Thermodynamics Lecture Notes: Chapter 11

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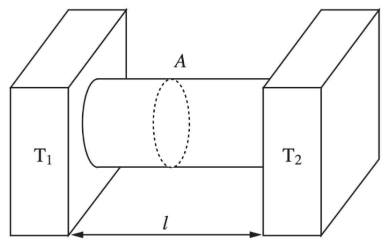
Contents

11 Heat transfer

11.1 Heat conduction and Fourier's law

Thermal conduction: Thermal energy is transferred through direct contact of materials with different temperatures, for example as shown in the Figure below.



The rate of energy transfer from object-1 to -2 is given by the Fourier's law:

$$\frac{Q}{t} = kA\frac{T_1 - T_2}{l} \tag{1}$$

where A is tyhe cross-section area of the object, l is the distance between the two ends, T_1 and T_2 are the temperatures of the obeject-1 and object-2 respectively, and, k is the thermal conductivity. Moreover, $\frac{Q}{t} > 0$ means that the thermal energy flows out from 1, then if $T_1 - T_2 > 0$, the thermal energy flows out from the object-1. If we chose the origin of the coordinate system to be the edge of the object-1, we have $\frac{dT}{dx} < 0$ for $T_1 > T_2$. Therefore, for an infinitesimal small length, this leads to

$$\frac{dQ}{dt} = -kA\frac{dT}{dx} \tag{2}$$

The material with large k are called thermal conductors and the ones with small k are called thermal insulators.

Two applications:

1. **Heat loss through windows**
   What is the heat flow rate through a glass window ($A = 2m \times 1.5m$) with a thickness of 3.2mm and $T_{inner} = 15°C$ and $T_{outer} = 14°C$?
   Apply Fourier's law:

   $$\frac{Q}{t} = kA\frac{T_1 - T_2}{l} = 0.84\frac{3 \times 1}{3.2 \times 10^{-3}}J/s = 790J/s = 680kcal/h \tag{3}$$

   Which ways exist to improve the insulation windows?

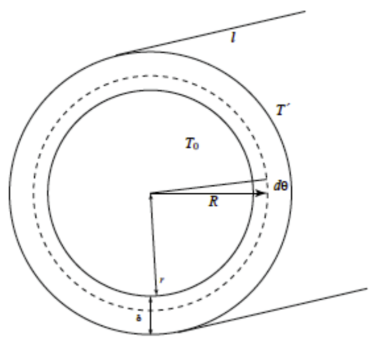
   - Double or triple glazing: This involves using two or three glass panes with air or inert gas (like argon or krypton) sealed between them. The gas acts as an insulator, and multiple panes provide additional barriers to heat transfer.
   - For double or triple-glazed windows, filling the space between the panes with inert gases like argon or krypton, which have lower thermal conductivity than air, can improve insulation.

- Using window frames with a thermal break, a material that reduces the flow of thermal energy between the interior and exterior of the window, can greatly improve insulation. Materials like fiberglass, vinyl, or wood, or metal frames with a thermal break, are commonly used.

2. **Hot water in cylindrical pipes**
A cylindrical pipe has inner radius r with a wall thickness of δ. The pipe carries hot water inside at a temperature of T_0. The temperature outside is T'. Let us consider a small wedge of the cylinder with a length l, as shown in the figure below. From the formula for the heat transfer, we know

$$\frac{dQ}{dt} = -kA\frac{dT}{dx} \tag{4}$$

which means that the rate of heat transfer is proportional to the temperature gradient, the direction of the heat is in R and area A at R is given by $l \cdot R \cdot d\theta$. Therefore it leads to the heat rate for this small wedge, $d(\frac{dQ}{dt})$, is given by



$$d\left(\frac{dQ}{dt}\right) = -k \cdot l \cdot R \cdot d\theta\frac{dT}{dR} \tag{5}$$

The total rate is then obtained by integrating the left side of the equation for θ from 0 to 2π and

$$\frac{dQ}{dt} = -2\pi k \cdot l \cdot R \cdot \frac{dT}{dR} \tag{6}$$

When the temperatures inside and outside remain constant, the temperature gradient remains constant, thus $\frac{\delta Q}{dt}$ is also constant, and it follows that $R \cdot \frac{dT}{dR} = c$, and

$$dT = \frac{c}{R}dR \rightarrow T = c\ln R + c' \tag{7}$$

where c and c' are unknown constants. Since at $R = r$, and $R = r + \delta$, $T = T'$, c can be obtained as

$$c = \frac{T' - T_0}{\ln\frac{r+\delta}{r}} \tag{8}$$

For $\frac{\delta}{r} \ll 1$, we obtain

$$\ln\frac{r+\delta}{r} = \ln(1 + \frac{\delta}{r}) \approx \frac{\delta}{r} \tag{9}$$

thus

$$\frac{dQ}{dt} = 2\pi k(T_0 - T')l\frac{r}{\delta} \tag{10}$$

Conclusion: the larger the radius of the pipe, the longer is the pipe, the thinner is the pipe, and the more heat is lost.

