

# Computational Neuroscience: Neuronal Dynamics of Cognition



## Continuum models: Cortical fields and perception

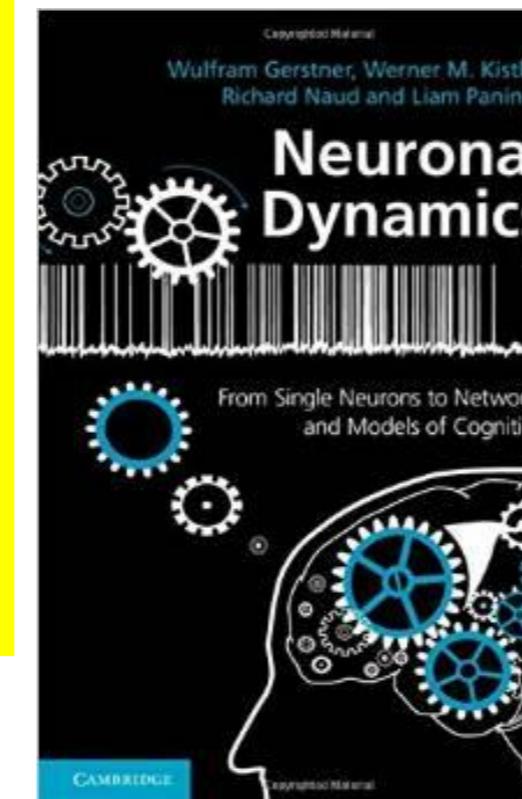
Wulfram Gerstner

EPFL, Lausanne, Switzerland

*Reading:*  
**NEURONAL DYNAMICS**

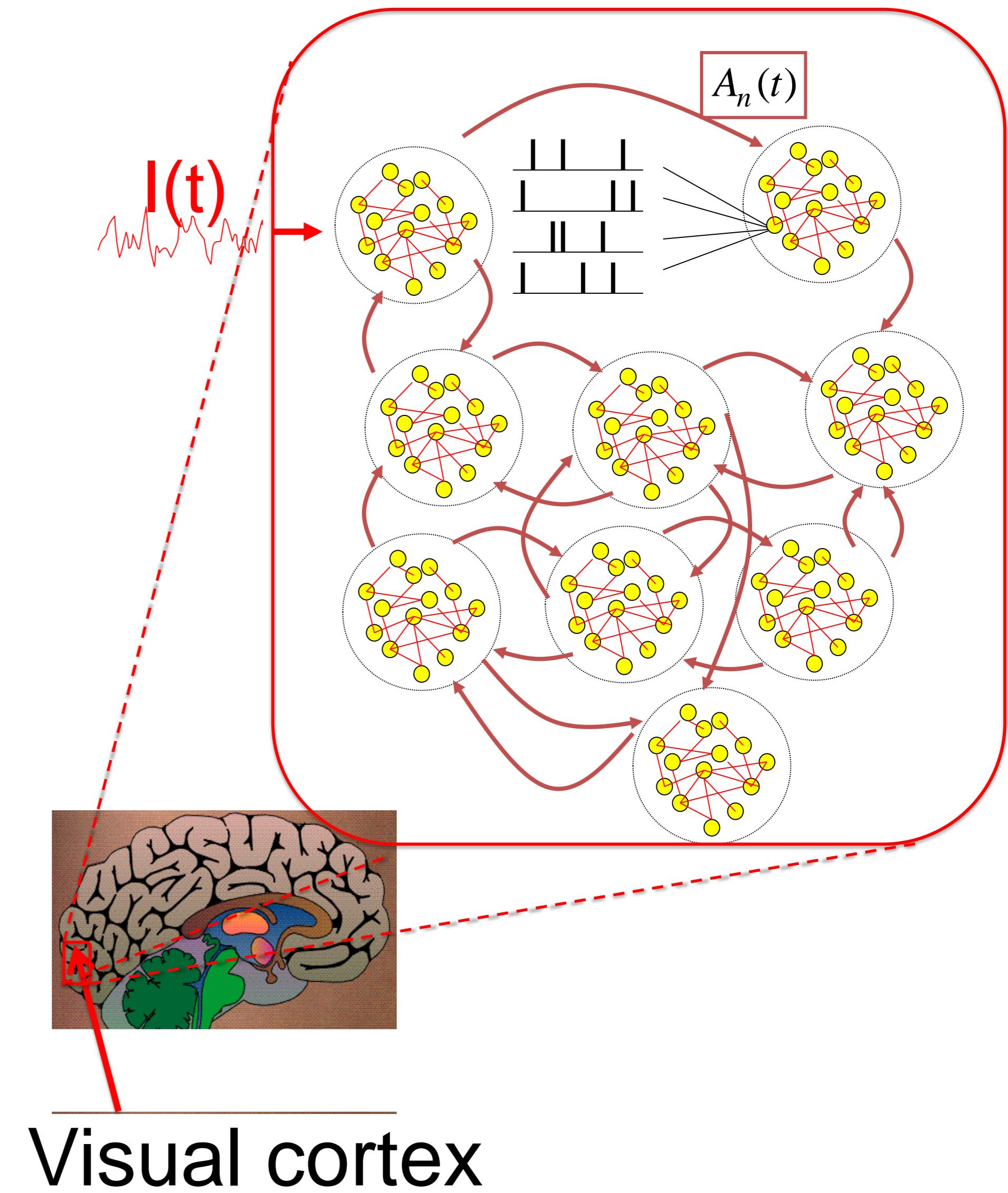
Ch. 18 +  
+Ch. 12.3.7+Ch 15.1-15.2.3

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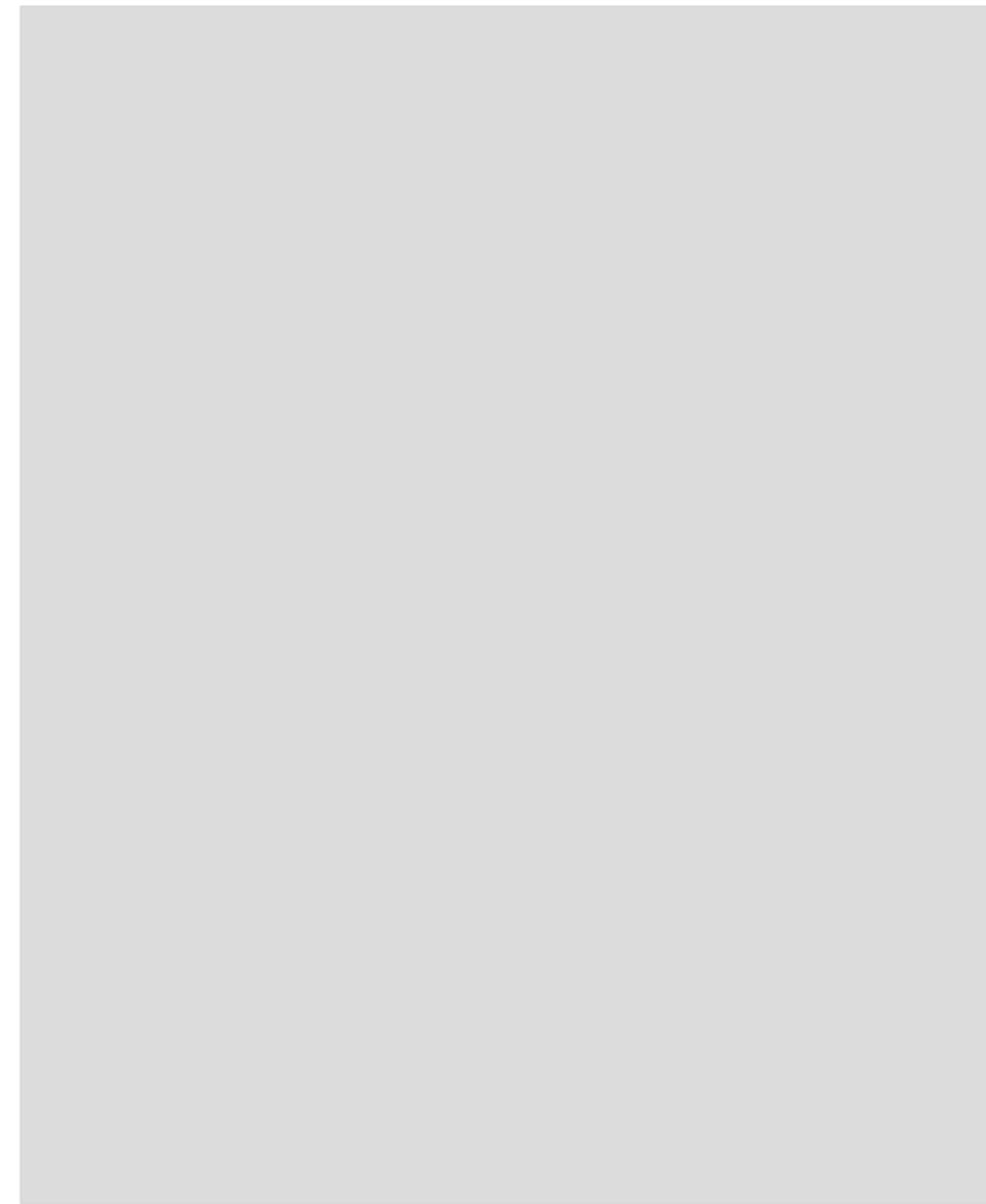
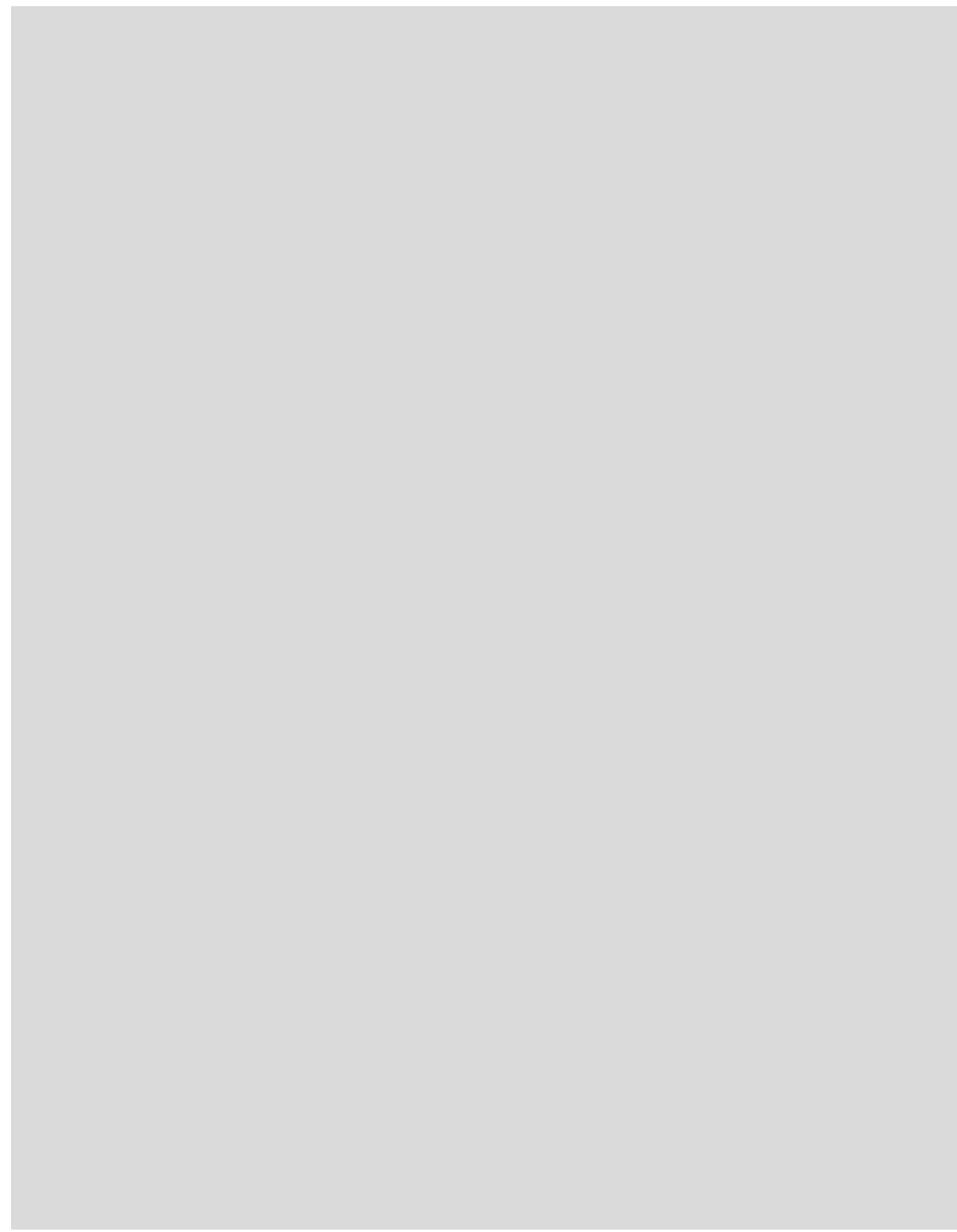
- 1. Aims and challenges**
  - review: mean-field arguments
- 2. Transients**
  - generalized integrate-and-fire model
  - transients can be sharp or slow
- 3. Spatial continuum (cortex)**
  - orientation columns
- 4. Spatial continuum (model)**
  - field equations
- 5. Solution types**
  - uniform solution
  - bump solution
- 6. Perception**
- 7. Head direction cells**

