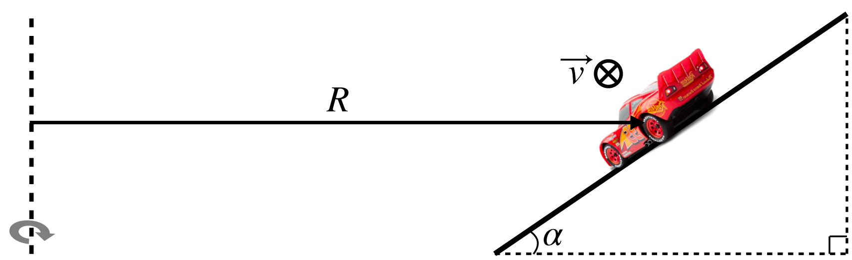
Problem Set 4

Circular motion

PHYS-101(en)

1. Circular motion: banked turn

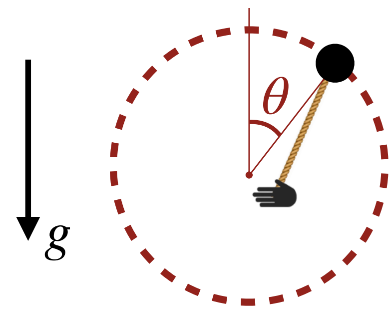
A car of mass m is going around a circular turn of radius R, that is banked at an angle α with respect to the ground. The coefficient of static friction between the tires and the road is μ_s. Let g be the magnitude of the gravitational acceleration. You may neglect kinetic friction (i.e. the car's tires do not slip).



1. At what speed should the car enter the banked turn if the road is very slippery (i.e. $\mu_s \to 0$) in order not to slide up or down the banked turn? Call this speed v_0.
2. What is the minimum speed v_{min} the car needs so that it does not slide **down** the banked turn? You can assume that $\mu_s < \tan\alpha$.
3. What is the maximum speed v_{max} the car can have so that it does not slide **up** the banked turn? You can assume that $\mu_s \tan\alpha < 1$.
4. Suppose the car enters the turn with a speed v such that $v_{max} > v > v_0$. Find an expression for the magnitude of the friction force.

2. Swinging ball

Sally swings a ball of mass m in a circle of radius R in an upright vertical plane by means of a massless string. The speed of the ball is constant and it makes one revolution every t_0 seconds.



1. Find an expression for the radial component of the tension in the string $T_\rho(\theta)$, where θ is the angle between the vertical and radial directions. Note that while the ball moves in a circle, Sally's hand cannot remain at the center of the circle, if a constant speed is to be maintained. Express your answer in terms of some combination of the parameters m, R, t_0, and the gravitational constant g.
2. Is there a range of values of t_0 for which this type of circular motion can **not** be maintained? If so, what is that range?

3. Spiral motion of a point mass

A point mass P with mass m is represented in polar coordinates. The motion of P is determined by the vector sum of the following two external forces acting on it

$$\vec{F}_1 = -mk^2\vec{r}$$

and

$$\vec{F}_2 = -2m\lambda\vec{v},$$

where $k > \lambda > 0$. Note that we neglect gravity. The force $\vec{F}_1$ is spring-like (i.e. proportional and opposite to $\vec{r}$, the displacement from the equilibrium position at the origin) and $\vec{F}_2$ is a viscous friction-type force (i.e. proportional and opposite to the velocity v).

In this problem you are given that:

- $\dot{\phi} \neq 0$, $\ddot{\phi} = 0$, and $\ddot{\rho} \neq 0$,
- the initial conditions at $t = 0$: $\phi = 0$ and $\rho = \rho_0$,
- the formulas for the velocity $\vec{v}$ and the acceleration $\vec{a}$ in polar coordinates:

$$\vec{v} = \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi}$$

$$\vec{a} = \left(\ddot{\rho} - \rho\dot{\phi}^2\right)\hat{\rho} + \left(\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi}\right)\hat{\phi},$$

- the solution of the equation $\dot{\rho} = -\lambda\rho$ has the form $\rho(t) = Ce^{-\lambda t}$, where C is an integration constant.

1. Represent the system graphically in polar coordinates.
2. Write down the equations of motion in the form of differential equations, without solving them.
3. From the equations of motion, determine
   - the radial position $\rho(t)$,
   - the angle $\phi(t)$ and use it to find $\rho(\phi)$ from $\rho(t)$, and
   - the speed of the particle $v(t)$.

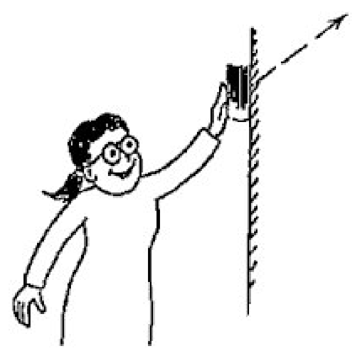
4. Circular motion of the earth

The earth is spinning about its axis with a period of 23 hours 56 min and 4 sec. The equatorial radius of the earth is 6.38×10^6 m. The latitude of Lausanne is $46°31'$N.

1. Find the velocity of a person at EPFL as they undergo circular motion about the earth's axis of rotation.
2. Find the person's centripetal acceleration.

5. Homework: Pushing a book against a wall

You are holding a book against a vertical wall by pushing it upwards with your hand. The angle between your force and the vertical is α (which is $< 90°$). The mass of the book is m and the coefficient of static friction is μ_s. There are two cases: if you push too hard the book will start to slide up and if you don't push hard enough the book will slide down.



1. Draw free body diagrams for both cases, when the book is just about to start sliding.
2. For both cases, calculate the magnitude of your force (as a function of α) to just prevent slipping.
3. Calculate the force (as a function of α) for which the friction becomes zero. Evaluate your result for $\alpha = 0°$ and $\alpha = 90°$.