Last Name .... First Name..................

Neural Networks and Biological Modelling Exam
3 July 2018

- Write your name in legible letters on top of this page.
- The exam lasts 160 min.
- Write **all** your answers in a legible way on the exam (no extra sheets).
- No documentation is allowed apart from 1 sheet A5 of **handwritten** notes.
- No calculator is allowed.
- Have your student card displayed before you on your desk.
- **Check that your exam has 14 pages**

Evaluation:

1. ..../6 pts
2. .... / 19 pts
3. .... / 8 pts
4. .... / 11 pts (this includes 3 bonus points)
5. .... / 8 pts

Total: .... / 52 pts

1 Biophysics of ion channels (6 points)

We consider the following model of an ion channel

$$I_{ion} = g_0\, x^p\, (u - E)$$

where u is the membrane potential. The parameters g_0, p and $E = 0$ are constants.

(a) What is the name of the variable E? ....

Why does it have this name, what does it signify (give answer in one short sentence)

....

....

number of points: ....../ 1

(b) The variable x follows the dynamics

$$\frac{dx}{dt} = -\frac{x - x_0(u)}{\tau}$$

where $x_0(u)$ is monotonically increasing and bounded between zero and one. Suppose that, at $t = 0$, we make a voltage step from a fixed value E to a new constant value u_0. Give the mathematical solution $x(t)$ for $t > 0$.

$x(t) =$ ....

number of points: ....../ 2

Experimental colleagues tell you that they are able to apply voltage steps as in (b) and that by measuring the current they want to determine the parameters g_0 and p of the ion channel in (a) and (b)

(c) How should they proceed to measure the parameter p? What would be different between the case p=1 and $p = 2$? You can sketch a little figure to illustrate your answer.

....
....
....

number of points: ....../ 2

(d) Under the assumption that $x_0(u)$ is bounded between zero and 1, how can they measure g_0?

....
....
....

number of points: ....../ 1

2 Phase Plane Analysis (19 points)

We study two interacting populations. For each population the firing rate is given by a step-like gain function $f(h)$ where h is the input potential and

$f(h) = 10\text{Hz}$ for $h < 0$ and $f(h) = 40\text{Hz}$ for $h \geq 0$.

The interaction between the two populations is given by the equations

$$\tau \frac{dh_1}{dt} = -h_1 + R\,I_1 + w f(h_1) - b f(h_2) \quad (1)$$

$$\tau \frac{dh_2}{dt} = -h_2 + R\,I_2 + w f(h_2) - b f(h_1) \quad (2)$$

The parameters are $w = 0.05\text{mV/Hz}$ and $b = 0.2\text{mV/Hz}$ and $\tau = 5\text{ms}$.

For the first part of the analysis we assume $R\,I_1 = R\,I_2 = 3\text{mV}$

(a) Calculate the nullcline defined implicitly by $dh_2/dt = 0$. Consider the four cases $h_1 < 0$ and $h_1 > 0$ combined with $h_2 < 0$ and $h_2 > 0$. Hint: despite the nonlinearities, you can assume that the solution is unique.

(a1) $h_1 < 0$

....

....

(a2) $h_1 > 0$

....

