Final exam
PHYS-101(en)
17 January 2025

Problem booklet

Guidelines:

- **Do NOT open this booklet before the start of the exam!**
- **Justify your answers.** Answers without a justification will not be counted.
- **Provide your answers in the designated spaces in your Ans booklet only.** Anything written outside those spaces will be ignored by the scanner.
- You may use scrap paper sheets to prepare your answers. However, only the answers in the Ans booklet will be scanned and graded.
- **Do not unstaple** the pages of the Ans booklet.
- **Write in black or blue ink only**. Red ink, friction pens (such as Frixion Ball) and pencils are not allowed because they cannot be properly scanned.
- Calculators, smartphones, tablets, or any other electronic devices are **not** allowed.
- Assistants will check your student ID card and have you sign an attendance sheet. Please have your Camipro card in front of you on your table, ready to be checked.
- Assistants are **not** allowed to answer any questions, except in cases of doubt about the meaning of a word or a phrase.
- **Formula sheet:** One handwritten double-sided A4 sheet. Formula sheets that do not conform to the rules may be confiscated. No other material is permitted.

1. Block on an inclined surface (17 points)

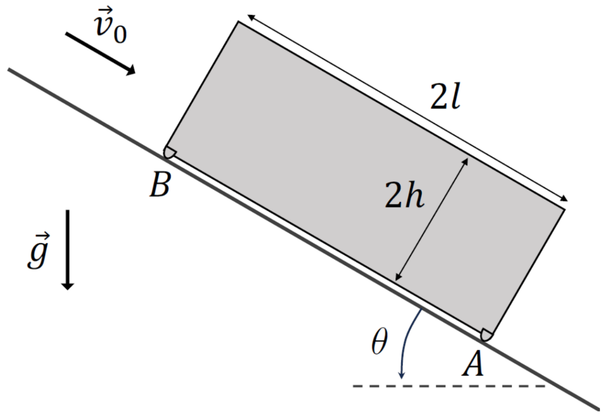

Figure 1: Block on an inclined surface

A solid block of uniform density, total mass m, and a rectangular profile (length $2\,l$, height $2\,h$) lies on a flat surface making an angle θ ($0 \leq \theta < \pi/2$) with the horizontal (see Fig. 1). The block makes contact with the surface only at the two points A and B and friction (either static, with coefficient μ_s, or kinetic, with coefficient μ_k) can act on A and B. You can neglect all other friction forces, including air drag.

To start with, consider the situation in which the block is released with an initial speed v_0 and travels a distance d. Assume that the block moves without rolling over (in other words, A and B *never* cease to be in contact with the surface).

a) Draw a diagram showing the forces acting on the block and their point of application. Clearly indicate your coordinate system.

b) Compute the speed v_f of the block after having moved a distance d. Express your answer in terms of v_0, d, g, θ, and μ_k.

c) What condition must μ_k fulfill for the block to slow down, i.e., to guarantee that $v_f < v_0$?

What condition must μ_k fulfill for the block to *not* come to a full stop, i.e., to guarantee that $v_f > 0$?

Show that these two conditions can be written as a single expression in the form $\tan(\theta) < \mu_k < \tan(\theta) + \lambda$, where λ is a function of v_0, d, g, and θ. Give the expression for λ.

d) Show that the magnitude of the normal force acting on point B is $|\vec{N}_B| = \frac{mg}{2}\cos(\theta)\left|1 - \frac{h}{l}\mu_k\right|$.

Hint: Consider the rotational state of the block around its center of mass.

e) Find the condition on μ_k for point B to always remain in contact with the surface. Notice that you can express your answer in terms of h and l only.

Now consider the situation in which the block is initially at rest ($v_0 = 0$) and does not roll over.

f) Show that $\tan(\theta) > \mu_s$ for the block to start sliding down the inclined plane.

