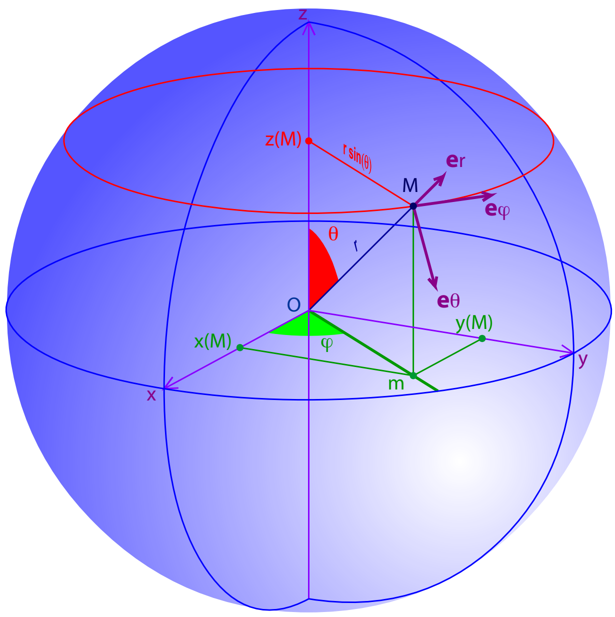
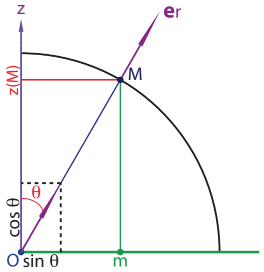
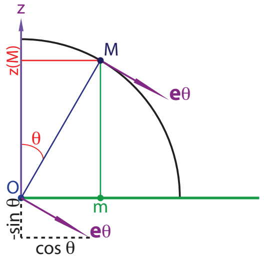
Coordonnées sphériques

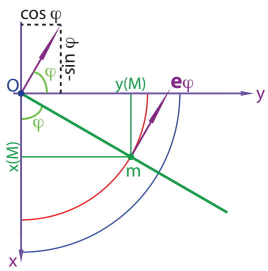




$$\vec{e}_r = \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$$



$$\vec{e}_\theta = \begin{pmatrix} \cos\theta\cos\phi \\ \cos\theta\sin\phi \\ -\sin\theta \end{pmatrix}$$



$$\vec{e}_\phi = \begin{pmatrix} -\sin\phi \\ \cos\phi \\ 0 \end{pmatrix}$$

<u>Dérivation des vecteurs $\vec{e}_r$, $\vec{e}_\theta$ et $\vec{e}_\phi$...</u>

$$\dot{\vec{e}}_r = \begin{pmatrix} \dot{\theta}\cos\theta\cos\phi - \dot{\phi}\sin\theta\sin\phi \\ \dot{\theta}\cos\theta\sin\phi + \dot{\phi}\sin\theta\cos\phi \\ -\dot{\theta}\sin\theta \end{pmatrix} = \dot{\theta}\vec{e}_\theta + \dot{\phi}\sin\theta\vec{e}_\phi$$

$$\dot{\vec{e}}_\theta = \begin{pmatrix} -\dot{\theta}\sin\theta\cos\phi - \dot{\phi}\cos\theta\sin\phi \\ -\dot{\theta}\sin\theta\sin\phi + \dot{\phi}\cos\theta\cos\phi \\ -\dot{\theta}\cos\theta \end{pmatrix} = -\dot{\theta}\vec{e}_r + \dot{\phi}\cos\theta\vec{e}_\phi$$

$$\dot{\vec{e}}_\phi = \begin{pmatrix} -\dot{\phi}\cos\phi \\ -\dot{\phi}\sin\phi \\ 0 \end{pmatrix} = -\dot{\phi}\sin\theta\vec{e}_r - \dot{\phi}\cos\theta\vec{e}_\theta$$

$$\overrightarrow{OM} = \vec{r} = r\vec{e}_r$$

$$\vec{v} = \dot{\vec{r}} = \dot{r}\vec{e}_r + r\dot{\vec{e}}_r = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_theta + r\dot{\phi}\sin\theta\vec{e}_\phi$$

$$\vec{a} = \dot{\vec{v}} = \ddot{r}\vec{e}_r + \dot{r}\dot{\vec{e}}_r + \dot{r}\dot{\theta}\vec{e}_\theta + r\ddot{\theta}\vec{e}_\theta + r\dot{\theta}\dot{\vec{e}}_\theta + \dot{r}\dot{\phi}\sin\theta\vec{e}_\phi + r\ddot{\phi}\sin\theta\vec{e}_\phi + r\dot{\phi}\dot{\theta}\cos\theta\vec{e}_\phi + r\dot{\phi}\sin\theta\dot{\vec{e}}_\phi$$

$$= \underbrace{\ddot{r}\vec{e}_r} + \underline{\dot{r}\dot{\theta}\vec{e}_\theta} + \dot{r}\dot{\phi}\sin\theta\vec{e}_\phi + \underline{\dot{r}\dot{\theta}\vec{e}_\theta} + \underline{r\ddot{\theta}\vec{e}_\theta} - \underbrace{r\dot{\theta}^2\vec{e}_r} + r\dot{\theta}\dot{\phi}\cos\theta\vec{e}_\phi + \dot{r}\dot{\phi}\sin\theta\vec{e}_\phi + r\ddot{\phi}\sin\theta\vec{e}_\phi$$

$$+ r\dot{\phi}\dot{\theta}\cos\theta\vec{e}_\phi + r\dot{\phi}\sin\theta(-\underbrace{\dot{\phi}\sin\theta\vec{e}_r} - \underline{\dot{\phi}\cos\theta\vec{e}_\theta})$$

$$a_r = \ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta \quad a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta \quad a_\phi = 2\dot{r}\dot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta + r\ddot{\phi}\sin\theta$$