So far: heat conduction in materials/solids.
Basic principle: coupling of neighbouring atoms so that oscillation energy can be transferred from x to $x + dx$.
Metals: special case due to free movable electrons
→ collisions among electrons and atoms contribute to the heat conduction
→ typically thermal conductors!

These processes are also possible in liquids and gases:

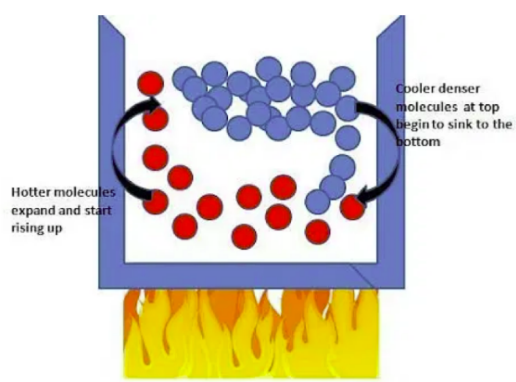
- Liquids: coupling between neighbouring atoms weaker → less efficient thermal conduction than in solids; e.g. water has a small conduction capability.
- Gases: Energy transfer via collisions between atoms and molecules. Even smaller conductivity than in liquids and solids due to low density.
  → Air or other gas good for insulation e.g. doubly glazed window, reduced heat losses!!

11.2 Heat Convection

Heat convection is an important heat transfer mechanisms for gases and liquids. A change of density with temperature and gravitational force can generate a flow of matter with different temperatures. Fluid flows, and thus, convection, needs to modelled (numerically) via complex fluid dynamics (hydrodynamic simulations).

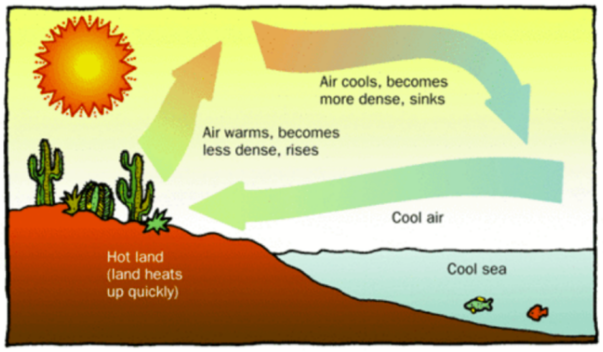
Examples:

- Water in a pot

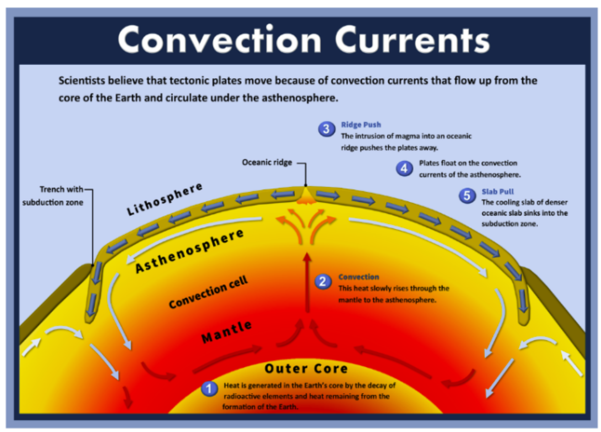


Convection Current

- Convection in the earth atmosphere



- Convection of lava inside planets – responsible for movement of tectonic plates.



11.3 Radiation

Heat transfer via radiation is an energy transfer without any medium, through electromagnetic waves. Radiation rate from an object with a temperature T is found to follow the Stefan-Boltzmann equation

$$\frac{Q}{t} = \epsilon \sigma A T^4 \tag{11}$$

where σ is the Stefan-Boltzmann constant, $\sigma = 5.67 \times 10^{-8} W/(m^2 K^4)$ and Q is the flow of thermal energy from the object. The constant ϵ is called the emissivity of the object, a number between 0 and 1: $\epsilon = 1$ corresponds to "black body".

An object can also absorb energy with radiation. When the body has a temperature T_1 is placed in an environment with a temperature T_2, the energy flow into the object is given by

$$\frac{Q}{t} = \epsilon \sigma A (T_1^4 - T_2^4) \tag{12}$$

i.e. if $T_1 = T_2$ there is no energy flow. If $T_1 > T_2$, we have $\frac{Q}{t} > 0$, thus thermal energy is flowing out from the body to the environment.