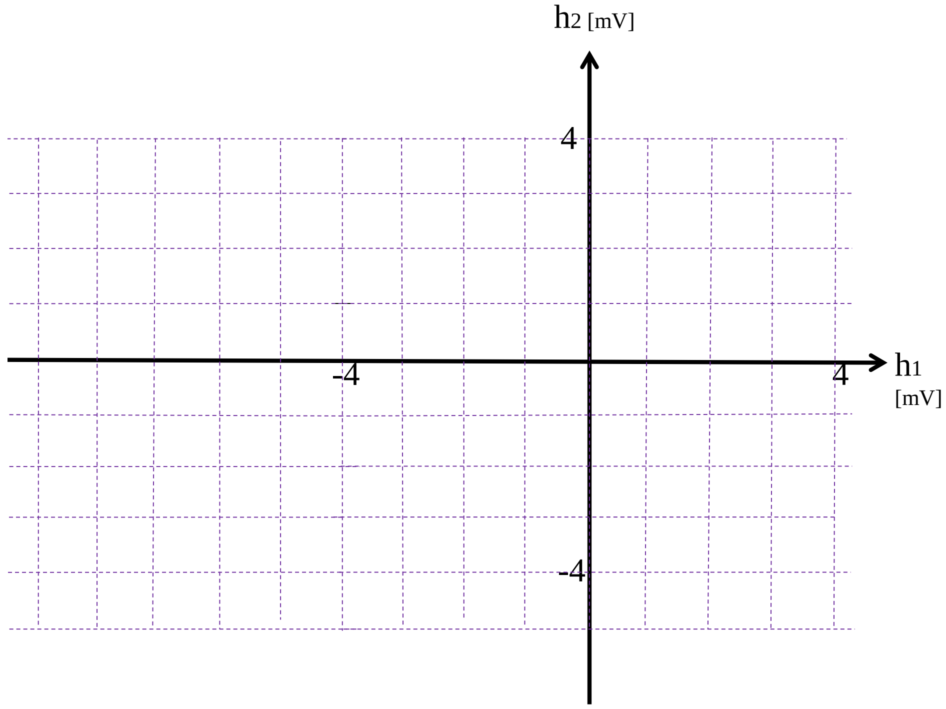
....

number of points: ....../ 2

Free space for your calculations, do not use to write down answers.

(b) **Plot the two nullclines in the figure here. Annotate the nullclines by writing h_1-nullcline and h_2-nullcline.**

number of points: ....../ 3



(c) In the above graph, add an arrow indicating the direction of flow at the point $(h_1, h_2) = (1, -1)$ and at $(h_1, h_2) = (2, -0.001)$.

number of points: ....../ 1

(d) Add, in the above graph, representative qualitative arrows indicating the flow in **at least six different regions** of the phase plane and **on all segments of the nullclines**

number of points: ....../ 4

(e) Add, in the above graph, a trajectory starting at $(h_1, h_2) = (1, -1)$.

number of points: ....../ 1

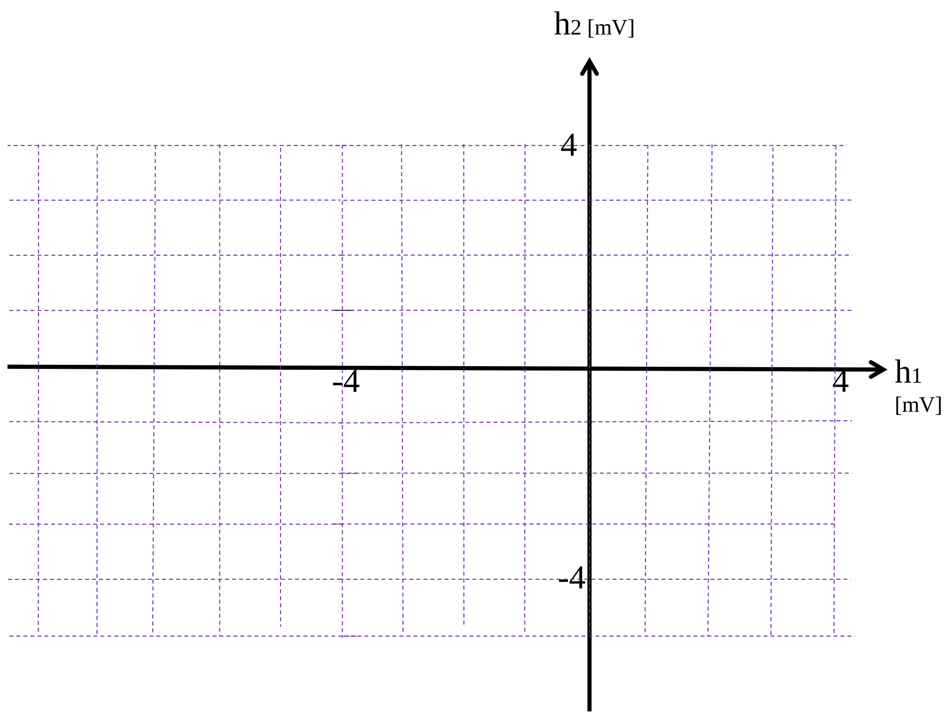
(f) Add, in the above graph, a trajectory starting at $(h_1, h_2) = (-5, 5)$.

