

# General Physics: Mechanics

## PHYS-101(en) Lecture 10a: Collisions



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# Today's agenda (Serway 9, MIT 15)

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1. Collisions
2. Elastic collisions
3. Inelastic collisions
4. More collisions

# Last few weeks - conserved quantities

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- Conservation of momentum:

*In a given inertial reference frame,  
the total momentum of an isolated system stays constant.*

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$$\sum \vec{p}_i = \sum \vec{p}_f \quad \Rightarrow \quad \sum m\vec{v}_i = \sum m\vec{v}_f$$

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- Conservation of energy:

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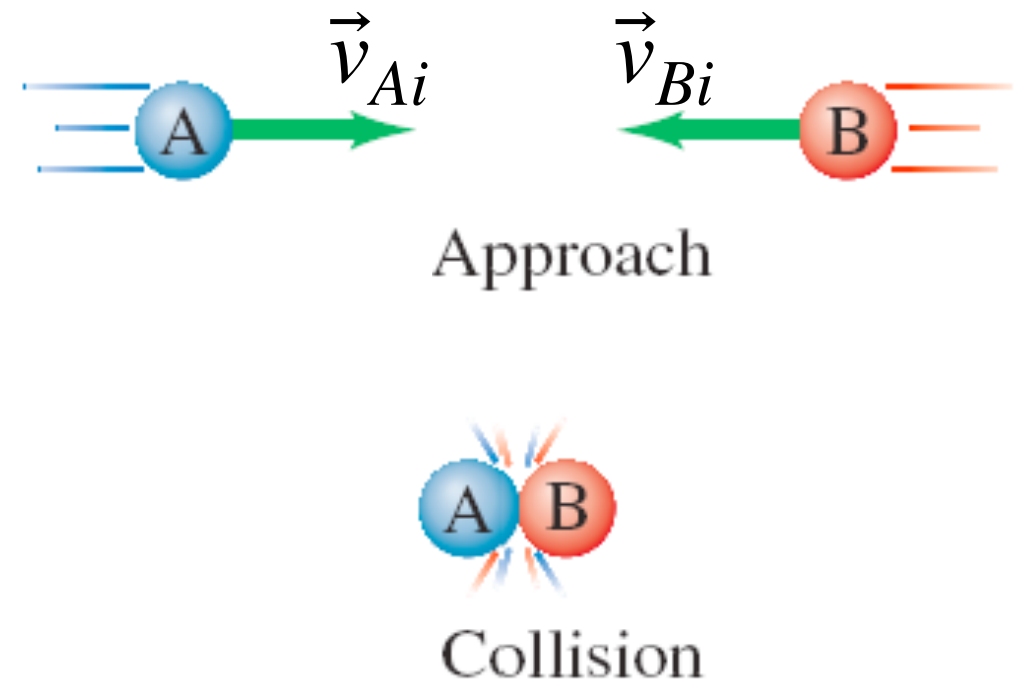
*If all forces doing work on a system are conservative, then its mechanical energy is conserved.*

$$\sum E_{mi} = \sum E_{mf} \Rightarrow \sum K_i + \sum U_i = \sum K_f + \sum U_f$$

Dropping things on an anvil

# Elastic versus inelastic collisions

- Throughout a collision:
  - **Momentum** is always conserved (when the net external force is zero or when using the impulse approximation)

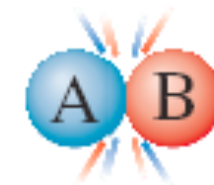


# Elastic versus inelastic collisions

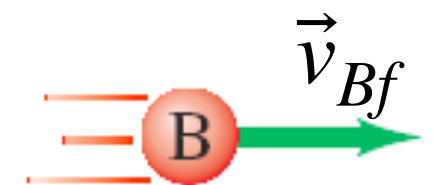
- Throughout a collision:
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Approach



Collision



If elastic

# Elastic versus inelastic collisions

- Throughout a collision:
  - **Momentum** is always conserved (when the net external force is zero or when using the impulse approximation)
  - **Kinetic energy** is conserved when the collision is **elastic** (i.e. no nonconservative work, no change in potential energy)
  - **Kinetic energy** is not conserved when the collision is **inelastic**



Approach



Collision



If elastic



If inelastic

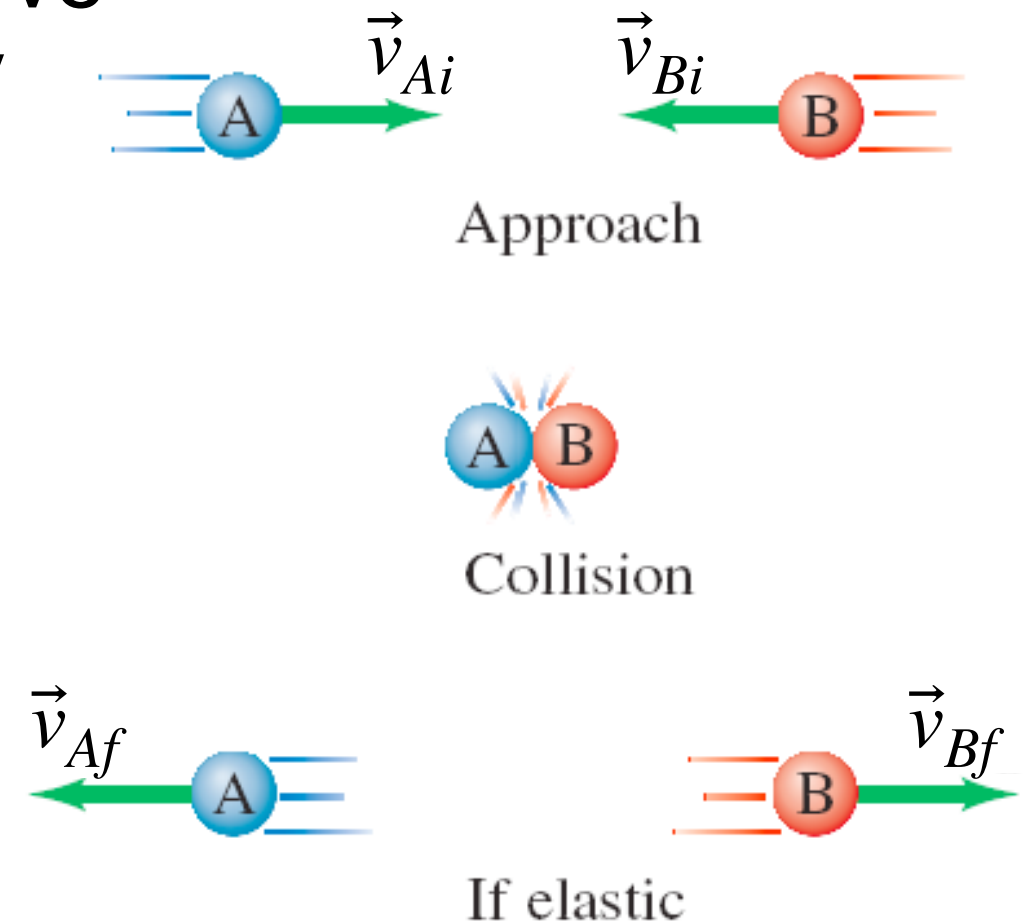
# Conceptual question

A ball is thrown at a wall. The ball bounces off and returns with a speed equal to the speed it had before colliding with the wall. Which of the following quantities are the same after the collision as they were before the collision?

- A. The kinetic energy of the ball.
- B. The momentum of the ball.
- C. Both the kinetic energy and the momentum of the ball.
- D. Neither the kinetic energy nor the momentum of the ball.

# Elastic collision in one dimension

- Remember *elastic collisions* conserve both momentum and kinetic energy





# Elastic collision in one dimension

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# DEMO (766): Elastic collision, same mass

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Cart A, with mass  $m$  moving with speed  $v_{Ai}$ , collides head-on with cart B of equal mass. What are the speeds of the two carts after the collision, assuming it is elastic?

