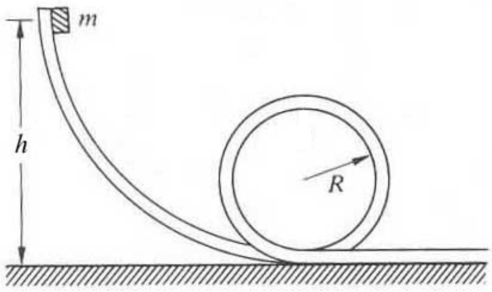
Problem Set 9

Potential energy, conservation of energy

PHYS-101(en)

1. Circular loop

An object of mass m is released from rest at a height h above the surface of a table. The object slides along the inside of a loop-the-loop track, consisting of a ramp and a circular loop of radius R (as shown in the figure). If the mass is just barely able to complete the loop without ever losing contact with the track, what height h did the object start at? Assume that the track is frictionless and neglect air resistance.



2. Two-body interaction

The force of interaction between a particle of mass m_1 and a second particle of mass m_2 separated by a distance r is given by an attractive gravitational force and a repulsive force that is proportional to r^{-3} with a proportionality constant C,

$$\vec{F}(r) = \left(-\frac{Gm_1m_2}{r^2} + \frac{C}{r^3}\right)\hat{r}.$$

Choose your zero point for potential energy at infinity and note that the differential line element in spherical coordinates is $d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}$.

1. If the masses start off an infinite distance apart and are then moved until they are a distance R apart, what is the potential energy difference $\Delta U = U(R) - U(\infty)$?
2. What is the distance R_0 between the two masses when they are in equilibrium? What is the value of the potential energy $U(R_0)$? Is this equilibrium stable or unstable?

3. A particle in Gaussian potential

A particle of mass m moves in one dimension. Its potential energy is given by

$$U(x) = -U_0 e^{-x^2/a^2},$$

where U_0 and a are positive constants.

1. Draw an energy diagram showing the potential energy $U(x)$, the kinetic energy $K(x)$, and the total energy $E < 0$ for the motion of a particle that is trapped between two turning points at $x = \pm a$.
2. Find the force $F(x)$ on the particle as a function of position x.
3. Find the particle's speed at the origin $x = 0$ such that, when it reaches either of the turning points at $x = \pm a$, it will reverse its motion.

4. Review: A smooth rope and a rough block

A uniform inextensible horizontal rope with a mass M and length L is attached to a block of mass m_b and is lying on a table. The opposite end of the rope is pulled horizontally with a force $\vec{F}_a$. The coefficient of kinetic friction between the block and table is μ_k, while friction between the rope and table can be neglected. Find the tension in the rope as a function of the distance from the block.