number of points: ....../ 1

Free space for your calculations, do not use to write down solutions/answers.

(g) Repeat steps (b) and (d), but consider now $R\,I_1 = -1\text{mV}$, whereas you keep $R\,I_2 = +3\text{mV}$ as before. Plot your result here:

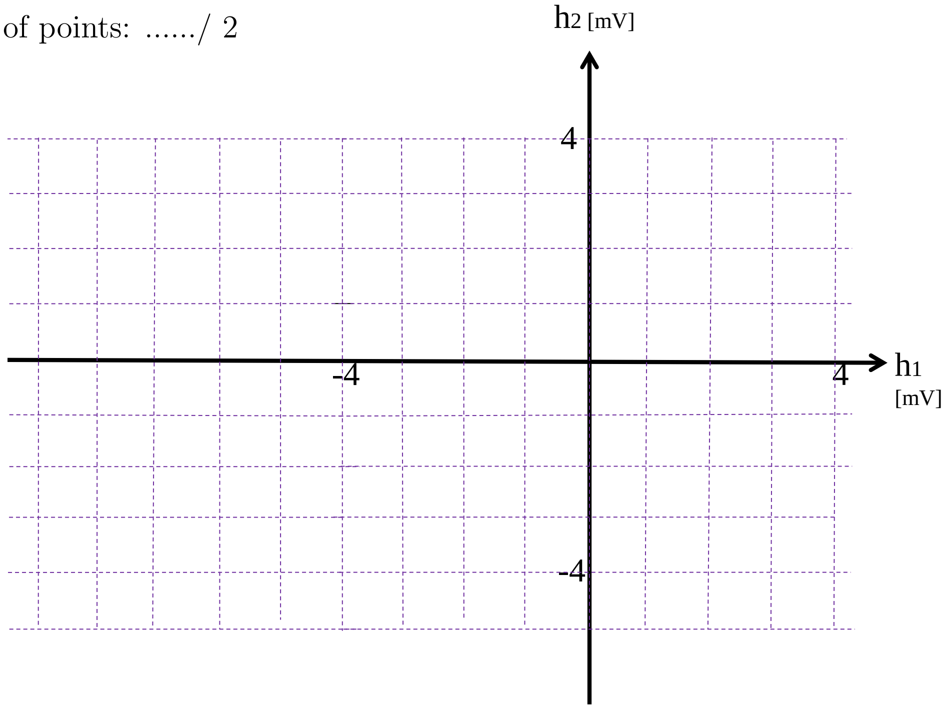
number of points: ....../ 2



Free space for your calculations, do not use to write down solutions/answers.

(h) Repeat steps (b) and (d), but consider now $R\,I_1 = -1\text{mV}$, and $R\,I_2 = -1\text{mV}$.
Plot your result here:

number of points: ....../ 2



(i) Relate your results to models of decision making:

The scenario in (h) corresponds to

....

....

The scenario in (g) corresponds to

....

....

The scenario in (b) corresponds to

....

....