A. Cart B is initially at rest ( $v_{Bi} = 0$ )

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B. Cart B is also moving with an initial velocity  $v_{Bi}$

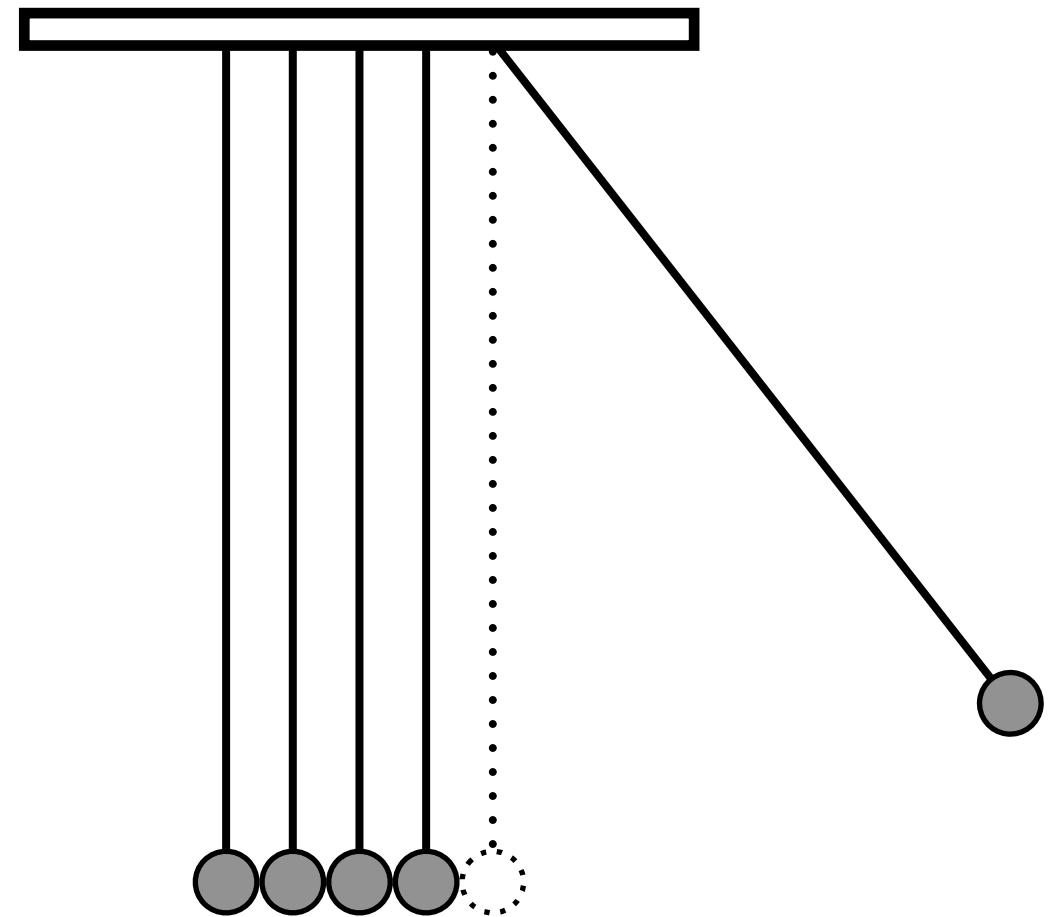
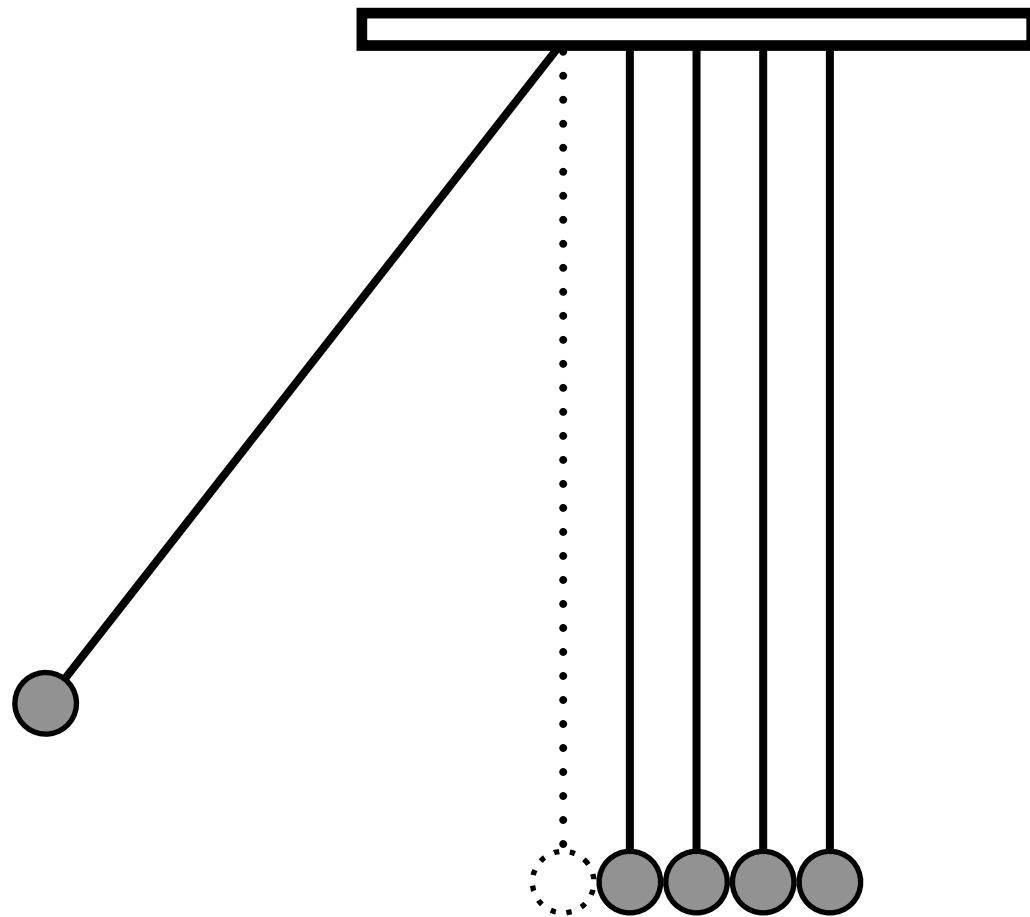
# DEMO (766): Elastic collision, same mass

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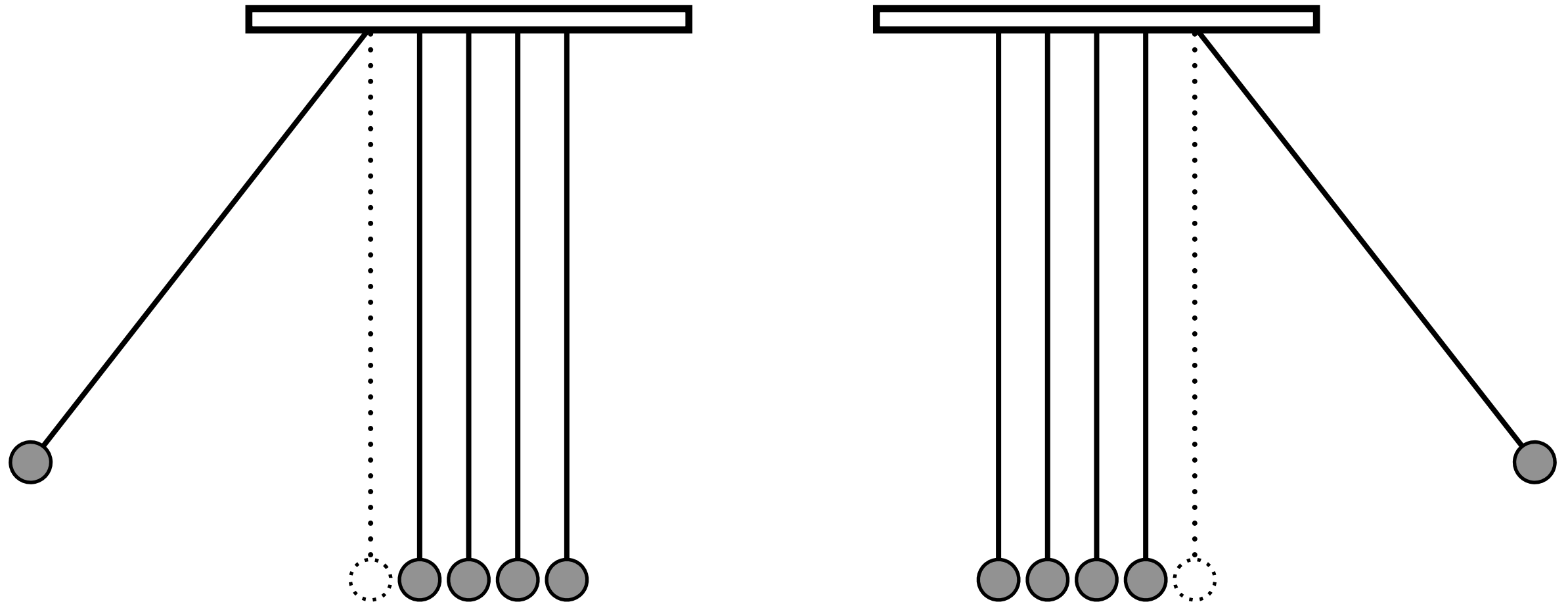
Cart A, with mass  $m$  moving with speed  $v_{Ai}$ , collides head-on with cart B of equal mass. What are the speeds of the two carts after the collision, assuming it is elastic?