# 1. Aims and Challenges



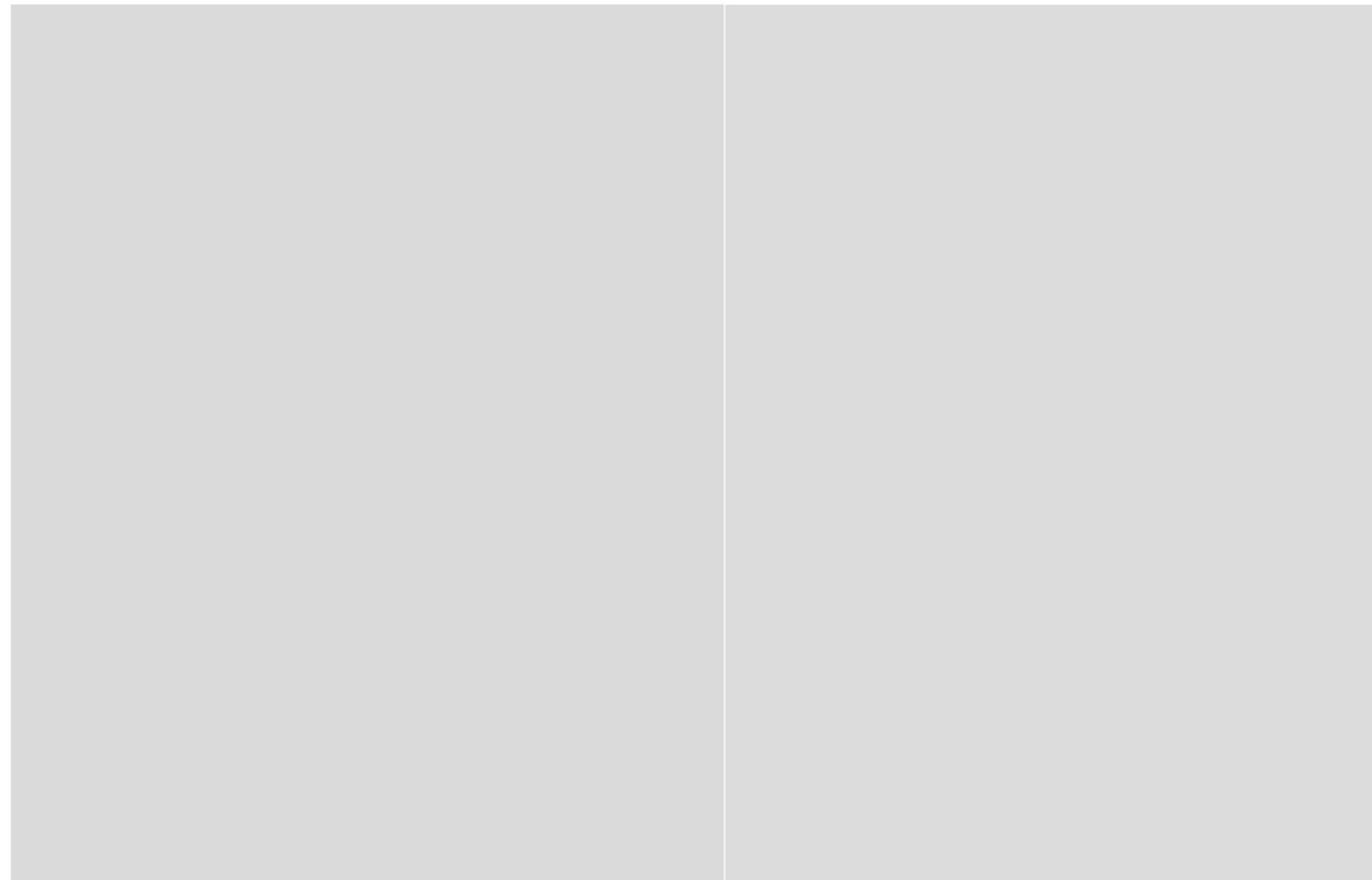
# 1. Aims and Challenges: Visual Perception

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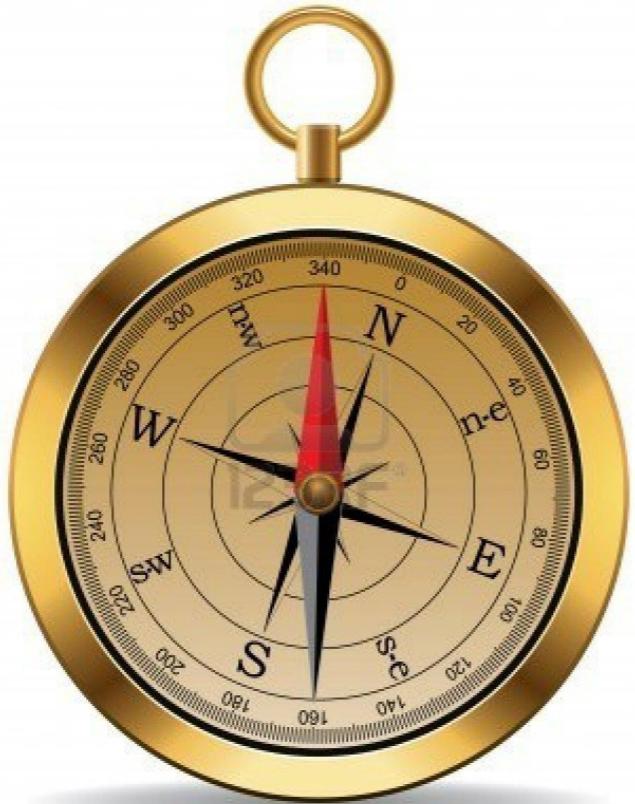


Visual Perception  
→weak contrasts  
→world is continuous

# 1. Aims and Challenges: sense of direction

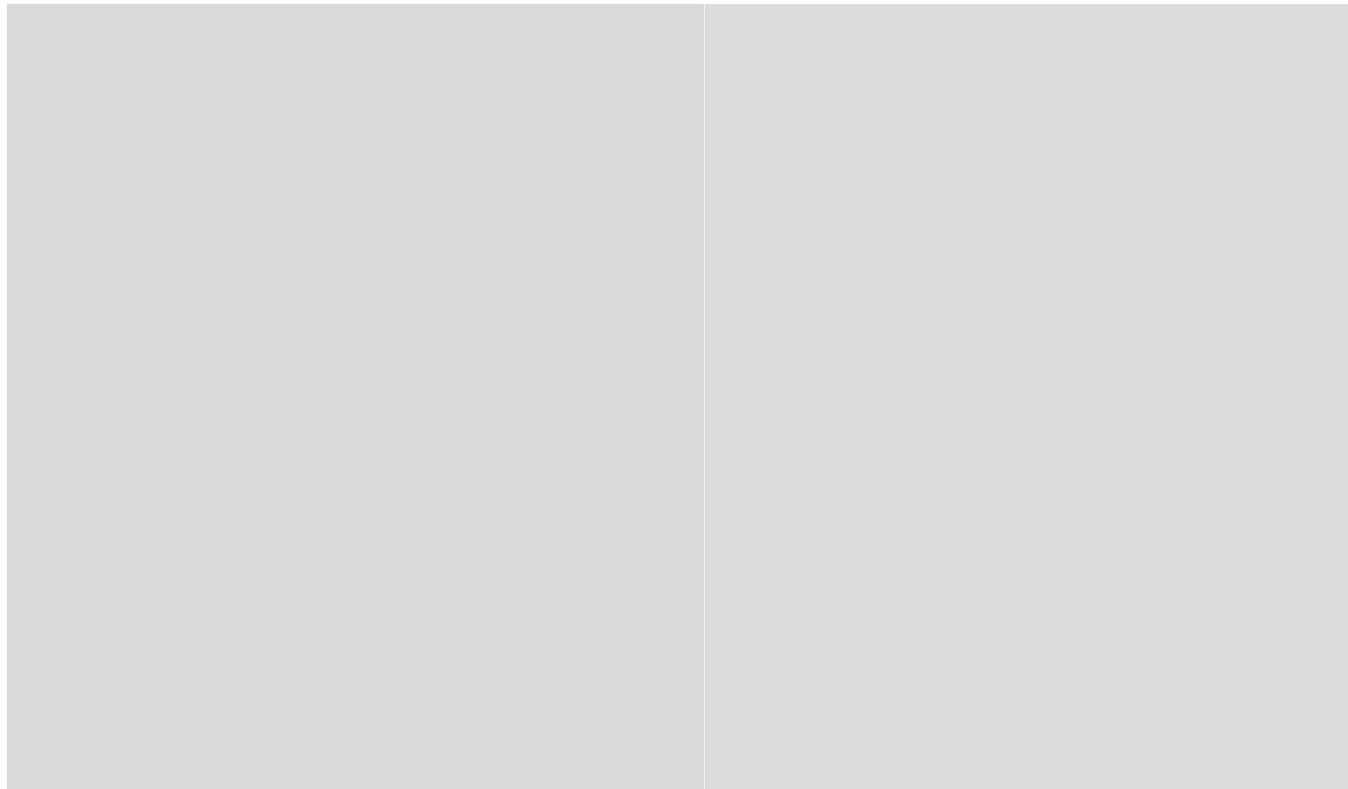
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Sense of direction  
→ internal compass

