General Physics II: Tutorial Material

Lecture 1 on Chapter 1 and 2 (Temperature)

1) The periodic table gives:

Atomic number	**Element**	**Mass number**	**Atomic mass**
13	Al	27	26.981539
26	Fe	56	55.934938

Which object has more atoms: 1 kg of iron (Fe) or 1 kg of aluminium (Al)?

Solution :

The atomic mass of aluminium is less than that of iron, i.e., one atom of Al has less mass than one atom of Fe. Therefore, there are more atoms in 1 kg of aluminium than in 1 kg of iron.

2) How many atoms are there in a 3g of aluminium?

Solution:

From the atomic mass of Al, 26.981539, the mass of one Aluminum atom is given by

$$26.981539u \times 1.66 \times 10^{-27} kg/u = 4.48 \times 10^{-26} kg \tag{1}$$

Therefore, the number of atoms in 3 g Aluminum is given by

$$\frac{3 \times 10^{-3}}{4.48 \times 10^{-26}} = 6.70 \times 10^{22} \tag{2}$$

3) Suppose system C is not in equilibrium with system A nor with system B. Does this imply that A and B are not in equilibrium? What can be said about the temperatures of A, B, and C?

Solution:

No, we do not know whether system A and B are in equilibrium or not. The temperature of system C is equal neither to A nor B. But we have no information on the temperature relation between A and B.

4) In an alcohol-in-glass thermometer, the alcohol column has length $11.82cm$ at $0.0°C$ and length 21.85 cm at $100.0°C$. What is the temperature if the column length has:

1. $18.70cm$?
2. $14.60cm$?

Solution:

The change of the temperature for 1 cm increase in the length of the alcohol

$$\frac{100°C - 0°C}{21.85cm - 11.82cm} = 9.97°C/cm \tag{3}$$

Therefore:

- $9.97°C/cm \times (18.70 - 11.82)cm + 0°C = 68.6°C$
- $9.97°C/cm \times (14.60 - 11.82)cm + 0°C = 27.7°C$

5) A flat bimetal strip consists of a strip of aluminium riveted to a strip of iron. When heated, the strip will bend. Which metal will be on the outside of the curve? Why?

Solution:

Aluminium has a larger thermal coefficient of linear expansion than that for iron. Therefore, Aluminium will be outside of the curve when heated.

6) The density of water at $4°C$ is $1.00 \times 10^3 kg/m^3$. What is water's density at $94°C$, assuming a constant coefficient of volume expansion, $210 \times 10^{-6}(°C)^{-1}$?

Solution:

The water with a volume of $1m^3$ at $4°C$ changes its volume at $94°C$ by

$$210 \times 10^{-6}(C°)^{-1} \times 1m^3 \times (94 - 4)C° = 0.0189m^3 \tag{4}$$

Thus, the volume is $1.0189m^3$. The density of the water at $94°C$ is then given by

$$\frac{1.00 \times 10^3 kg/m^3}{1.0189} = 0.981 \times 10^3 kg/m^3 \tag{5}$$

7) There is a 10 cm long bar made by a material with a coefficient of linear expansion to be $10^{-1}/C°$ at $0°C$. If we warm up the bar to $5°C$, how long will be the bar? If we warm the bar by another $5°C$ ($5°C$ to $10°C$), how long will be the bar? If we warm the 10 cm long bar at $0°C$ to $10°C$ directly, how long it will be? How do we understand the result?

Solution:

Warming up that bar from $0°C$ to $5°C$, the length of the bar will become

$$l_5 = 10 \times [1 + 10^{-1} \times (5 - 0)] = 15cm \tag{6}$$

and waming up by another $5°C$,

$$l_{10} = 15 \times [1 + 10^{-1} \times (10 - 5)] = 22.5cm \tag{7}$$

By heating up the bar directly from $0°C$ to $10°C$, we obtain

$$l_{10}^{direct} = 10 \times [1 + 10^{-1} \times (10 - 0)] = 20cm \tag{8}$$

i.e., two processes to warm up the bar to the same temperature give different lengths which is a paradox. This indicates the expression using a linear coefficient is an approximation to calculate the elongation. This can be understood by reformulating the first case as

$$l = l_0 \lim_{n\to\infty} \left[1 + c\frac{(T_f - T_i)}{n}\right]^n \approx l_0 \left[1 + c(T_f - T_i) + \text{higher order}\right] \tag{9}$$

where c is the linear expansion coefficient and T_i and T_f are the initial and final temperature, then the linear approximation can be valid only for $c(T_f - T_i) \ll 1$. For this particular case,

$c(T_f - T_i) = 1$, thus the linear approximation does not really work. But note that for most metal solids the linear expansion coefficient is of the order of $10^{-5} 1/°C$, meaning in that case the approximate expansion formula would be good enough for the given temperature change.