- When objects with equal mass collide elastically in 1D, they simply swap velocities

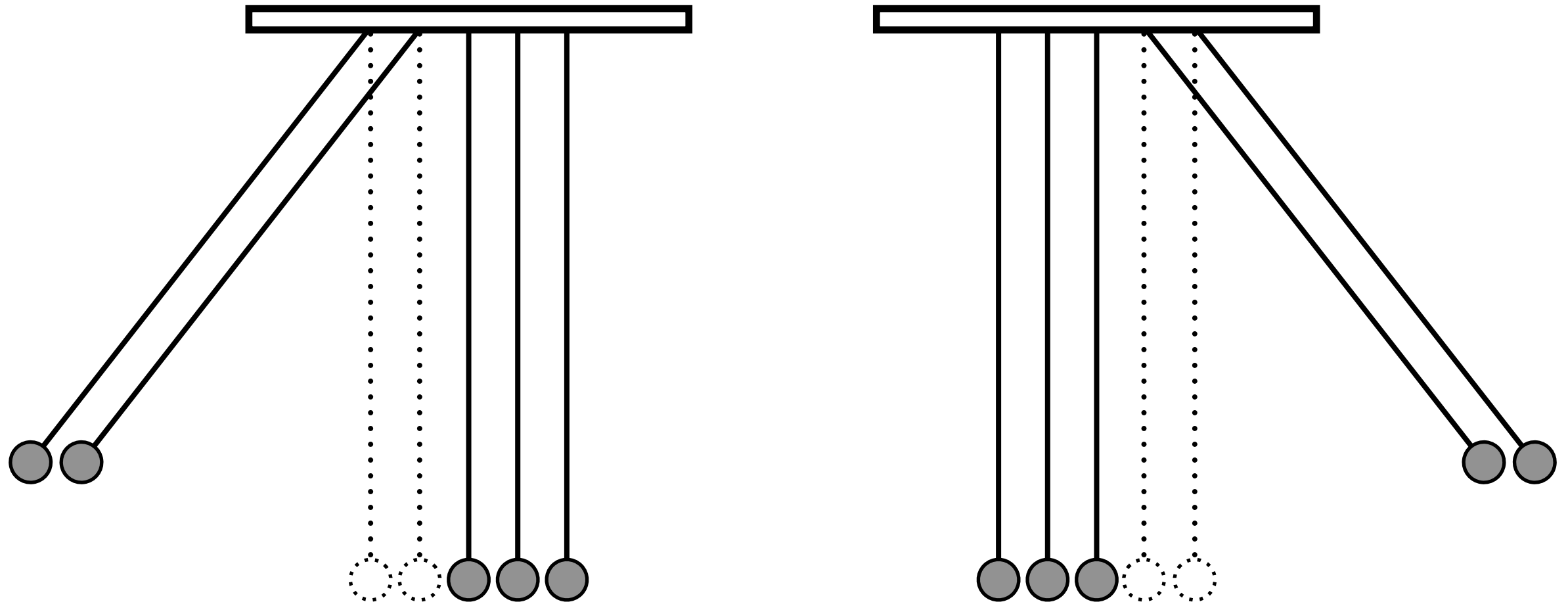
# DEMO (89): Newton's cradle



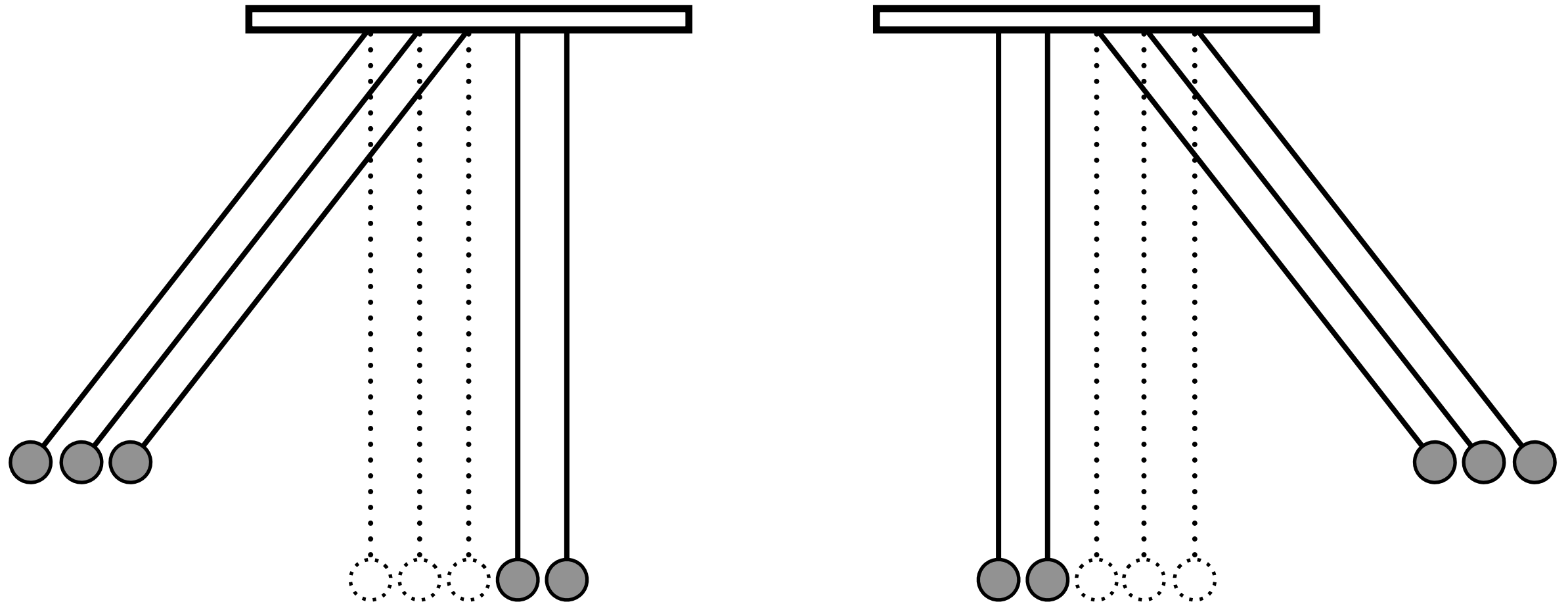
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# DEMO (766): Elastic collision, different mass

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Cart A, with mass  $m_A$  moving with speed  $v_{Ai}$ , experiences a head-on elastic collision with cart B, which has mass  $m_B$  and is at rest.

