Neural Networks and Biological Modeling

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Question set 10

Exercise 1: Firing statistics

Take maximum 20 minutes for this exercise, then switch to the next one:
Consider a stochastic spike generation process in discrete time. The probability of generating a spike in a time Δt is $P_{\Delta t} = \nu \Delta t$. Hence when we take the limit of Δt to 0 the expected value of the quantity $S(t) = \sum_f \delta(t - t_k^f)$ is:

$$\langle S(t) \rangle = \lim_{\Delta t \to 0} \frac{P_{\Delta t}(t)}{\Delta t} = \nu \; ; \; \text{for}\, t > 0.$$

Consider the probability of having two spikes in different time bins around t and t'. Define $\langle S(t)S(t') \rangle$ in a similar fashion, and show that it is equal to $\nu\delta(t - t') + \nu^2$.

Exercise 2: Poisson neuron

We consider a neuron that fires stochastically. Its firing rate is described by a Poisson process of rate ρ. In other words, in every small time interval Δt, the probability that the neuron fires is given by $\rho\Delta t$.

2.1 What is the probability that the neuron does *not* fire during a time of arbitrarily large length t? Hint: Consider first the probability of not firing during a short interval Δt.

2.2 Suppose that the neuron has fired at time t_0. Calculate the distribution of intervals $P(s)$, i.e., the probability density that the neuron fires its next spike after a time s.

2.3 Suppose that the neuron is driven by some input. For $t < t_0$, the input is weak, so that its firing rate is $\rho_0 = 2$Hz. For $t_0 < t < t_1 = t_0 + 100$ms, the input is strong and the neuron fires at $\rho_1 = 20$Hz.

(i) Calculate the interval distributions for weak and strong stimuli.

(ii) What is the probability of having a "burst" consisting of two intervals of less than 20 ms each if the input is weak/strong?

(iii) Suppose that the onset time t_0 of the strong input is unknown; can an observer, who is looking at the neuron's output, decide whether the input is weak or strong?

Exercise 3: Stochastic spike arrival

Consider a neuron with a passive membrane,

$$\tau \frac{du}{dt} = -(u - u_{\text{rest}}) + RI(t) \tag{1}$$

3.1 The neuron receives synaptic input at a rate ν such that

$$I(t) = q \sum_f \delta(t - t^f)\,. \tag{2}$$

Calculate the average value of membrane potential as a function of the presynaptic rate ν, assuming stochastic (Poisson) spike arrival.

Hint: Integrate Eq. 1 keeping explicitly the δ-function. Under the assumption of stochastic spike arrival we have $\left\langle \sum_f \delta(t - t^f) \right\rangle = \nu$.

3.2 Calculate the average value of membrane potential as a function of the presynaptic rate ν if the current coming from the presynaptic activity is:

$$I(t) = \sum_f \alpha(t - t^f)\,. \tag{3}$$

Hint: As before, integrate Eq. 1 keeping the δ-function explicit.

Exercise 4: Homework

4.1 The poisson neuron has a probability to fire in a very small interval Δt equal to $\nu \Delta t$. What will be the probability to observe exactly k spikes in the time interval $T = N\Delta t$ ($P_k(T)$)? Start with the probability to observe k events in N slots (the binomial distribution):

$$P(k, N) = \frac{N!}{k!(N-k)!} p_1^k p_2^{N-k}$$

where p_1 and p_2 are the probabilities to spike and to remain silent in one Δt slot respectively. Take the continuous time limit with Stirling's approximation ($N! \approx (N/e)^N$ for large N) to obtain the Poisson distribution:

$$P_k(T) = \frac{(\nu T)^k}{k!} e^{-\nu T}$$

Verify that this distribution predicts an average number of spikes $\langle k \rangle = \nu T$.

4.2 Suppose that a Poisson neuron with a constant rate of 20 Hz emits in a trial of 5 second duration 100 spikes at times $t^{(1)}$, $t^{(2)}$, ...$t^{(100)}$. The experiment is repeated such that a second spike train with a duration of 5 seconds is observed.

What is the percentage of spikes that coincide between the first and second trial with a precision of $\pm$2ms? More generally, what percentage of spikes coincide between two trials of a Poisson neuron with arbitrary rate ρ_0 under the assumption that trials are sufficiently long?