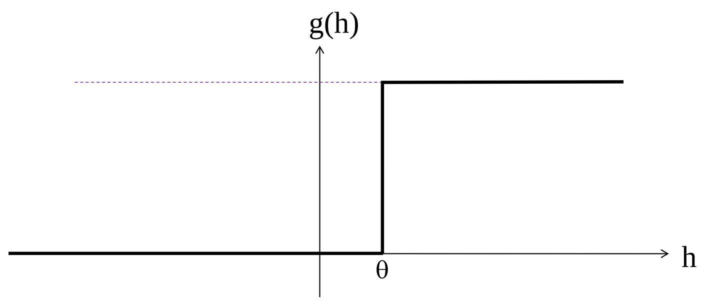
number of points: ....../ 3

3 Nonlinear self-coupled population and Analysis of 1-dim differential Equations (8 points)

We consider a population of neurons interacting with itself. The population dynamics is described by the rate equation:

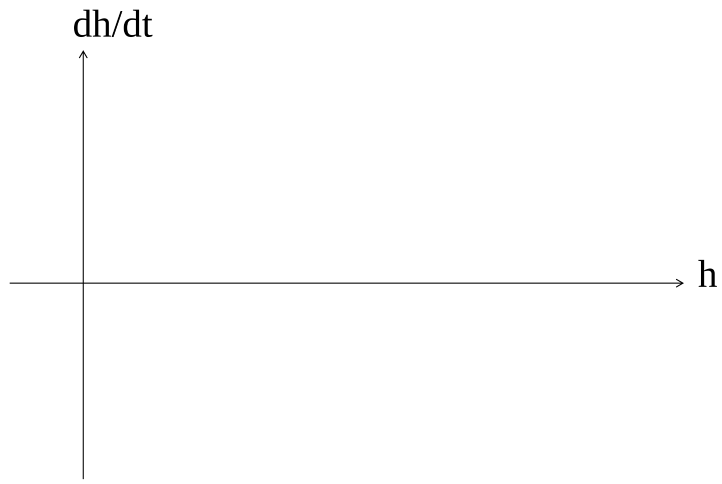
$$\tau \frac{d}{dt} h = -h + \tau \beta g(h) + R\, I_0 \tag{3}$$

where β is a parameter and $g(h)$ the firing rate, given by a step function with $g(h) = 1/\tau$ for $h > \theta$ and zero otherwise (see Figure).



(a) Assume $R\, I_0$ =0 and $\beta > \theta$. Draw dh/dt as a function of h in the space below.

number of points: ....../ 2

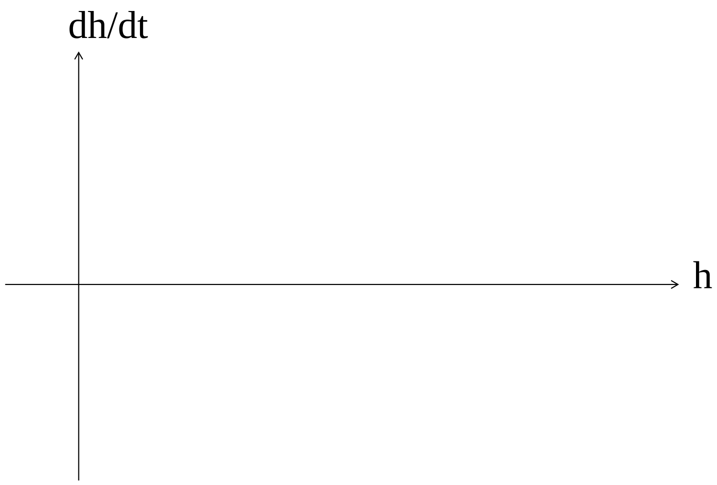


(b) Draw flow arrows for the one-dimensional system in your figure above. If there is a stable fixed point or several ones, mark the fixed point(s) with a letter 's'.

