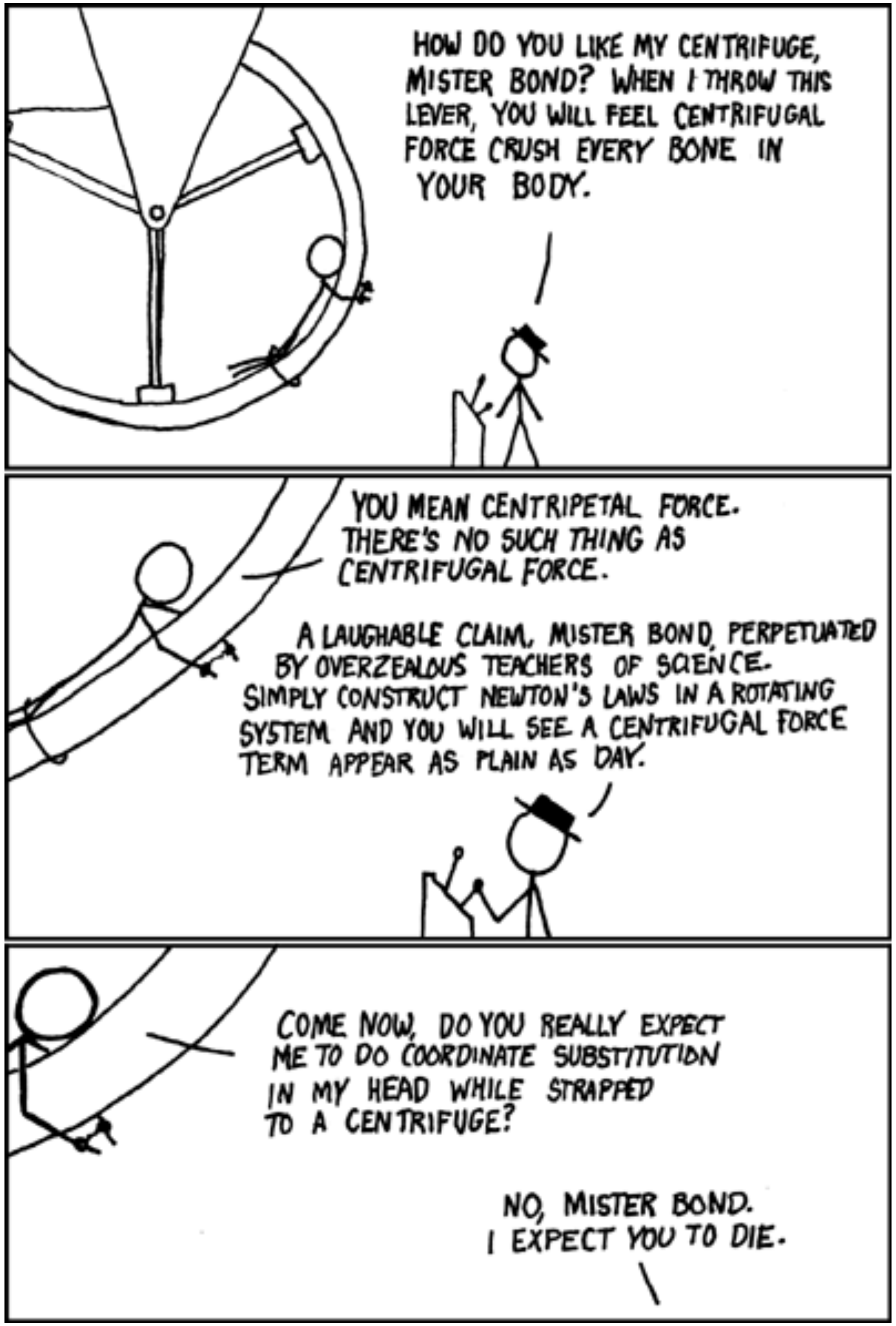
General Physics: Mechanics

EPFL Swiss Plasma Center

PHYS-101(en)

Lecture 5a:
Non-inertial reference frames, constraints



xkcd.com/123

Dr. Marcelo Baquero
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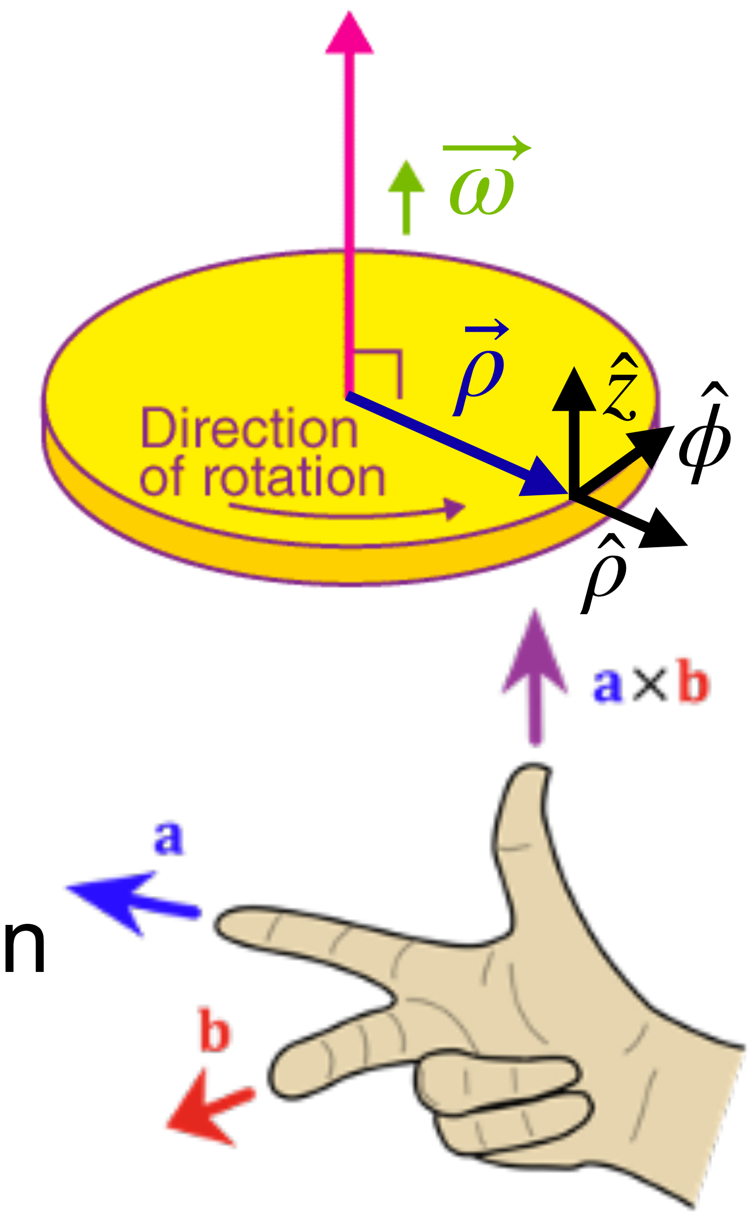
October 7th, 2024

Today’s agenda (Serway 6.3, MIT 8)

1. **Derivation of forces in non-inertial reference frames**
2. Applications of Newton’s laws
   - Ropes and pulleys
   - Example to understand constraints

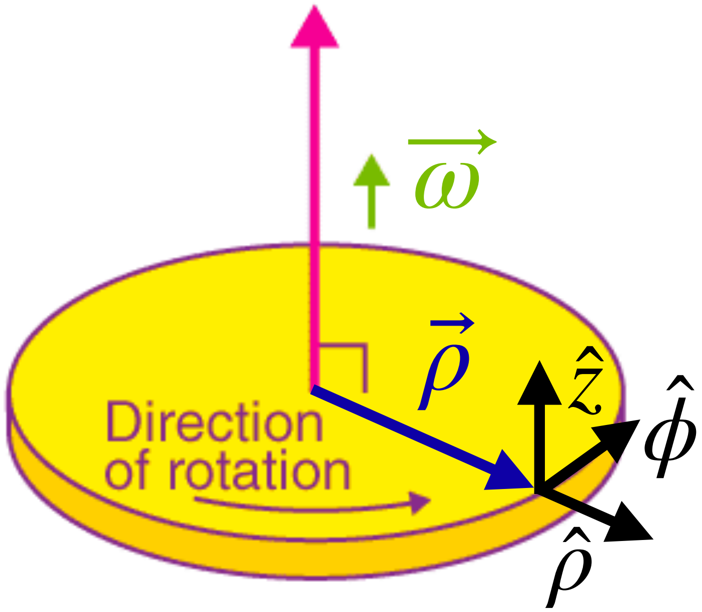
Review: Angular velocity and acceleration

- The magnitudes of the angular velocity and angular acceleration are defined as $\omega = \dot{\phi}$ and $\alpha = \ddot{\phi}$
- Their directions can be found using $\overrightarrow{\omega} = (\vec{\rho} \times \vec{v}_\phi)/\rho^2$ or $\overrightarrow{\alpha} = (\vec{\rho} \times \vec{a}_\phi)/\rho^2$ together with the right hand rule
- Often (but not always!) in the $\pm\hat{z}$ direction
- If we only care about the magnitudes:



Review: Angular velocity and acceleration

- For such a cylindrical coordinate system:



$$d\hat{\rho}/dt = \omega\hat{\phi} \quad \Leftrightarrow \quad d\hat{\rho}/dt = \vec{\omega} \times \hat{\rho}$$

$$d\hat{\phi}/dt = -\omega\hat{\rho} \quad \Leftrightarrow \quad d\hat{\phi}/dt = \vec{\omega} \times \hat{\phi}$$

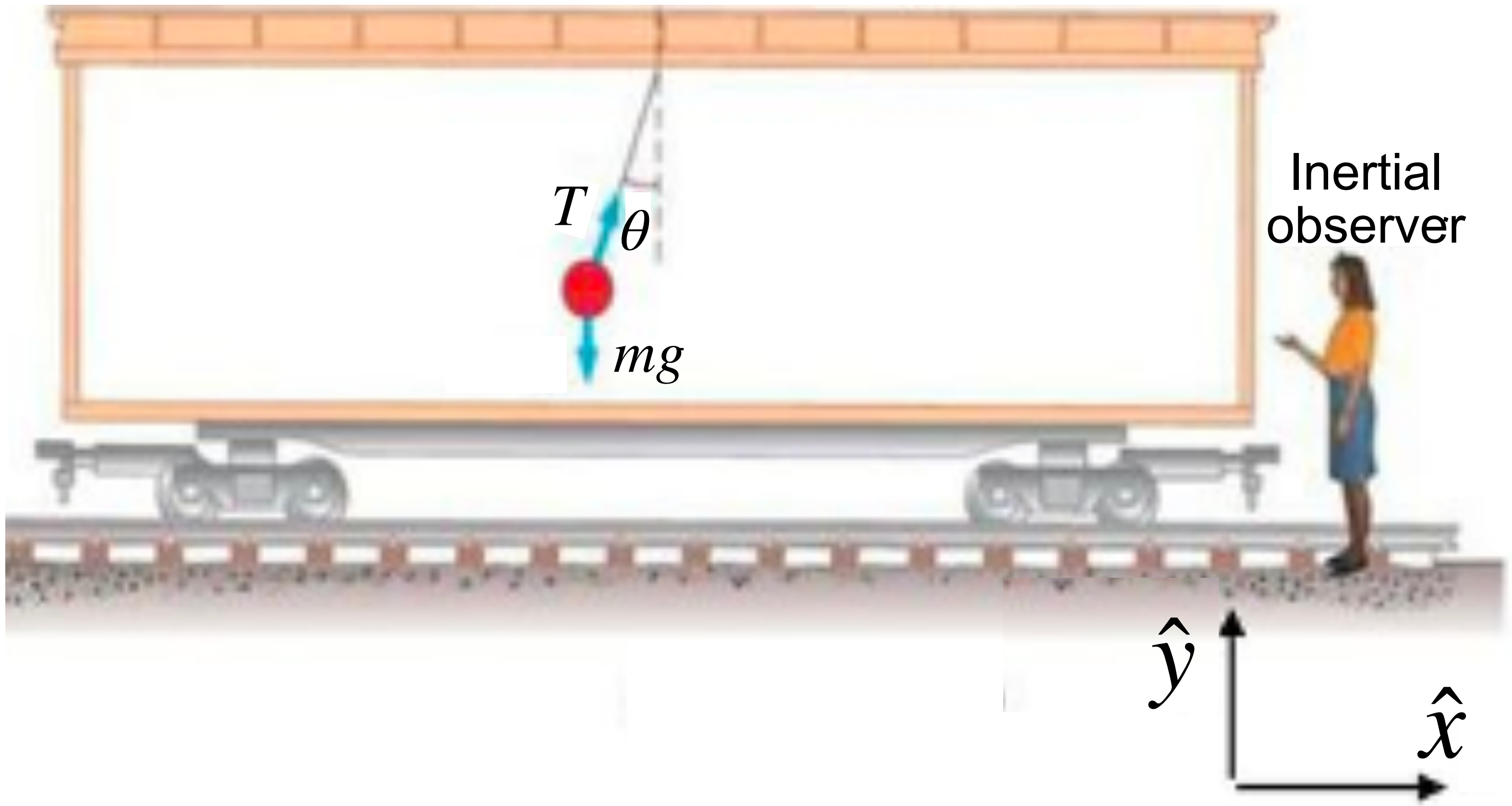
$$\vec{a}_{cent} = -\rho\omega^2\hat{\rho} \quad \Leftrightarrow \quad \vec{a}_{cent} = \vec{\omega} \times \left(\vec{\omega} \times \vec{\rho}\right)$$

DEMO (613)

Foucault pendulum

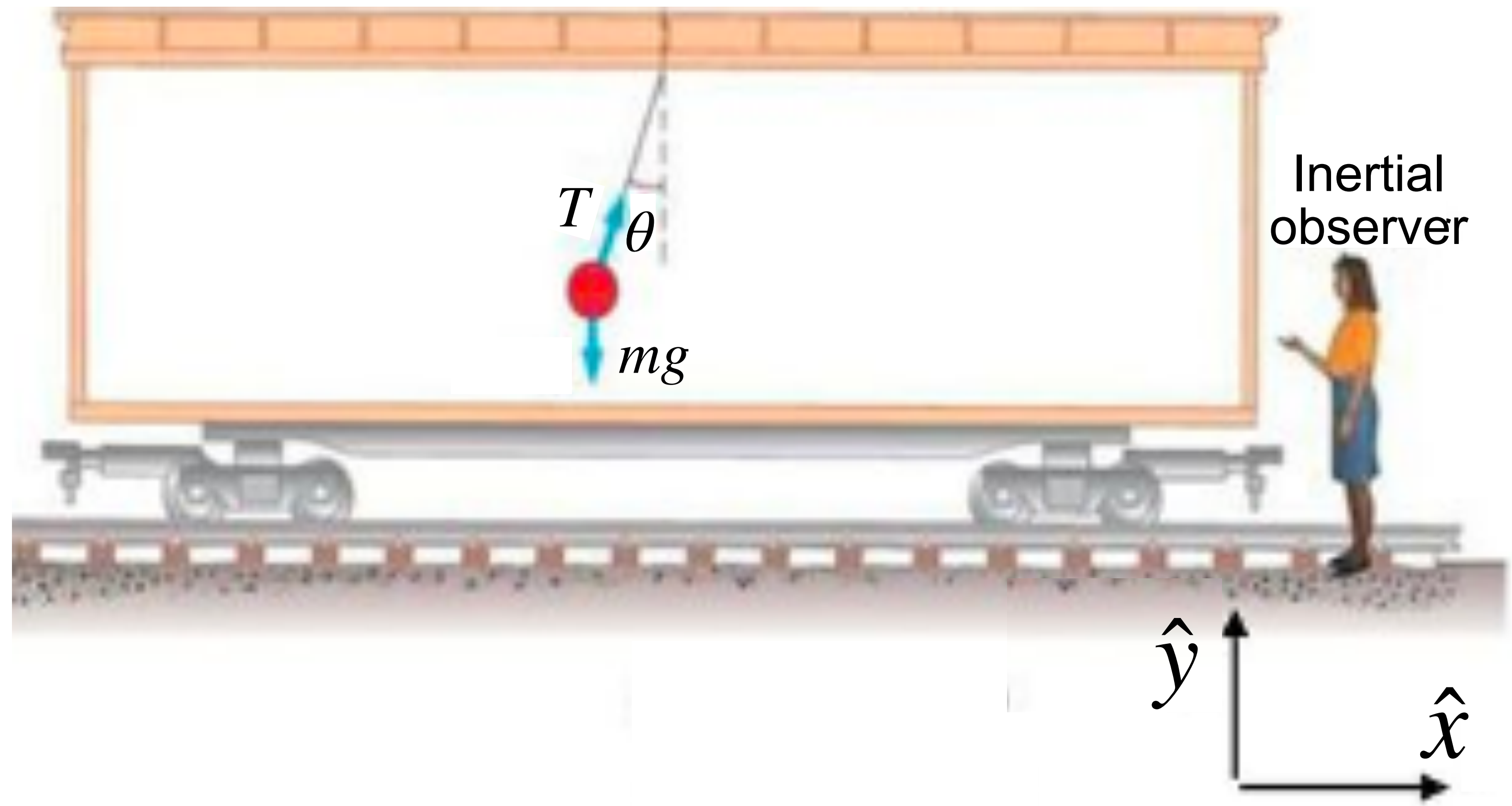
Inertial and non-inertial reference frames