Applications

- **Cooling by radiation**

  A person (skin temperature of 34 °C) is sitting in a locker room (black walls) at 15 °C, what is the heat loss rate of the person (if $\epsilon = 0.7$, and $A_{person} = 15\ m^2$)?

  $\rightarrow$ Apply SB law with $T_1 = 34$ °C = 307 K, and $T_2 = 15$ °C = 288 K $\rightarrow \frac{Q}{t} = 120W$

- **Two tea pots**

  Assume we have one ceramic tea pot with $\epsilon = 0.7$ and one shiny tea pot with $\epsilon = 0.1$. Both can be assumed to be a cube with length of 10cm, and both are filled with 0.75 l of 95 °C water.
  A) What is the heat loss rate?
  B) What is the drop in temperature after 30min?

  – A) Area of tea pot exposed to heat: $A = 5 \times (0.1m)^2 = 0.05m^2$

  $$\rightarrow \dot{Q} = \epsilon\sigma\ 0.05m^2 \times ((368K)^4 - (293K)^4) \approx 30\epsilon W \tag{13}$$

  $\rightarrow 20W$ for ceramic pot (radiates away more energy), and 3 W for shiny pot

  – B) For computing the drop in T after 30min, ignore the contribution from the pot.

  $$\begin{aligned} &\frac{dQ}{dt} = mc\frac{dT}{dt} \rightarrow \frac{dT}{dt} = \frac{\dot{Q}}{mc} = \frac{30\epsilon J/s}{0.75kg \times 4186J/(kg\ ^\circ C)} \\ &\frac{dT}{dt} \approx 0.01\epsilon\ ^\circ C/s \end{aligned} \tag{14}$$

  Since 30 min correspond to 1800 s: change in T of the ceramic pot is 12 °C, and for the shiny pot, it is 2 °C; thus, in a ceramic pot water cools down faster due to radiation.

- **Heating by radiation from the Sun**

  No usage of previous equation, because no uniform T_2 of environment. Instead, the Sun can be considered as a point source: 1350 J strikes the earth atmosphere per second and per square meter, and on a clear day, $\sim 1000\ W/m^2$ reaches the surface of the earth:

θ is the angle between sun rays and the line perpendicular to the area A $\to A\cos(\theta)$ is the effective area at right angle to sun rays.

$$\to \dot{Q} = 1000W/m^2 \epsilon A \cos\theta \tag{15}$$

What is the rate of energy absorption from the sun by a person ($\epsilon = 0.7$, 2m x 0.4m) lying on the beach ($\theta \sim 30$degree)?

$$\dot{Q} = 1000W/m^2 \times 0.7 \times 0.8m^2 0.87 = 500W \tag{16}$$

How can a person reduce the absorbed energy rate?
$\to$ wear something light colored to have a smaller ϵ!

Importance of the Stefan-Boltzmann law in Astrophysics:

Stars can be approximately considered as black-body radiators ($\to \epsilon = 1$). The giant star Betelgeuse (in orion, visible by eye, 640 ly away from us) has an energy loss rate of 10000 that of the sun, and a (surface) temperature roughly half of that of the sun. Given that the radius of the sun is $7 \cdot 10^8 m$, what is the radius of Betelgeuse?

$$\begin{aligned} &\text{surface area of the sun} = A_{sun} 4\pi R_s^2 \\ &\text{surface area of the Betelgeuse} = A_{sun} 4\pi R_B^2 = \frac{\dot{Q}}{\sigma T_B^4} \end{aligned} \tag{17}$$

Then we can compute the ratio of the radiation rate:

$$\begin{aligned} \frac{\dot{Q}_B}{\dot{Q}_S} &= \frac{R_B^2 T_B^4}{R_S^2 T_S^4} = 10^4 \to \frac{R_B^2}{R_S^2} = 10^4 \frac{T_S^4}{T_B^4} \\ R_B \sqrt{10^4 \frac{T_S^4}{T_B^4} R_S^2} &= 10^2 \left(\frac{T_S}{T_B}\right)^2 R_S \end{aligned} \tag{18}$$

Therefore, $R_B = 400 R_S \approx 3 \cdot 10^{11} m$.

$\to$ If Betelgeuse was our "Sun", it would envelope the earth!!
Betelgeuse is in a special stage, being a super-giant star: stopped H-burning in core, instead

higher element burning in core and H-burning in shell.
→ lots of energy released → higher radiation pressure (against gravity) so that radius increases, expected to explode as a supernova once fusion processes have ceased.

Note: Our Sun in a state of H-core burning since 4.6 billion years, total life time $\sim 10Gyrs$ → continue for at least 5 Gyrs.

But: when H fuel exhausted → red giant phase and radius inflation due to H-shell burning for a few 100 million years close to or beyond the earth orbit.

Afterwards, gravity takes over and the sun will collapse to a white dwarf (degenerate electron pressure acting against gravity).