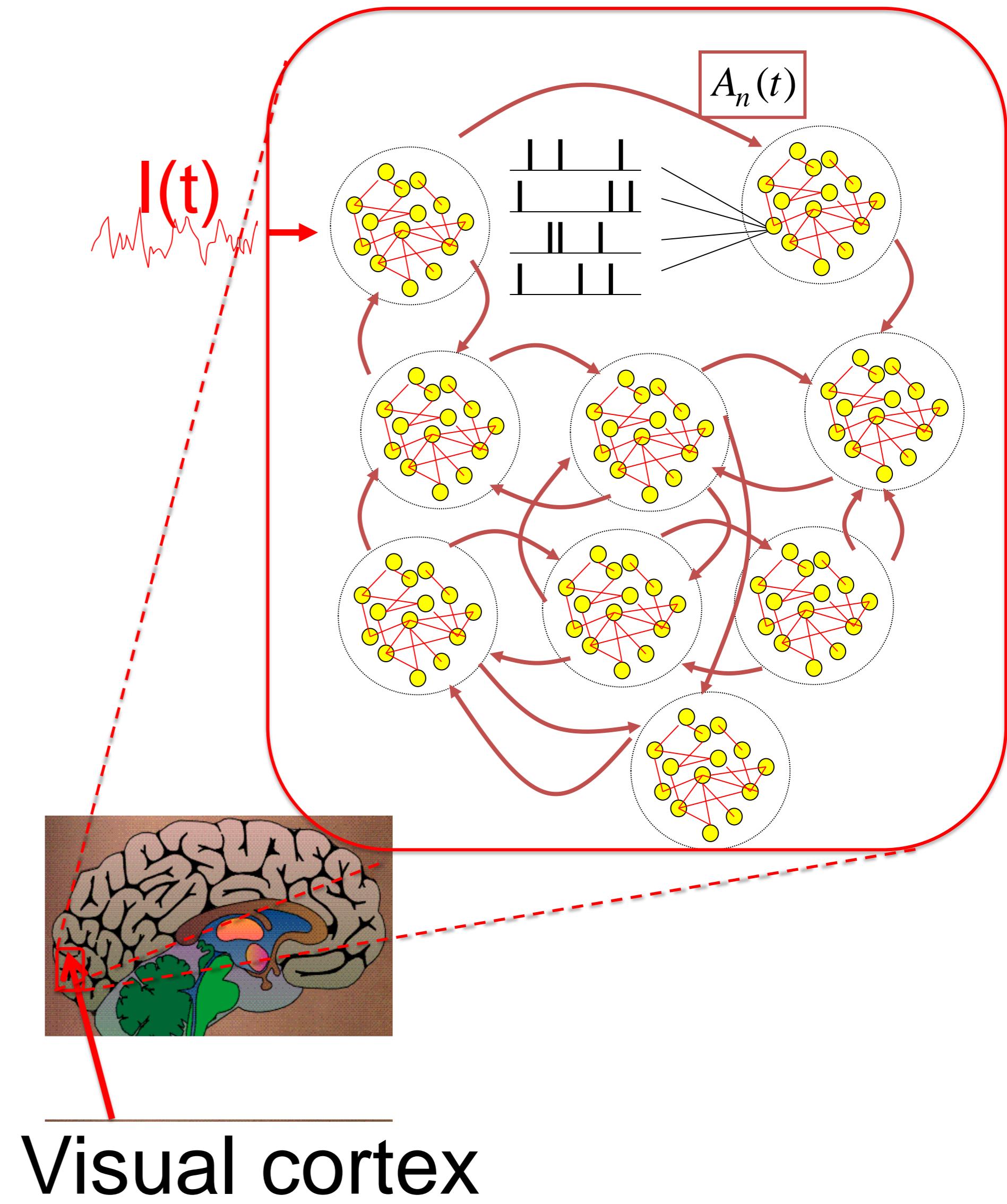


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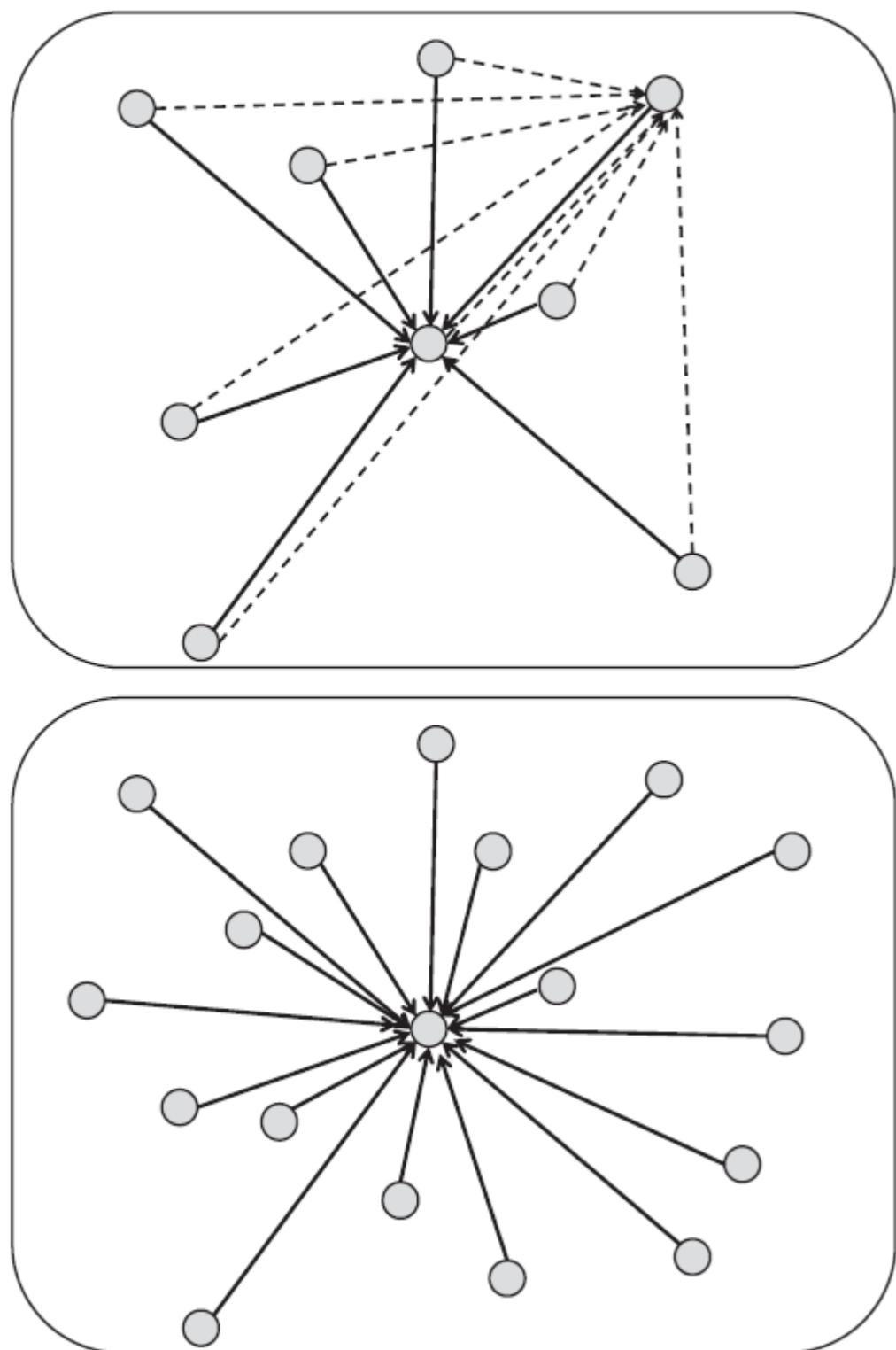
Sense of direction  
→ internal compass



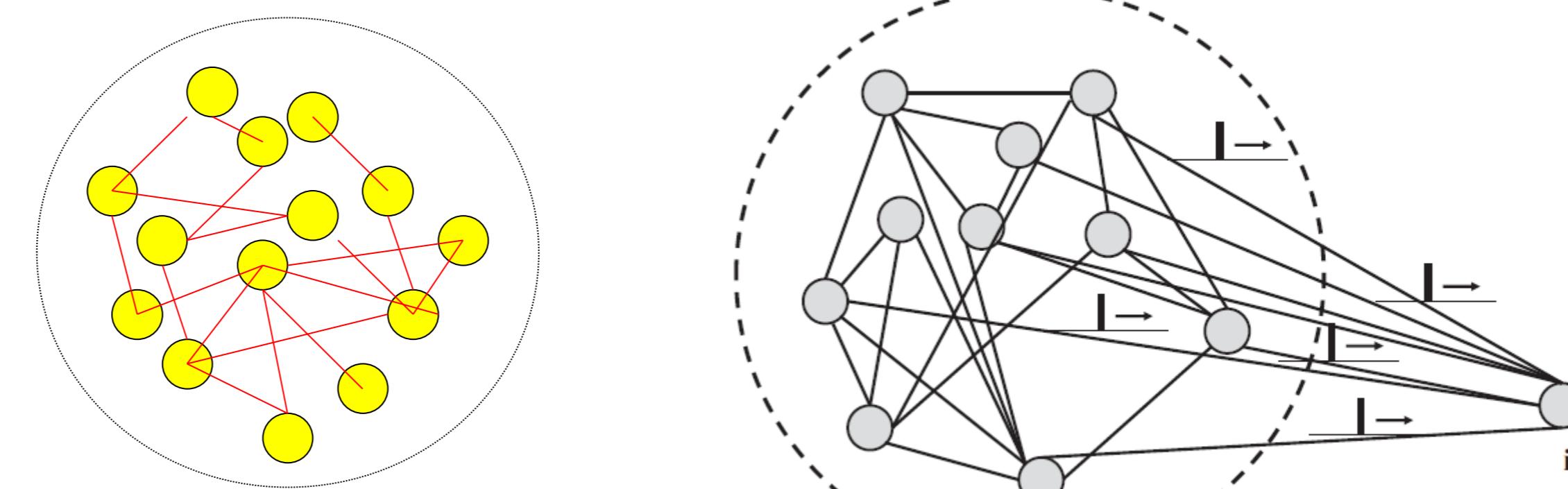
Visual Perception  
→ weak contrasts  
→ world is continuous



## 1. review: mean-field arguments



# Single population full connectivity



All neurons receive the same  
total input current ('mean field')

# 1. Review: mean-field arguments

All neurons receive the same total input current ('mean field')

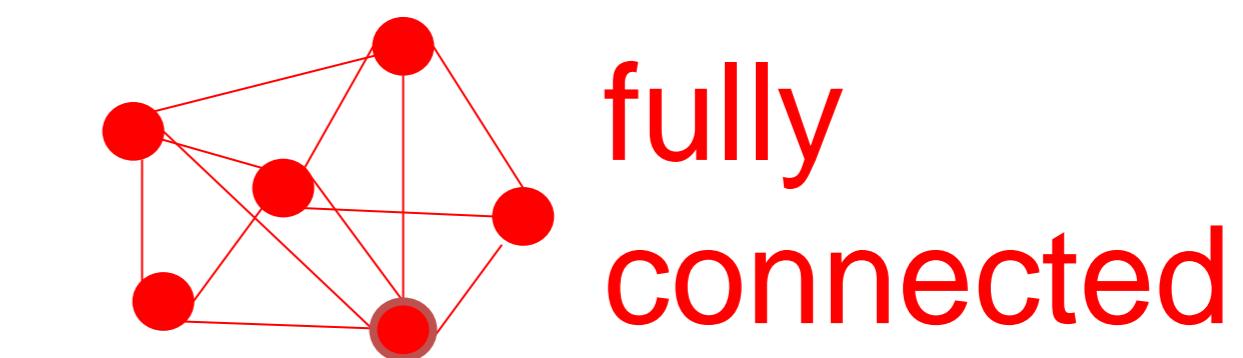
$$I(t) = J_0 q A(t) + I^{ext}(t)$$

$$I_i(t) = J_0 \int \alpha(s) \underbrace{A(t-s) ds}_{\text{Ultra-short current pulse}} + I^{ext}(t)$$

