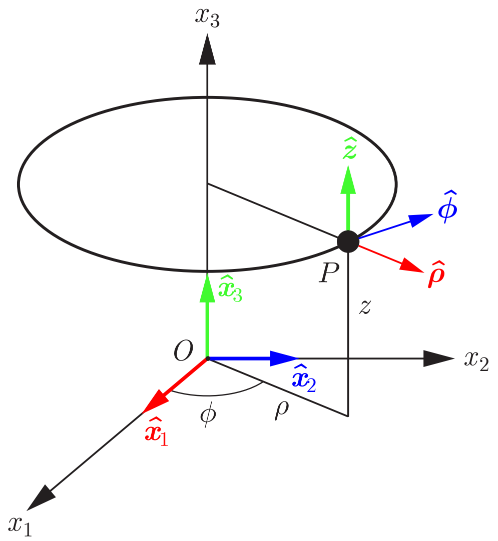
Formulaire

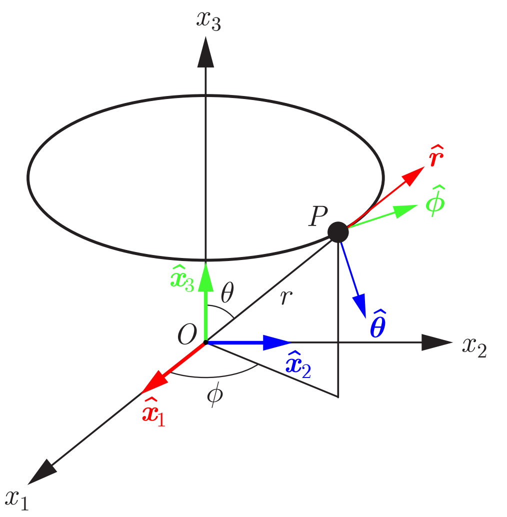
Point matériel

Coordonnées cylindriques :



$$\boldsymbol{r} = \rho\,\hat{\boldsymbol{\rho}} + z\,\hat{\boldsymbol{z}}$$
$$\boldsymbol{v} = \dot{\rho}\,\hat{\boldsymbol{\rho}} + \rho\dot{\phi}\,\hat{\boldsymbol{\phi}} + \dot{z}\,\hat{\boldsymbol{z}}$$
$$\boldsymbol{a} = \left(\ddot{\rho} - \rho\dot{\phi}^2\right)\hat{\boldsymbol{\rho}} + \left(\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi}\right)\hat{\boldsymbol{\phi}} + \ddot{z}\,\hat{\boldsymbol{z}}$$

Coordonnées sphériques :



$$\boldsymbol{r} = r\,\hat{\boldsymbol{r}}$$
$$\boldsymbol{v} = \dot{r}\,\hat{\boldsymbol{r}} + r\dot{\theta}\,\hat{\boldsymbol{\theta}} + r\dot{\phi}\sin\theta\,\hat{\boldsymbol{\phi}}$$
$$\boldsymbol{a} = \left(\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta\right)\hat{\boldsymbol{r}} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta\right)\hat{\boldsymbol{\theta}} + \left(r\ddot{\phi}\sin\theta + 2\dot{r}\dot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta\right)\hat{\boldsymbol{\phi}}$$

Mouvement relatif :

$$\boldsymbol{v}_a(P) = \boldsymbol{v}_a(A) + \boldsymbol{v}_r(P) + \boldsymbol{\Omega} \times \boldsymbol{AP}$$
$$\boldsymbol{a}_a(P) = \boldsymbol{a}_a(A) + \boldsymbol{a}_r(P) + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{AP}) + 2\,\boldsymbol{\Omega} \times \boldsymbol{v}_r(P) + \dot{\boldsymbol{\Omega}} \times \boldsymbol{AP}$$

Formules de Poisson : où $\boldsymbol{r}_r(P) = \boldsymbol{AP}$

$$\dot{\hat{\boldsymbol{u}}} = \boldsymbol{\Omega} \times \hat{\boldsymbol{u}}$$ où $\hat{\boldsymbol{u}}$ est un vecteur unitaire quelconque