A. What are the final velocities of the carts?

# DEMO (766): Elastic collision, different mass

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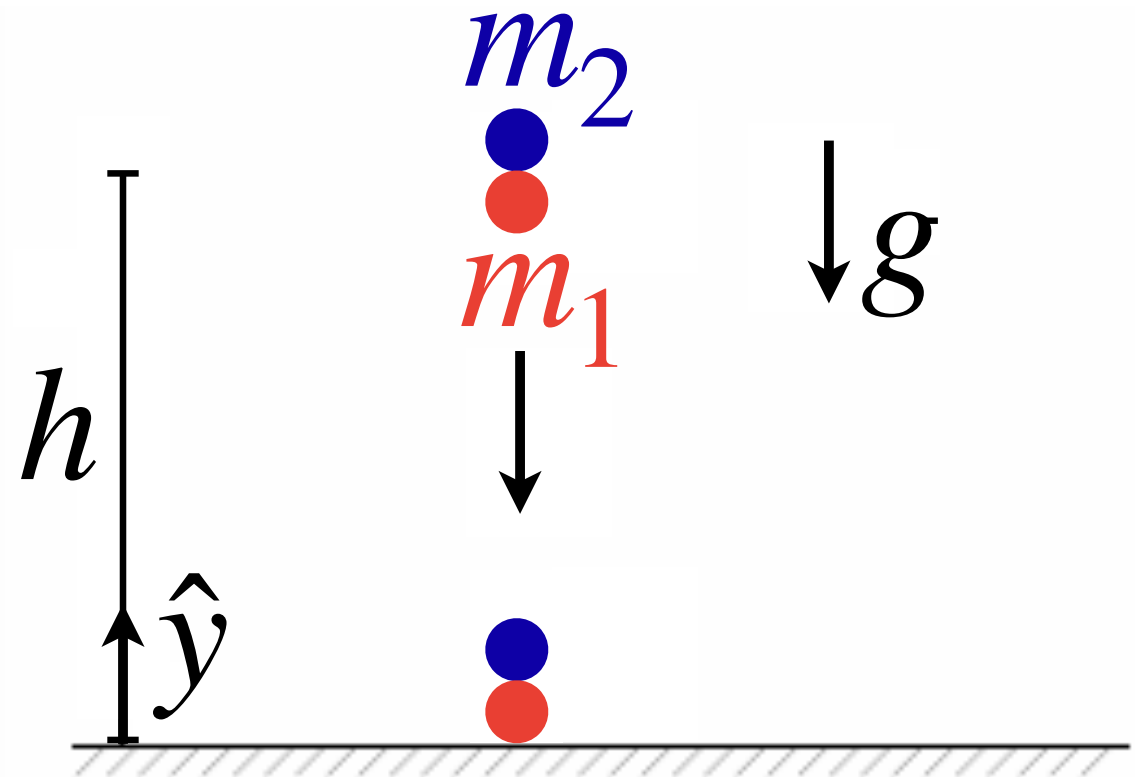
Cart A, with mass  $m_A$  moving with speed  $v_{Ai}$ , experiences a head-on elastic collision with cart B, which has mass  $m_B$  and is at rest.

B. What if  $m_A$  is much *larger* than  $m_B$ ?

C. What if  $m_A$  is much *smaller* than  $m_B$ ?

# Example

Two small balls are dropped from the same height  $h$ , one on top of the other. Ball 2 is on top, while ball 1 is below and is much more massive with  $m_1 \gg m_2$ . First, ball 1 collides with the ground at speed  $v_0$  and rebounds elastically. Then, as ball 1 starts to move upward, it collides elastically with ball 2 which is still moving downwards also with speed  $v_0$ . What is the **relative** speed between the two balls after they collide?

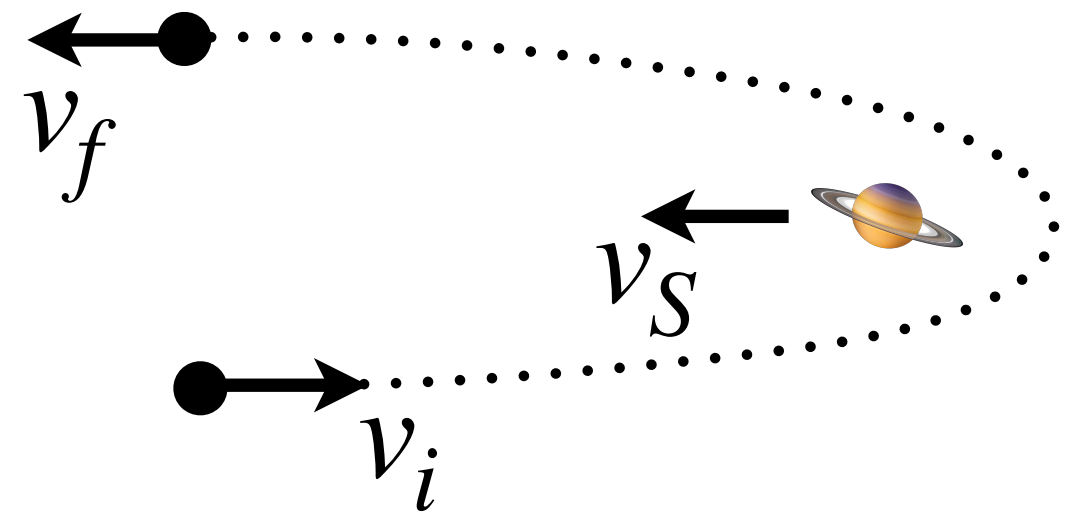


Seismic accelerator

# Conceptual question

A small spacecraft with speed  $v_i$  approaches Saturn, which is moving in the opposite direction at speed  $v_S$ . Due to gravitational interactions with Saturn, the spacecraft swings around Saturn and heads off in the direction opposite to its approach. After it is far enough away to be effectively free of Saturn's gravity, the final speed of the spacecraft  $v_f$  is...

- A.  $v_i - v_S$ .
- B.  $v_i + 2v_S$ .
- C.  $v_i - 2v_S$ .
- D.  $v_i + v_S$ .
- E.  $2v_i - v_S$ .



# Inelastic collisions

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- A “perfectly inelastic” collision is when the objects stick together afterwards, so there is one final velocity



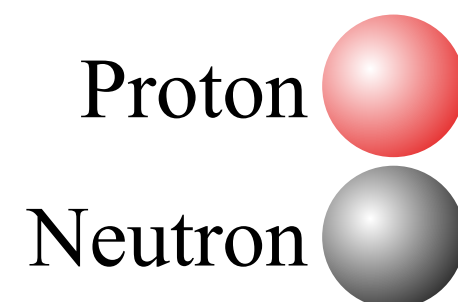
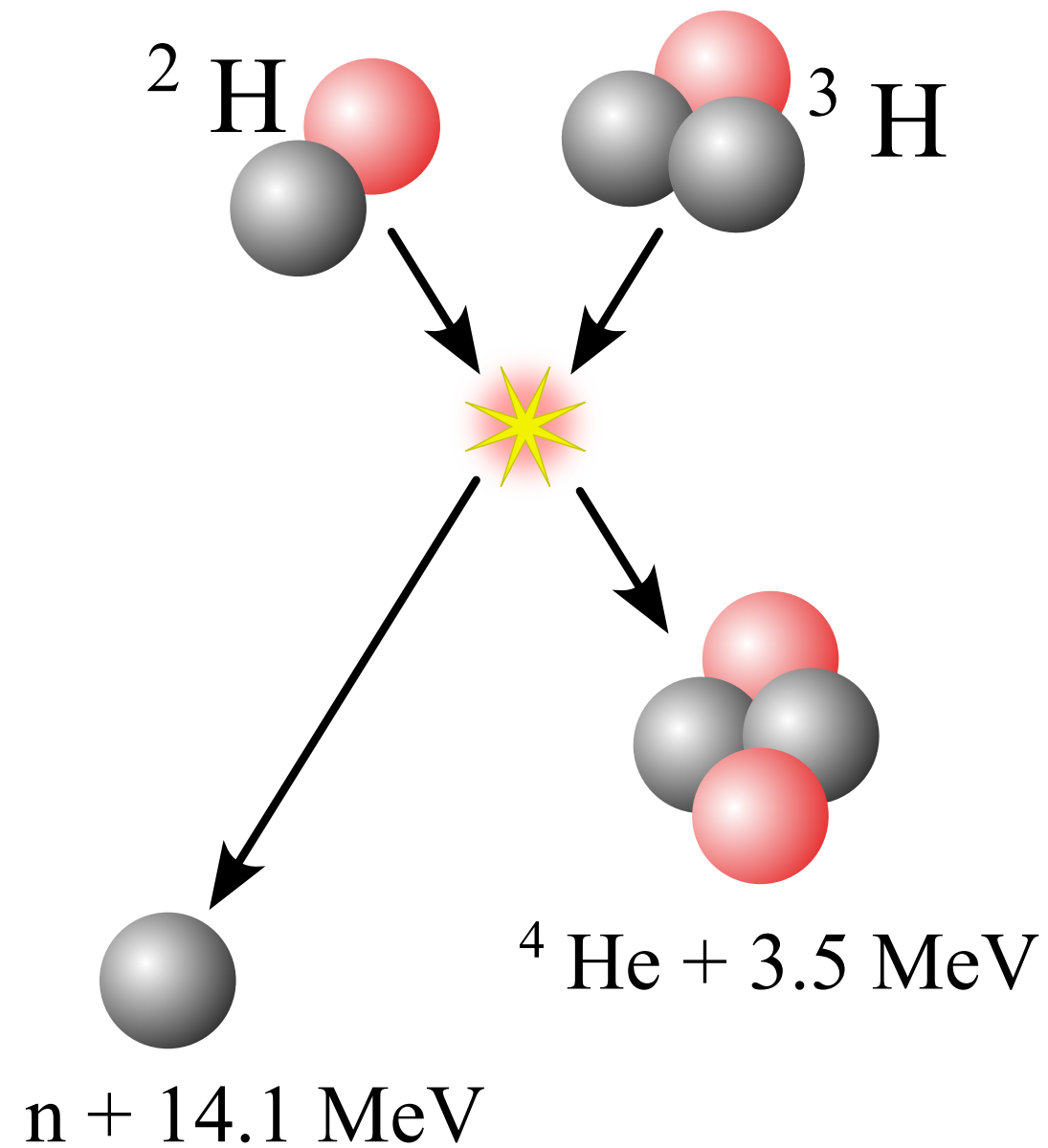
# Inelastic collisions