- A train is accelerating



Inertial and non-inertial reference frames

- A train is accelerating
- An inertial observer beside the train attributes the acceleration of the ball to the tension force



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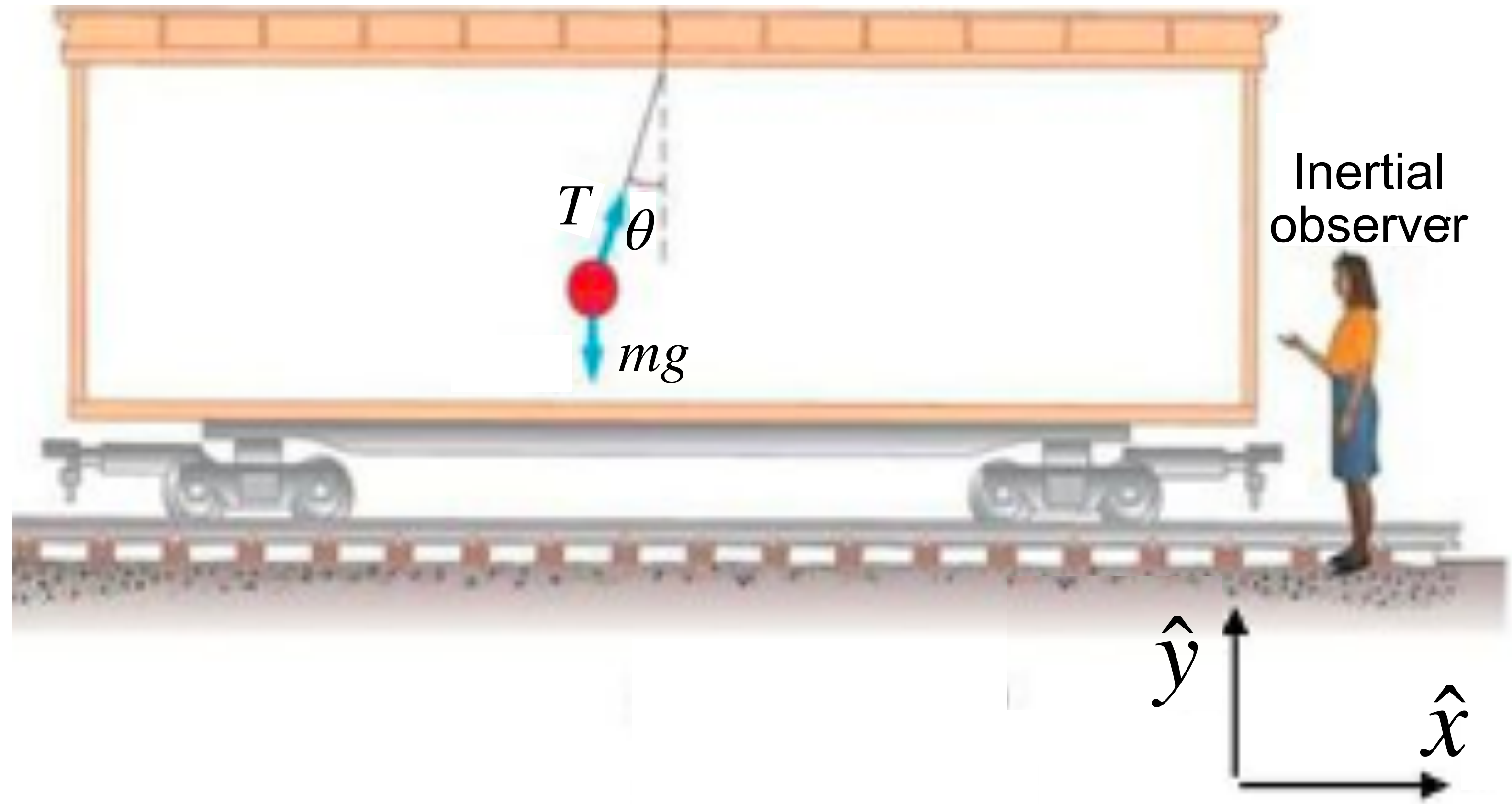
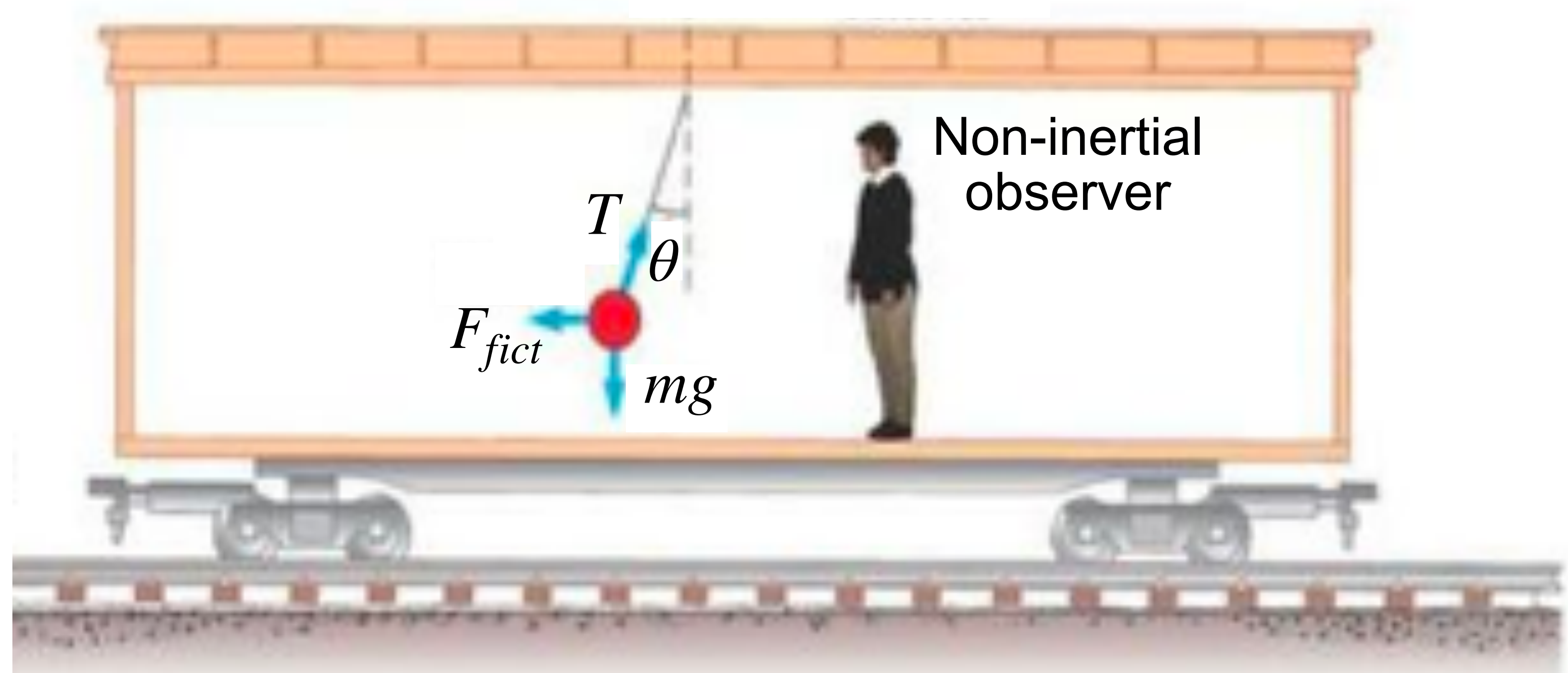
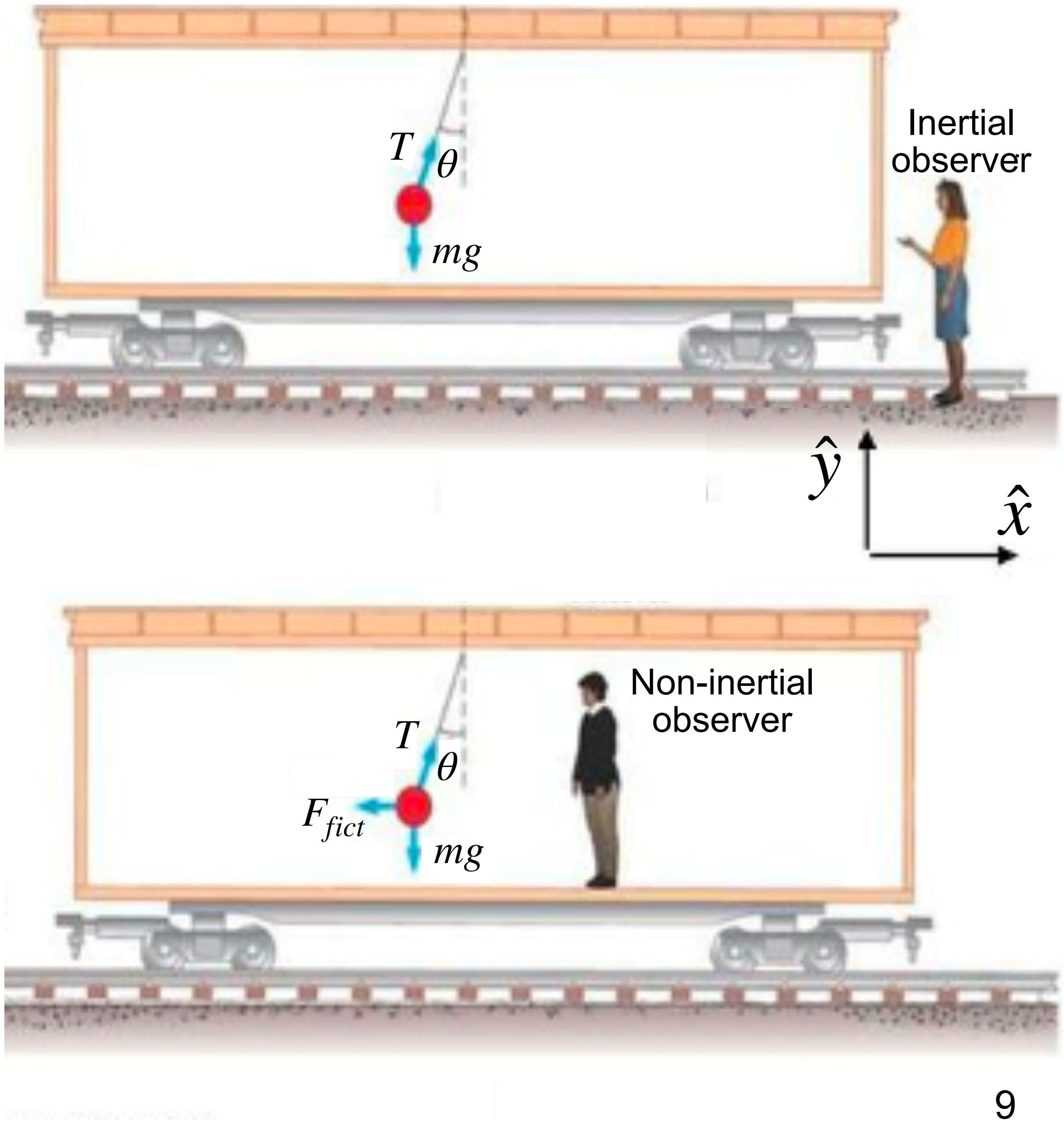




Inertial and non-inertial reference frames

- A train is accelerating
- An inertial observer beside the train attributes the acceleration of the ball to the tension force
- An non-inertial observer on the train sees the ball at rest, so there is no net force
- The deflection from vertical is attributed to a **fictitious** (or inertial) force



DEMO (230 and 483)

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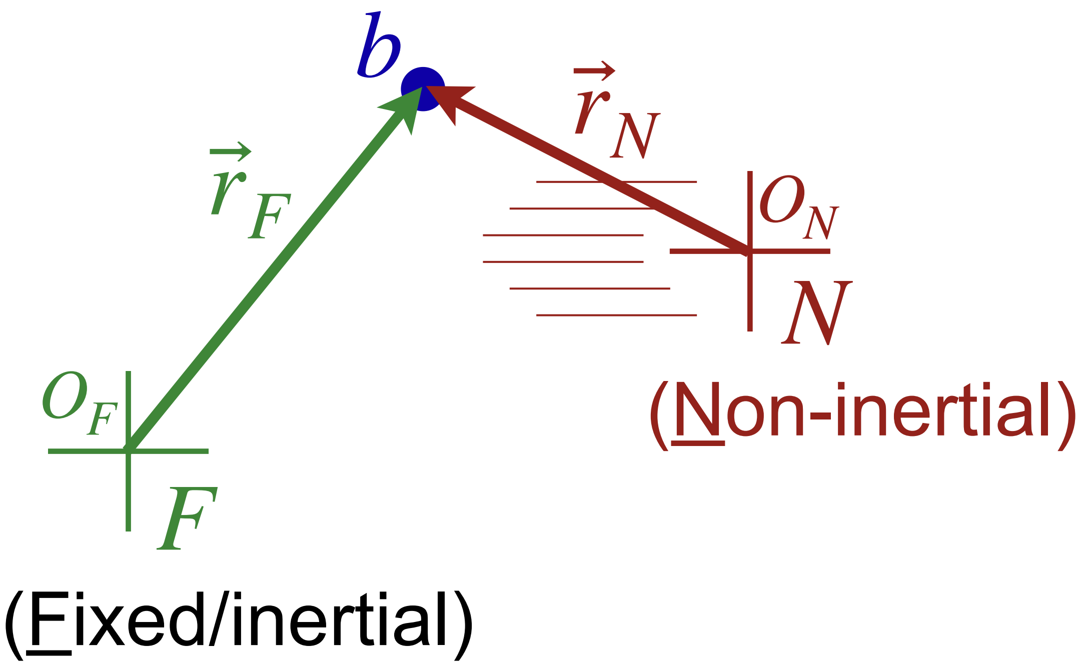
Translating balloon

