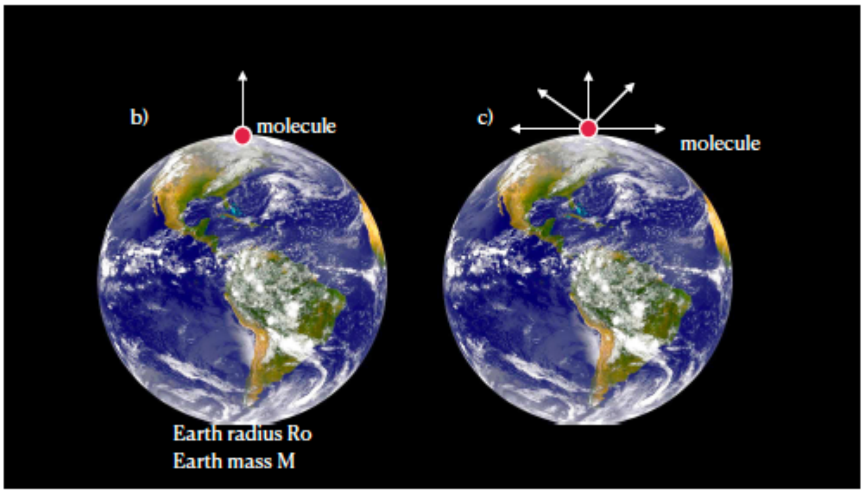
General Physics II at EPFL -- Thermodynamics

Mock exam I

1. *Statistical physics:* molecules escaping the Earth into space

At the earth surface, there are air molecules moving in random directions. The earth's gravitational pull prevents most air molecules from escaping into space. However, the pull is not infinitely strong. In this problem, you will calculate how likely it is that air molecules do escape.



a) Suppose the air molecules can be approximated as ideal gas. What is the average kinetic energy of one gas molecule as a function of the temperature? And what is the probability $P(v_z)$ that a gas molecule has a certain speed perpendicular to the earth surface (v_z) at a given temperature T? Make a schematic sketch of this probability P versus the speed.

b) The potential energy of a molecule of mass m on the surface of the earth is $-mMG/R_0$, where M is the mass of the earth, G is the gravitational constant, and R_0 is the radius of the earth. Suppose a particle can only escape if it is sufficiently fast in the direction away from the center of the earth as indicated in the figure above on the left. The condition, which the kinetic energy of an air molecule must satisfy to escape from the surface of the earth into space, is that the kinetic energy must be larger than the potential energy:

$$\frac{1}{2}mv_z^2 > \frac{mMG}{R_0} \tag{85}$$

Thus, the escape speed for a particle is:

$$v_{z,esc} > \sqrt{\frac{2MG}{R_0}} \tag{86}$$

How probable is it for a molecule at temperature T to satisfy this condition and to escape from earth? Supply a numerical answer. Use the numerical values below. Instruction: Set $\frac{mv_z^2}{2k_BT}$ to x^2.

c) Suppose that as long as the particle has enough kinetic energy in any direction away from the earth, even if it is not entirely perpendicular to the surface of the earth as in b), it will escape (see figure on the right). This means that the absolute escape speed (irrespective of the direction) is given by:

$$v_{esc} > \sqrt{\frac{2MG}{R_0}} \tag{92}$$

To compute the probability that a particle will escape in this case, one has to switch from cartesian to spherical coordinates ($dx\ dy\ dz = r^2 dr\ d\phi\ \sin\theta d\theta$), and integrate the probability above over half a sphere, i.e. over θ, $\phi =$ and v (again, v is the absolute value of a velocity vector):

$$P(v) = \sqrt{\frac{m}{2\pi k_B T}}^{3} \int_{v_{esc}}^{\infty} e^{-\frac{mv^2}{2k_BT}} v^2 dv \int_0^{\frac{\pi}{2}} \sin\theta d\theta \int_0^{2\pi} d\phi = \tag{93}$$

$$= \sqrt{\frac{m}{2\pi k_B T}}^{3} \int_{v_{esc}}^{\infty} e^{-\frac{mv^2}{2k_BT}} 2\pi v^2 dv \tag{94}$$

Calculate this probability $P(v)$ (by computing the last integration over v) and provide a numerical answer. Instruction: Do the same substitution as in c), $\frac{mv^2}{2k_BT} = x^2$.

The following quantities may be helpful:

$\frac{mMG}{R_0 k_B T} \approx 25$

$\frac{1}{\sqrt{\pi}} \int_1^\infty e^{-t^2} dt = 8 \cdot 10^{-2}$ $\quad \frac{1}{\sqrt{\pi}} \int_2^\infty e^{-t^2} dt = 2 \cdot 10^{-3}$ $\quad \frac{1}{\sqrt{\pi}} \int_3^\infty e^{-t^2} dt = 1 \cdot 10^{-5}$

$\frac{1}{\sqrt{\pi}} \int_4^\infty e^{-t^2} dt = 8 \cdot 10^{-9}$ $\quad \frac{1}{\sqrt{\pi}} \int_5^\infty e^{-t^2} dt = 8 \cdot 10^{-13}$ $\quad \frac{1}{\sqrt{\pi}} \int_6^\infty e^{-t^2} dt = 1 \cdot 10^{-17}$

$\frac{1}{\sqrt{\pi}} \int_1^\infty t^2 e^{-t^2} dt = 1 \cdot 10^{-1}$ $\quad \frac{1}{\sqrt{\pi}} \int_2^\infty t^2 e^{-t^2} dt = 1 \cdot 10^{-2}$ $\quad \frac{1}{\sqrt{\pi}} \int_3^\infty t^2 e^{-t^2} dt = 1 \cdot 10^{-4}$

$\frac{1}{\sqrt{\pi}} \int_4^\infty t^2 e^{-t^2} dt = 1 \cdot 10^{-7}$ $\quad \frac{1}{\sqrt{\pi}} \int_5^\infty t^2 e^{-t^2} dt = 2 \cdot 10^{-11}$ $\quad \frac{1}{\sqrt{\pi}} \int_6^\infty t^2 e^{-t^2} dt = 4 \cdot 10^{-16}$

2. *Heat*: Water and Ice

Consider an isolated system with a 10 kg ice block in 20 kg of water, which is in a thermal equilibrium state. In the following, assume that the specific heat and heat of fusion of the ice are 0.5kcal/(kg · C) and 80kcal/kg, respectively, and specific heat of the water is 1kcal/(kg · C). Assume an atmospheric pressure.

1) What is the temperature of the system?

2) If we add 20 kg of water at 90°C to the system, what will be the temperature of the system after reaching its equilibrium and what are the constituents of the system?