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  - Impose  $v_{Af} = v_{Bf} = v_f$  instead of energy conservation

# Inelastic collisions

- *Inelastic collisions* do not conserve kinetic energy, but still conserve momentum (as long as the system is isolated)
- Energy can be lost to potential energy or thermal energy (i.e. heat) due to friction
- A “perfectly inelastic” collision is when the objects stick together afterwards, so there is one final velocity
  - Impose  $v_{Af} = v_{Bf} = v_f$  instead of energy conservation
- Kinetic energy can even be gained through an inelastic collision!
- “How?!”, you ask...

# Fusion!

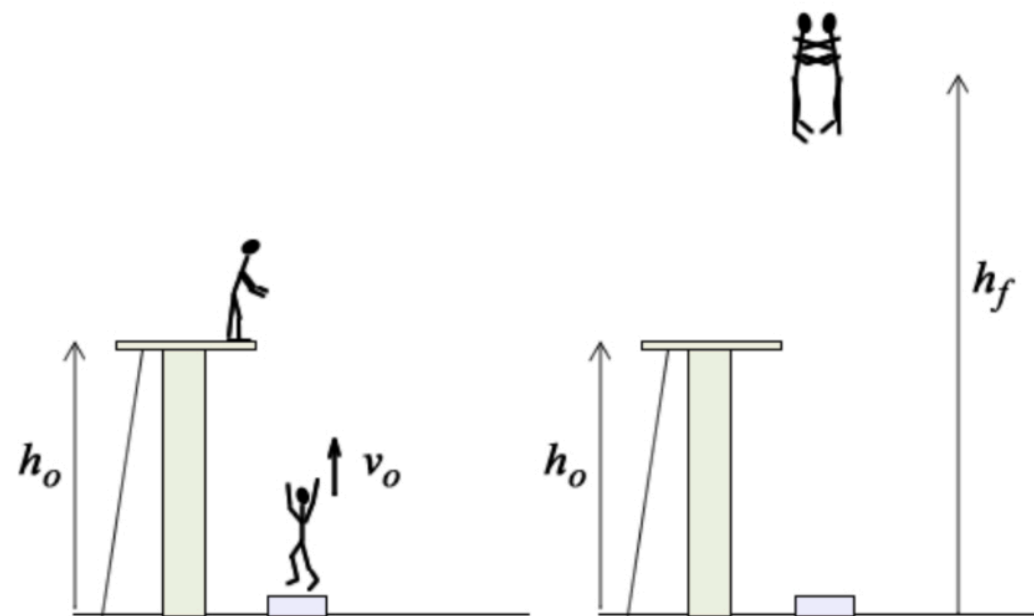


# We've already seen inelastic collisions

## 1. Acrobat and clown

An acrobat of mass  $m_A$  jumps upwards off a trampoline with an initial speed  $v_0$ . At a certain height, he grabs a clown of mass  $m_C$ , who is standing stationary at the edge of a platform. They then fall together. Assume that the time it takes for the acrobat to grab the clown is very short.

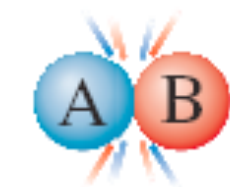
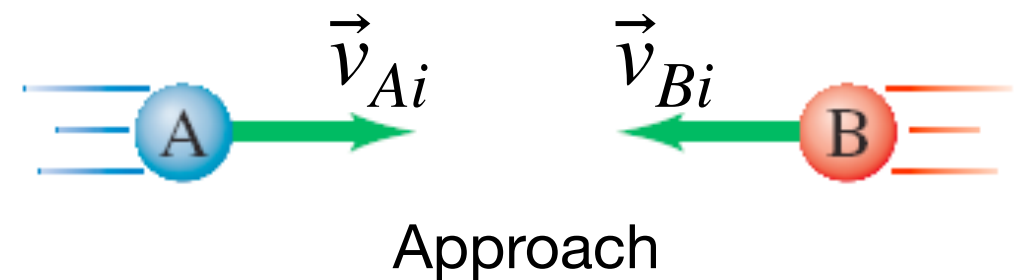
**Problem set 7**



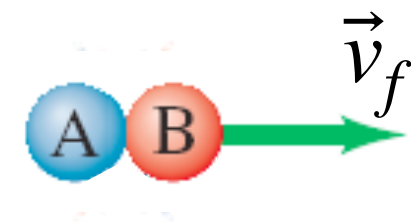
What is the maximum height  $h_f$  reached by the acrobat and clown? Write your answer in terms of some or all of the following:  $m_A$ ,  $m_C$ ,  $g$ ,  $h_0$ , and  $v_0$ .

# Perfectly inelastic collision in one dimension

- Inelastic collisions conserve momentum, but **not** kinetic energy (so we need an additional constraint)
- In this case, we can find a completely general solution for the final velocities in terms of the initial velocities



Collision



If perfectly inelastic

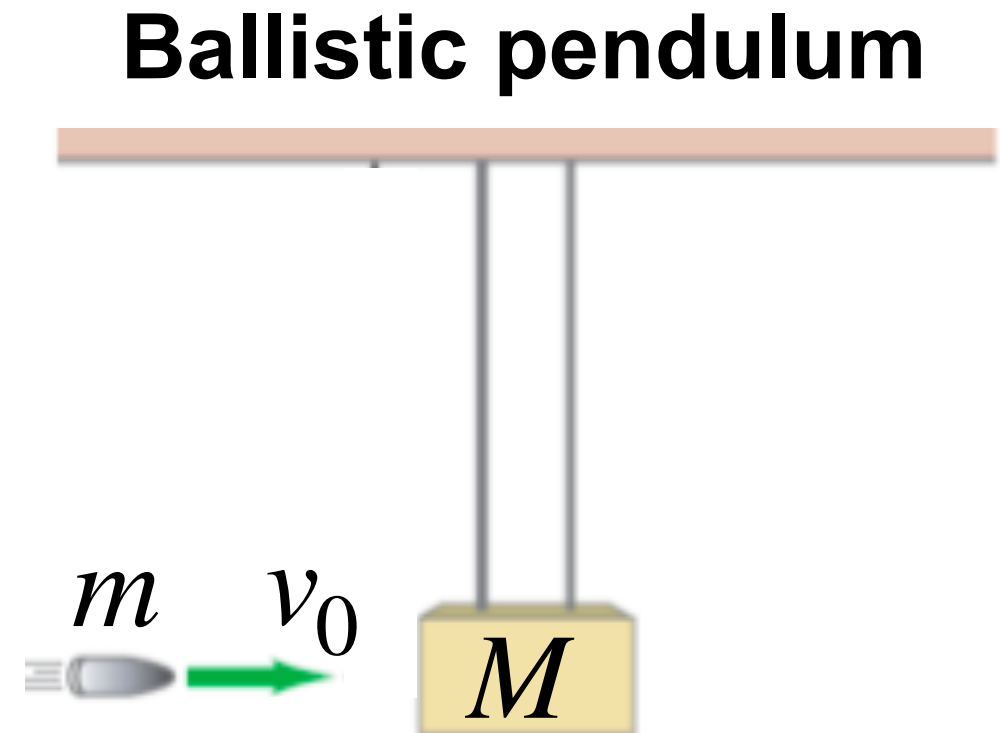
# DEMO (766): Perfectly inelastic collision

