General Physics II: Tutorial Material

Lecture 7 (Mock Exam I)

1) A steel wire is wrapped over a block of ice with two heavy weights attached to the end of the wire. The wire passes through a block of ice without cutting the block in two. The ice melts under the wire and the water freezes again above the wire. The wire is considered a rigid rod of negligible mass lying on the ice block with an area of contact A. The two weights of mass M each are hanging at both ends of the wire. The entire system is at atmospheric pressure P_0 and the ice is held at a temperature $T_m - \Delta T$ where T_m is the melting temperature at atmospheric pressure. The molar latent heat of melting of ice is l, the molar volume of water v_l and the molar volume of ice v_s. Determine the minimal mass M of each weight for this experiment to succeed, i.e., for the wire to pass through the ice block.

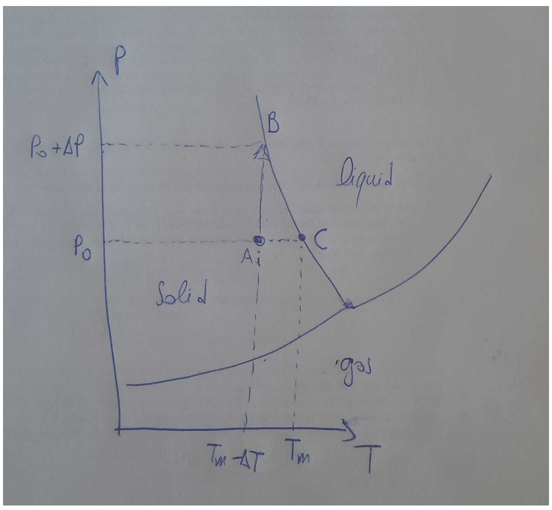
Solution : The process of ice melting due to the pressure exerted by the weights is represented by a vertical line on the $P(T)$ diagram. The pressure variation between the atmospheric pressure P_0 and the pressure $P_0 + dP$ at ice melting is expressed as:

$$\Delta P = \int_{P_0}^{P_0+\Delta P} dP = \int_{T_m}^{T_m-\Delta T} \frac{dP}{dT} dT \tag{1}$$

Using the Clausius-Clapeyron relation, where the ice latent heat of melting l is considered constant, the pressure variation ΔP is expressed as:

$$\Delta P = -\frac{l}{v_s - v_l} \int_{T_m}^{T_m-\Delta T} \frac{dT}{T} = \frac{l}{v_s - v_l} \ln\left(\frac{T_m}{T_m - \Delta T}\right) \tag{2}$$

Helpful remark: When the wire melts the ice, we go vertically from A to B in the figure below. But to know the position of B, we need to pass through the phase transition curve. In that case, we use C as initial point because it is the one we know:



Then, the pressure variation dP that allows ice to melt is equal to the pressure exerted by the minimal weight of the two masses on the area of contact A between the wire and the ice block:

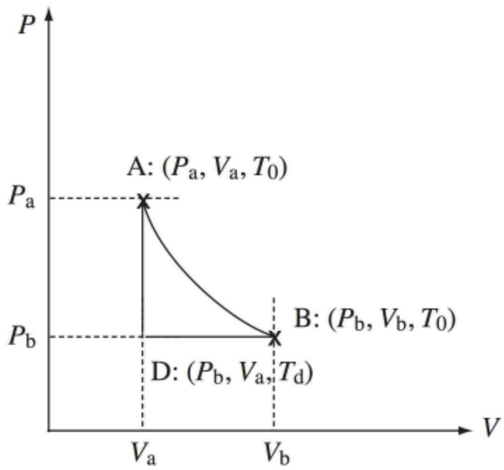
$$\Delta P = \frac{2Mg}{A} \tag{3}$$

Equating both expressions for ΔP and solving for M, we obtain the minimal value for the mass M of each weight:

$$M = \frac{Al}{2g(v_s - v_l)} \ln\left(\frac{T_m}{T_m - \Delta T}\right) \tag{4}$$

2) Let us consider an isothermal change of the state $A(V_A, P_A, T_0) \to B(V_B, P_B, T_0)$ in a closed system, where $V_A < V_B$.

1. Calculate the work and heat exchange of the system taking this path.
2. The system now takes another path from A to B, namely an isovolumetric (=isochoric) process from A to D first, followed by an isobaric process from D to B. Calculate the work and heat for the two processes. Is the total heat of the path equal to the total work of the path? Is the result expected, and why?



Solution :

1. From the ideal gas law:

$$P = \frac{nRT_0}{V}$$

The work for the isothermal process is given by:

$$W = \int_{V_a}^{V_b} P\,dV = \int_{V_a}^{V_b} \frac{nRT_0}{V} dV = nRT_0 \ln\left(\frac{V_b}{V_a}\right) > 0 \tag{5}$$