Linearly accelerating non-inertial frame

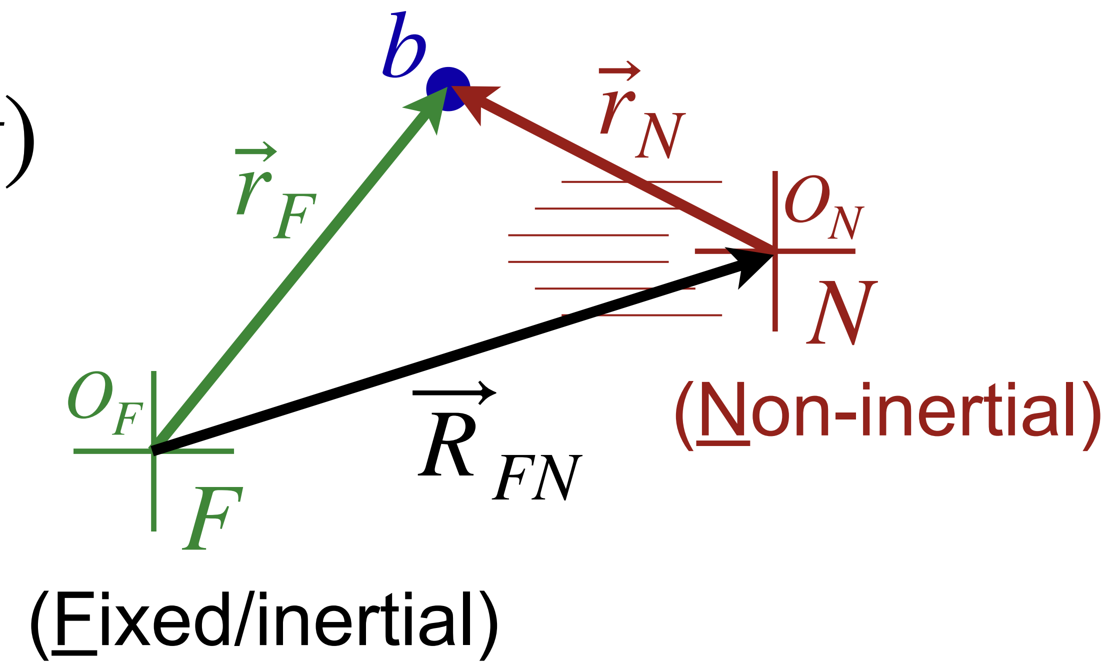
- Take reference frame N, which is **accelerating** in a line



Linearly accelerating non-inertial frame

- Take reference frame N, which is **accelerating** in a line

$$\vec{r}_N(t) = \vec{r}_F(t) - \overrightarrow{R}_{FN}(t)$$



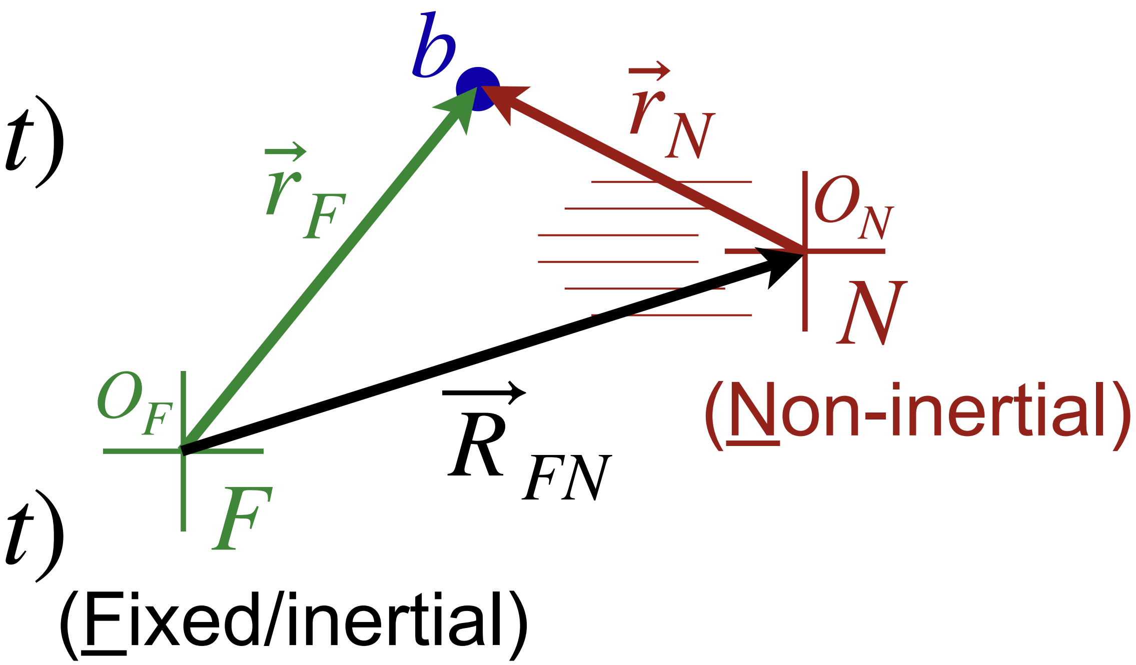
Linearly accelerating non-inertial frame

- Take reference frame N, which is **accelerating** in a line

$$\vec{r}_N(t) = \vec{r}_F(t) - \overrightarrow{R}_{FN}(t)$$

- Take derivative in time

$$\vec{v}_N(t) = \vec{v}_F(t) - \overrightarrow{V}_{FN}(t)$$



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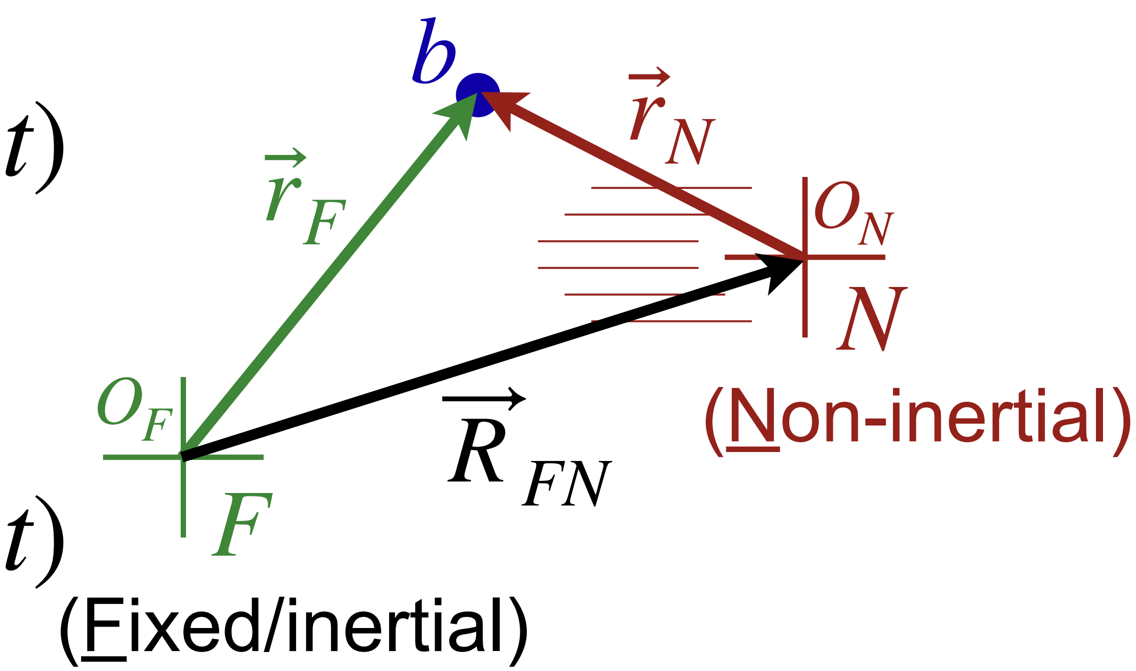

- Take derivative in time

$$\vec{v}_N(t) = \vec{v}_F(t) - \overrightarrow{V}_{FN}(t)$$

- Take derivative in time again

$$\vec{a}_N(t) = \vec{a}_F(t) - \overrightarrow{A}_{FN}(t)$$

- Multiply by mass of ball to get $\sum \overrightarrow{F}_N = \sum \overrightarrow{F}_F - m_b \overrightarrow{A}_{FN}$

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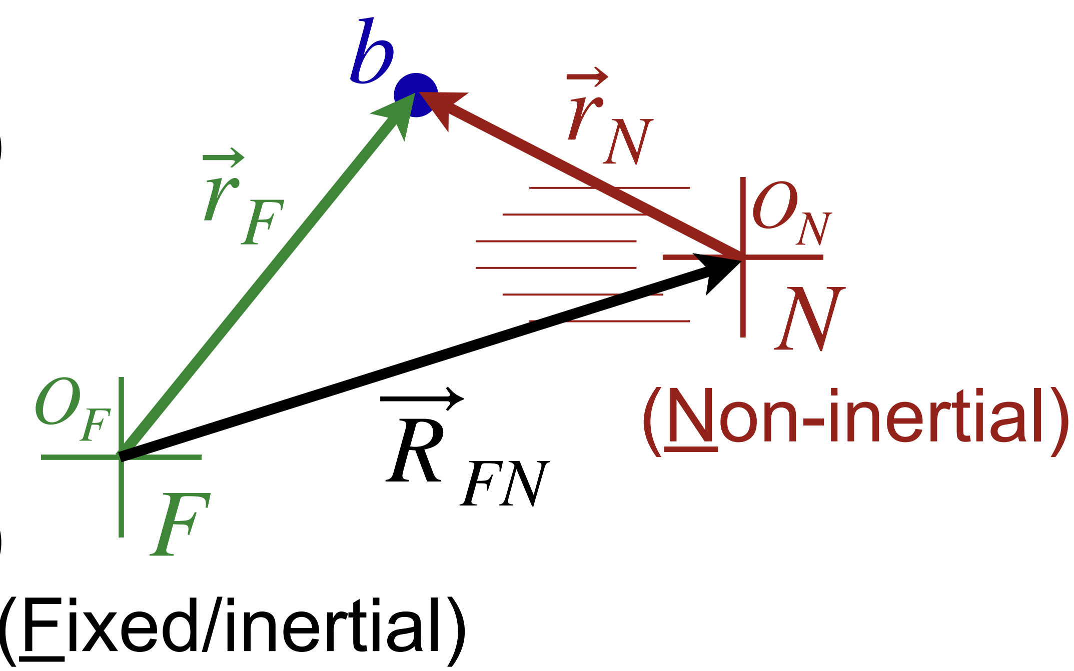

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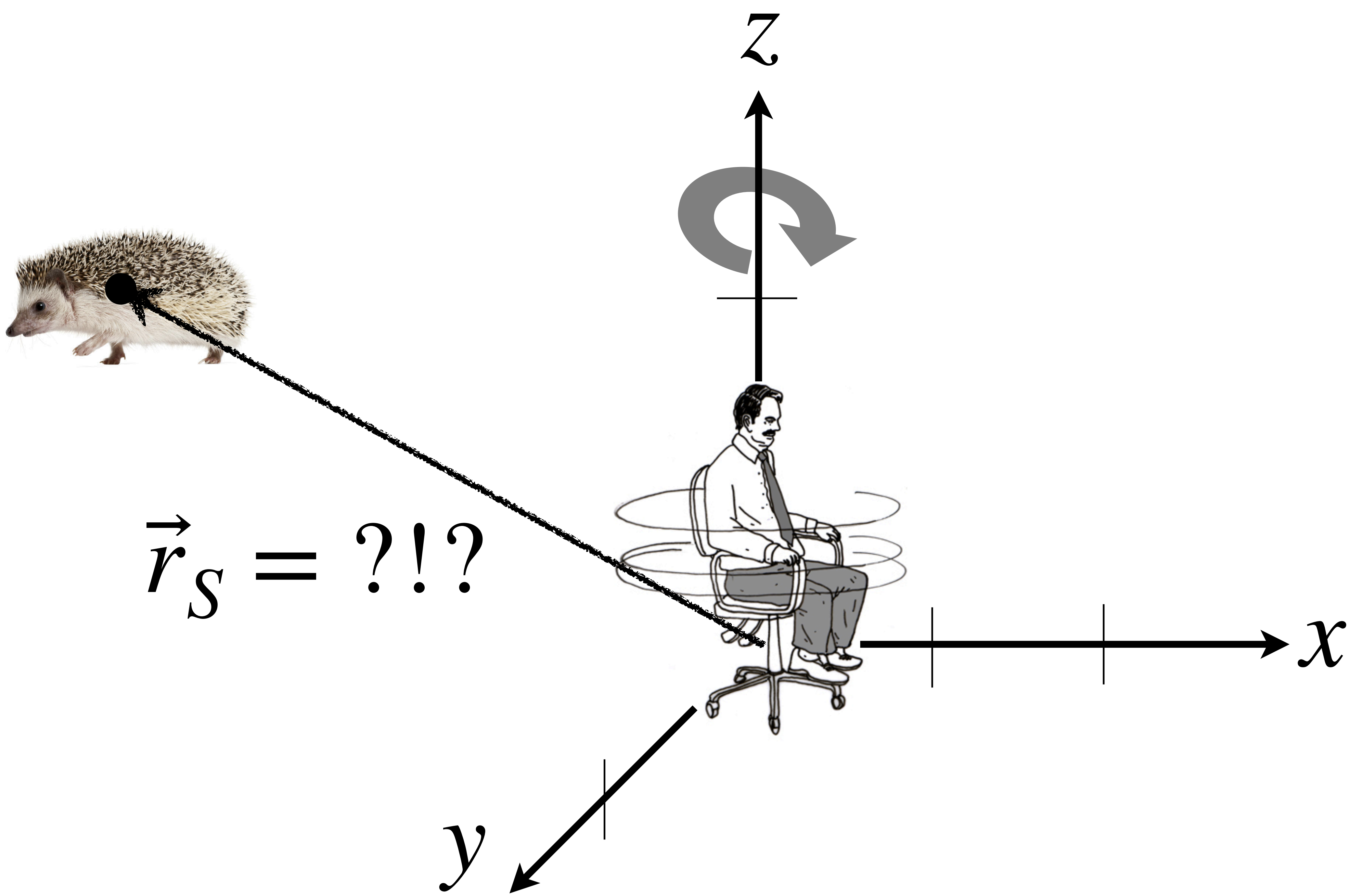
- Take derivative in time again

$$\vec{a}_N(t) = \vec{a}_F(t) - \overrightarrow{A}_{FN}(t)$$

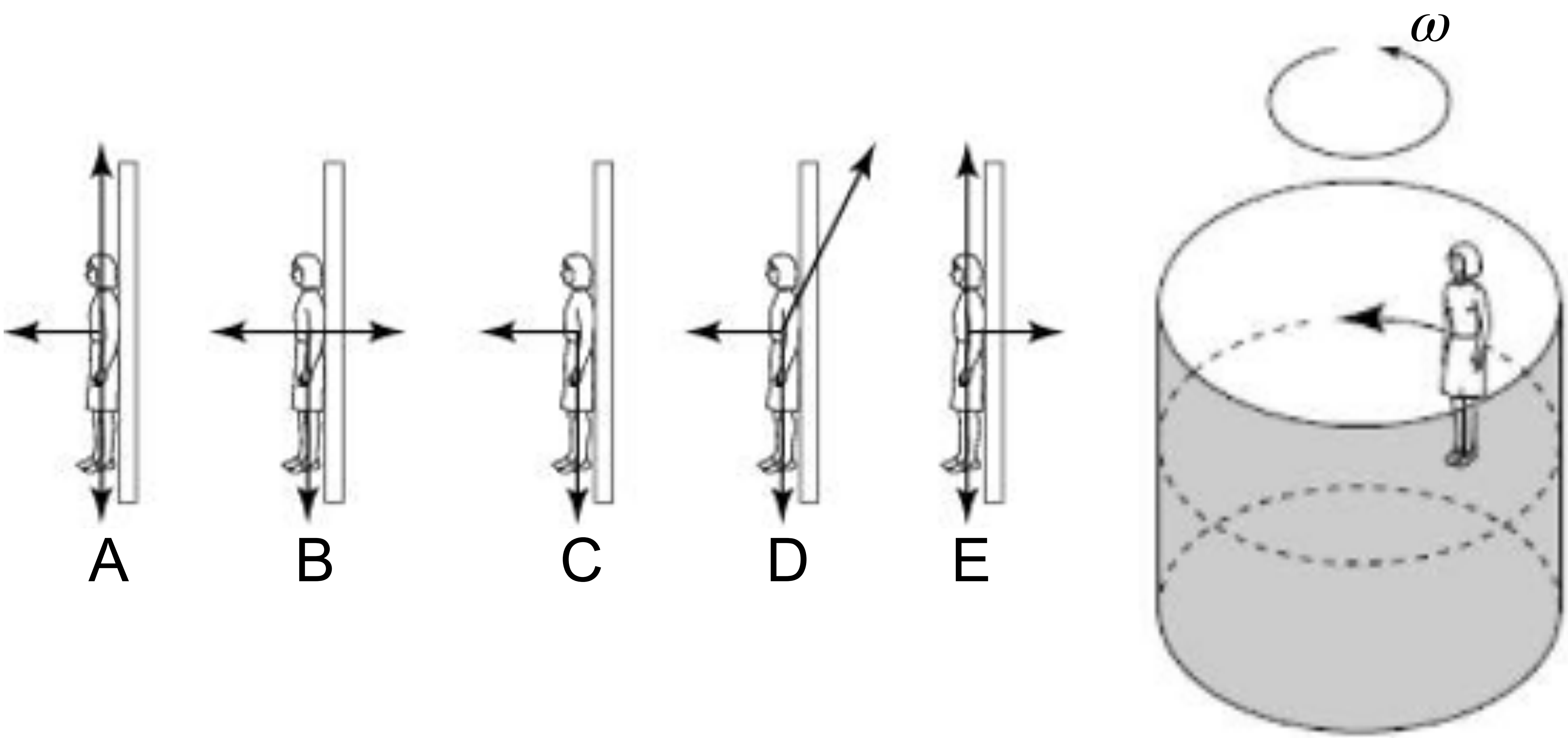
- Multiply by mass of ball to get $\sum \overrightarrow{F}_N = \sum \overrightarrow{F}_F - m_b \overrightarrow{A}_{FN}$
- In frame N, we see fictitious forces $\overrightarrow{F}_{fict} = -m_b \overrightarrow{A}_{FN}$

What about rotating reference frames?