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Cart A, with mass  $m_A$  moving with speed  $v_{Ai}$ , collides head-on with cart B, which has mass  $m_B$  and is at rest. What are the speeds of the two carts after the collision, assuming it is perfectly inelastic?

# How to measure the speed of a bullet?

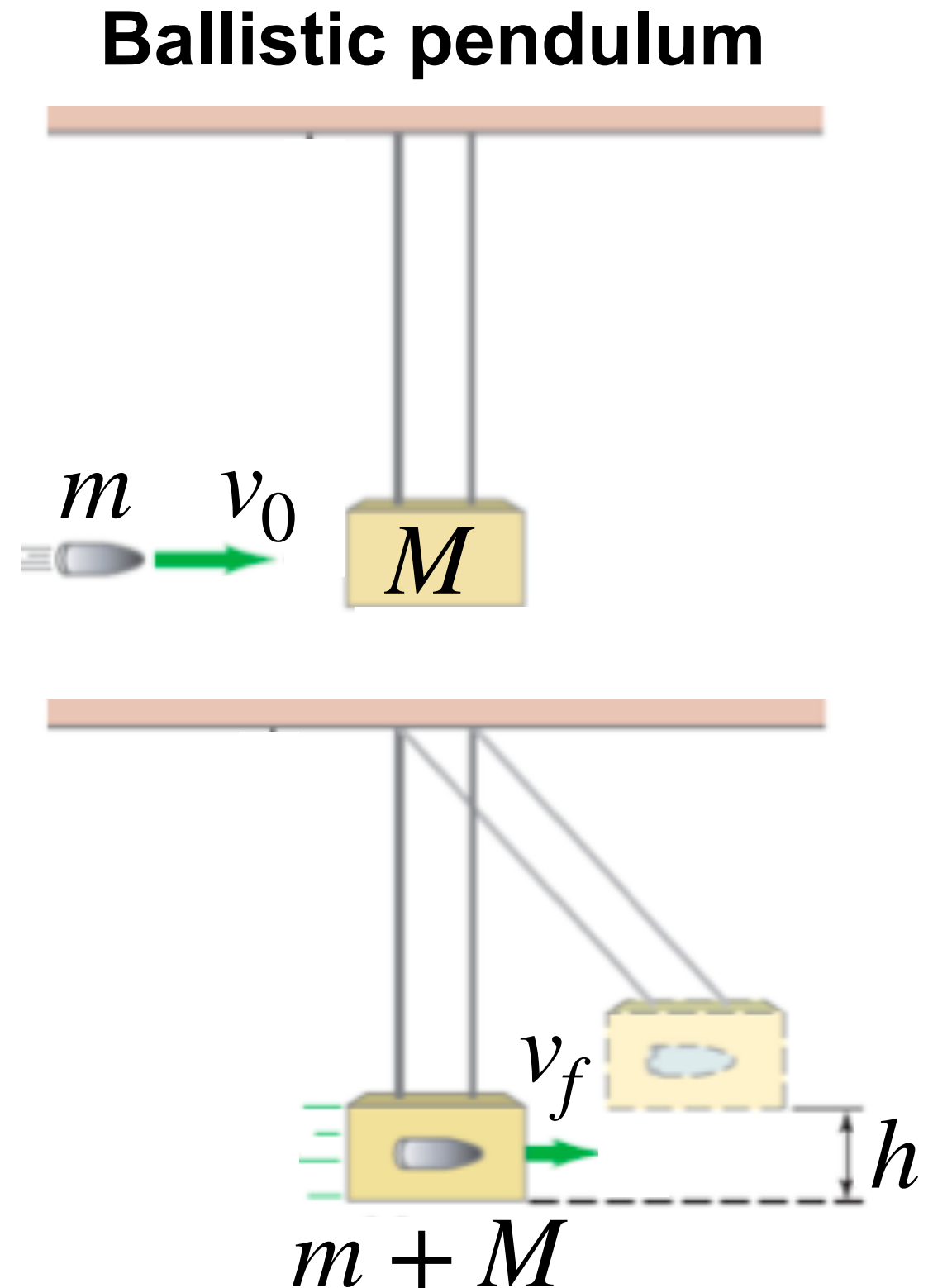
Any projectile, of mass  $m$ , is fired into a large block of mass  $M$ , which is suspended like a pendulum. As a result of the collision, the pendulum and projectile swing up together to a maximum height  $h$ .



# How to measure the speed of a bullet?

Any projectile, of mass  $m$ , is fired into a large block of mass  $M$ , which is suspended like a pendulum. As a result of the collision, the pendulum and projectile swing up together to a maximum height  $h$ .

Calculate the initial horizontal speed of the projectile  $v_0$  from the maximum height  $h$ .

