the PV diagram of the Atkinson cycle.

b) In which subprocesses is the (reversible) entropy change positive and in which negative? Note that it can be helpful to draw the TS diagram. Why is a negative entropy change not violating the second law?

c) Now suppose that the adiabatic expansion process (4 to 5) is replaced by an isothermal expansion process at constant temperature T_4, and assume to have a van der Waals gas instead of ideal gas, with $P = \frac{nRT}{V-nb} - a\frac{n^2}{V^2}$ and with $E_{int} = C_V T - a\frac{n^2}{V}$. Compute the work done in this process, heat transfer and its entropy change - all quantities as a function of V_4, V_5, and T_4. Also indicate whether the heat transfer and the change in entropy is positive or negative.

To practice a bit more TD potentials:

4) To practice a bit more TD potentials: Pressure in a Soap Bubble: A soap bubble is a system consisting of two subsystems. Subsystem (f) is the thin film and the subsystem (g) is the gas enclosed inside the film. The surrounding air is a thermal bath. The equilibrium is characterized by the minimum of the free energy F of the system. The differential of the free energy dF reads:

$$dF = -(S_g + S_f)dT + 2\gamma dA - (P - P_0)dV, \tag{2}$$

where a is the surface area of the soap film and V the volume of the bubble. The parameter γ is called the surface tension. It characterizes the interactions at the interface between the liquid and the air. Since the soap film has two such interfaces, there is a factor 2 in front of the parameter γ. The surface tension γ is an intensive variable that plays an analogous role for a surfacic system as the pressure P for a volumic system. However, the force due to pressure of a gas is exerted outwards whereas the force due to the surface tension is exerted inwards. This is the reason why the signs of the corresponding two terms in dF differ. The term $P - P_0$ is the pressure difference between the Pressure P inside the bubble and the atmospheric pressure P_0. Consider the bubble to be a sphere of radius r and show that

$$P - P_0 = \frac{4\gamma}{r} \tag{3}$$

5) Gibbs-Helmholtz equations: Show that

a)

$$E_{\text{int}}(S, V) = -T^2 \frac{\partial}{\partial T}\left(\frac{F(T, V)}{T}\right) \tag{4}$$

where $T = T(S, V)$ is to be understood as a function of S and V.

b)

$$H(S,P) = -T^2 \frac{\partial}{\partial T}\left(\frac{G(T,P)}{T}\right) \tag{5}$$

where $T = T(S,P)$ is to be understood as a function of S and P.