Conceptual question

A rider in a "barrel of fun" finds herself stuck with her back to the wall. Which diagram correctly shows the forces acting on her as seen from an fixed (inertial) reference frame?



Video conceptual question

responseware.eu
Session ID: epflphys101en

Rotationally accelerating non-inertial frame

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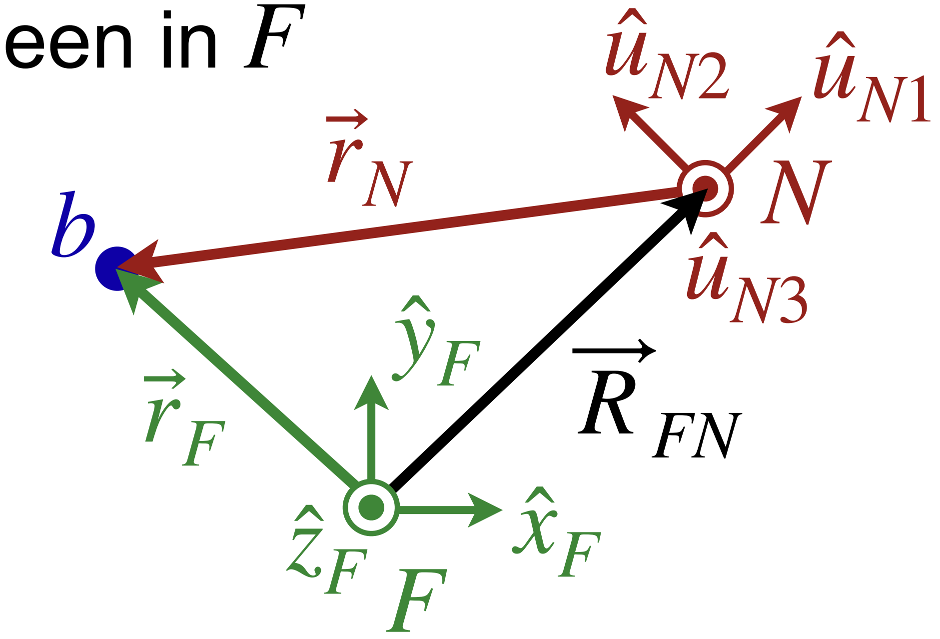
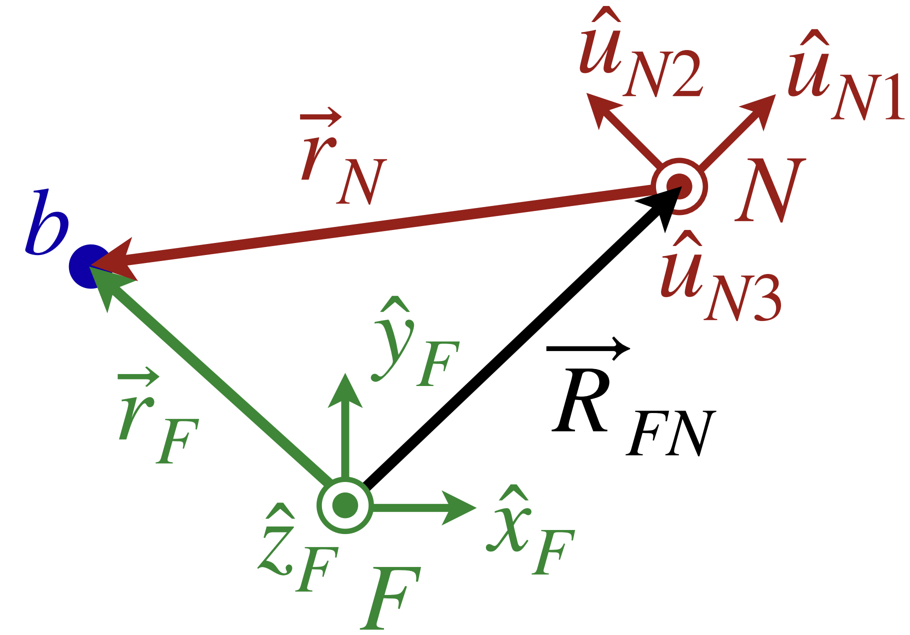
- $\vec{r}_F$ is the position of the ball, as seen in the Fixed inertial frame F
- $\vec{r}_N$ is the position of the ball, as seen in the Non-inertial frame N
- $\overrightarrow{R}_{FN}$ is the position of the origin of N, as seen in F

$$\vec{r}_F = \overrightarrow{R}_{FN} + \vec{r}_N$$

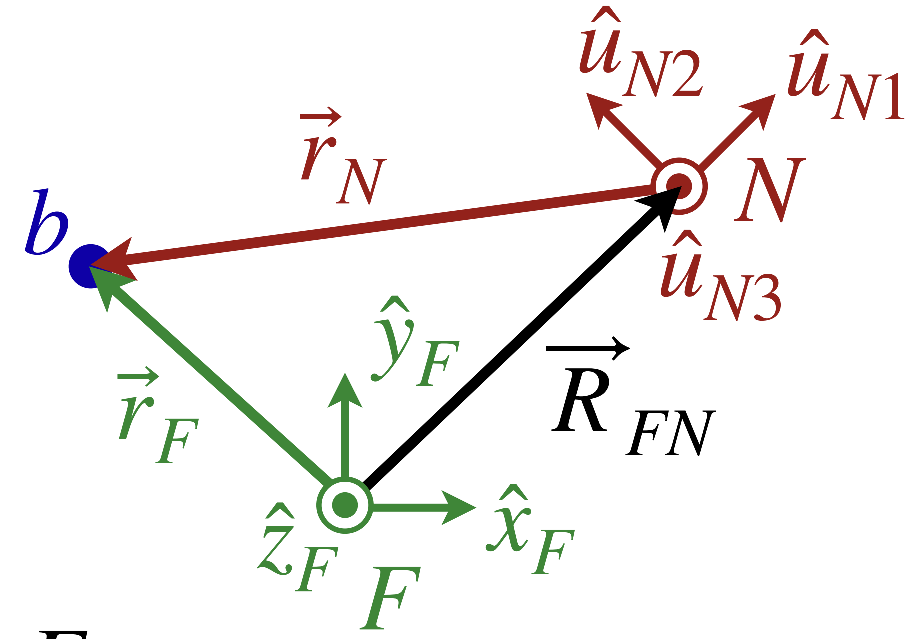


Rotationally accelerating non-inertial frame



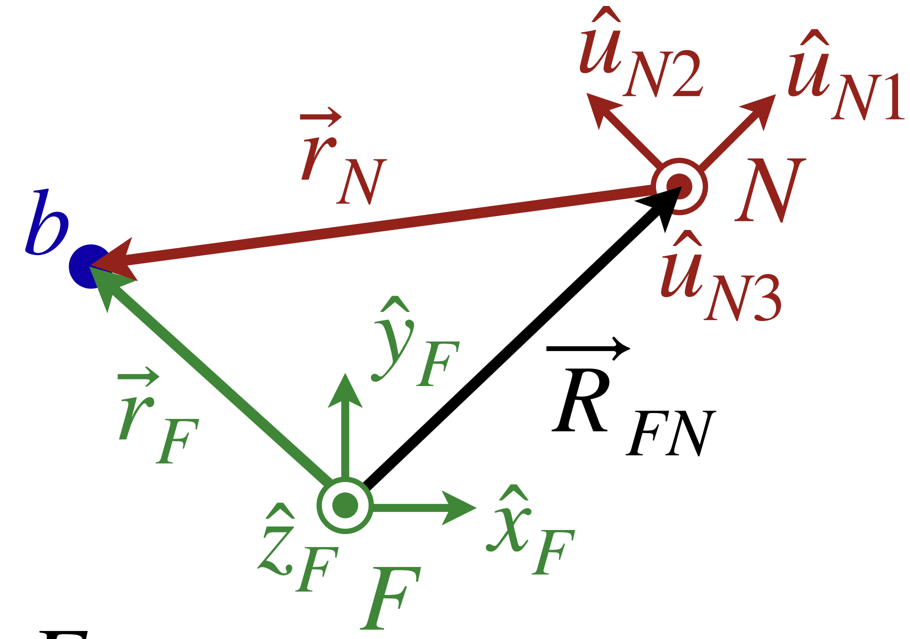
Rotationally accelerating non-inertial frame

- $\vec{v}_F$ is the velocity of the ball, as seen in F
- $\vec{v}_N$ is the velocity of the ball, as seen in N
- $\overrightarrow{V}_{FN}$ is the velocity of the origin of N, as seen in F
- The last term is the rotation of N, as seen in F



$$\vec{v}_F = \overrightarrow{V}_{FN} + \vec{v}_N + \sum_{j=1}^{3} r_{Nj}\dot{\hat{u}}_{Nj}$$

Rotationally accelerating non-inertial frame



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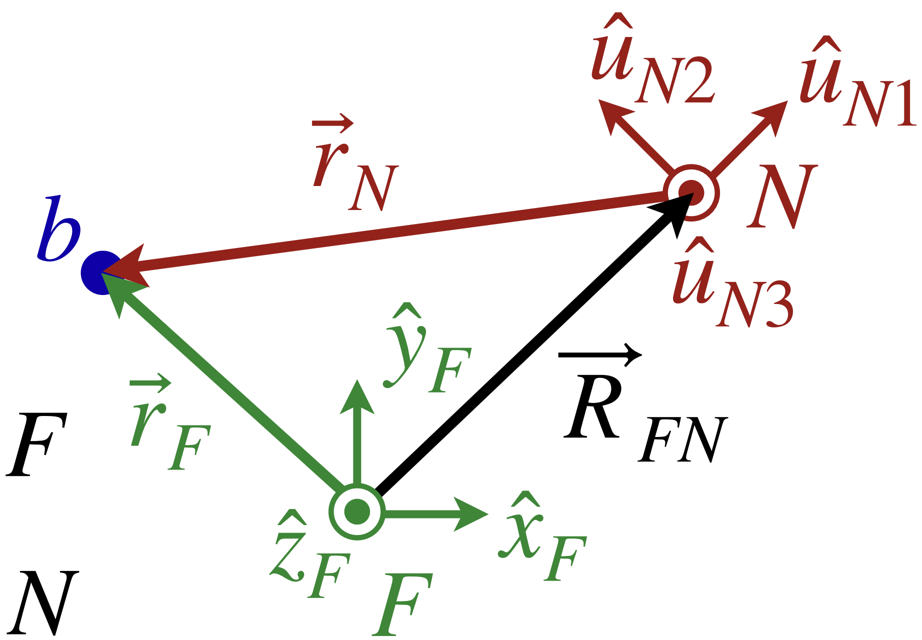
$$\vec{v}_F = \overrightarrow{V}_{FN} + \vec{v}_N + \sum_{j=1}^{3} r_{Nj}\dot{\hat{u}}_{Nj}$$

- Differentiate this expression with respect to time to obtain

$$\vec{a}_F = \overrightarrow{A}_{FN} + \left(\vec{a}_N + \sum_{j=1}^{3} v_{Nj}\dot{\hat{u}}_{Nj}\right) + \left(\sum_{j=1}^{3} v_{Nj}\dot{\hat{u}}_{Nj} + \sum_{j=1}^{3} r_{Nj}\ddot{\hat{u}}_{Nj}\right)$$

Rotationally accelerating non-inertial frame

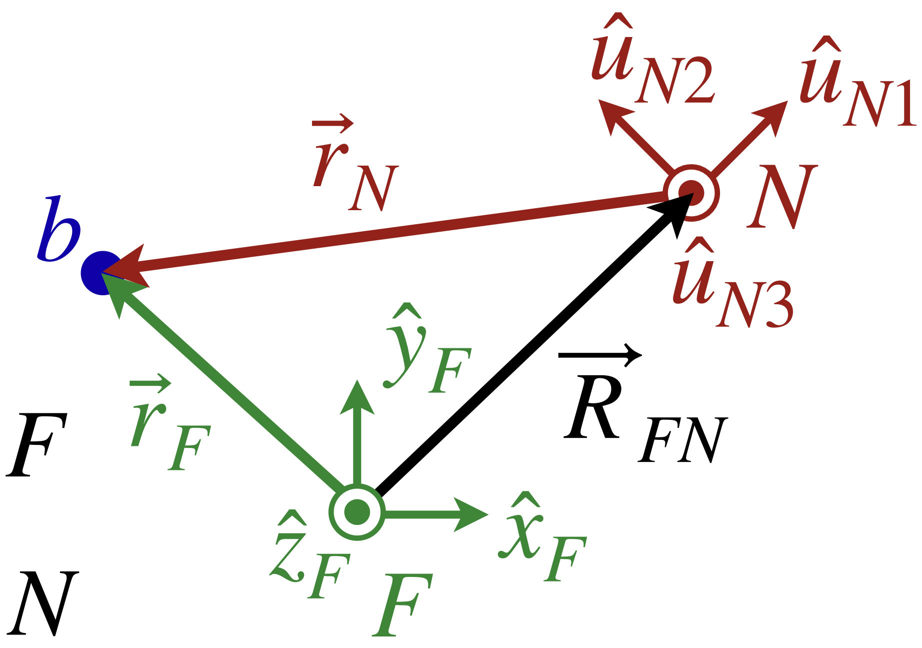
$$\vec{a}_F = \vec{a}_N + \overrightarrow{A}_{FN} + 2\sum_{j=1}^{3} v_{Nj}\dot{\hat{u}}_{Nj} + \sum_{j=1}^{3} r_{Nj}\ddot{\hat{u}}_{Nj}$$



- $\vec{a}_F$ is the acceleration of the ball, as seen in F
- $\vec{a}_N$ is the acceleration of the ball, as seen in N
- $\overrightarrow{A}_{FN}$ is the acceleration of the origin of N, as seen in F
- Final two terms are due to the rotation of N, as seen in F

Rotationally accelerating non-inertial frame

$$\vec{a}_F = \vec{a}_N + \overrightarrow{A}_{FN} + 2\sum_{j=1}^{3} v_{Nj}\dot{\hat{u}}_{Nj} + \sum_{j=1}^{3} r_{Nj}\ddot{\hat{u}}_{Nj}$$



