General Physics II: Tutorial Material

Lecture 4 on Chapter 5 (Statistical thermodynamics)

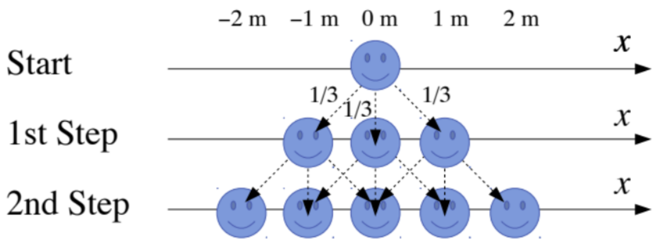
1) The rms speed of molecules in a gas at $20.0°C$ is to be increased by 2.0%. To what temperature must it be raised?

2) If you double the mass of the molecules in a gas, is it possible to change the temperature to keep the velocity distribution from changing? If so, how much change do you need to make to the temperature?

3) There are four coins with two faces, head and tail. Each coin has 50% probability to show head and 50% probability to show tail, when tossed individually. When we toss the four coins together:

1. How many head-tail configurations are there if we can distinguish individual coins? What are the probabilities for those configurations?
2. How many head-tail configurations are there if we cannot distinguish individual coins? Which configuration has the highest probability to be realized?

4) A drunken person is standing at $x = 0m$. When the drunken person makes one step, the person may go to the left (negative direction in x by 1m), remain at the same position or to the right (positive direction in x by 1 m) with a same probability (1/3 for each).



1. What is the probability for the drunken person to be at x = -8m, -7 m, -6 m, -5 m, -4 m, -3 m, -2 m, -1 m, -0 m, 1 m, 2 m, 3 m, 4 m, 5 m, 6 m, 7 m and 8 m after 1, 2, 4 and 7 steps?
2. What are the mean values, $\langle x \rangle$, and the rms, x_{rms}, for x after 1, 2, 4 and 7 steps? Figure 1 below show four probability distributions in x following the Gauss distribution, $G(x)$, given by
$$G(x) = \frac{1}{\sqrt{2\pi x_{rms}^2}} \exp\left[-\frac{(x - \langle x \rangle)^2}{2x_{rms}^2}\right] \quad (1)$$
with x_{rms} equal to those obtained for 1, 2, 4 and 7 steps above, but not necessarily in this order.
3. Find out which Gauss distributions of Fig. 1 belong to which steps.
4. Superimpose the probability distributions of the x position of the drunken person on the Gauss distribution of corresponding steps. What kind of conclusion can you draw from comparing the distributions?

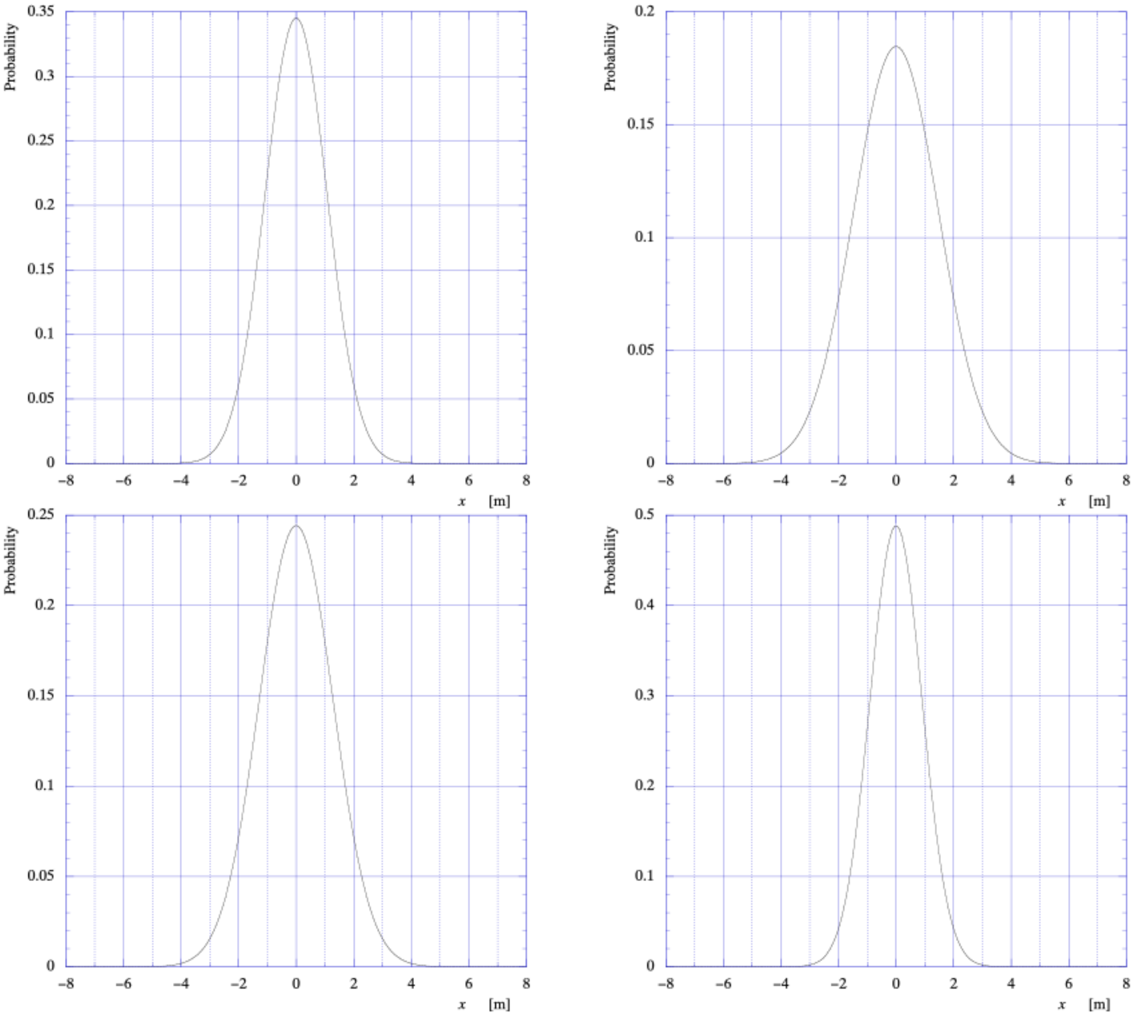


Figure 1: Gauss distributions of task 4-3.

5) There are N indistinguishable gas molecules uniformly distributed in a box with a volume V. Consider a small region in the box with a volume V_1.

1. What is the probability to find any but only one molecule in this region?
2. What is the probability to find any n molecules in this region?
3. What is the average number of molecules, $\langle n \rangle$, and its standard deviation $\Delta n \equiv \sqrt{\langle n^2 \rangle - \langle n \rangle^2}$, where $\langle n^2 \rangle$ is the average of n^2, in this region?
4. If N is of the order of the Avogadro number, i.e. about 10^{24}, and the volume of the considered region is about 1% of the total volume, how large is $\Delta n / \langle n \rangle$? What does it mean?

NB: The following formula might be useful:

$$\begin{aligned}
(p+q)^M &= \sum_{m=0}^{M} \frac{M!}{m!(M-m)!} p^m q^{M-m} \\
\sum_{m=0}^{M} m \frac{M!}{m!(M-m)!} p^m q^{M-m} &= Mp(p+q)^{M-1} \\
\sum_{m=0}^{M} m^2 \frac{M!}{m!(M-m)!} p^m q^{M-m} &= Mp(p+q)^{M-1} + M(M-1)p^2(p+q)^{M-2}
\end{aligned} \tag{2}$$