General Physics II: Tutorial Material

Lecture 10 (Chapter 9, Thermal machines)

1) Braking cycle
A system is made up of a vertical cylinder which is sealed at the top and closed by a piston at the bottom. A valve A controls the intake of gas at the top and an exhaust valve B (also at the top) is held back by a spring that exerts a constant pressure P2 on the valve. The system goes through the following processes:

- $0 \to 1$: the piston is at the top of the cylinder; Valve A opens up and the piston is lowered into it so that some of the gas at atmospheric pressure $P_0 = P_1$ is added to the cylinder. The gas is at room temperature T_1. Valve B is closed. The maximum volume occupied by the incoming gas is V_1.
- $1 \to 2$: Valve A is now closed and the piston moves upward, fast enough so that the process can be considered adiabatic. Valve B remains closed as long as the pressure during the rise of the piston is lower than P_2. As the piston continues its rise, the gas reaches pressure $P_2 = 10P_1$, at a temperature T_2 in a volume V_2. Assume a reversible adiabatic process.
- $2 \to 3$: As the piston keeps moving up, valve B opens up, the pressure is $P_3 = P_2$ and the gas is released in the environment while valve A still remains closed until the piston reaches the top, where $V_3 = V_0 = 0$.
- $3 \to 0$: Valve B closes and valve A opens up. The system is ready to start over again.

Analyze this cycle by using the following instructions:

a) Draw the P-V diagram for the 3 processes that the system is undergoing.

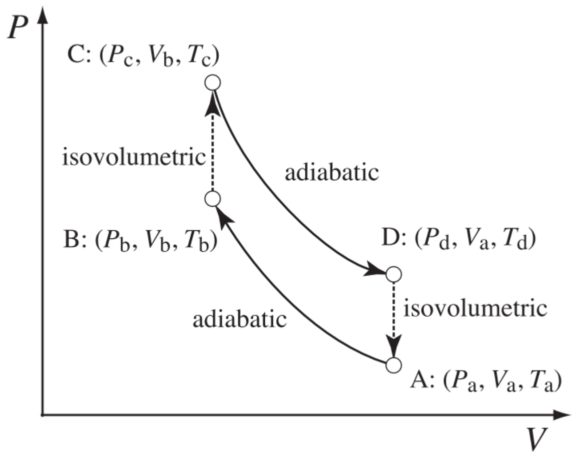
b) Determine the temperature T_2 and volume V_2.

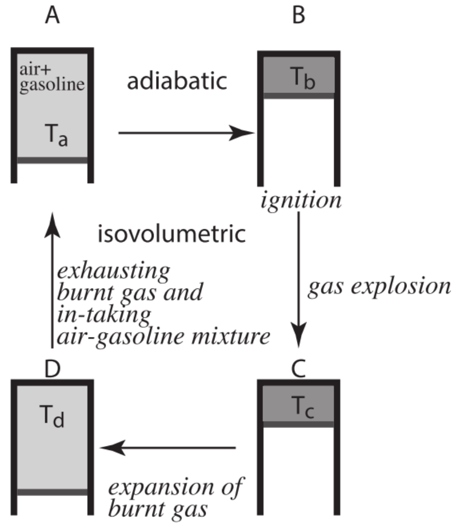
c) Find the work W performed per cycle.

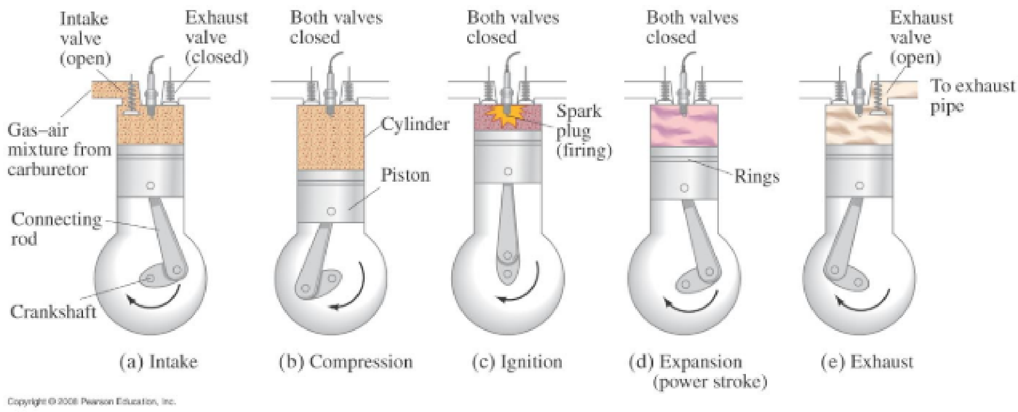
Numerical application:

$$V_0 = V_3 = 0,\ P_0 = P_1 = 1 \times 10^5 \text{ Pa},\ V_1 = 0.25 \text{ L},\ T_1 = 27°\text{C},\ \gamma = 1.4 \tag{1}$$

2) For the Otto Cycle shown in the figures below, calculate the efficiency of the Otto cycle engine and compare with that of the Carnot cycle engine, $\epsilon_{Carnot} = 1 - \frac{T_a}{T_c}$, where T_a and T_c are the lowest and highest temperature of the system, respectively. Which one of the two engines is more efficient?







3) In the Otto cycle, the volume ratio in the expansion, $C \to D$, is identical to that for the compression, $A \to B$, and is given by V_a/V_b. Some hybrid cars use Atkinson cycle where the volume ratios are different. This is realised by changing the timing of exhaust or/and intake and also called Miller cycle. In order to compare its performance with the Otto cycle, we consider the Miller cycle to use the same volume of air-gasoline mixture gas, V_a, for the adiabatic compression and the condition for ignition, i.e. the adiabatic compression of the air-gasoline mixture starts at $A(P_a, V_a, T_a)$.

The Miller cycle shown in the P-V plot below is the following:

i) At A, the piston is somewhere in the middle of the cylinder. The volume, V_a, is filled with the air-gasoline mixture and all the valves are closed.

ii) The piston moves up to the top (B) and the gas is ignited and explodes ($B \to C$).

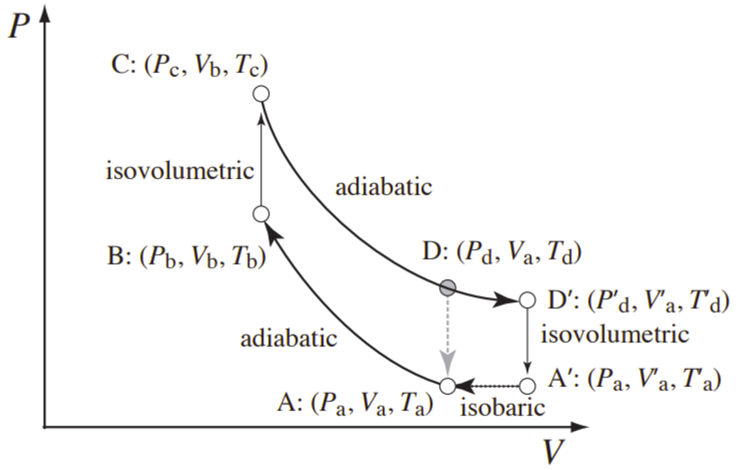
iii) The piston is pushed down ($C \to D'$) and reaches the lowest position of the cylinder (D').

iv) The piston goes up to exhaust the burnt gas (exhaust valve open) and goes down to take in the air-gasoline mixture (intake valve open), which corresponds to the isovolumetric reduction of the pressure, $D' \to A'$, where at A' the cylinder is back at the lowest position.

v) When the piston starts to move up, the intake valve is still open, thus isobaric compression starts till arriving at A where the in-take valve closes.

vi) Back to the original state and ready for the next cycle.

Figure below is the P-V plot for a Miller cycle, together with an equivalent Otto cycle.



Exercise 3.png

Show that an engine with Miller cycle ($A \to B \to C \to D' \to A' \to A$) is more efficient than that with Otto cycle ($A \to B \to C \to D \to A$).

4) A heat pump is used to warm up a room at temperature T_1 by transferring thermal energy from outside at temperature T_2, where $T_1 > T_2$ i.e. the outside is colder than the room, using work done to the heat pump. Show that a heat pump is more economical than heating the room directly with the work by computing the efficiency ("Coefficient of Performance") of the heat pump using the Carnot cycle.