index  $i$  disappears

$$w_{ij} = \frac{J_0}{N}$$

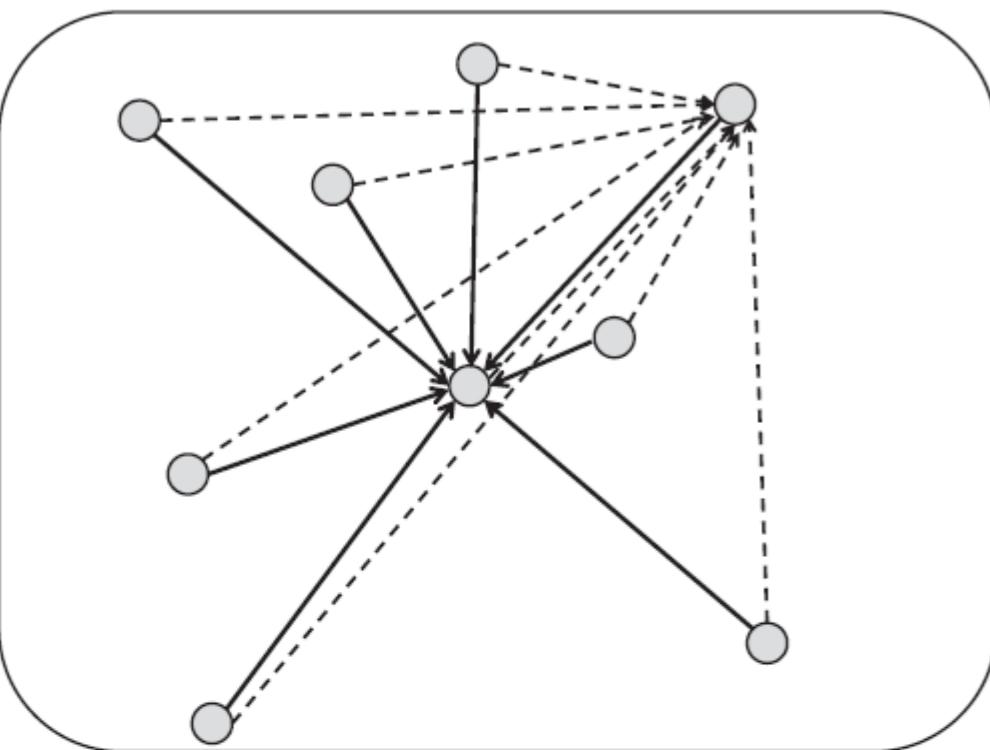
$$I_i^{net}(t) = \sum_j \sum_f w_{ij} \underbrace{\alpha(t - t_j^f)}_{\text{All spikes, all neurons}} + I^{ext}$$



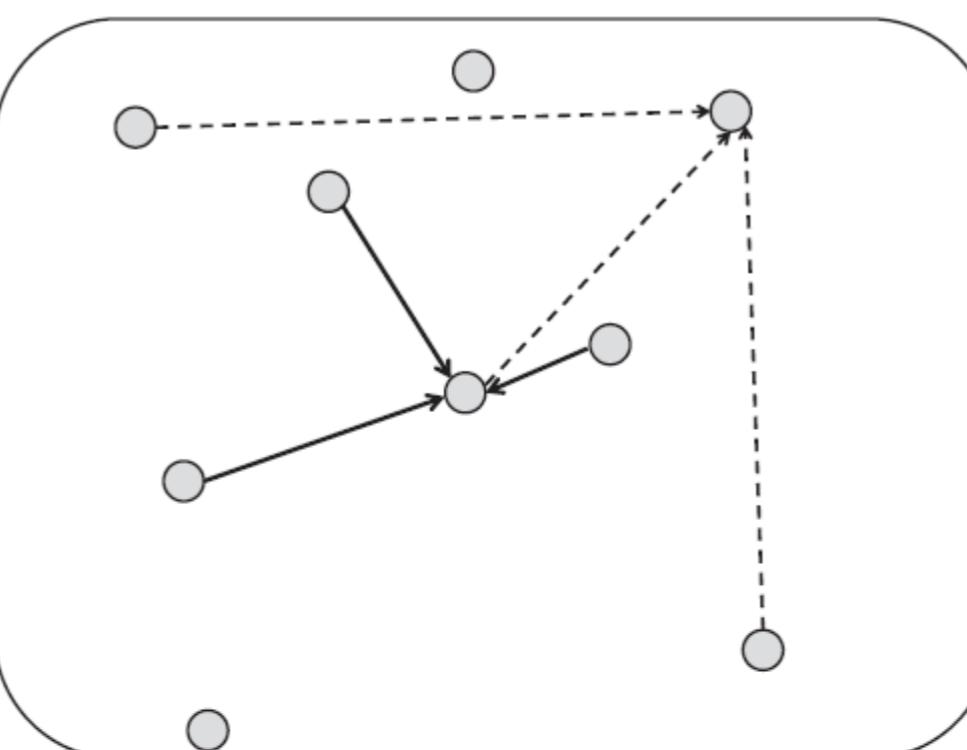
fully connected

# 1. Review: mean-field also works for random coupling

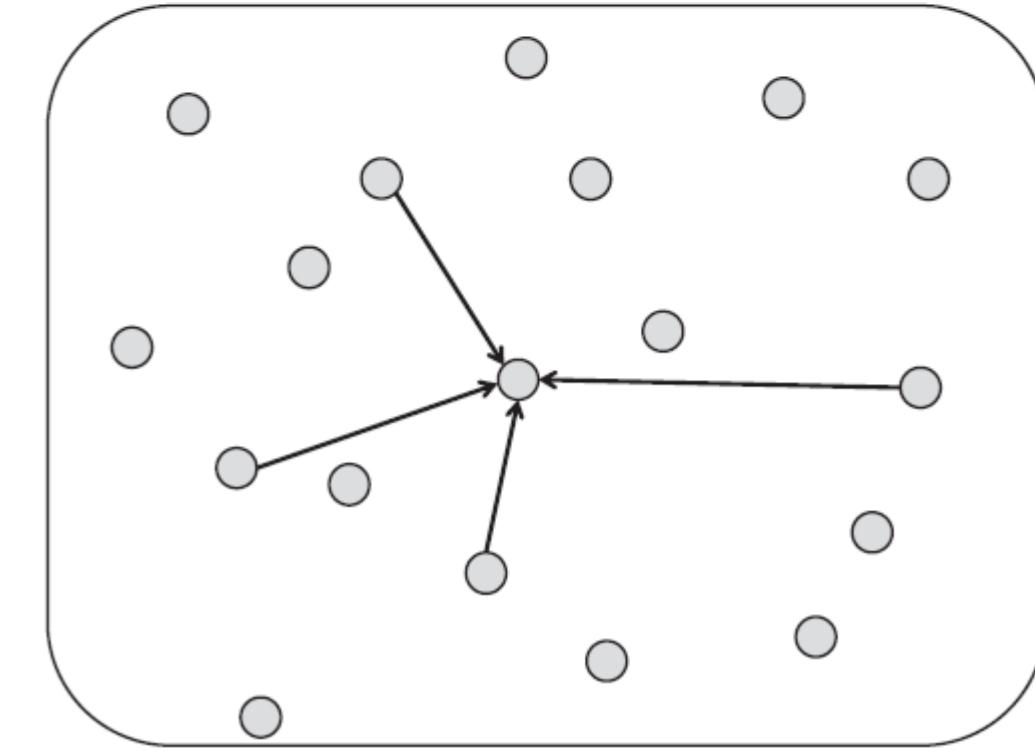
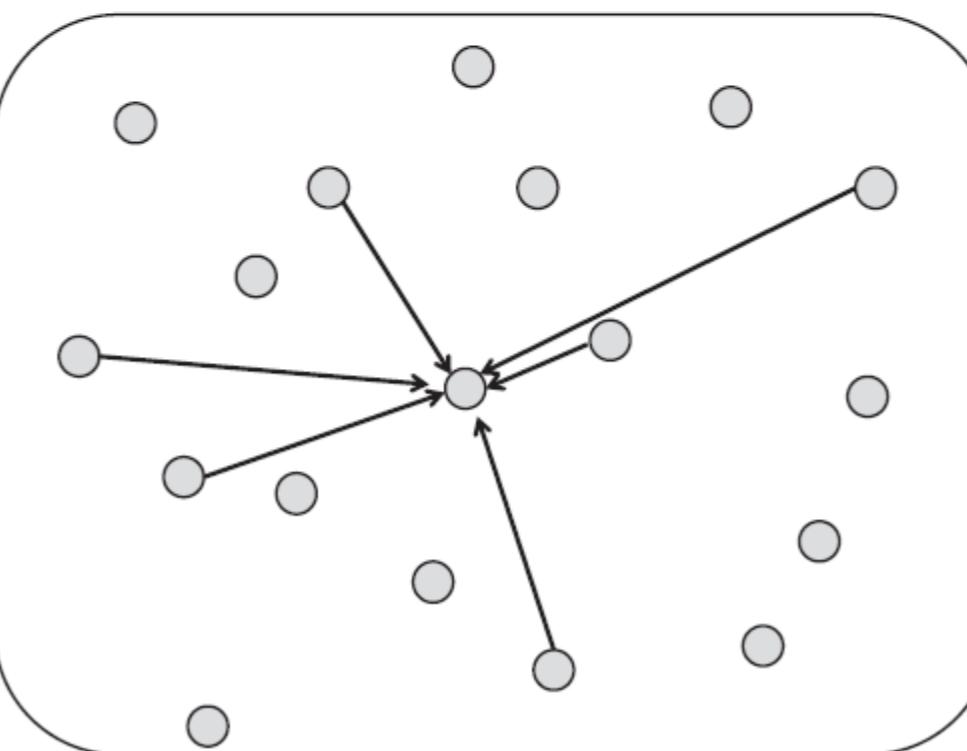
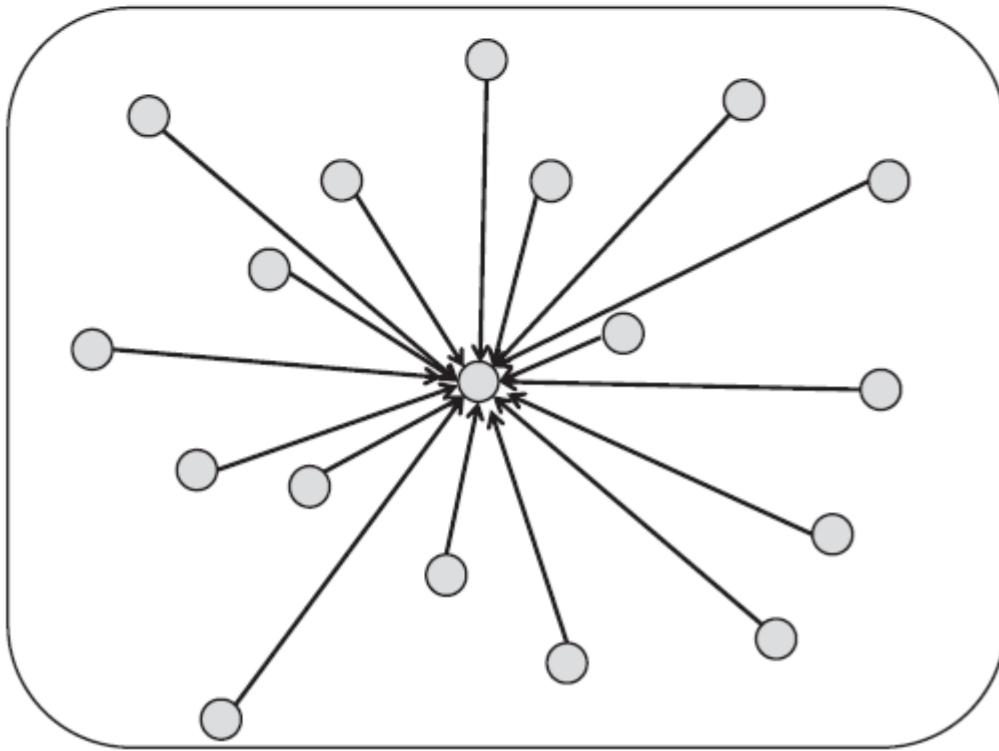
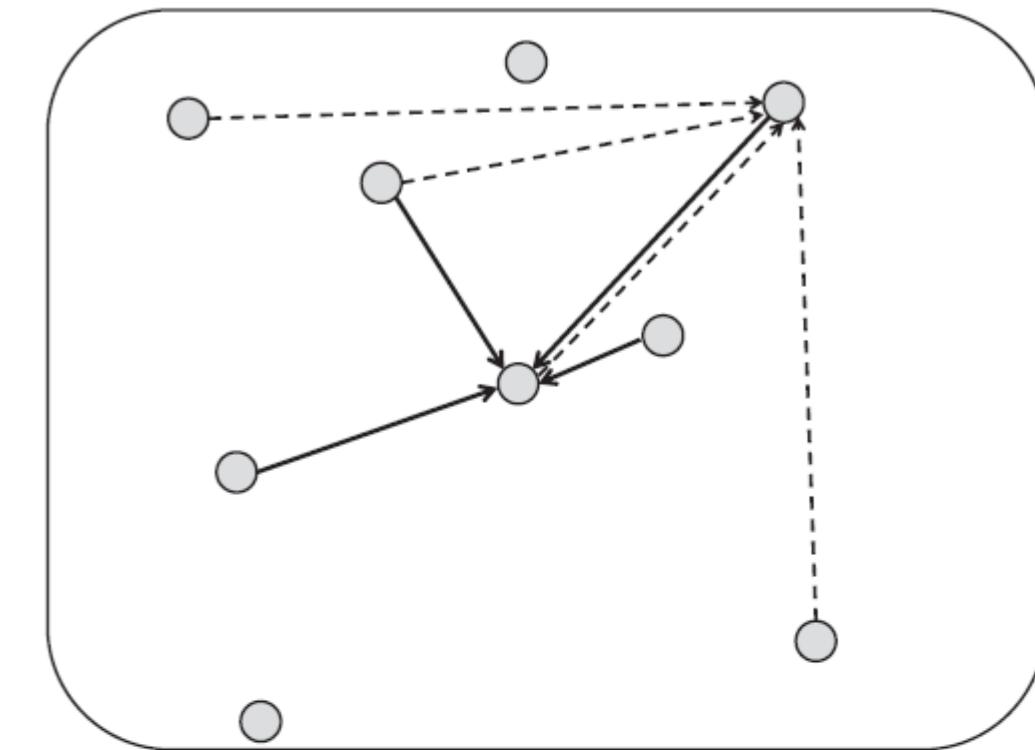
full connectivity