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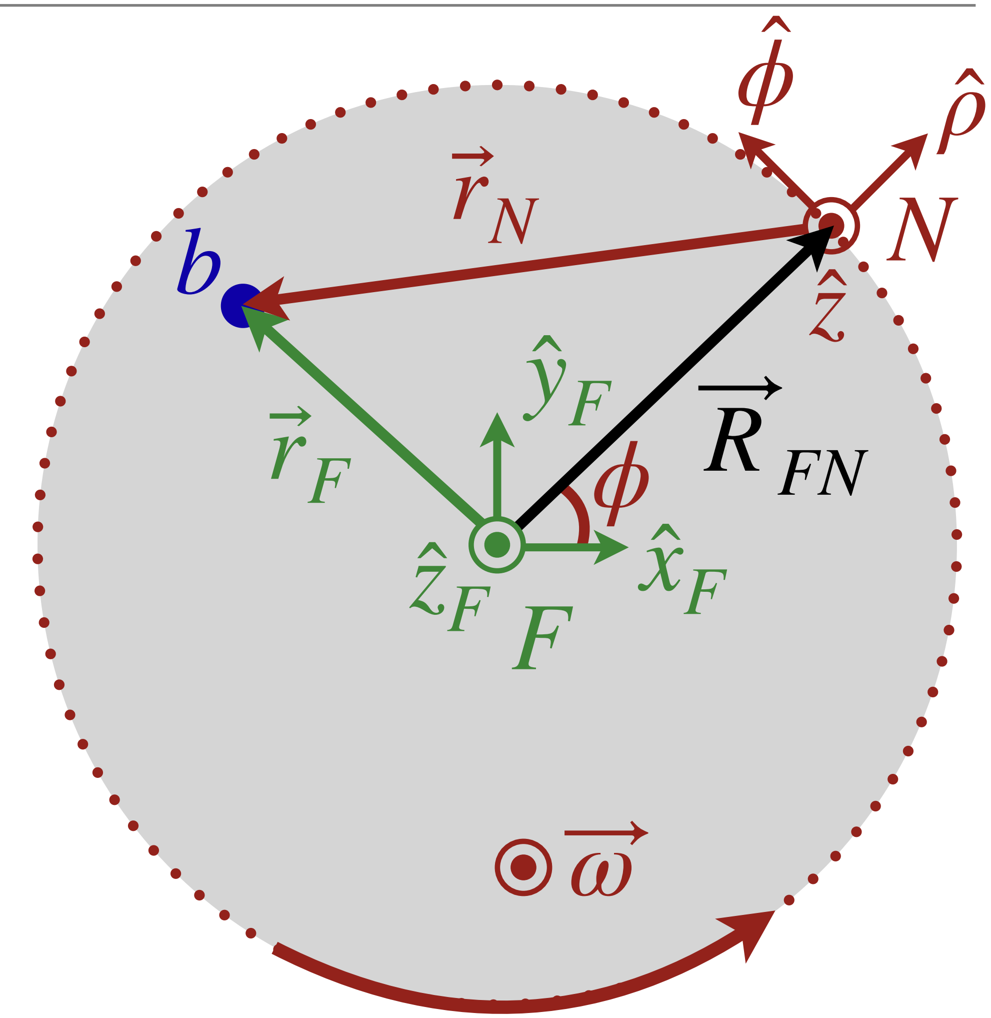
- Final two terms are due to the rotation of N, as seen in F

- Multiply by the mass of the ball, use Newton's 2nd law, and rearrange to see

$$\Sigma\overrightarrow{F}_N = \Sigma\overrightarrow{F}_F - m_b\overrightarrow{A}_{FN} - 2m_b\sum_{j=1}^{3} v_{Nj}\dot{\hat{u}}_{Nj} - m_b\sum_{j=1}^{3} r_{Nj}\ddot{\hat{u}}_{Nj}$$

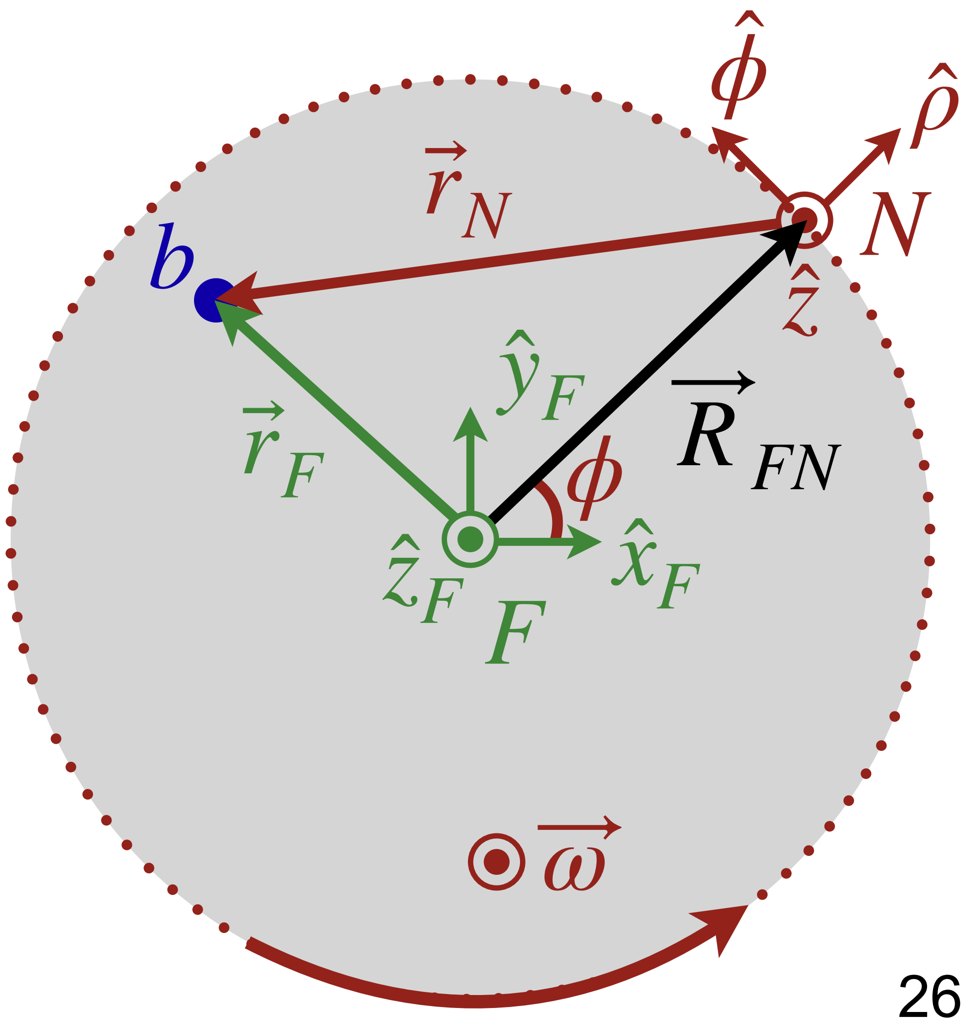
Modeling the merry-go-round

- Let $\left(\hat{u}_{N1}, \hat{u}_{N2}, \hat{u}_{N3}\right) \rightarrow \left(\hat{\rho}, \hat{\phi}, \hat{z}\right)$



Modeling the merry-go-round

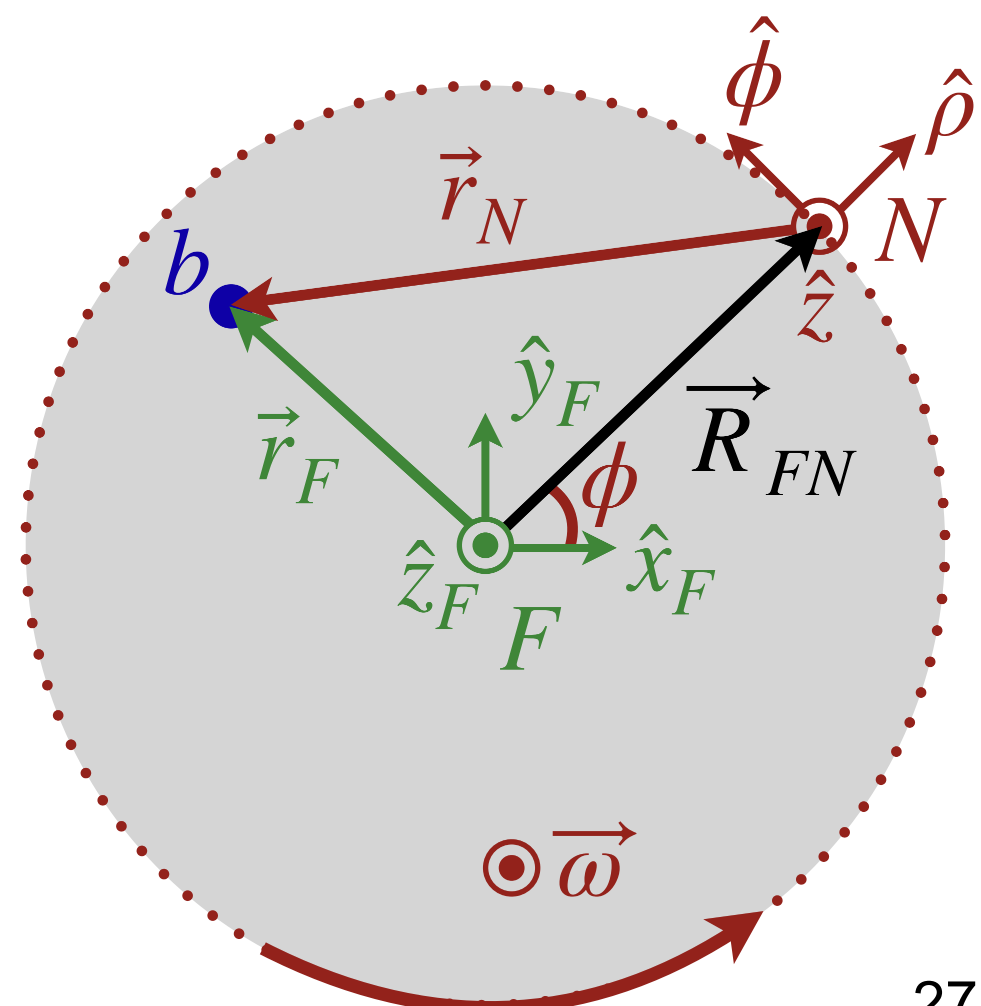
$$\Sigma \overrightarrow{F}_N = \Sigma \overrightarrow{F}_F - m_b \overrightarrow{A}_{FN} - 2m_b \overrightarrow{\omega} \times \vec{v}_N - m_b \overrightarrow{\alpha} \times \vec{r}_N - m_b \overrightarrow{\omega} \times \left(\overrightarrow{\omega} \times \vec{r}_N \right)$$



Modeling the merry-go-round

$$\Sigma\overrightarrow{F}_N = \Sigma\overrightarrow{F}_F - m_b\overrightarrow{A}_{FN} - 2m_b\overrightarrow{\omega}\times\vec{v}_N - m_b\overrightarrow{\alpha}\times\vec{r}_N - m_b\overrightarrow{\omega}\times\left(\overrightarrow{\omega}\times\vec{r}_N\right)$$

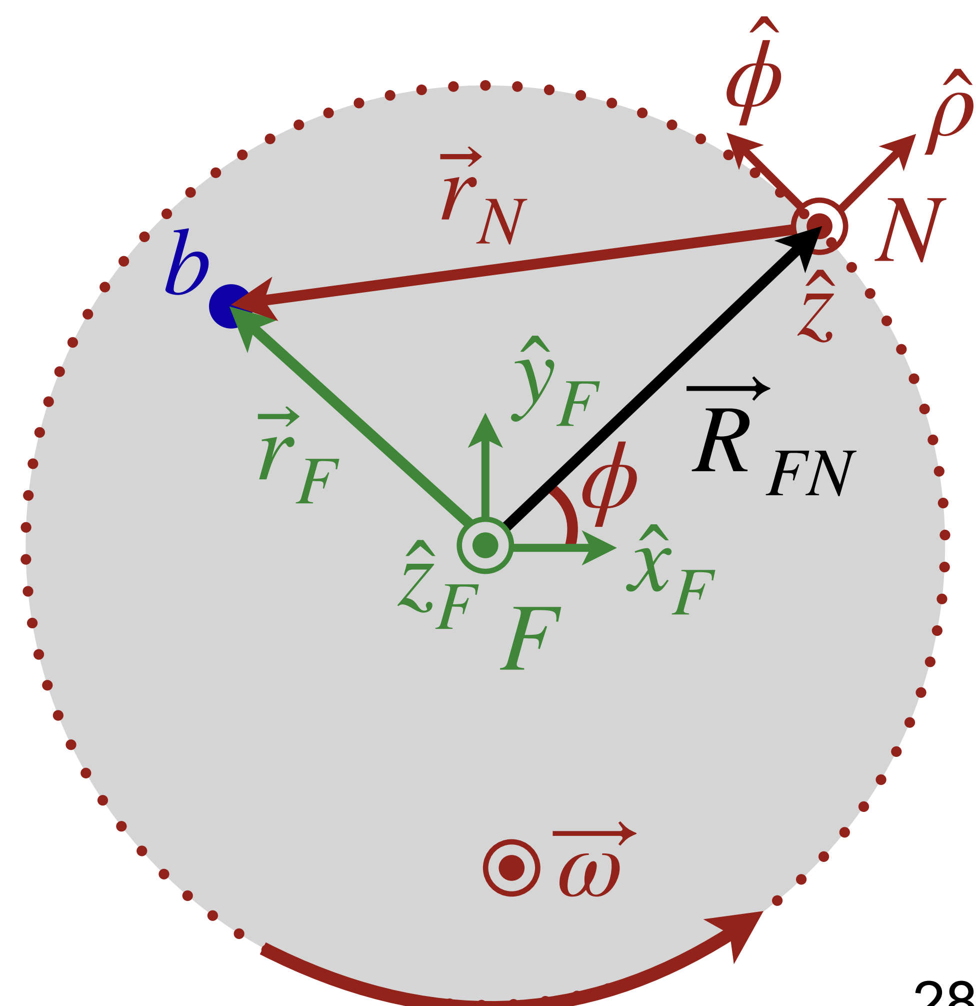
- $\Sigma\overrightarrow{F}_N$ are the forces *seen* in the non-inertial reference N
- $\Sigma\overrightarrow{F}_F$ are the forces *seen* in the fixed inertial reference F



Modeling the merry-go-round

$$\Sigma\overrightarrow{F}_N = \Sigma\overrightarrow{F}_F - m_b\overrightarrow{A}_{FN} - 2m_b\overrightarrow{\omega}\times\vec{v}_N - m_b\overrightarrow{\alpha}\times\vec{r}_N - m_b\overrightarrow{\omega}\times\left(\overrightarrow{\omega}\times\vec{r}_N\right)$$

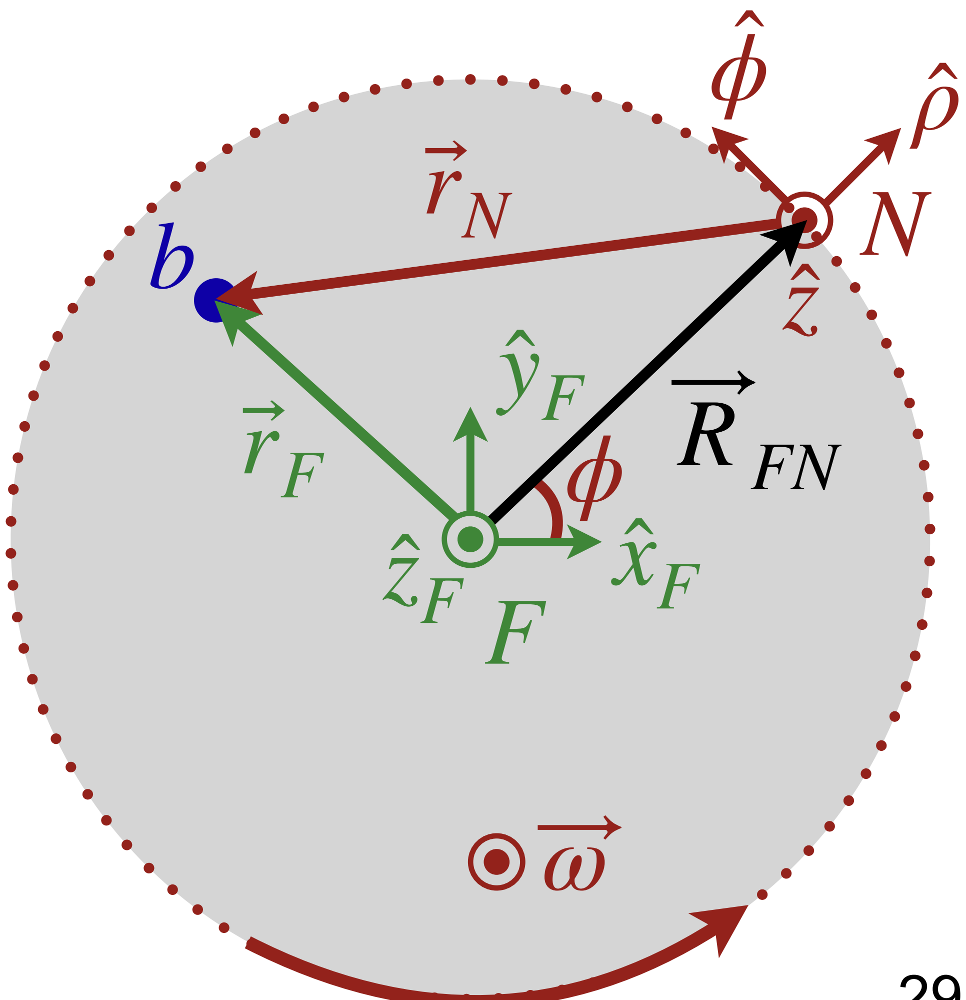
- $\Sigma\overrightarrow{F}_N$ are the forces *seen* in the non-inertial reference N
- $\Sigma\overrightarrow{F}_F$ are the forces *seen* in the fixed inertial reference F
- $-m_b\overrightarrow{A}_{FN}$ is the fictitious force associated with the *translational* motion of the origin of N, as seen in F