2. Curling (20 points)

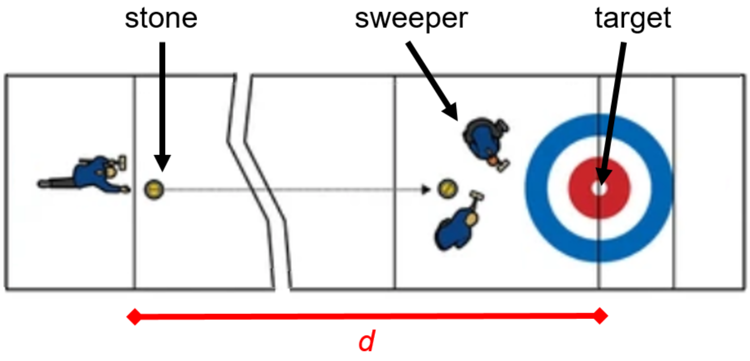

Figure 2: Curling

You and your friends are playing a game of curling. In curling (see Fig. 2), two teams take turns sliding "stones" of mass m on a flat horizontal surface covered with ice, also known as the "curling sheet", towards a target located at a distance d from where the players release the stones. Each team also has two "sweepers" who may sweep the ice in front of the stone to reduce the friction of the ice.

In this problem, we use a simplified model of curling in which the motion of the stones is entirely one-dimensional and the stones cannot rotate.

Notice that the coefficient of kinetic friction μ_k of the stones with the curling sheet is *non-zero*. In other words, you may *not* neglect the friction of ice in this problem. You may, nevertheless, neglect air drag.

Initially, you and your teammates agree to do *no* sweeping. You plan how you want to proceed before sliding your stone:

a) Draw a diagram of the problem, clearly indicating your choice of reference frame. Show all forces acting on the stone after it is released.

b) If the stone is released at a time $t = 0$ with an initial speed v_0, derive an expression for the velocity $\vec{v}$ of the stone as a function of time.

   Give an expression valid for all time $t \geq 0$.

c) Using the velocity as a function of time, as well as the position as a function of time, find the initial speed v_0 required for the stone to stop on the target. Give your answer in terms of μ_k, g, and d.

d) In that case (when the stone stops on the target), determine the work done on the stone by friction, W_f.

   Then, compute the total change of kinetic energy of the stone and verify that it is equivalent to W_f. Why can this equality be expected for the motion of the stone?

Your teammates are skeptical of your calculations and decide to sweep (going against the agreement!) after you have let go of the stone. They sweep for one third of the distance to the target, thereby reducing the coefficient of friction by half in that part of the path.

e) Using the "Work – Kinetic energy" theorem, find the location at which your stone comes to a stop.

By your last turn, one of the opposing team's stones is blocking the target at a distance $0.9\,d$ from the release point. In order to win the game, you want to slide your stone and make it collide with your opponent's. You make sure that the collision is *perfectly inelastic* by putting a small amount of glue (of negligible mass) on the side of your stone, such that both stones stick together upon impact.

If you release the stone with the correct initial speed, v_n, your stone will stop exactly on the target, while the stone of the opposing team will be slightly displaced. This is a risky move, so you must avoid any errors in the calculations if you do not want to lose the match!

This time your teammates do fulfill the agreement of *no* sweeping.

f) Show that the velocity of the two sticking stones right after the collision has the same direction as your stone right before the collision and half the speed.

   Assume that the interaction time is short enough for the impulse approximation to hold. Why is this hypothesis necessary?

g) Determine the speed that the two sticking stones must have right after the collision to come to a stop on the target. Give your answer in terms of μ_k, g, and d.

h) Compute v_n, the initial speed that you must give your stone to win the game, in terms of μ_k, g, and d.