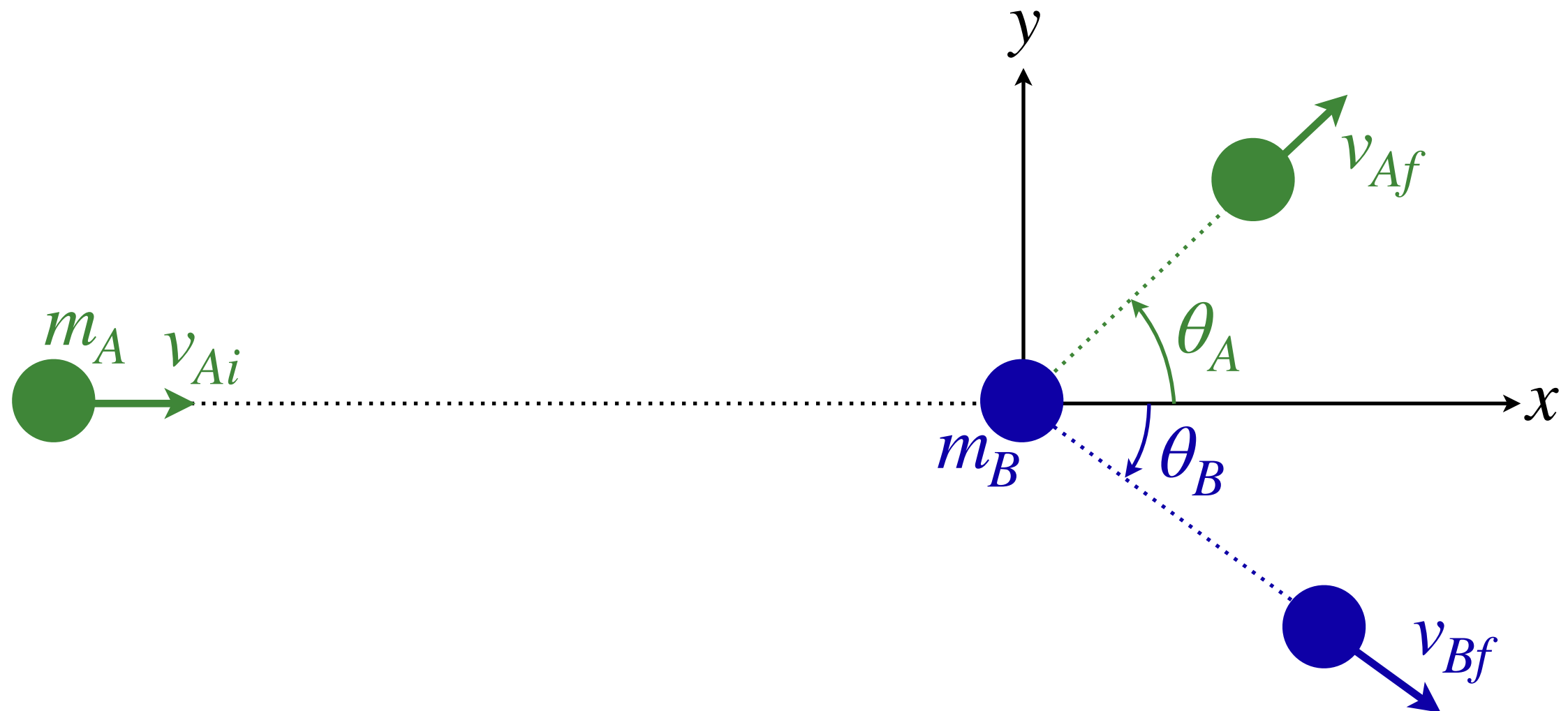




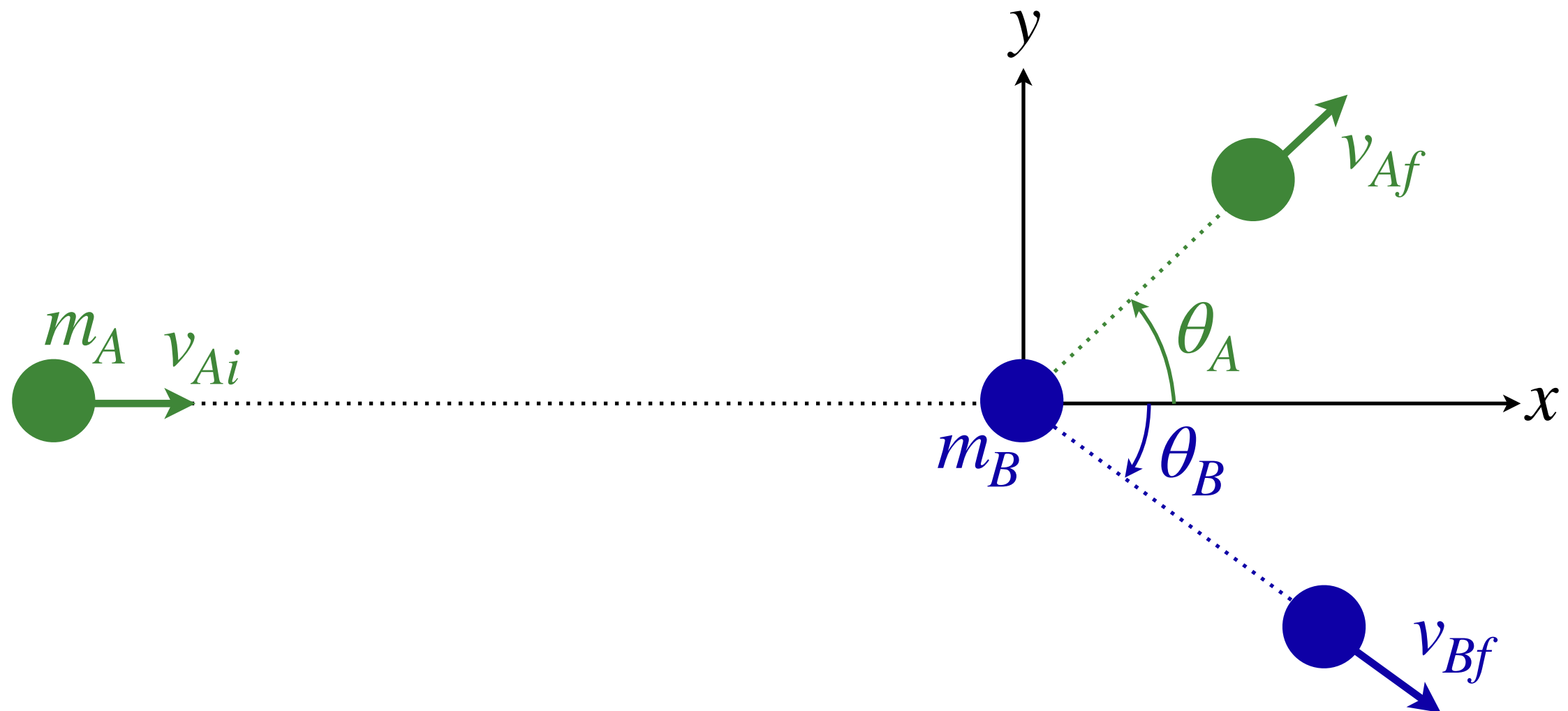
# How to measure the speed of a bullet?

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# Collisions in two or three dimensions



# Collisions in two or three dimensions



- Still apply conservation of momentum and kinetic energy (or alternative condition), but significantly more math

# 2D elastic collision, same mass

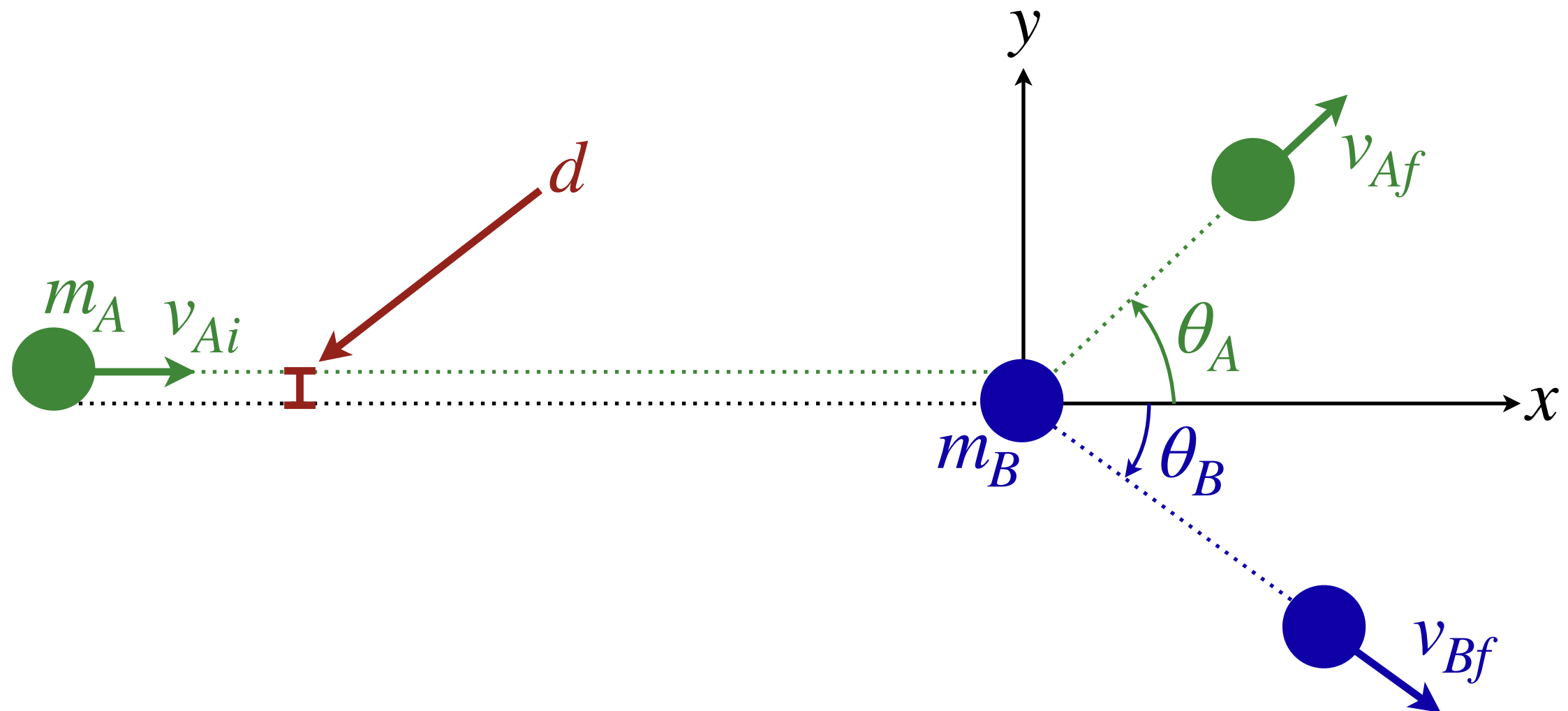
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A ball of mass  $m$  moving with speed  $v_{Ai}$  without friction on a horizontal surface collides elastically with another ball of an equal mass, at rest. After the collision, what is the angle between their velocities  $\theta_A + \theta_B$ ?