random: prob  $p$  fixed



random: number  $K$  of inputs fixed



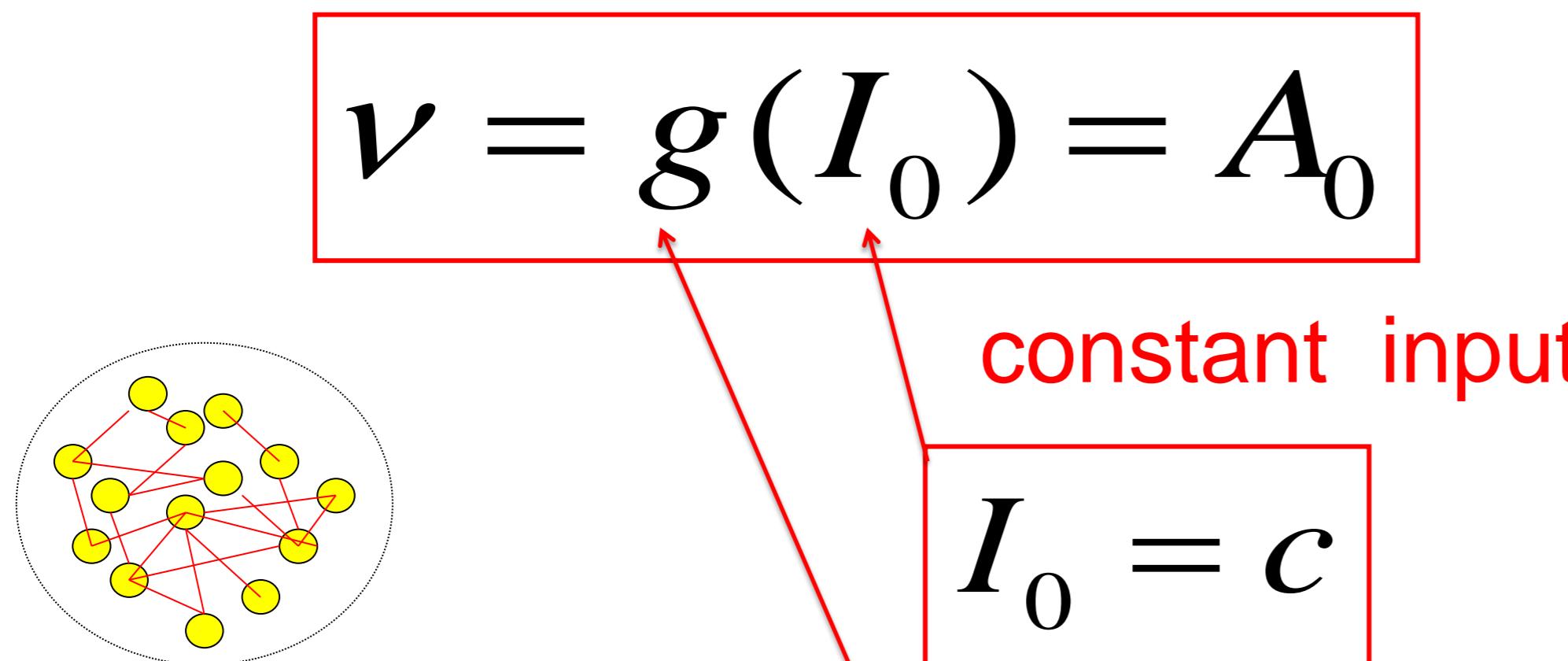
*Image: Gerstner et al.  
Neuronal Dynamics (2014)*

# 1. Review: stationary state/asynchronous activity

Homogeneous network

All neurons are identical,

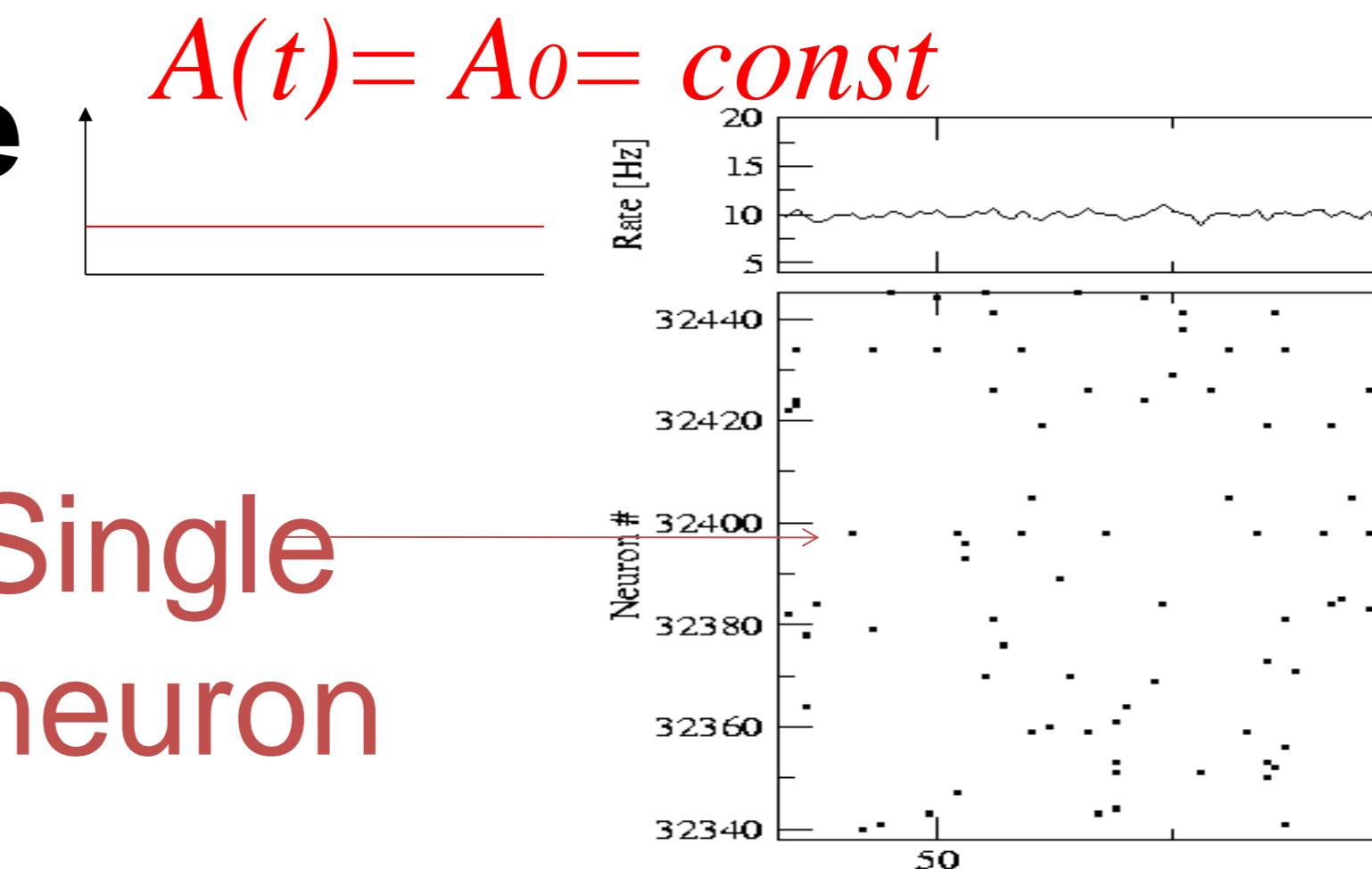
**Single neuron rate = population rate**



Gain function at appropriate noise level

frequency (single neuron)  $\nu = 1/\langle s \rangle$

rate=1/(meanInterval)



# 1. Review : mean-field arguments for homogeneous population

- single neuron is driven by the ‘population activity’ of all others
- all neurons in populations receive the same input
- mean-field arguments work for fully connected and randomly connected populations
- in the **stationary** state, the single neuron firing rate is equal to the ‘population activity’ of a homogeneous population
- in the **stationary** state, ‘population activity’ can be predicted by
  - (i) single neuron gain function (f-I curve)
  - (ii) external input
  - (iii) intra-population coupling strength
- in the **stationary** state, choice of neuron model irrelevant (apart from gain function/f-I curve)