Modeling the merry-go-round

$$\Sigma\overrightarrow{F}_N = \Sigma\overrightarrow{F}_F - m_b\overrightarrow{A}_{FN} - 2m_b\overrightarrow{\omega}\times\vec{v}_N - m_b\overrightarrow{\alpha}\times\vec{r}_N - m_b\overrightarrow{\omega}\times\left(\overrightarrow{\omega}\times\vec{r}_N\right)$$

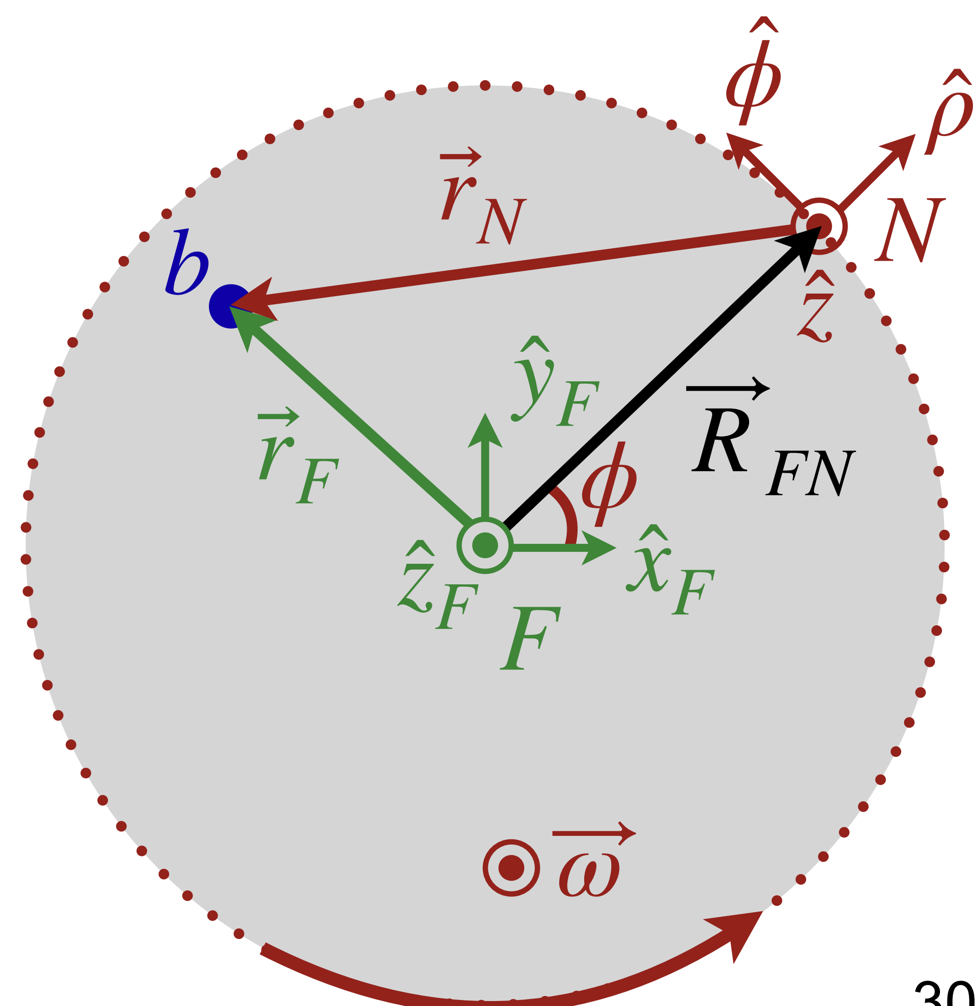
- $\Sigma\overrightarrow{F}_N$ are the forces *seen* in the non-inertial reference N
- $\Sigma\overrightarrow{F}_F$ are the forces *seen* in the fixed inertial reference F
- $-m_b\overrightarrow{A}_{FN}$ is the fictitious force associated with the *translational* motion of the origin of N, as seen in F
- Next is the Coriolis term ($\vec{v}_N$ is the velocity of the ball as seen in N)



Modeling the merry-go-round

$$\Sigma\overrightarrow{F}_N = \Sigma\overrightarrow{F}_F - m_b\overrightarrow{A}_{FN} - 2m_b\overrightarrow{\omega}\times\vec{v}_N - m_b\overrightarrow{\alpha}\times\vec{r}_N - m_b\overrightarrow{\omega}\times\left(\overrightarrow{\omega}\times\vec{r}_N\right)$$

- $\Sigma\overrightarrow{F}_N$ are the forces *seen* in the non-inertial reference N
- $\Sigma\overrightarrow{F}_F$ are the forces *seen* in the fixed inertial reference F
- $-m_b\overrightarrow{A}_{FN}$ is the fictitious force associated with the *translational* motion of the origin of N, as seen in F
- Next is the Coriolis term ($\vec{v}_N$ is the velocity of the ball as seen in N)
- Next is the Euler term ($\overrightarrow{\alpha}$ is the angular acceleration of N and $\vec{r}_N$ is the position of the ball as seen in N)
- Last is the centrifugal term



Modeling the merry-go-round

- Thrower defines frame N and we only care about horizontal motion
- In F, the horizontal motion of the ball is straight $\Rightarrow \Sigma \overrightarrow{F}_F = 0$
- $\overrightarrow{A}_{FN} = -R_{FN}\omega^2\hat{\rho}$ is centripetal because, when viewed from F, the origin of N is undergoing circular motion
- Initially, the ball leaves the thrower's hand with $\vec{v}_N = -v_{N0}\hat{\rho}$
- Initially, $\vec{r}_N = 0$ because the ball starts from the origin of N (i.e. the thrower)

$$\Sigma \overrightarrow{F}_N = \Sigma \overrightarrow{F}_F - m_b \overrightarrow{A}_{FN} - 2m_b \overrightarrow{\omega} \times \vec{v}_N - m_b \overrightarrow{\alpha} \times \vec{r}_N - m_b \overrightarrow{\omega} \times \left(\overrightarrow{\omega} \times \vec{r}_N\right)$$

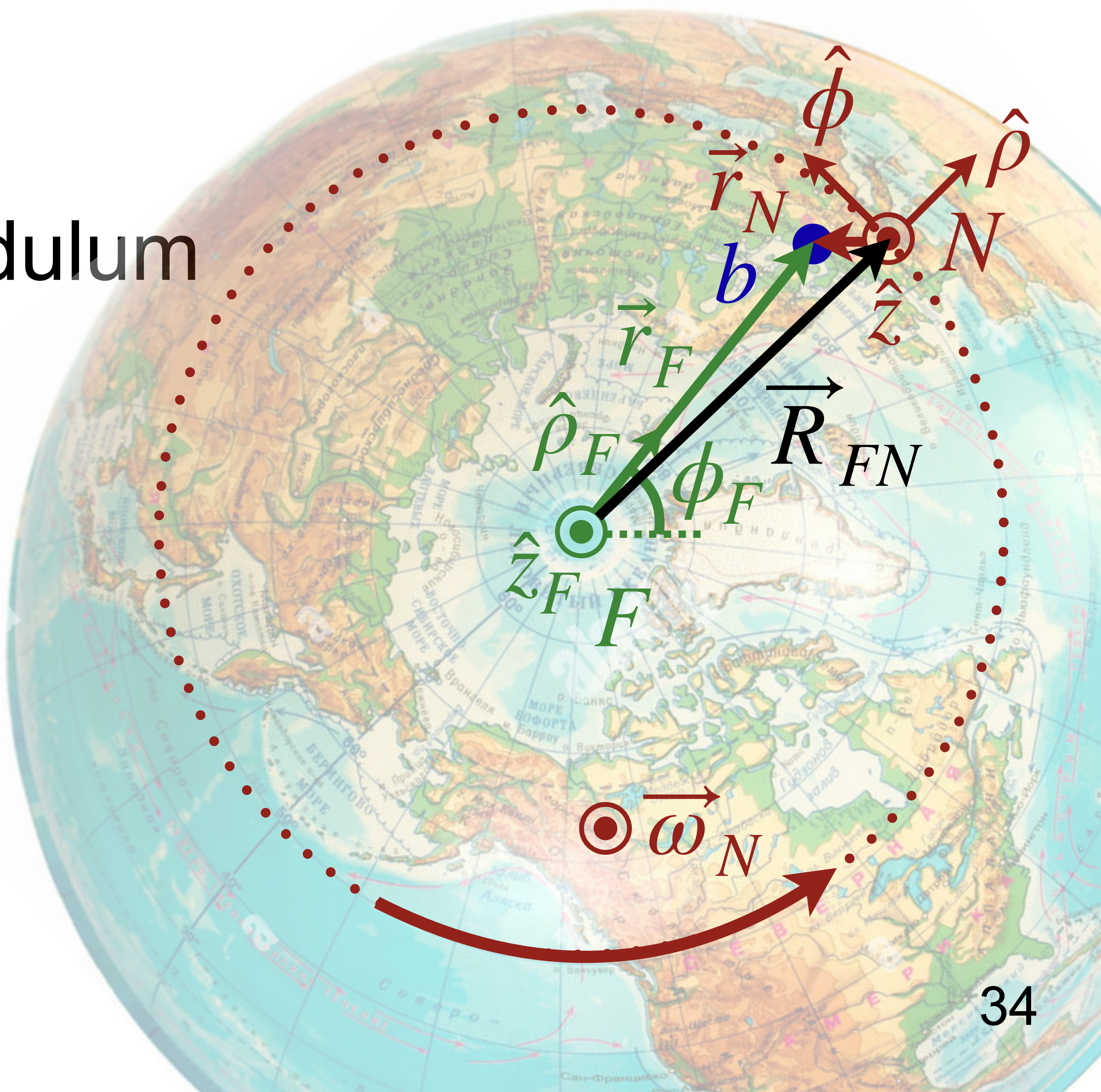
Video conceptual solution

DEMO (171)

Coriolis effect

DEMO (613)

Foucault pendulum



Today’s agenda (Serway 6.3, MIT 8)

1. Derivation of forces in non-inertial reference frames
2. **Applications of Newton’s laws**
   - **Ropes and pulleys**
   - (Example to understand constraints)

Ropes: an ancient and awesome tool

- A rope transmits a force of tension along its length
- Tension arises as a reaction against opposing forces applied to the rope

Ropes: an ancient and awesome tool

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- A rope transmits a force of tension along its length
- Tension arises as a reaction against opposing forces applied to the rope
- Fundamentally caused by molecular bonds that prevent the rope from being torn apart
- The difference in tension between two parts of a static rope is equal to the tangential force applied along the rope between them

$$\Delta T = \sum F_{||}$$

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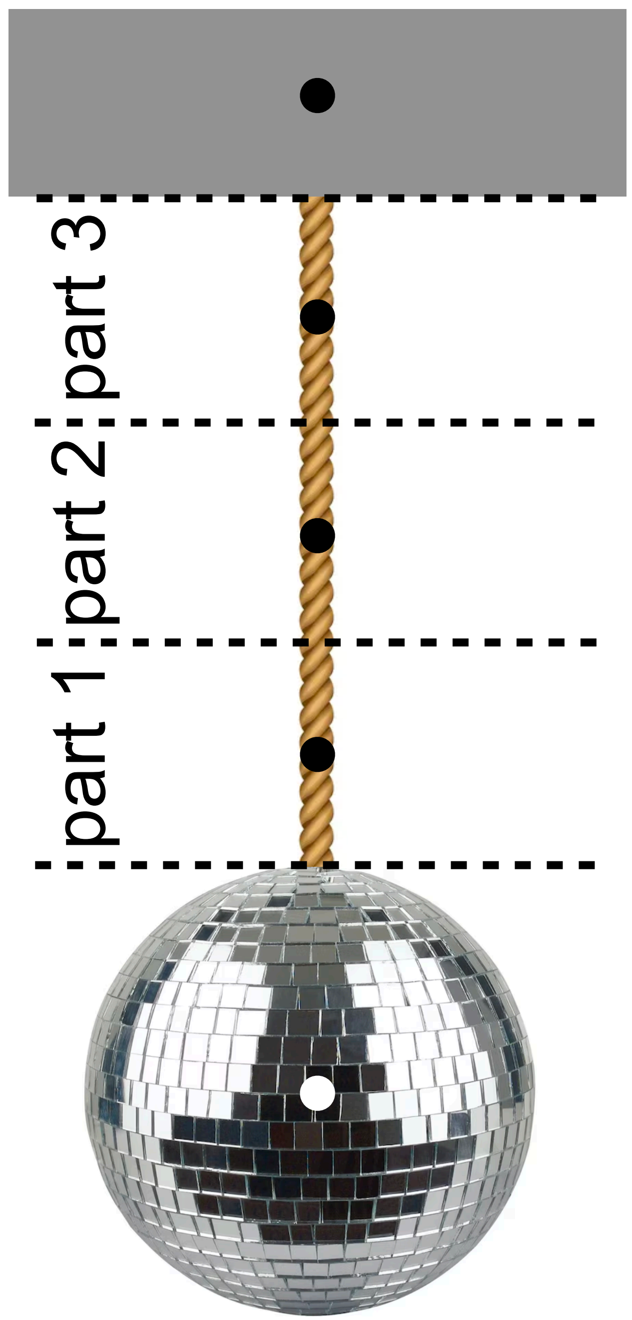