# 2D elastic collision, same mass

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# 2D elastic collision, same mass



- To solve for the 4 unknowns ( $v_{Af}$ ,  $v_{Bf}$ ,  $\theta_A$ ,  $\theta_B$ ), we need a fourth equation
- Knowing the “impact parameter”  $d$ , allows you to directly determine  $\theta_B$

# DEMO (763)

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Billiard table

# Conceptual question

Cart A is at rest. An identical cart B is moving to the right and collides **elastically** with cart A. After the collision, which of the following is true

- A. Carts A and B are both at rest.
- B. Cart B stops and cart A moves to the right with speed equal to the original speed of cart B.
- C. Cart A remains at rest and cart B bounces back with speed equal to its original speed.
- D. Cart A moves to the right with a speed slightly less than the original speed of cart B and cart B moves to the right with a very small speed.



# Conceptual question

Cart A is at rest. An identical cart B, moving to the right, collides **inelastically** with cart A. They stick together. After the collision, which of the following is true.

- A. Carts A and B are both at rest.
- B. Carts A and B move to the right with a speed less than cart B's original speed.
- C. Carts A and B moves to the right with speed greater than Cart B's original speed.
- D. Cart B stops and cart A moves to the right with speed equal to the original speed of cart B.

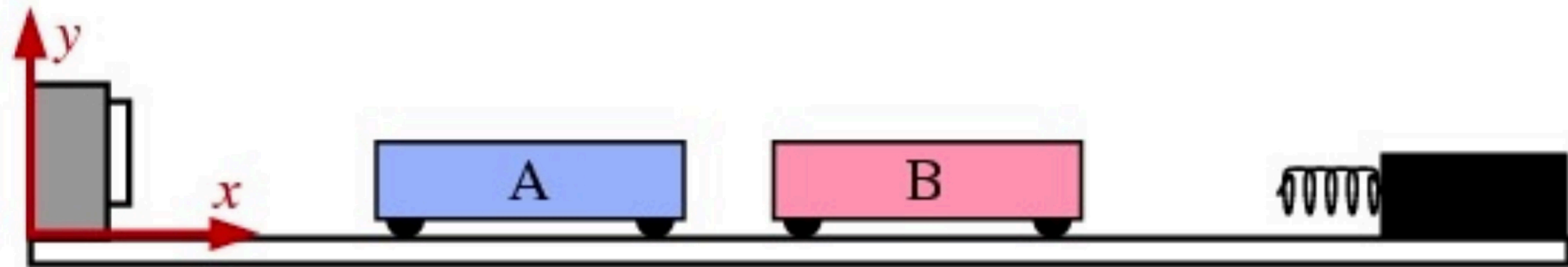
# Conceptual question

An explosion splits an object initially at rest into two pieces of unequal mass. Which piece has the greater kinetic energy?

- A. The less massive piece
- B. The more massive piece
- C. They both have the same kinetic energy
- D. There is not enough information to tell

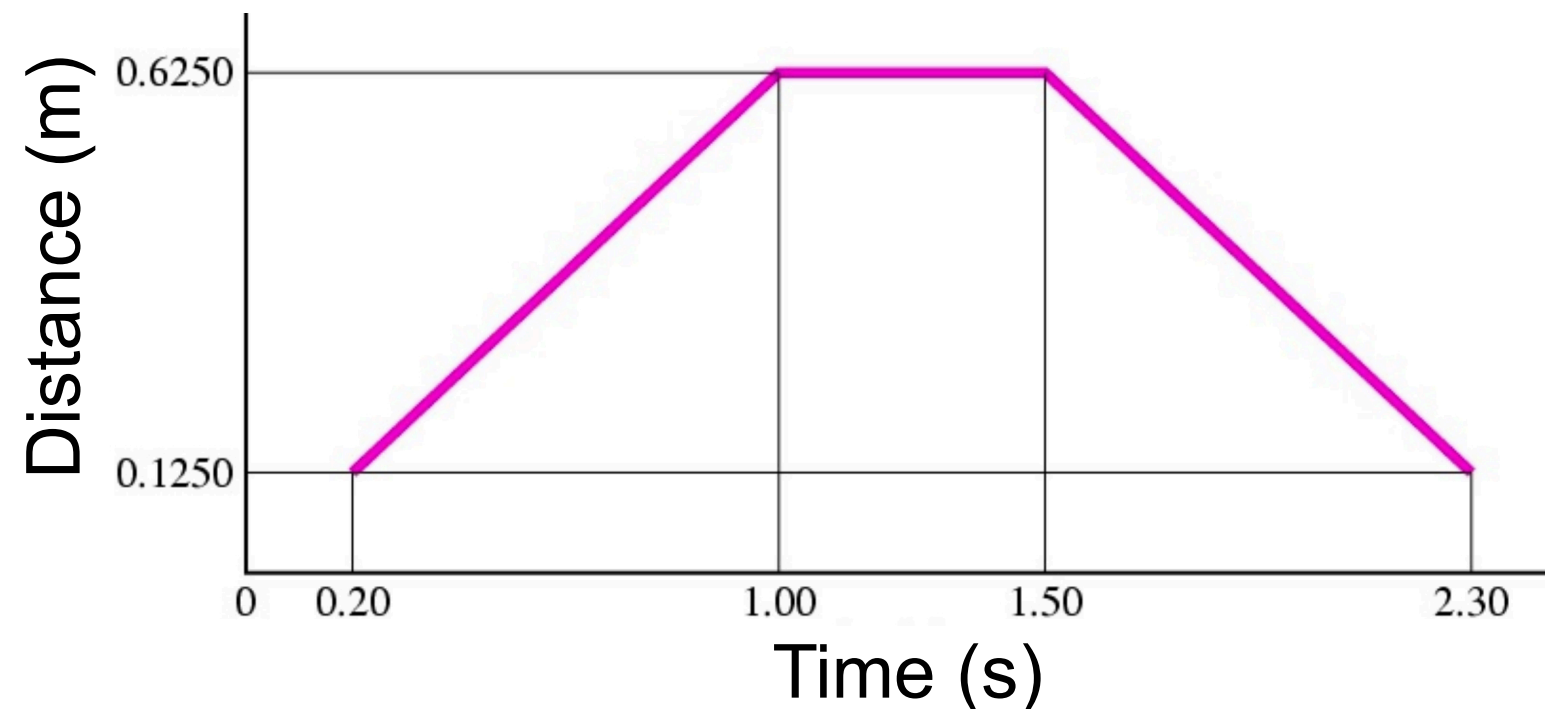
# Conceptual question

An experimental setup to study the collision of two carts:



In the experiment, cart A rolls to the right, away from the motion sensor at the left end of the track, and hits cart B, which is at rest. The graph below shows the distance from the motion sensor to cart A as a function of time. Which objects collide at time  $t = 1.5$  s?

- A. Cart B and the spring.
- B. Cart B and the motion sensor.
- C. Carts A and B.
- D. Cart A and the spring.
- E. Cart A and the motion sensor.



# Summary

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- As long as the system is *isolated*, momentum is conserved in any collision
- If the collision is *elastic*, kinetic energy is also conserved
  - *Elastic* means that the nonconservative work is zero and there is no change in the potential energy
- If the collision is *inelastic*, kinetic energy is not conserved, so you need to find some other condition
- If the collision is *perfectly inelastic*, the objects stick together, so the other condition is to set the final velocities of the objects to be equal

# See you tomorrow for fusion reactions