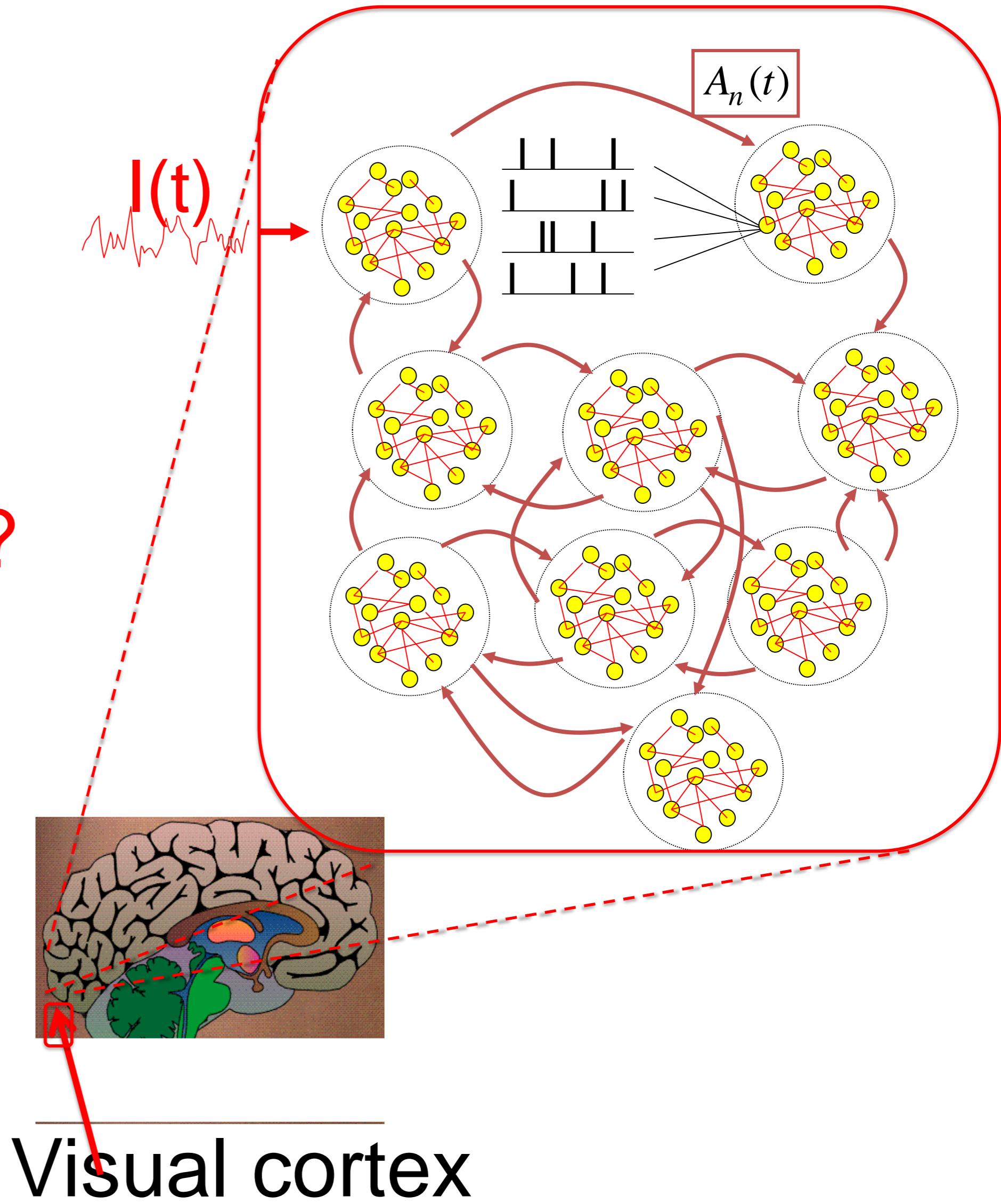
# 1. Aims and challenges

## Mathematical aims:

- beyond stationary states  
→ **transients?**
- more than one population  
→ **how many? continuum?**

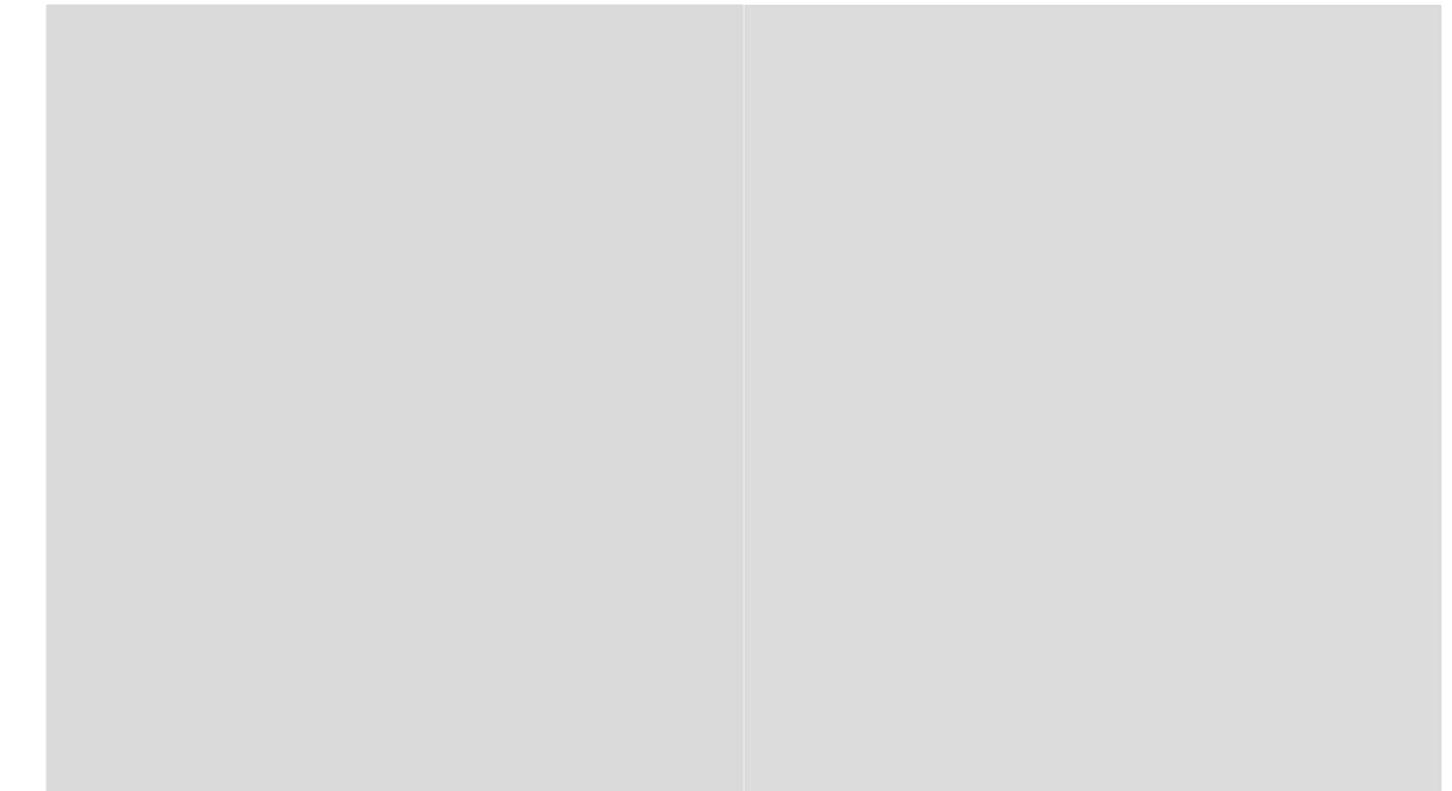
## Cognitive Modeling aims:

- functional consequences  
→ **visual perception?**  
→ **sense of direction?**



# 1. Aims and challenges: compass and perception

Sense of direction  
→ internal compass



Visual Perception  
→ weak contrasts  
→ world is continuous

# Computational Neuroscience: Neuronal Dynamics of Cognition



**Continuum models:**

**Cortical fields and perception**

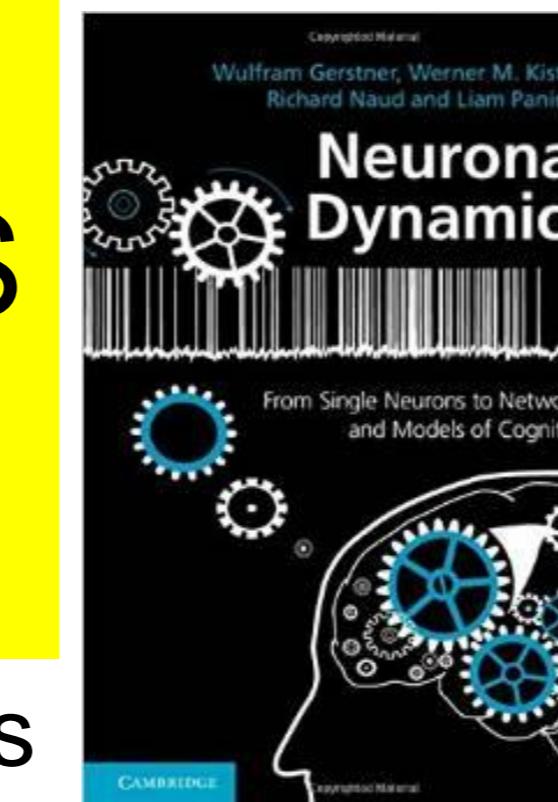
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## 1. Aims and challenges

- review: mean-field arguments

## 2. Transients

- generalized integrate-and-fire model
- transients can be sharp or slow

## 3. Spatial continuum (cortex)

- orientation columns

## 4. Spatial continuum (model)

- field equations

## 5. Solution types

- uniform solution
- bump solution

## 6. Perception

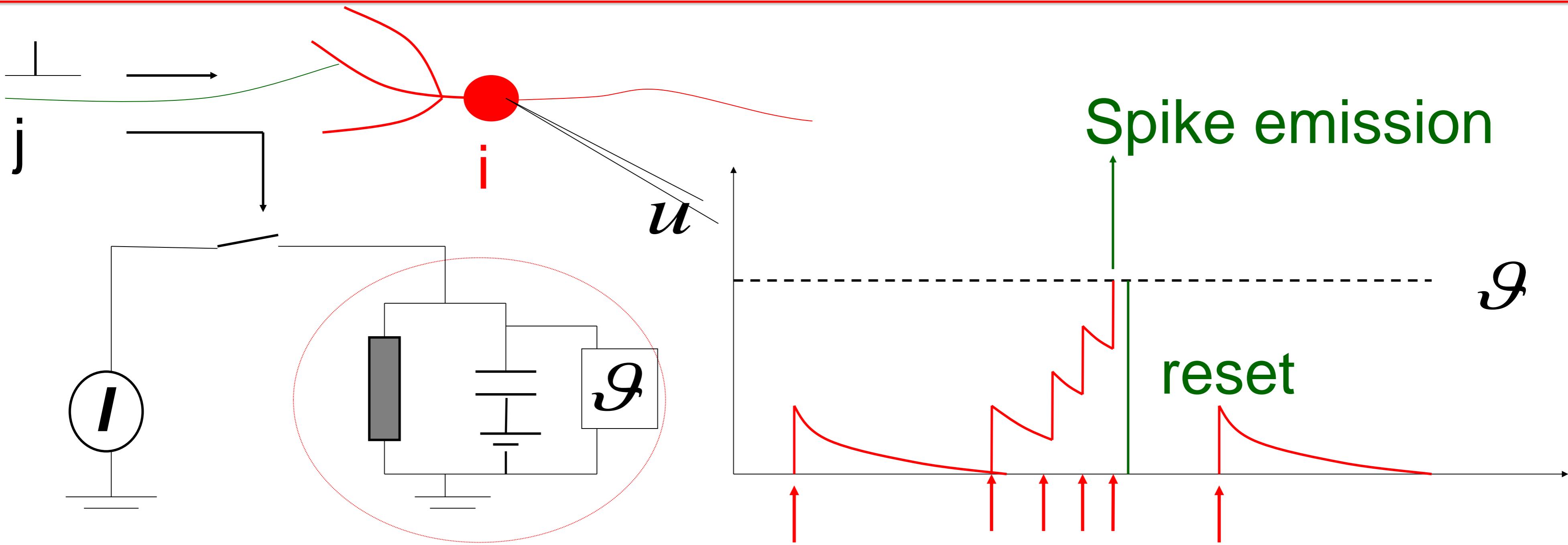
## 7. Head direction cells

## 2. Aims of this section: Transients

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- beyond stationary states  
→ transients?
- but then neuron model matters!  
→ introduce **generalized integrate-and-fire models**:
  - Spike Response Model (SRM)
  - Generalized Linear Model (GLM)

