

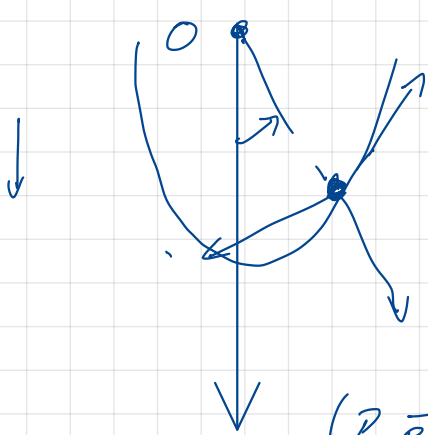
Glissière hémisphérique

Lagrange I

γ

Ph. Neillhaupt.





$$(r, \vec{e}_r, \vec{e}_\theta, \vec{e}_\phi)$$

Vitesse du point P:

$$\vec{OP} = r \vec{e}_r$$

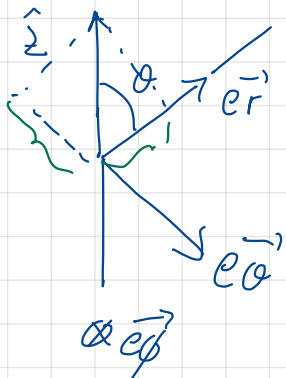
$$\dot{\vec{OP}} = \dot{r} \vec{e}_r + r \dot{\vec{e}}_r$$

$$= \dot{r} \vec{e}_r + r \dot{\theta} \vec{e}_\theta + r \sin\theta \dot{\phi} \vec{e}_\phi$$

$\vec{\omega}$ du repère:

$$\vec{\omega} = \omega \hat{z} + \dot{\theta} \vec{e}_\theta + \dot{\phi} \vec{e}_\phi$$

$$= \omega \cos\theta \vec{e}_r - \omega \sin\theta \vec{e}_\theta + \dot{\phi} \vec{e}_\phi$$



$$\hat{z} = \begin{bmatrix} \vec{e}_r & \vec{e}_\theta \end{bmatrix} \begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix}$$

(par projections)

$$\vec{\omega} \wedge \vec{r} =$$

$$\dot{\vec{OP}} = \begin{bmatrix} \dot{r} \vec{e}_r + r \dot{\theta} \vec{e}_\theta + r \sin\theta \dot{\phi} \vec{e}_\phi \end{bmatrix} = \begin{bmatrix} \dot{r} \vec{e}_r + r \dot{\theta} \vec{e}_\theta + r \sin\theta \dot{\phi} \vec{e}_\phi \end{bmatrix}$$

On aurait pu poser directement $\dot{\phi} = \omega$ dans les coordonnées sphériques.

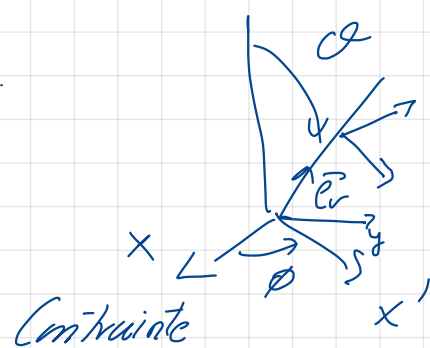
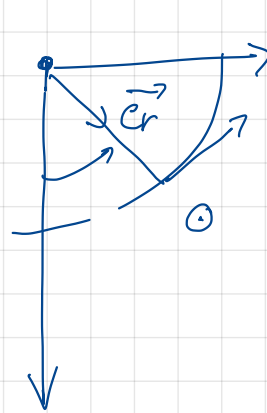
Energie cinétique

$$T = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\dot{r}^2 + r^2 \dot{\theta}^2 + r^2 \sin^2\theta \dot{\phi}^2 \right)$$

$$\frac{1}{2} m \left(\dot{r}^2 + r^2 \dot{\theta}^2 + r^2 \sin^2\theta \dot{\phi}^2 \right)$$

contrainte $r = R \Rightarrow \dot{r} = 0$

$$T = \frac{1}{2} m \left(R^2 \omega^2 \sin^2\theta + R^2 \dot{\theta}^2 \right)$$



(contrainte)

$$\phi(t) =$$

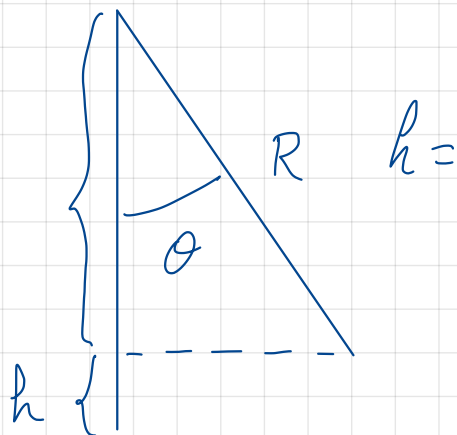
avec $\omega =$

$$\omega = [\text{rad/s}]$$

Energie potentielle:

$$mg h = mg$$

$$= mg - mg$$



$$L = T - V = \frac{1}{2} m (\quad)$$

- +

1 coordonnée généralisée θ

$$\frac{d}{dt} (\quad) - \quad = 0$$

=

=

$$\frac{d}{dt} (\quad) =$$

$\Rightarrow$ dynamique

(Remarque c'est
un moment cinétique
mom. Inertie $\times$ vit. angulaire)

= 0

interprétation points d'équilibre : $\theta = 0$ $\dot{\theta} = 0$

$$\omega\vartheta = \frac{\varphi}{K\omega^2}$$