number of points: ....../ 2

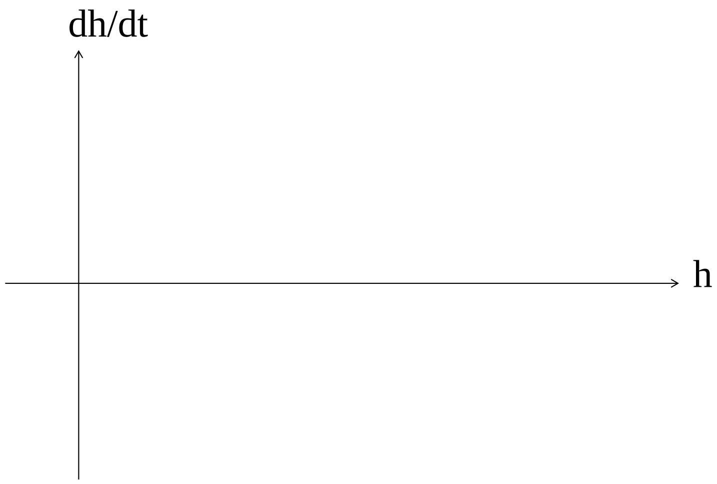
(c) Repeat the same steps as in (a) and (b) for the case $R\,I_0$ =0 and $\beta < \theta$ in the space below:

number of points: ....../ 1



(d) Repeat the same steps as in (a) and (b) for the case $R\,I_0 > 0$ and $\beta > \theta$ in the space below:

number of points: ....../ 1



(e) What is the minimal value of the input current so that the fixed point pattern in (d) is different from that in (a)?

....

Interpret your answer in terms of the neurons interacting in the population described by Eq. (3).

....

number of points: ....../ 2

4 Stochastic Spike Arrivals and Spiking Probability (11 points, includes 3 bonus points)

There are certain types of neurons that communicate with 'doublets' of spikes: Whenever the firing threshold is crossed, the neuron emits two spikes, at a fixed interval t_0. (Later we will use t_0=5ms).

Suppose that **each of the two spikes of the doublet** causes a postsynaptic depolarization of standard exponential shape:

(i) Upon spike arrival the membrane potential of the postsynaptic neuron jumps by an amount b (later we will use b=1mV).

(ii) Thereafter it decays with a time constant τ_m. (Later we will use $\tau_m = 10$ms.)

Consider a postsynaptic neuron that receives input from K presynaptic neurons where each of the presynaptic neuron fires doublets of spikes. Suppose that, in each of the K neurons, a **spike doublet occurs stochastically at a constant rate** ν_0. (Later we consider ν_0=1Hz.)

The total input potential of the postsynaptic neuron is

$$u(t) = \sum_{k=1}^{K} \sum_{f} \alpha(t - t_k^f) \tag{4}$$

where α is defined above by the sentences explaining the shape of the depolarization and t_k^f denotes the spike arrival times.

(a) Determine the mean input potential of the postsynaptic neuron. (Assuming K presynaptic neurons, each one generating spike doublets at rate ν_0.) Express your result as an explicit function of the parameters t_0, b, τ_m, ν_0, K; no integral signs should be left.

$< u >=$

....

number of points: ....../ 2

(b1) Evaluate the mean input potential for $t_0 = 5ms; b = 1mV; \tau_m = 10ms; K = 1000; \nu_0 = 1Hz$. Pay attention to the units

$< u >=$ ....

number of points: ....../ 1

(b2) Knowing that the typical distance between resting potential and threshold is in the range of 20-30 mV, do you expect the neuron to fire *regularly* when driven with the input spikes as defined above?

Yes/No because ....

number of points: ....../ 1

(c) How would the mean input potential change if instead of each neuron firing doublets at rate ν_0, it would fire single spikes at rate $\nu = 2\nu_0$?

....

Give an intuitive or mathematical reason.

....

number of points: ....../ 2

(d) How would the standard deviation of the input potential change if instead of each neuron firing doublets at rate ν_0, it would fire single spikes at rate $\nu = 2\nu_0$?

....

Give an intuitive or mathematical reason. You may want to consider the limiting cases $t_0 \to 0$ and $t_0 \gg \tau_m$.

....

number of points: ....../ 2

(e) **BONUS. Only start if you are done with the rest of the exam.**

(e1) Determine mathematically the variance of the input potential of the postsynaptic neuron. Express your result as a function of the parameters t_0, b, τ_m, ν_0, K; no integral signs should be left.

