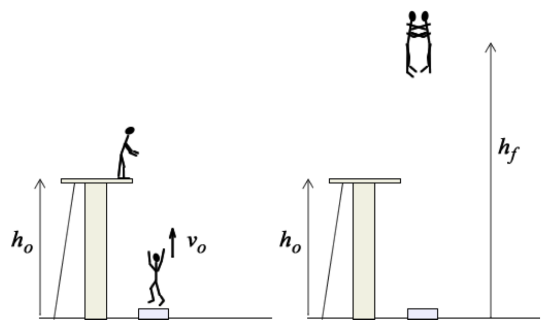
Problem Set 7

Momentum and continuous mass transfer

PHYS-101(en)

1. Acrobat and clown

An acrobat of mass m_A jumps upwards off a trampoline with an initial speed v_0. At a height h_0, the acrobat grabs a clown of mass m_C, who is standing stationary at the edge of a platform. They then travel upwards together. Assume that the time it takes for the acrobat to grab the clown is very short.



What is the maximum height h_f reached by the acrobat and clown? Write your answer in terms of some or all of the following: m_A, m_C, g, h_0, and v_0.

2. Falling raindrop

A raindrop of initial mass m_0 starts from rest and falls under the influence of gravity. Assume that as the raindrop travels through a stationary cloud, it gains mass at a rate proportional to the momentum of the raindrop,

$$\frac{dm_r}{dt} = km_r v_r,$$

where m_r is the instantaneous mass of the raindrop, v_r is the instantaneous velocity of the raindrop, and k is a constant with units of m^{-1}. You may neglect air resistance.

1. Derive a differential equation for the raindrop's acceleration dv_r/dt in terms of k, v_r, and the acceleration due to gravity g.
2. What is the terminal speed $v_{r\infty}$ of the raindrop? Express your answer in terms of k and g.

3. Falling chain

A uniform chain of mass M and length L is suspended vertically with its lowest end touching a scale. The chain is released and falls onto the scale. What is the force measured by the scale when a length of chain D has fallen? Assume that the individual links of the chain are infinitesimally small and let g denote the acceleration due to gravity.

4. Homework: Rocket with changing mass

A rocket at rest on the ground is launched vertically. It travels upwards and consumes fuel at a constant rate of D (in units of mass per time), which is ejected downwards in the form of gas with a constant speed of u relative to the rocket.

Determine the speed of the rocket when the fuel is exhausted, given that the total fuel mass is m_t and the total mass of the rocket (including fuel) is M at take-off. Determine the altitude of the rocket at the moment that the fuel is exhausted. You may neglect air resistance and consider the acceleration due to gravity g to be constant.

Values: $M = 100$ tons, $u = 3000$ m/s, $m_t = 80$ tons, $D = 500$ kg/s, $g = 10$ m/s^2

Hint 1: $\vec{F}_{net}^{ext} = d\vec{p}/dt$ (i.e. generalized Newton's second law)

Hint 2: $\int \ln(x)dx = x(\ln(x) - 1) + \text{constant}$