Note that from the ideal gas law, we have $P_a > P_b$. Since the temperature between A and B does not change, there is no change in the internal energy and the first law of thermodynamics gives $\Delta E_{int} = Q - W = 0$, thus

$$Q = W = nRT_0 \ln\left(\frac{V_b}{V_a}\right) \tag{6}$$

2. Since no change is made in volume for $A \to D$:

$$W_{AD} = 0 \tag{7}$$

In a isovolumetric process we have $Q_{AD} = nC_V \Delta T$.
The temperature of D is given by the ideal gas law to be

$$T_d = \frac{V_a P_b}{nR} \tag{8}$$

and since

$$T_0 = \frac{V_a P_a}{nR} \tag{9}$$

$T_0 > T_d$ i.e. the temperature decreases. The temperature difference is given by

$$\Delta T = T_d - T_0 = \frac{V_a(P_b - P_a)}{nR} \tag{10}$$

and heat by

$$Q_{AD} = nC_V \Delta T = \frac{V_a(P_b - P_a)}{R} C_V \tag{11}$$

For D to B, the work is given by

$$W_{DB} = P_b \Delta V = P_b(V_b - V_a) \tag{12}$$

and heat

$$Q_{DB} = nC_P \Delta T = \frac{P_b(V_b - V_a)}{R} C_P \tag{13}$$

Therefore, the total work is given by:

$$W = W_{AD} + W_{DB} = P_b(V_b - V_a) \tag{14}$$

and total heat:

$$Q = Q_{AD} + Q_{DB} = P_b(V_b - V_a) \tag{15}$$

where $C_P = C_V + R$ and $P_a V_a = P_b V_b = nRT_0$ are used.

The result shows that $Q = W$ as expected, since the change in the internal energy does not depend on the path, but given only by the difference in the internal energies of the final and of the initial states.

3) Work as a process-dependent Quantity: three processes are performed on a gas from a state given by P_1, V_1 to a state $(P_2, V_2\ )$ given

1. an isochoric process followed by an isobaric process
2. an isobaric process followed by an isochoric process
3. a process where PV remains constant.

Compute for the three processes the work performed on the gas from the initial to the final state. Determine the analytical results first, and then give numerical values in Joules ($P_1 = P_0$= 1bar; $V_1 = 3V_0$; $P_2 = 3P_0$; $V_2 = V_0 = 1$l).
Note: drawing these processes in a PV diagram can help.

Solution : There is no work performed on the gas during an isochoric process, only during the isobaric process or during the process where PV remains constant

1. Since the isochoric process does not produce work, the total work performed by the gas by an isochoric process followed by an isobaric process is given by:

$$\begin{aligned} W &= \int_{V_1}^{V_2} PdV = P_2 \int_{V_1}^{V_2} dV = 3P_0 \int_{3V_0}^{V_0} dV \\ &= 3P_0(V_0 3V_0) = -6P_0V_0 = -600J \end{aligned} \tag{16}$$

   Since the final volume V_2 is smaller than the initial volume V_1, work is done ON the system ($\Delta E = Q - W$, if W is negative, then the internal energy increases!)

2. The work performed on the gas by an isobaric process followed by an isochoric process is given by:

$$\begin{aligned} W &= \int_{V_1}^{V_2} PdV = P_1 \int_{V_1}^{V_2} dV = P_0 \int_{3V_0}^{V_0} dV \\ &= P_0(V_0 - 3V_0) = -2P_0V_0 = -200J \end{aligned} \tag{17}$$

3. The work performed on the gas by a process where PV remains constant is given by:

$$\begin{aligned} W &= \int_{V_1}^{V_2} PdV = P_1V_1 \int_{V_1}^{V_2} \frac{dV}{V} = 3P_0V_0 \int_{3V_0}^{V_0} \frac{dV}{V} \\ &= 3P_0V_0 \log\left(\frac{V_0}{3V_0}\right) = -3P_0V_0\log(3) = -330J \end{aligned} \tag{18}$$

4) Special exercise: Breaking of Copper due to cooling
Let's imagine we heat a piece of a copper pipe to 300°C and fasten it tightly at the ends so that it can't contract upon cooling. The tensile strength (=maximum stress a material can withstand while being stretched or pulled before breaking) of copper is 230 MN/m^2, and the Yung's modulus E for Copper is 110 GN/m^2. At what temperature will the pipe break as it cools down?

Solution: *The length change Δl that would occur upon cooling by ΔT, if the pipe were not clamped, is compensated by an equal elongation due to the thermal stress F/A. This is linked to the length change Δl via the Yung's modulus $E = (F/A)/(\Delta l/l)$.*

First, calculate change of length of copper pipe if it wasn't fixed:

$$\Delta l_1 = \alpha l \Delta T$$

Due to a thermal stress F/A, the pipe would be elongated by Δl_2:

$$\Delta l_2 = l\frac{F/A}{E}$$

The length change Δl_1 caused by cooling compensates for the length change Δl_2 caused by the tensile stress, so that their sum equals zero. Thus, we can calculate ΔT:

$$\Delta l_1 + \Delta l_2 = 0 \;\rightarrow\; \alpha l \Delta T + l\frac{F/A}{E} = 0$$

Thus:

$$\Delta T = -\frac{F/A}{\alpha E} = -\frac{230 \cdot 10^6 N/m^2}{(17 \cdot 10^6 1/K) \cdot (110 \cdot 10^9 N/m^2)} = -123K = -123°C$$

Add this value to the initial temperature. This results in the final temperature, at which the copper pipe, clamped at the ends, will break due to cooling:

$$T_f = T_1 + \Delta T = 300°C - 123°C = 177°C$$

Comment: The hot water pipes in houses are therefore never rigidly clamped, as they are heated during soldering. Moreover, temperature differences as high as those assumed here do not occur: On the one hand, the flowing water cannot be colder than 0 °C, and on the other hand, 60 °C is not usually exceeded with hot water. In addition, modern heating systems work with even lower flow temperatures. Therefore, the effect described here is not to be feared with proper installation and normal operation.