Solide indéformable

Cinématique :

$$\boldsymbol{V}_Q = \boldsymbol{V}_P + \boldsymbol{\Omega} \times \boldsymbol{PQ}$$
$$\boldsymbol{A}_Q = \boldsymbol{A}_P + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{PQ}) + \dot{\boldsymbol{\Omega}} \times \boldsymbol{PQ}$$

Théorèmes de transfert du moment cinétique :

$$\boldsymbol{L}_O = \boldsymbol{OP} \times M\,\boldsymbol{V}_G + \boldsymbol{L}_P\ , \qquad \boldsymbol{L}_O = \boldsymbol{OG} \times M\,\boldsymbol{V}_G + \boldsymbol{L}_G\ , \qquad \boldsymbol{L}_P = \boldsymbol{PG} \times M\,\boldsymbol{V}_G + \boldsymbol{L}_G$$

Théorèmes de transfert des moments de force :

$$\sum \boldsymbol{M}_O = \boldsymbol{OP} \times M\boldsymbol{A}_G + \sum \boldsymbol{M}_P\ , \qquad \sum \boldsymbol{M}_O = \boldsymbol{OG} \times M\boldsymbol{A}_G + \sum \boldsymbol{M}_G$$
$$\sum \boldsymbol{M}_P = \boldsymbol{PG} \times M\boldsymbol{A}_G + \sum \boldsymbol{M}_G$$

Théorème du moment cinétique :

$$\sum \boldsymbol{M}_O^{\text{ext}} = \dot{\boldsymbol{L}}_O\ , \qquad \sum \boldsymbol{M}_G^{\text{ext}} = \dot{\boldsymbol{L}}_G\ , \qquad \sum \boldsymbol{M}_C^{\text{ext}} = \dot{\boldsymbol{L}}_C\ , \qquad \sum \boldsymbol{M}_P^{\text{ext}} = \dot{\boldsymbol{L}}_P + \boldsymbol{V}_P \times M\,\boldsymbol{V}_G$$

Moment cinétique et moments d'inertie :

$$\boldsymbol{L}_G = \mathsf{I}_G\,\boldsymbol{\Omega} = \sum_{i=1}^{3} I_{G,i}\,\Omega_i\,\hat{\boldsymbol{e}}_i = I_{G,1}\,\Omega_1\,\hat{\boldsymbol{e}}_1 + I_{G,2}\,\Omega_2\,\hat{\boldsymbol{e}}_2 + I_{G,3}\,\Omega_3\,\hat{\boldsymbol{e}}_3 \qquad \text{où} \qquad I_{G,i} = \sum_\alpha m_\alpha\, r_{\alpha,i}^2$$

Théorème de Huygens-Steiner :

$$I_{A,i} = I_{G,i} + M\,d^2 \qquad \mathbf{I_A} = \mathbf{I_G} + M \begin{pmatrix} \gamma_y^2 + \gamma_z^2 & -\gamma_x\,\gamma_y & -\gamma_x\,\gamma_z \\ -\gamma_x\,\gamma_y & \gamma_x^2 + \gamma_z^2 & -\gamma_y\,\gamma_z \\ -\gamma_x\,\gamma_z & -\gamma_y\,\gamma_z & \gamma_x^2 + \gamma_y^2 \end{pmatrix}$$

Quantité de mouvement et énergie cinétique :

$$\boldsymbol{P} = M\,\boldsymbol{V}_G\ , \qquad T = \frac{1}{2}\,M\,\boldsymbol{V}_G^2 + \frac{1}{2}\left(I_{G,1}\,\Omega_1^2 + I_{G,2}\,\Omega_2^2 + I_{G,3}\,\Omega_3^2\right)$$

Théorème de l'énergie cinétique :

$$\Delta T_{1\to 2} = \sum_\alpha W_{1\to 2}\left(\boldsymbol{F}_\alpha^{\text{ext}}\right) = \sum_\alpha \int_{\mathcal{C}_{1\to 2}} \boldsymbol{F}_\alpha^{\text{ext}} \cdot d\boldsymbol{r}_\alpha$$