3. Dumbbell, pulley and block (23 points)

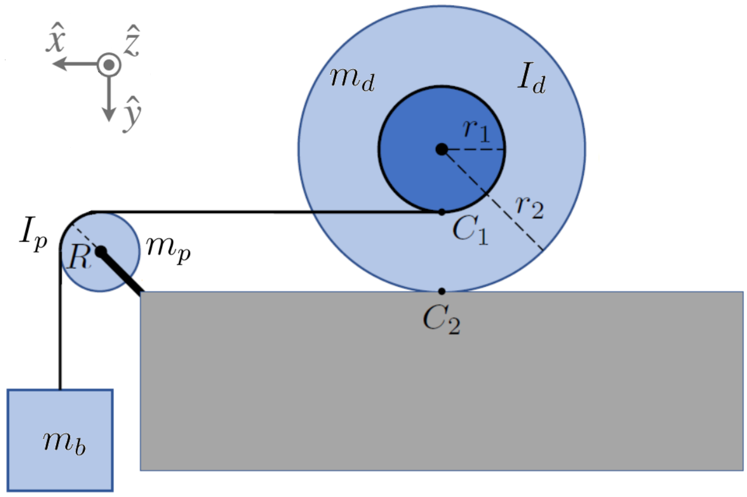

Figure 3: Dumbbell, pulley and block

Consider a dumbbell (*haltère*) consisting of a cylindrical handle (*poignée*) of radius r_1 and two discs of radius r_2, one at each end of the handle. The discs are fixed to the handle, making the dumbbell a rigid body of total mass m_d and total moment of inertia I_d about its axis (which passes through its center of mass and coincides with the axes of the handle and the discs).

The dumbbell is initially at rest, but is nevertheless free to roll without slipping on the flat horizontal surface of a table, as shown in Fig. 3. The point of contact between the dumbbell and the table is labeled C_2.

An inextensible, massless rope is wound (*enroulée*) around the handle and attached to a block of mass m_b, passing over a pulley of radius R, mass m_p and moment of inertia I_p with respect to its axis of rotation. The vertical portion of the rope has tension T_V and the horizontal portion has tension T_H. The horizontal portion loses contact with the dumbbell handle at point C_1.

You can assume that the rope does not slip on the pulley or the handle. There is no air drag.

a) Draw diagrams showing all the forces, as well as their point of application, acting on the dumbbell, the suspended mass m_b and the pulley.

   Use the coordinate system shown in Fig. 3.

   Remember that the “no slipping” condition means that there may be friction acting at C_2. What kind of friction is it and in which direction does it act? Why?

b) Show that the magnitude of the acceleration of the dumbbell's center of mass (CM) is related to T_H through $|\vec{a}_{CM}| = T_H\, r_2\,(r_2 - r_1)/\big(I_d + m_d\, r_2^2\big)$.

   In what direction does the CM of the dumbbell accelerate?

c) Show that the speed of the rope and the speed of the CM of the dumbbell are related by $v_{rope} = (1 - \frac{r_1}{r_2})\, v_{CM}$.

   Does the rope wind up (*s'enroule*) around the handle or does it unwind (*se déroule*)? Why?

   Hint: What is the tangential velocity at point C_1 with respect to the axis of the dumbbell? What about with respect to the table? How does the velocity at C_1 relate to the velocity of the rope, given that the rope does not slip on the handle?

d) If a_{by} is defined as the vertical component of the acceleration of m_b, find a_{by} in terms of T_V.

e) Show that the angular acceleration of the pulley is $\vec{\alpha}_p = \frac{R}{I_P}(T_V - T_H)\hat{z}$.

f) How is $|\vec{\alpha}_p|$ related to the acceleration of m_b?

Find an expression for T_H in terms of a_{by} and the parameters m_b, g, I_p, and R.

g) Argue that the acceleration of m_b and of the CM of the dumbbell are related through $a_{by} = (1 - \frac{r_1}{r_2})\, a_{CM}$.

Then, using the results obtained in the preceding parts, find $\vec{a}_{CM}$ (the acceleration of the CM of the dumbbell) in terms of the parameters of the problem (m_b, m_d, g, I_d, I_p, r_1, r_2, and R).