General Physics II: Tutorial Material

Lecture 3 on Mathematical Notes

1 Balloon

A helium-filled balloon escapes a child's hand at sea level where the atmosphere is in 1 atm and 20.0°C. When it reaches an altitude of 3600 m where the temperature is 5.0°C and the pressure 0.68 atm, how will its volume compare to that at the sea level.

2 Air bubble

An air bubble with a diameter of 3.60 mm was created at the bottom of the lake, which is 2.5m deep. When the bubble reached the surface of the lake, where the temperature is 27°C, the diameter of the air bubble became 4.00 mm. The pressure of the atmosphere at the surface of the lake was 1 atm. What is the temperature of the water at the bottom of the lake? The density of the water is 1×10^3 kg and the gravitational acceleration constant g is 9.80 m/s^2, and assume that the air behaves as an ideal gas.

3 Hydrogen in outer space

In our outer space, the density of matter is 1 atom per cm^3. It is dominated by the hydrogen atom and at a temperature of 2.7 K. What is the rms-speed of those hydrogen atoms? What is the pressure there in the unit of atm?

4 Vaccum pressure

The lowest pressure attainable using the best available vacuum technique is about $10^{-12} N/m^2$. At such a pressure, how many molecules are there per cm^3 at 0°C?

5 rms-speed of molecules

Show that the rms-speed of gas molecules is given by $v_{rms} = \sqrt{3P/\rho}$, where P and ρ are the pressure and density of the gas respectively.

6 The bicycle pump

A bicycle pump takes a volume ΔV of air at atmospheric pressure $\mathrm{P_0}$ and constant temperature $\mathrm{T_0}$ and compresses it so that it enters a tyre that has a volume $\mathrm{V_0}$. The air inside the tyre is initially at atmospheric pressure $\mathrm{P_0}$ and can be considered as an ideal gas. Determine the number of times N the user has to pump air into the tyre to reach a pressure $\mathrm{P_f}$. Assume that the pump is designed such that the air in the tyre is always at temperature $\mathrm{T_0}$.
Compute N for $\mathrm{V_0} = 50$ l, Delta V = 1.2 l, and $\mathrm{P_f} = 2.5\mathrm{P_0}$.

7 Basic mathematical concepts: Differentials of state functions

NOTE: Some of you may not be yet familiar with partial differentials – then please ignore this exercise until you have it in your math course, or you ask your TA!

7.1 Differentials of state functions

Consider the function $f(x, y) = y\exp(ax) + xy + bx\ln y$ where a and b are constants.
a) Calculate $\frac{\partial f(x,y)}{\partial x}$, $\frac{\partial f(x,y)}{\partial y}$, and $df(x, y)$
b) Calculate $\frac{\partial^2 f(x,y)}{\partial x \partial y}$

7.2 Differentials of the ideal gas equation

Consider the equation of state for ideal gas for the following tasks (PV = nRT).
a) Calculate the differential dP(T,V)
b) Calculate

$$\frac{\partial}{\partial T}\left(\frac{\partial P(T,V)}{\partial V}\right) \text{ and } \frac{\partial}{\partial V}\left(\frac{\partial P(T,V)}{\partial T}\right)$$

c) Cyclic rule for ideal gas: Calculate

$$\frac{\partial P(T,V)}{\partial T}\frac{\partial T(P,V)}{\partial V}\frac{\partial V(T,P)}{\partial P}$$

7.3 State Function: Rubber Cord

A rubber cord of length L, which can be described as a state function L(T, F) of the temperature T of the cord and of the forces of magnitude F applied at each end to stretch it. The two physical properties of the cord are:

a) the Young modulus, which can be expressed as $E = \frac{L}{A}\left(\frac{\partial L}{\partial F}\right)^{-1}$ with A being the cross section area. Note that this is the same (but mathematical) definition as in the lecture notes only for small changes and solved for E.

b) the thermal expansion coefficient α, which can be expressed as $\alpha = \frac{1}{L}\frac{\partial L}{\partial T}$.
Determine how much the length of the cord varies if its temperature changes by dT and at the same time the force F changes by dF. Assume that $dT << T$ and $dF << F$. Express dL in terms of E and α.