8) A ruler was calibrated to the correct length measurement at temperature, T_0. With this ruler, the two sides of a rectangular metal sheet are measured to be a'_1 and b'_1 at temperature, T_1.

1. What is the true surface area of the metal sheet at temperature T_1?
2. What will be the measured surface area of the metal sheet at temperature T_2, with the same ruler?

Note that the coefficient of linear expansion for the material used for the ruler is α_r and that for the metal sheet, α_s, and both are very small.

Solution :

1. Length of a bar made of the same metal as the ruler, l_0 at temperature T_0, becomes $l_1 = l_0[1 + \alpha_r(T_1 - T_0)]$ at temperature T_1. By denoting a_1 to be the true length at temperature T_1, the measured length by the ruler at temperature T_1 can be expressed as

$$a'_1 = \frac{a_1}{l_1} l_0 \tag{10}$$

Therefore, the true length is

$$a_1 = a'_1[1 + \alpha_r(T_1 - T_0)], \tag{11}$$

Similarly,

$$b_1 = b'_1[1 + \alpha_r(T_1 - T_0)] \tag{12}$$

and the true surface area of the metal sheet at temperature T_1 is given by

$$S_1 = a_1 b_1 \approx a'_1 b'_1[1 + 2\alpha_r(T_1 - T_0)] \tag{13}$$

2. The true length of the two sides of the metal sheet at temperature, T_2, is given by

$$a_2 = a_1[1 + \alpha_s(T_2 - T_1)] \approx a'_1[1 + \alpha_r(T_1 - T_0) + \alpha_s(T_2 - T_1)] \tag{14}$$

and

$$b_2 \approx b'1[1 + \alpha_r(T_1 - T_0) + \alpha_s(T_2 - T_1)] \tag{15}$$

The ruler at temperature T_2 will measure them to be

$$a'_2 = a_2[1 + \alpha_r(T_0 - T_2)] \approx a'_1[1 + \alpha_r(T_1 - T_2) + \alpha_s(T_2 - T_1)] \tag{16}$$

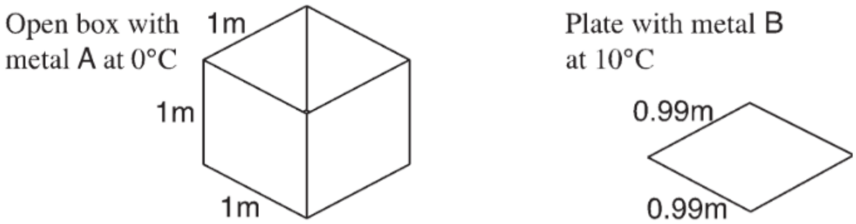
and

$$b'2 \approx b'_1[1 + \alpha_r(T_1 - T_2) + \alpha_s(T_2 - T_1)] \tag{17}$$

Therefore, the measure surface area of the metal sheet at temperature T_2 is given by

$$S'_2 = a'_2 b'_2 \approx a'_1 b'_1[1 + 2\alpha_r(T_1 - T_2) + 2\alpha_s(T_2 - T_1)]. \tag{18}$$

9) Coefficients of linear expansion for the metal A and metal B are $10^{-5}/°C$ and $5 \times 10^{-5}/°C$, respectively. A box with a dimension of $1m \times 1m \times 1m$ at $0°C$ is made of the five sheets of metal A without a top. There is a plate with a dimension of $0.99m \times 0.99m$ at $10°C$ made of the metal B. When both the box and plate are kept at the same temperature, what is the minimum temperature at which the metal B plate can be used to close the box completely? Note that the thickness of the metal plates can be neglected.



Solution:

Let us denote the temperature of the room to be $T°C$. The opening of the box at this temperature is given by $l_{box} \times l_{box}$, where

$$l_{box} = 1 \times [1 + 10^{-5} \times (T - 0)] \tag{19}$$

With the same temperature, the side of the plate, l_{plate}, becomes

$$l_{plate} = 0.99 \times [1 + 5 \times 10^{-5} \times (T - 10)] \tag{20}$$

In order the metal B plate to close the box completely, $l_{box \leq l_{plate}}$. Therefore, the minimum temperature can be obtained by solving

$$1 \times [1 + 10^{-5} \times (T - 0)] = 0.99 \times [1 + 5 \times 10^{-5} \times (T - 10] \tag{21}$$

It follows that

$$T \approx 265.70°C \tag{22}$$