## 2. Leaky Integrate-and-Fire Model



$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

linear

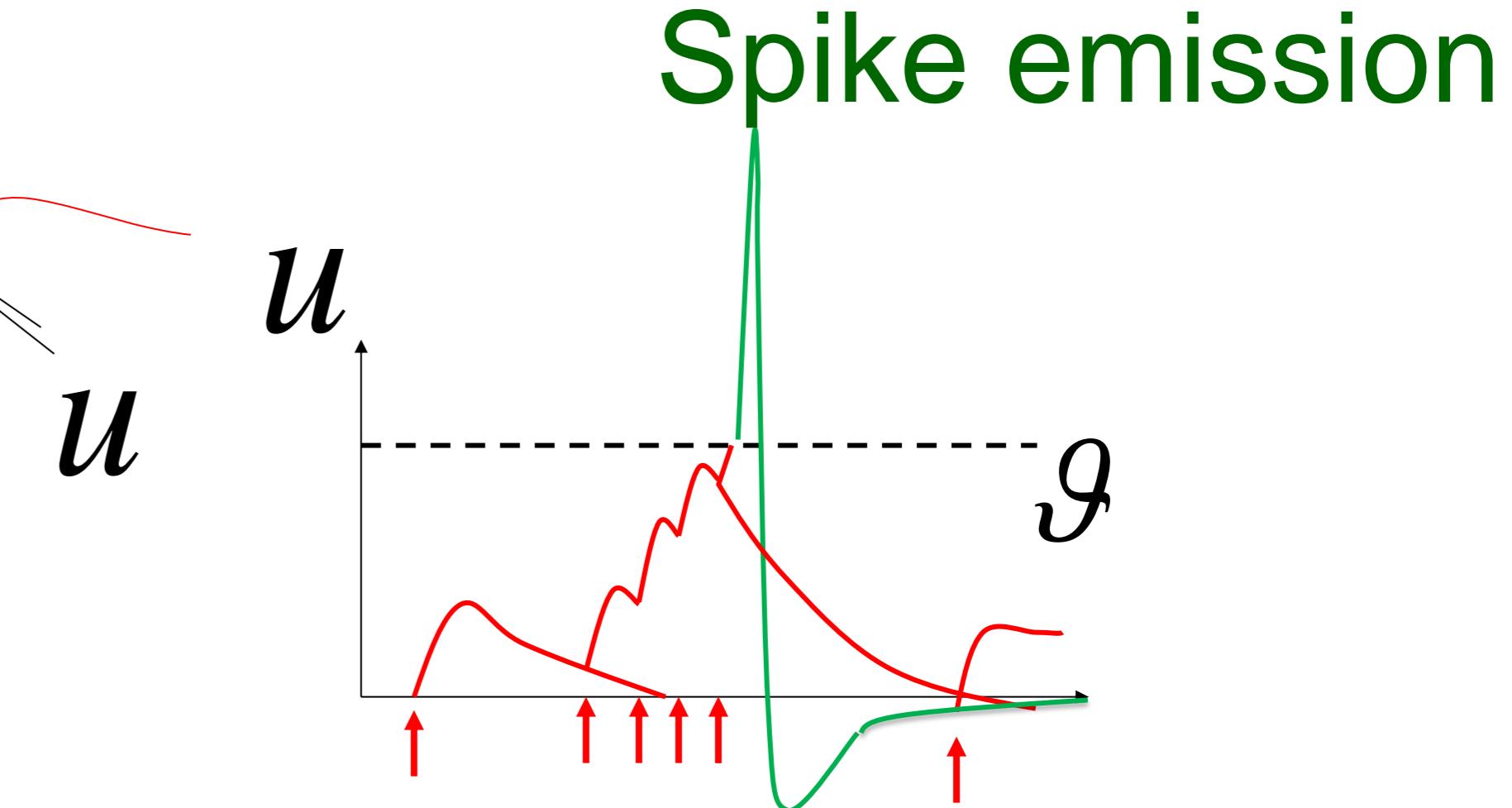
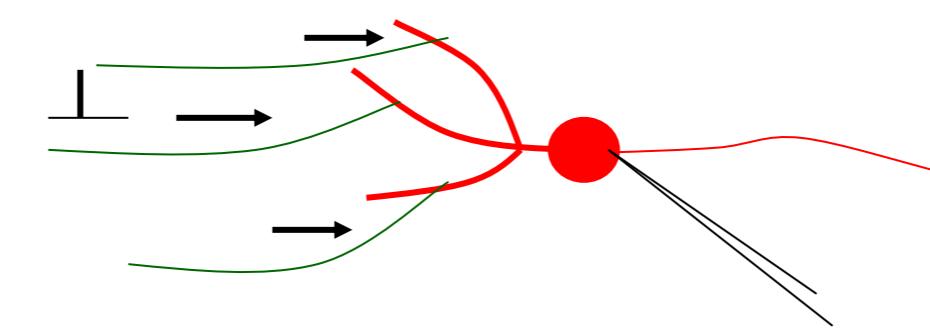
$$u(t) = \vartheta \Rightarrow \text{Fire+reset} \quad u \rightarrow u_r$$

threshold

## 2. Generalized Integrate-and-Fire Model

### Leaky Integrate-and-Fire Model:

*passive membrane*  
+ *threshold*  
+ *reset*



Input spike causes an EPSP  
= excitatory postsynaptic potential

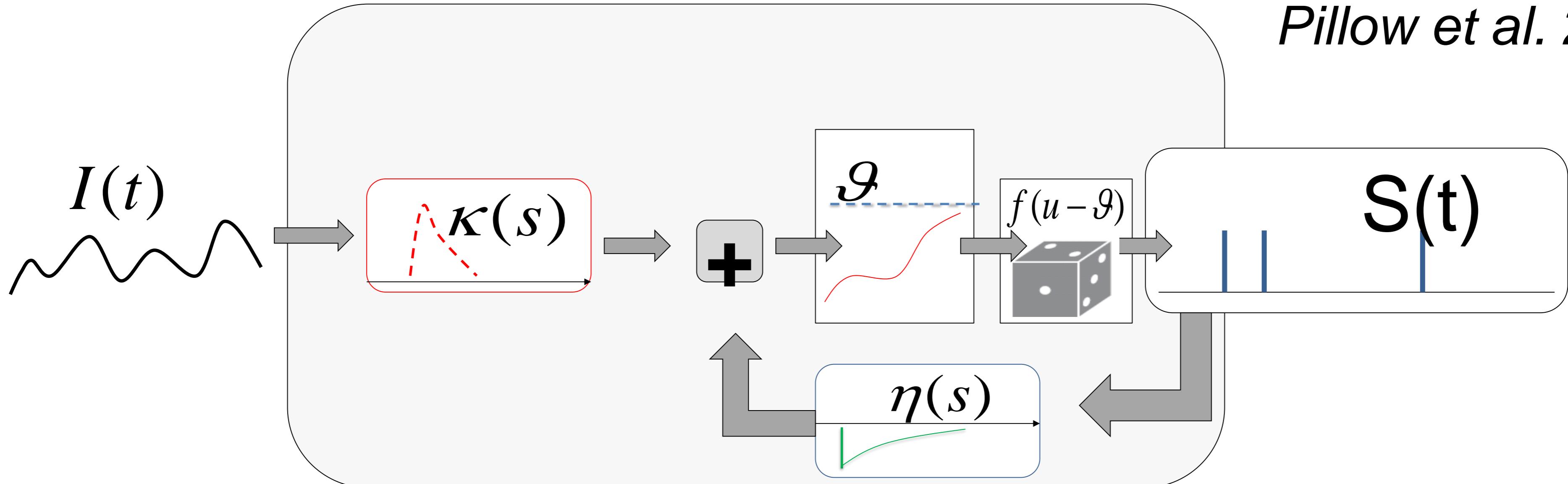
- output spikes are events
- generated at threshold
- after spike: reset/refractoriness



add  $\eta(s)$  (spike afterpotential)

# Spike Response Model (SRM) Generalized Linear Model (GLM)

Gerstner et al.,  
1992,2000  
Truccolo et al., 2005  
Pillow et al. 2008



**potential**  $u(t) = \int \underline{\eta(s)} S(t-s) ds + \int_0^\infty \underline{\kappa(s)} I(t-s) ds + u_{rest}$

**firing intensity**  $\rho(t) = f(u(t) - \vartheta)$

(escape noise) e.g.  $\rho(t) = \rho_0 \exp\left[\frac{u(t) - \vartheta}{\Delta}\right]$

## 2. Leaky Integrate-and-Fire Model: input potential

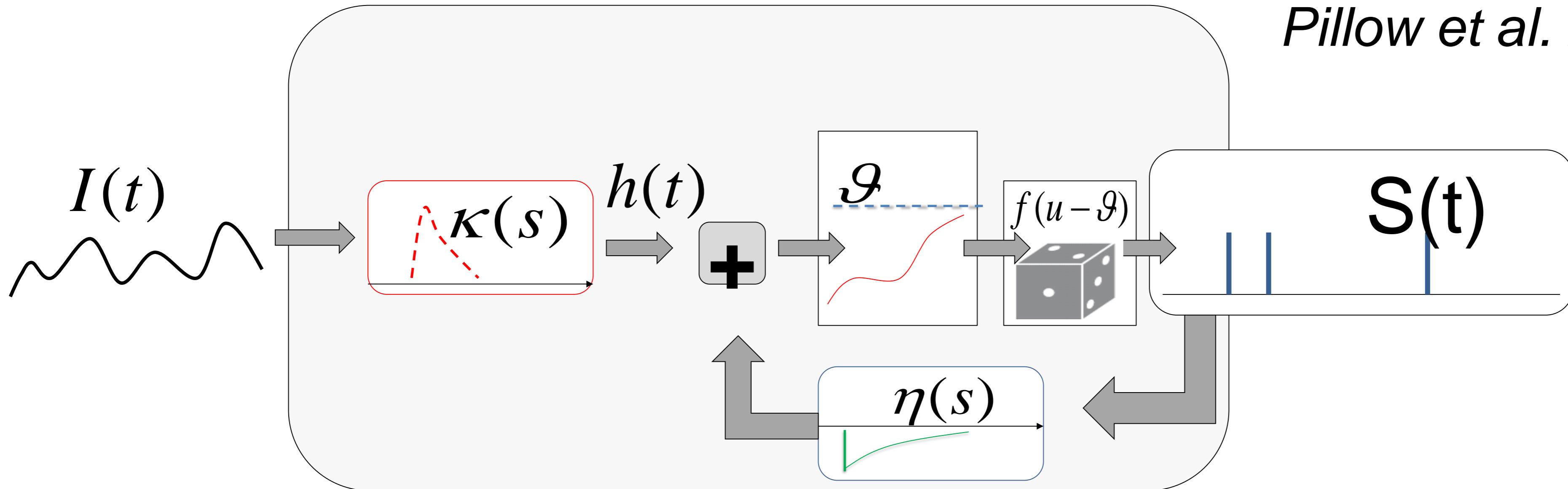
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$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