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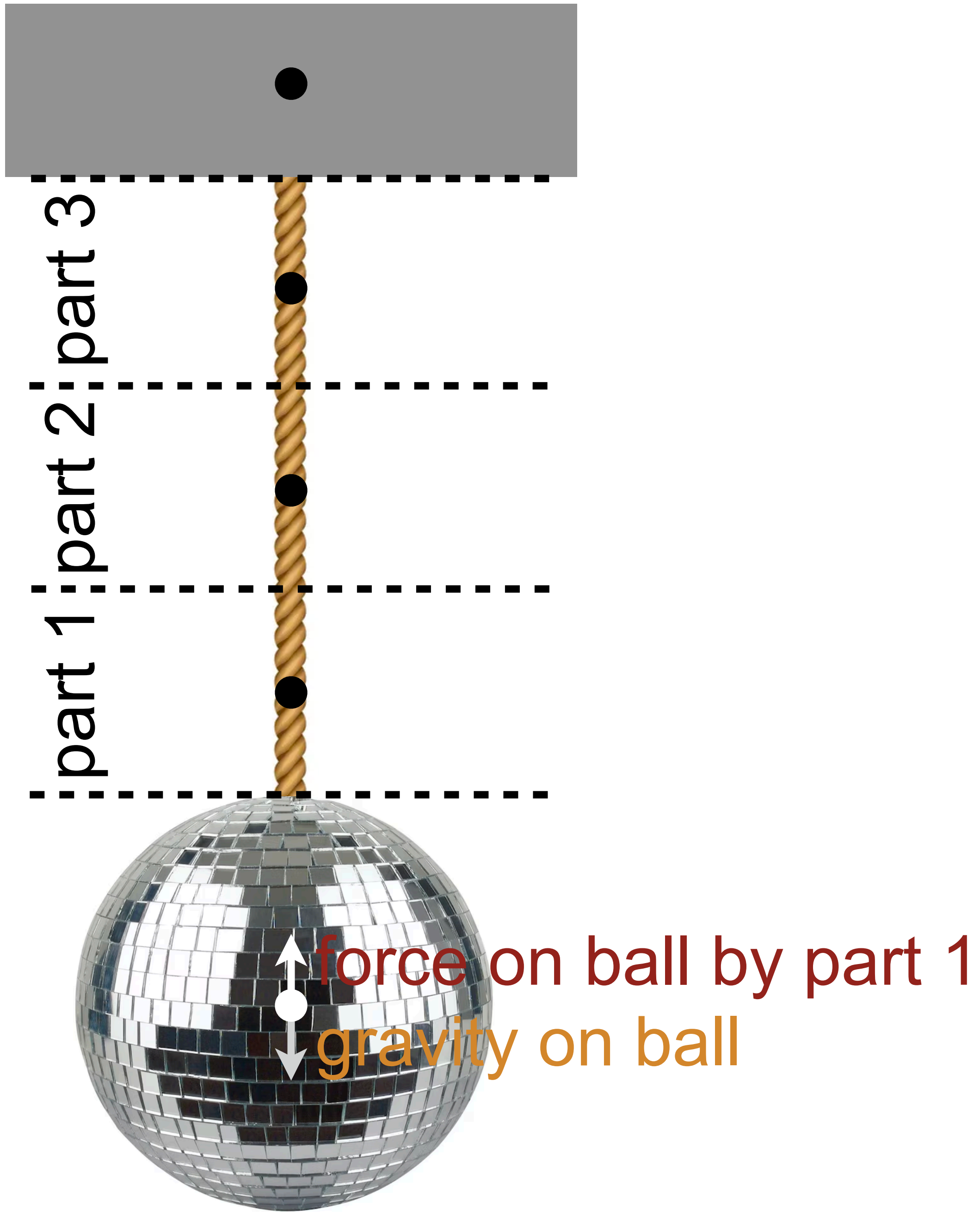

Ropes: an ancient and awesome tool

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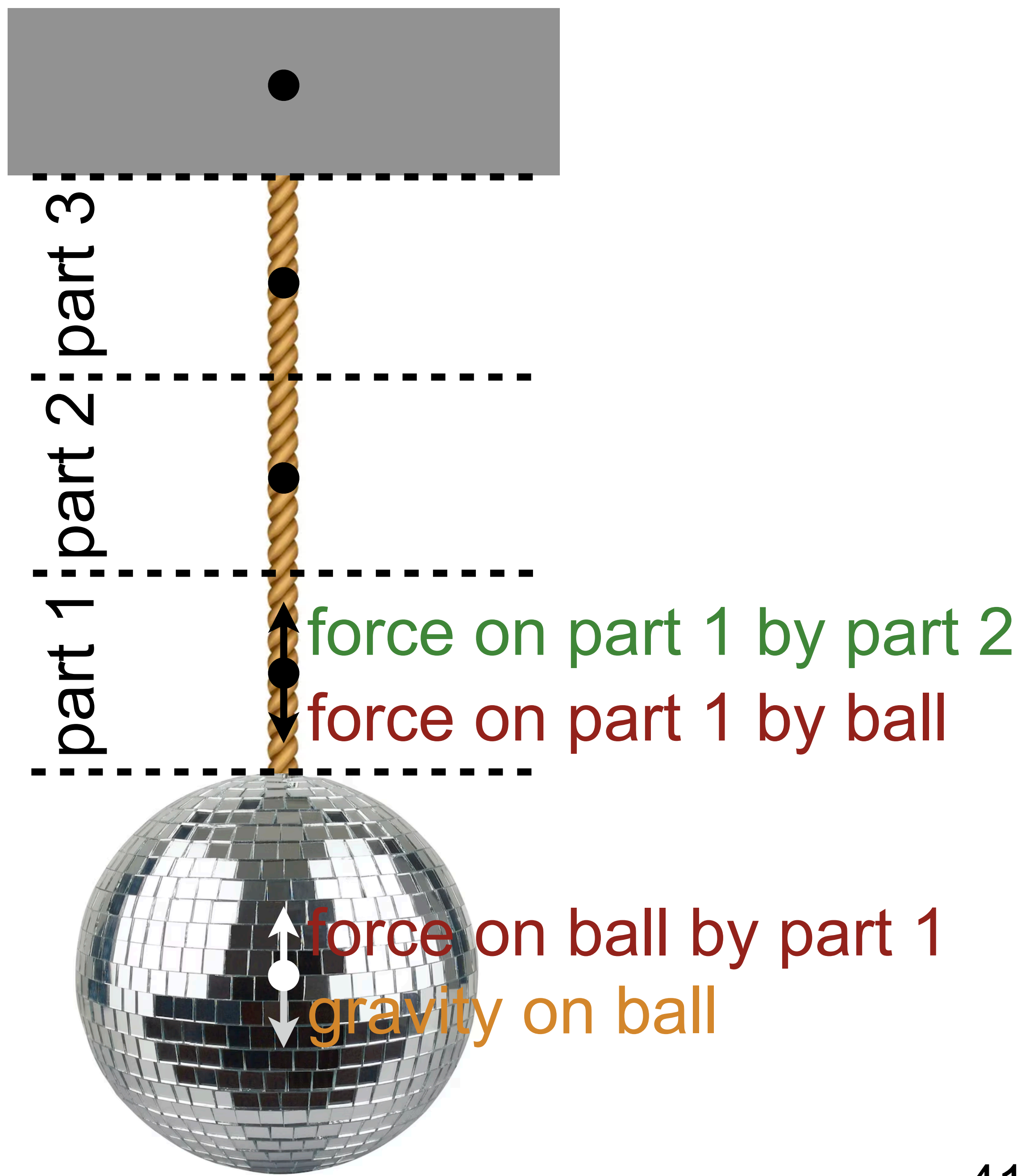

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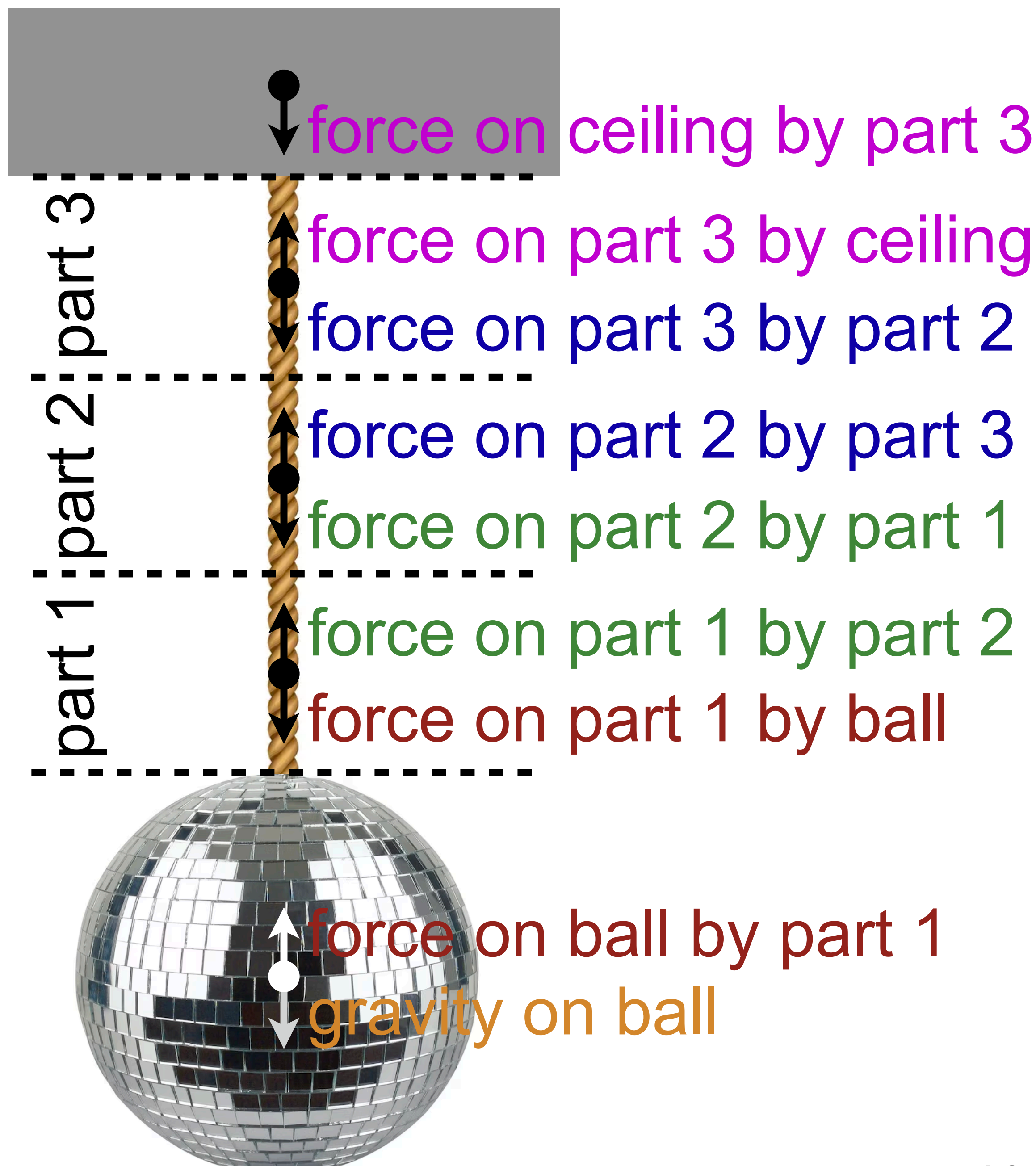

Conceptual question

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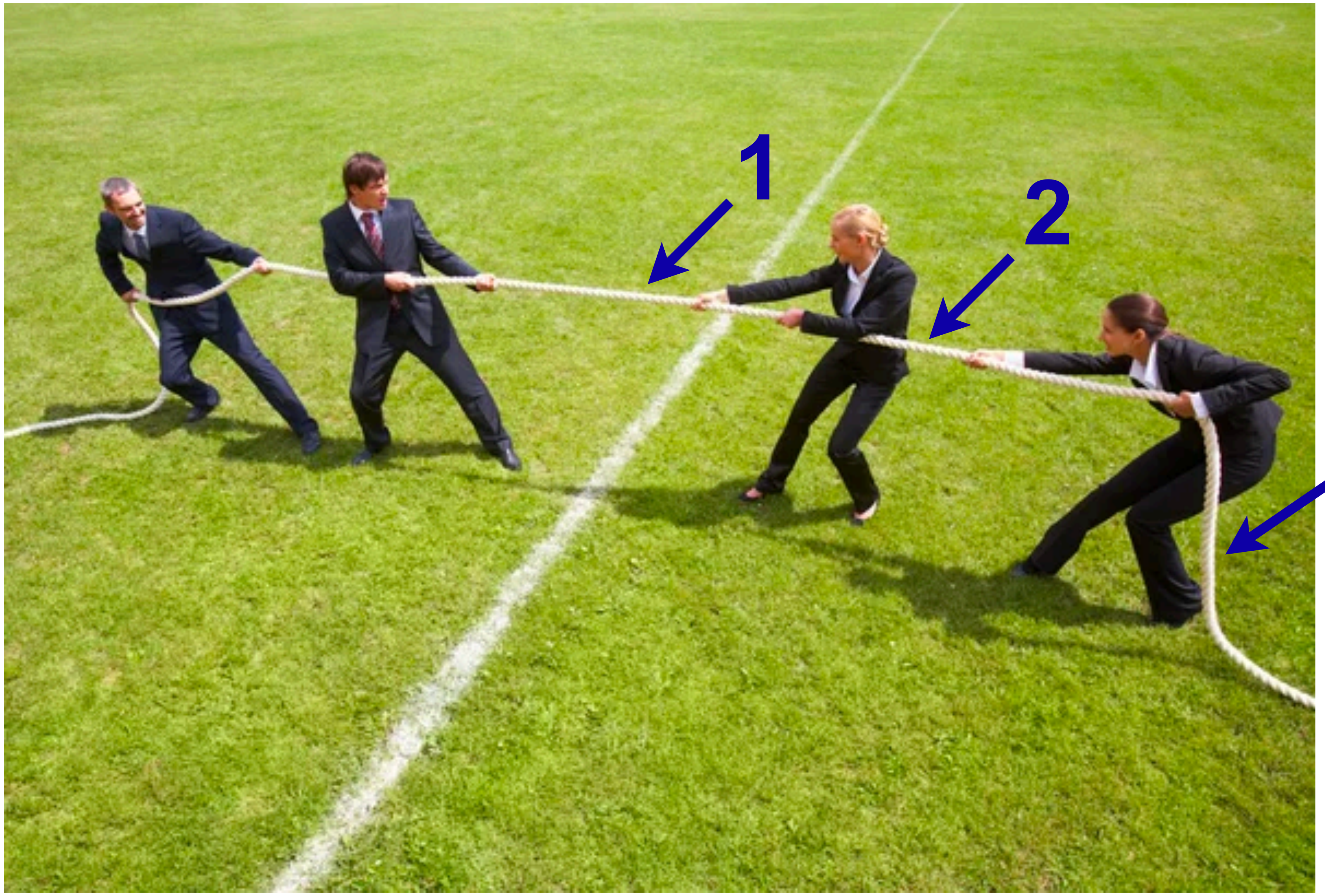
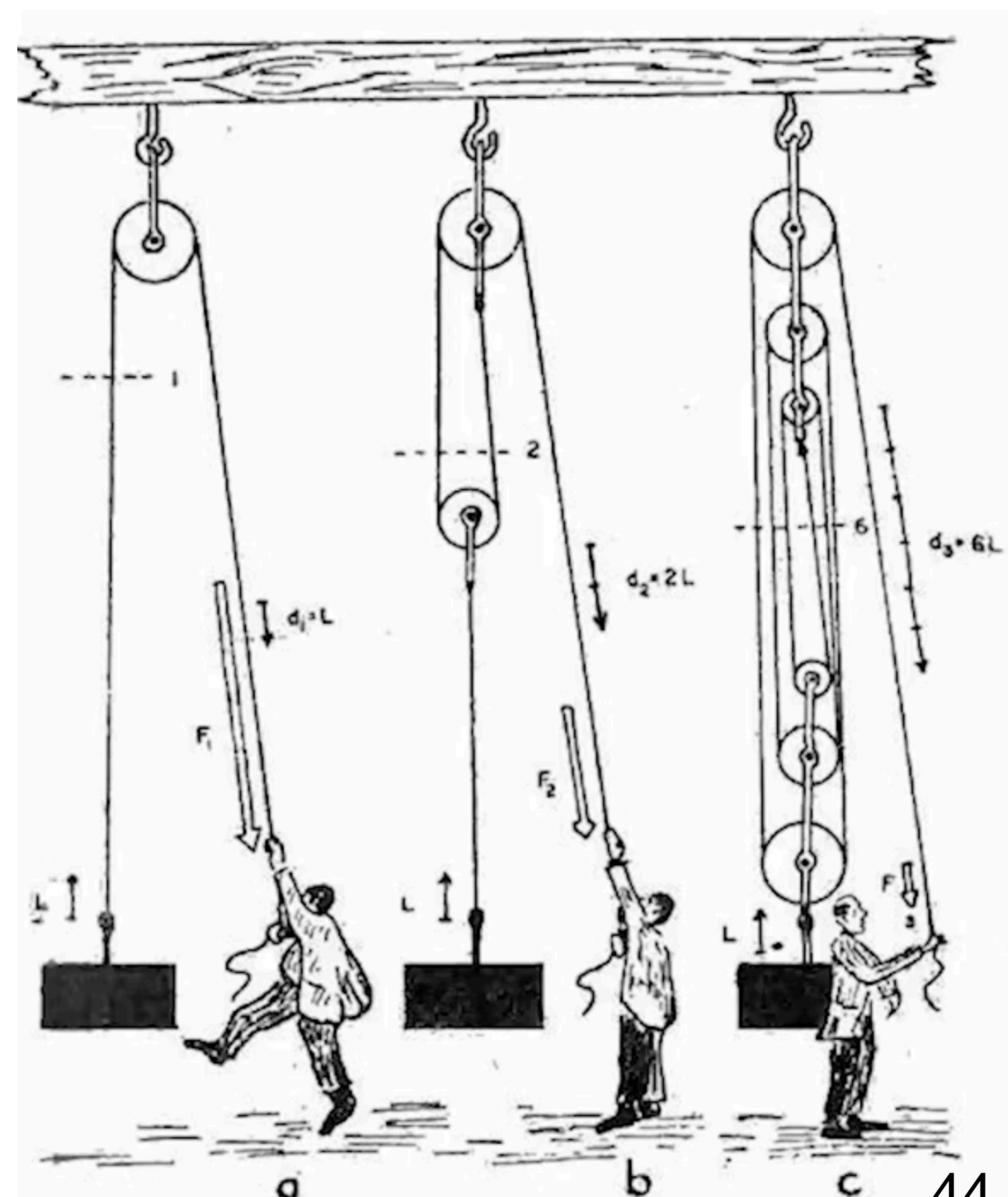
These business-type people are playing the game tug-of-war. Suppose each person is pulling with the same force F. What is the tension at points 1, 2, and 3 respectively?