$< (\Delta u)^2 >=$ ....

number of points: ....../ 2

(e2) Using your results of (a) and (e), evaluate the standard deviation of the membrane potential for $t_0 = 5ms; b = 1mV; \tau_m = 10ms; K = 1000; \nu_0 = 1Hz$. Pay attention to the units.

$< (\Delta u(t))^2 >^{0.5}=$ ....

number of points: ....../ 1

5 Mean-field models and separation of time scale (8 points)

We consider two populations of neurons. The first population is excitatory and has a population firing rate r.

The second population consists of N fully connected inhibitory spiking neurons that are connected with each other with identical negative weights characterized by a parameter $w_{II} < 0$. Each neuron also receives rate input from the first population characterized by a weight parameter $w_{IE} > 0$.

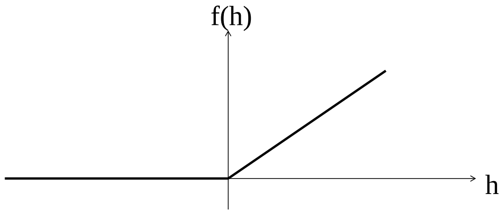
The membrane potential of a neuron i in the inhibitory population is given by

$$\tau_I \frac{d}{dt} h_i = -h_i + \tau\, w_{IE}\, r + \tau \frac{1}{N} w_{II} \sum_{k=1}^{N} \sum_{n} \delta(t - t_k^n) \tag{5}$$

where t_k^n is the nth firing time of neuron k. The parameter τ has units of time. Given the membrane potential h_k, neuron k fires stochastically (inhomogeneous Poisson process) with stochastic intensity

$$\rho_k(t) = f(h_k(t)) \tag{6}$$

where $f(h) = h/\tau$ for $h > 0$ and $f(h) = 0$ otherwise (see Figure).



(a) Rewrite the right-hand-side of Eq. (5) in terms of the population activity $A(t)$.

$\tau_I \frac{d}{dt} h_i =$

....

number of points: ....../ 1

(b) Consider two different neurons of the inhibitory population, say $i = 17$ and $i = 966$. Do they have the same or different values for the membrane potential? Give a reason:

The same/different because

....

....

number of points: ....../ 1

(c) Now we consider the limit $N \to \infty$. Use Eq. (6) as well as your result from (a) and write down a differential equation for the population activity $A(t)$. You may assume $h_i > 0$ for all i.

$\frac{d}{dt}A(t) =$

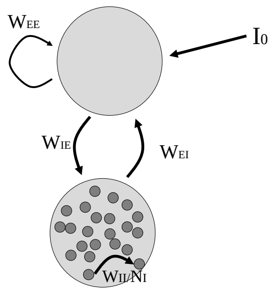
....

number of points: ....../ 2

Space for your calculations. Do not write any answers here.

Exam questions continue on the next page

(d) The excitatory population receives input spikes from all neurons of the inhibitory population as well as external input I_0 with input resistance R.



The membrane potential of the excitatory population is given by

$$\tau_E \frac{d}{dt} h_E = -h_E + R\, I_0 + \tau_E w_{EE} g(h_E) + \tau_E \frac{1}{N} w_{EI} \sum_{k=1}^{N} \sum_{n} \delta(t - t_k^n) \tag{7}$$

and its firing rate by $r = g(h_E)$.

Assume a separation of time scales $\tau_I \ll \tau_E$. Use your results from (c) and show that the two populations can be described by a one-dimensional differential equation

$$\tau_E \frac{d}{dt} h_E = -h_E + \tau_E\, \beta\, g(h_E) + h_0 \tag{8}$$

(d1) What is the value of the parameter β ?

β= ....

number of points: ....../ 3

(d2) What is the value of the parameter h_0?

h_0= ....

number of points: ....../ 1

(space for your calculations)