A. $0, F, 2F$
B. $2F, F, 0$
C. $F, F, 0$
D. $2F, 2F, 0$



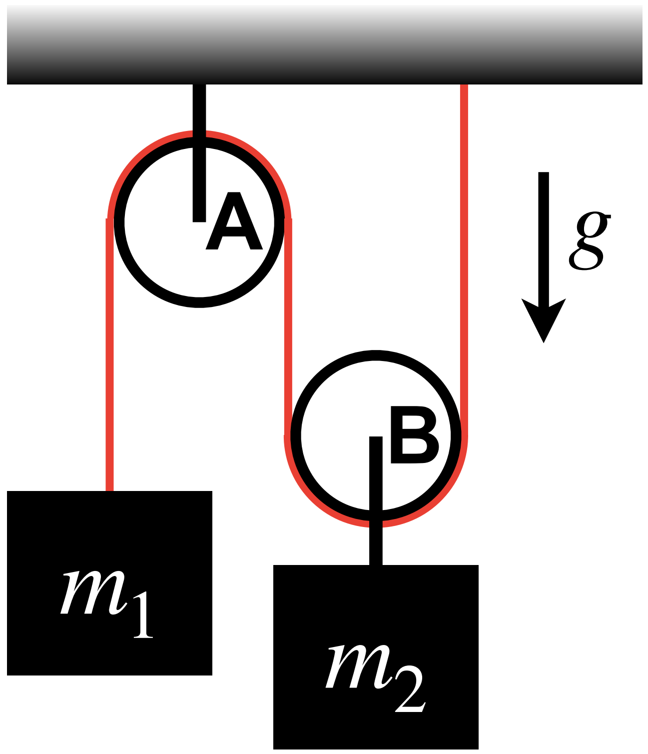
Pulleys: also an ancient and awesome tool

- All pulleys do is redirect force (if they are massless and frictionless)
- A single pulley allows you to better use your body weight
- More complicated arrangements create *mechanical advantage*
- Enables an input force to be multiplied, at the cost of requiring greater movement



Constraint conditions in a pulley system

A massless, inextensible rope is attached to the ceiling and wound through two massless, frictionless pulleys (A and B), from which two masses m_1 and m_2 are hung, as shown below. Find the tension in the rope and the acceleration of both masses. Does m_1 go up or does m_2?



Constraint conditions in a pulley system

EPFL
Swiss Plasma Center

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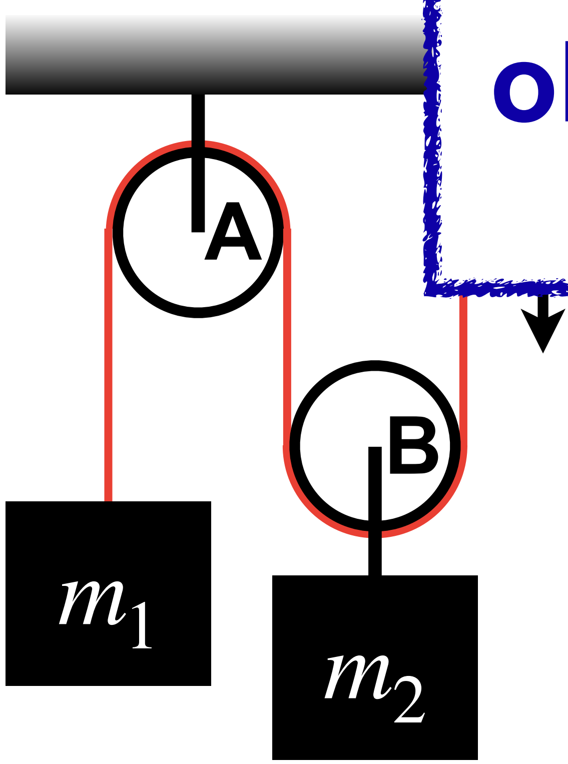

A *constraint condition* is a requirement on the motion of objects due to the geometry of the system

Constraint conditions in a pulley system

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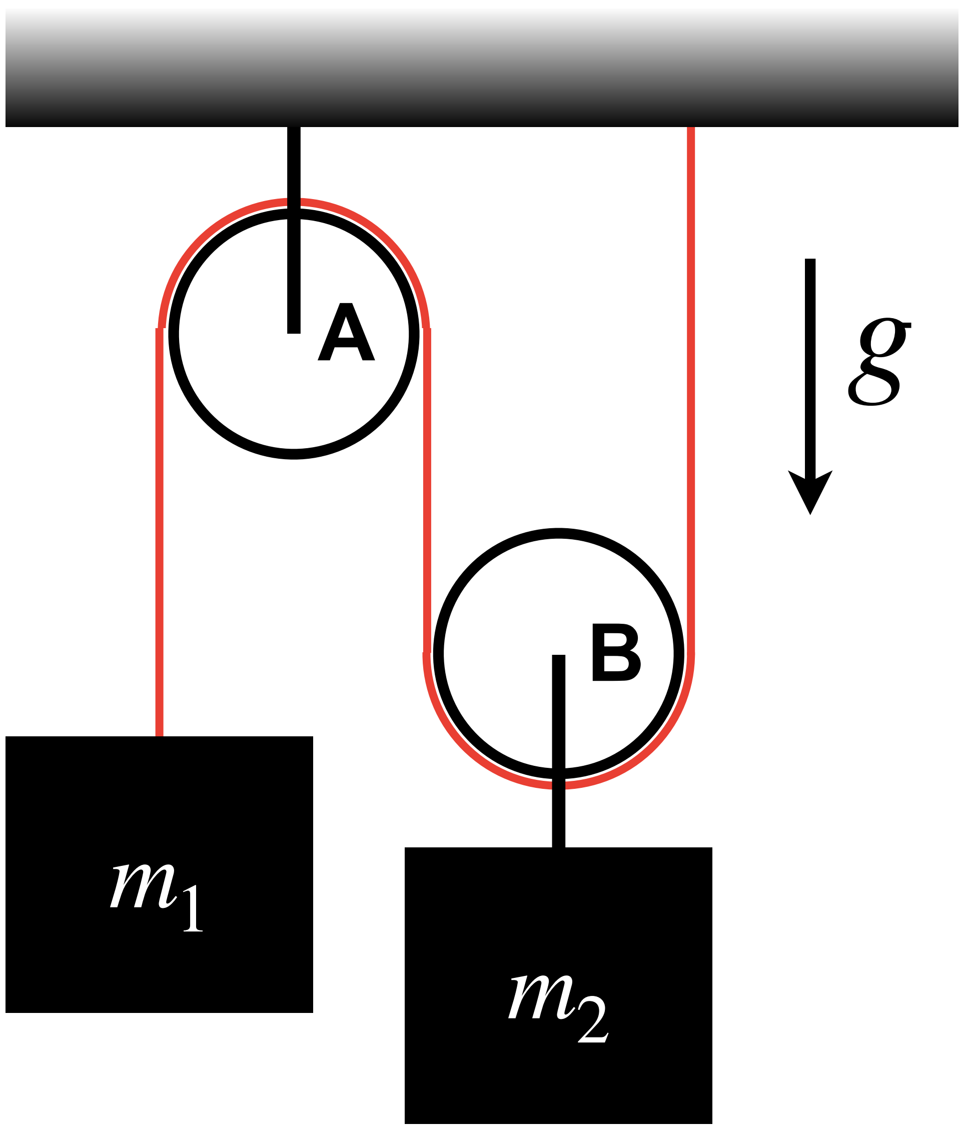

Constraint conditions in a pulley system

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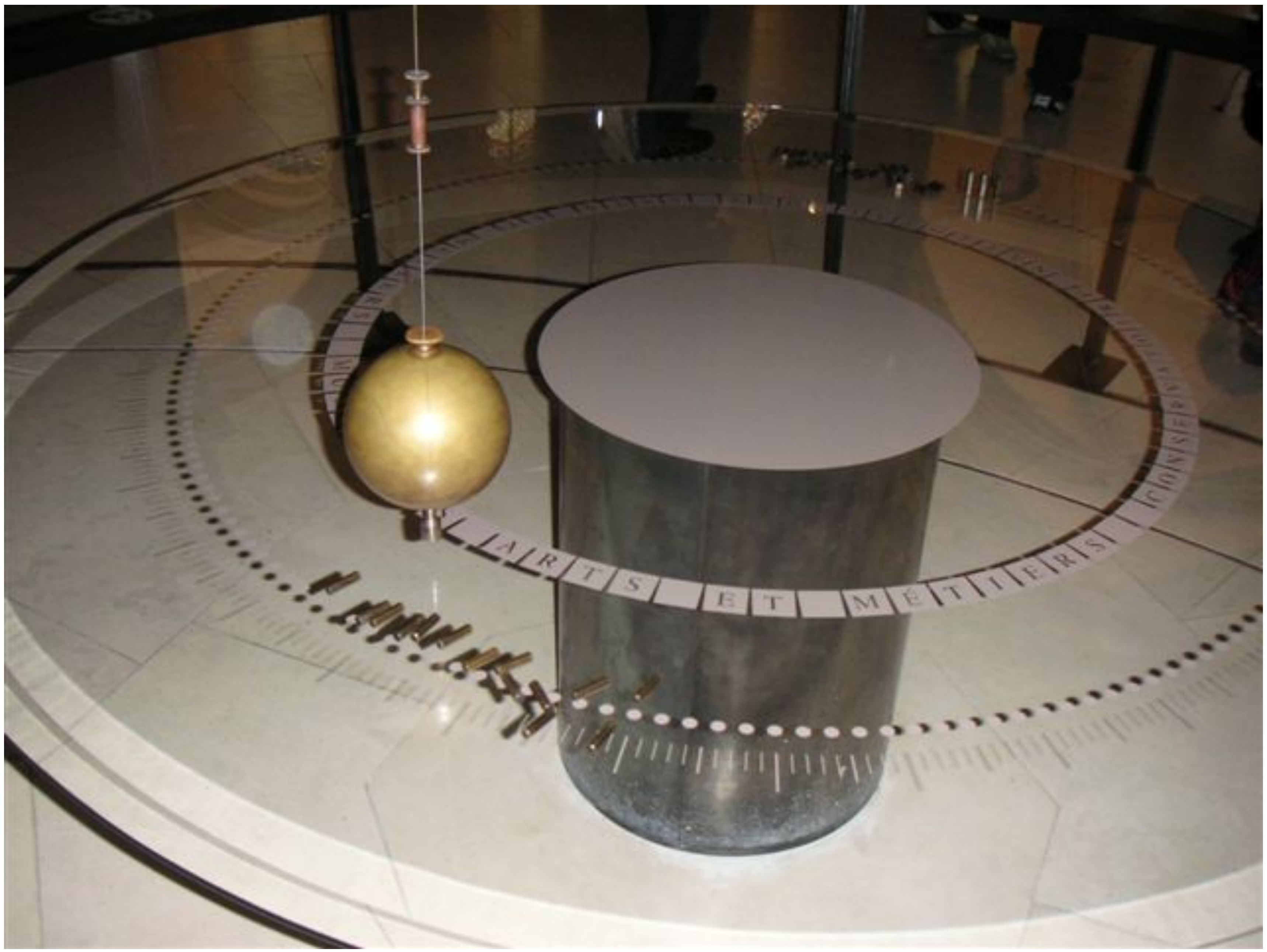
See you tomorrow!



Conceptual question

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Swiss Plasma Center

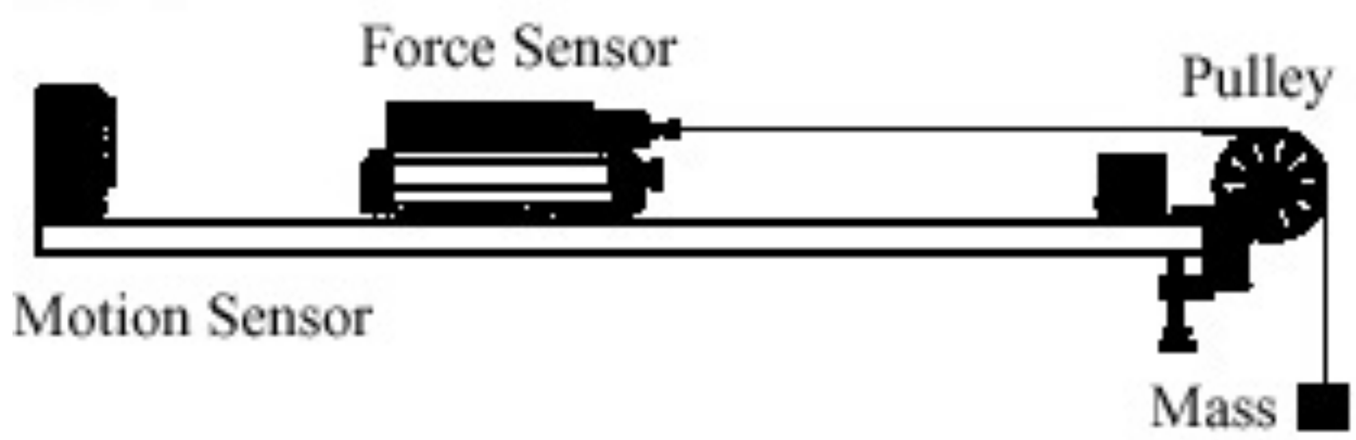
In the 17th century, Otto von Güricke, a physicist in Magdeburg, fitted two hollow bronze hemispheres together and removed the air from the resulting sphere with a pump. Two eight-horse teams could not pull the halves apart even though the hemispheres fell apart when air was readmitted. Suppose von Güricke had tied both teams of horses to one side and bolted the other side to a heavy tree trunk. In this case, the tension on the hemispheres would be…

A. twice…
B. exactly the same as…
C. half…

what it was before.

Conceptual question

A force sensor on a cart is attached via a string to a hanging weight. The cart is initially held. When the cart is allowed to move, the tension in the string…



A. increases.
B. stays the same.
C. decreases.
D. Cannot be determined.

Conceptual question

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Block 1 is constrained to move along a rough plane inclined at angle ϕ to the horizontal. It is connected, with a massless inextensible rope that passes over a massless pulley, to a bucket (block 2) to which sand is gradually added. The system is initially at rest.

What happens to the tension in the rope just after the block 1 begins to slip upward?

A. It increases.
B. It decreases.
C. It stays the same.