$$u(t) = u_{rest} + \text{input potential} + \text{reset potential}$$

# Spike Response Model (SRM) Generalized Linear Model (GLM)

Gerstner et al.,  
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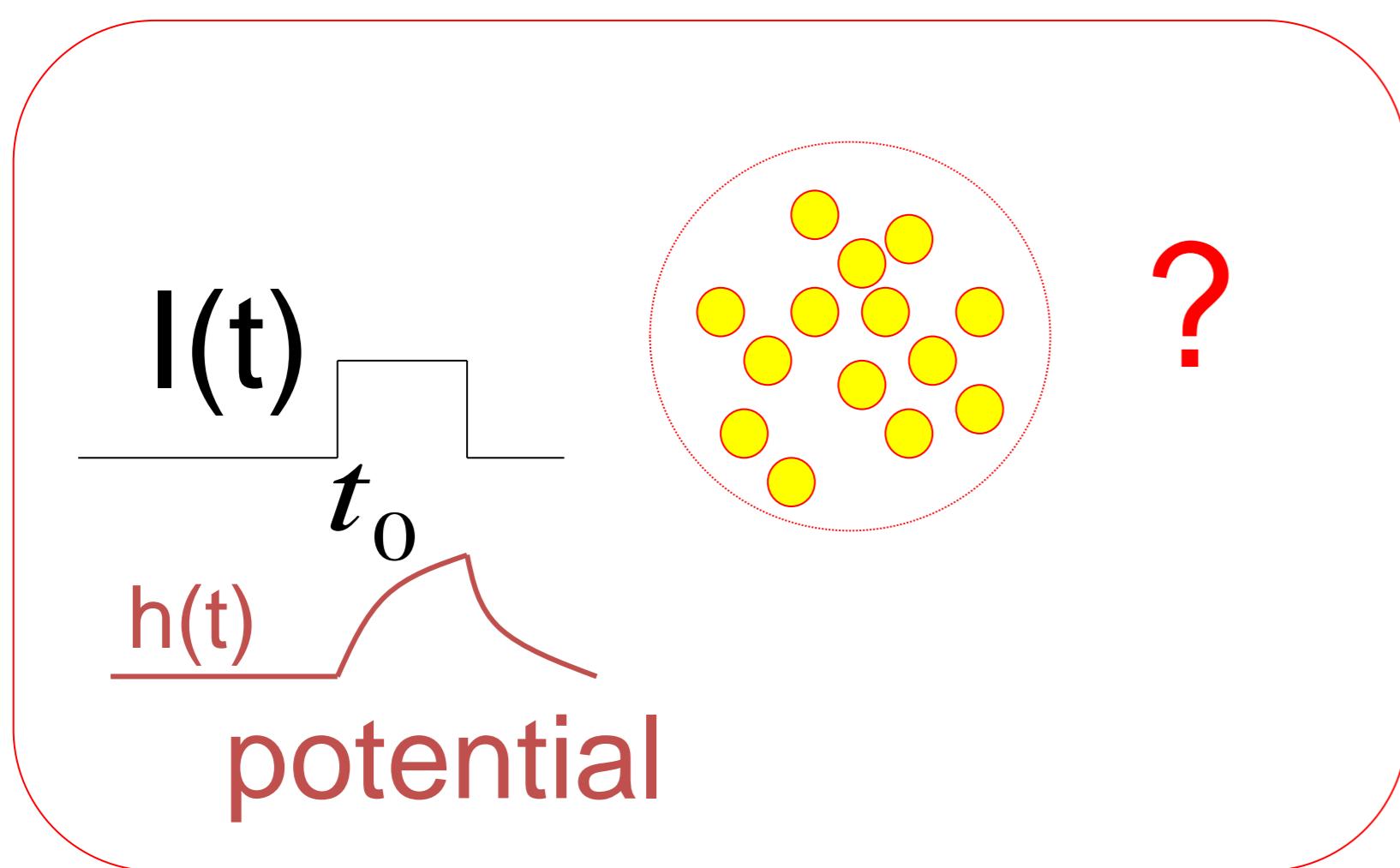


**potential**  $u(t) = u_{rest} + \int_0^{\infty} \underline{\kappa(s)} I(t-s) ds + \int \underline{\eta(s)} S(t-s) ds$

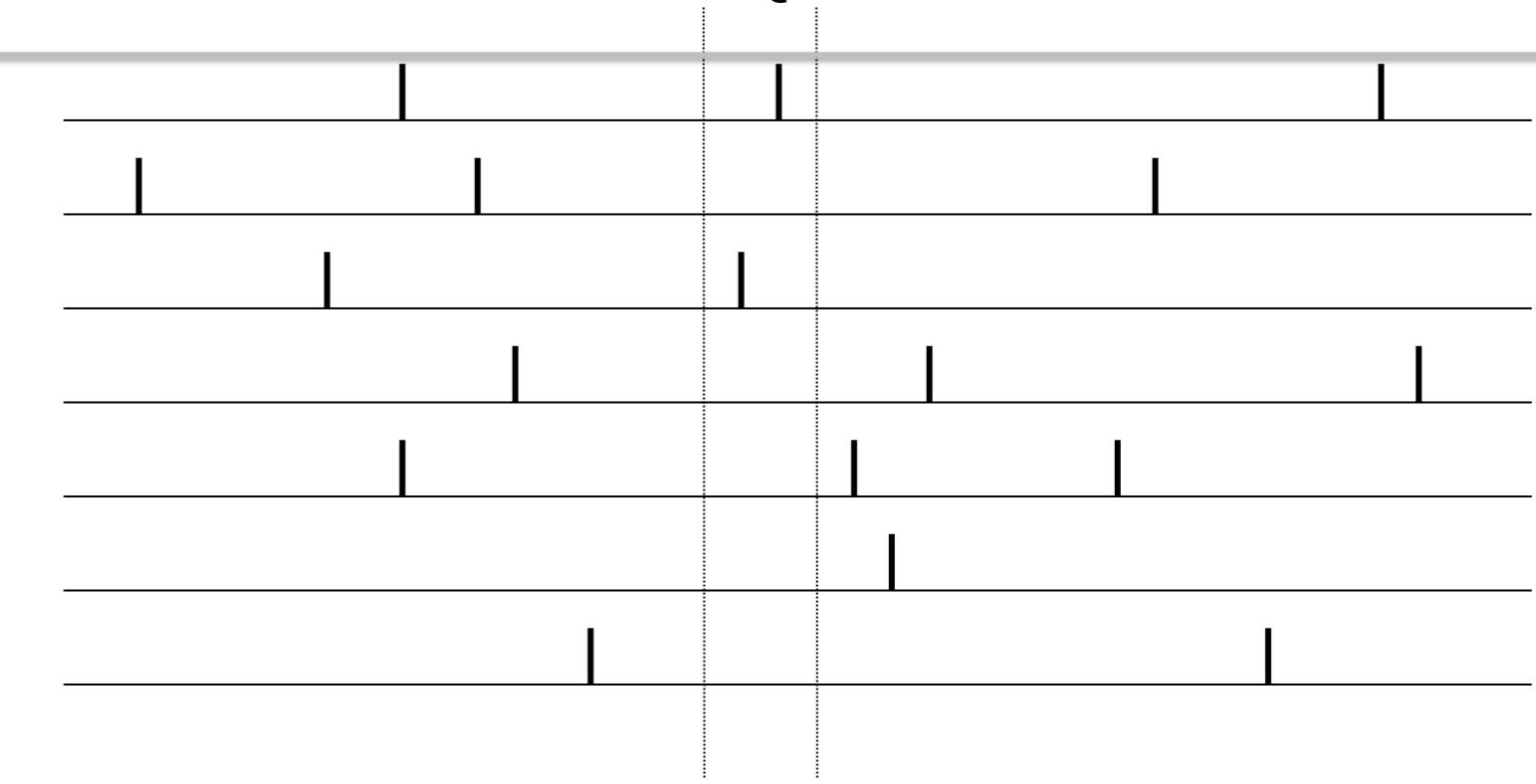
$\underline{\kappa(s) I(t-s)}$   
 $\underline{\eta(s) S(t-s)}$   
 $h(t)$

$u(t) = u_{rest} + \text{input potential} + \text{reset potential}$

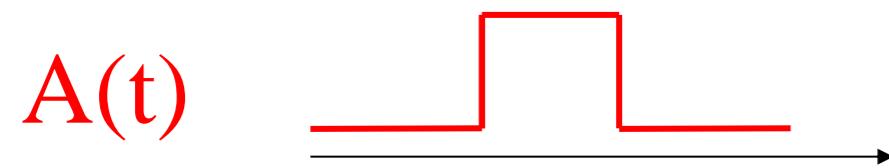
## 2. Transients in a population of **uncoupled** neurons



population  
activity



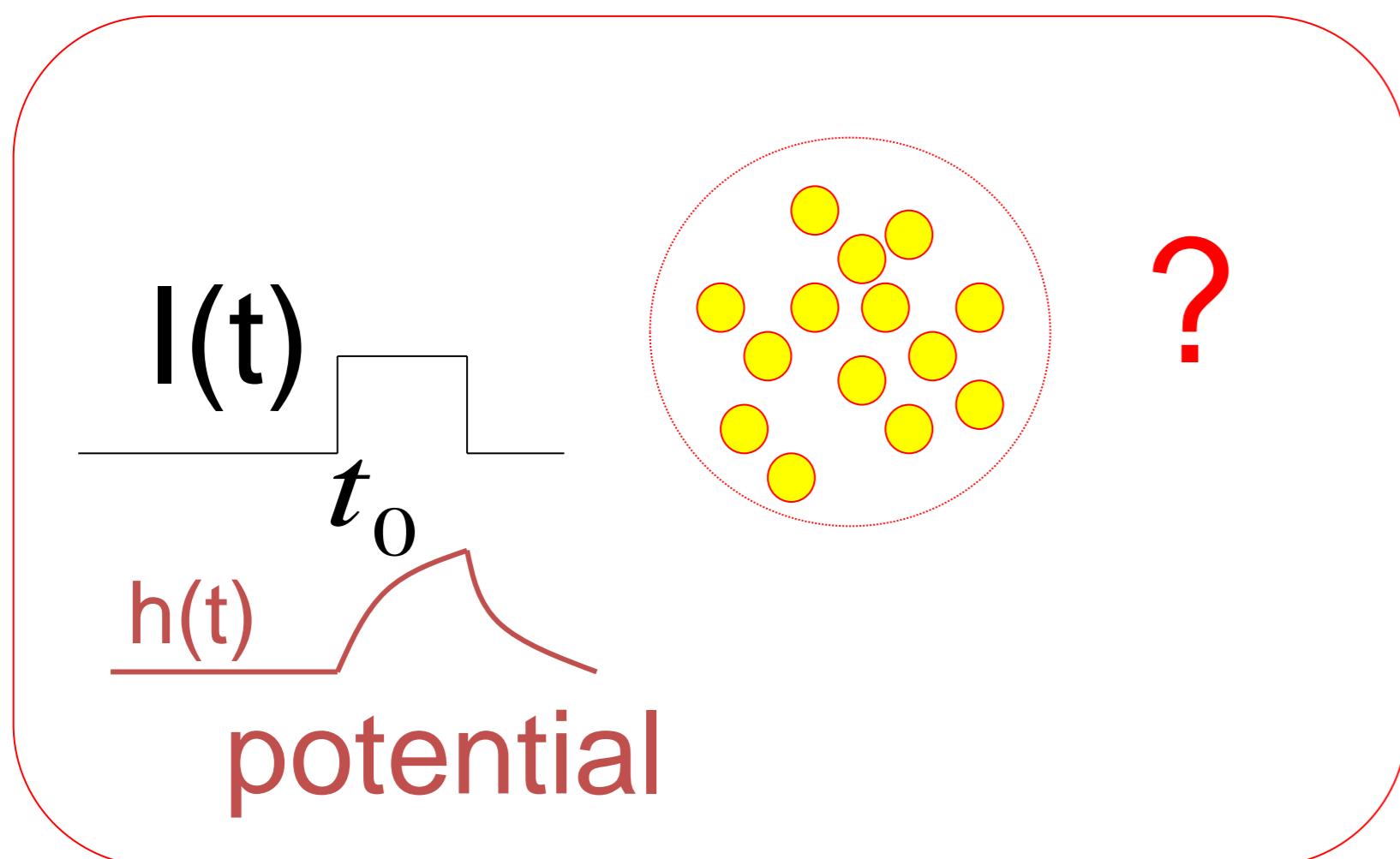
$$A(t) = \frac{n(t - \frac{\Delta t}{2}; t + \frac{\Delta t}{2})}{N \Delta t}$$



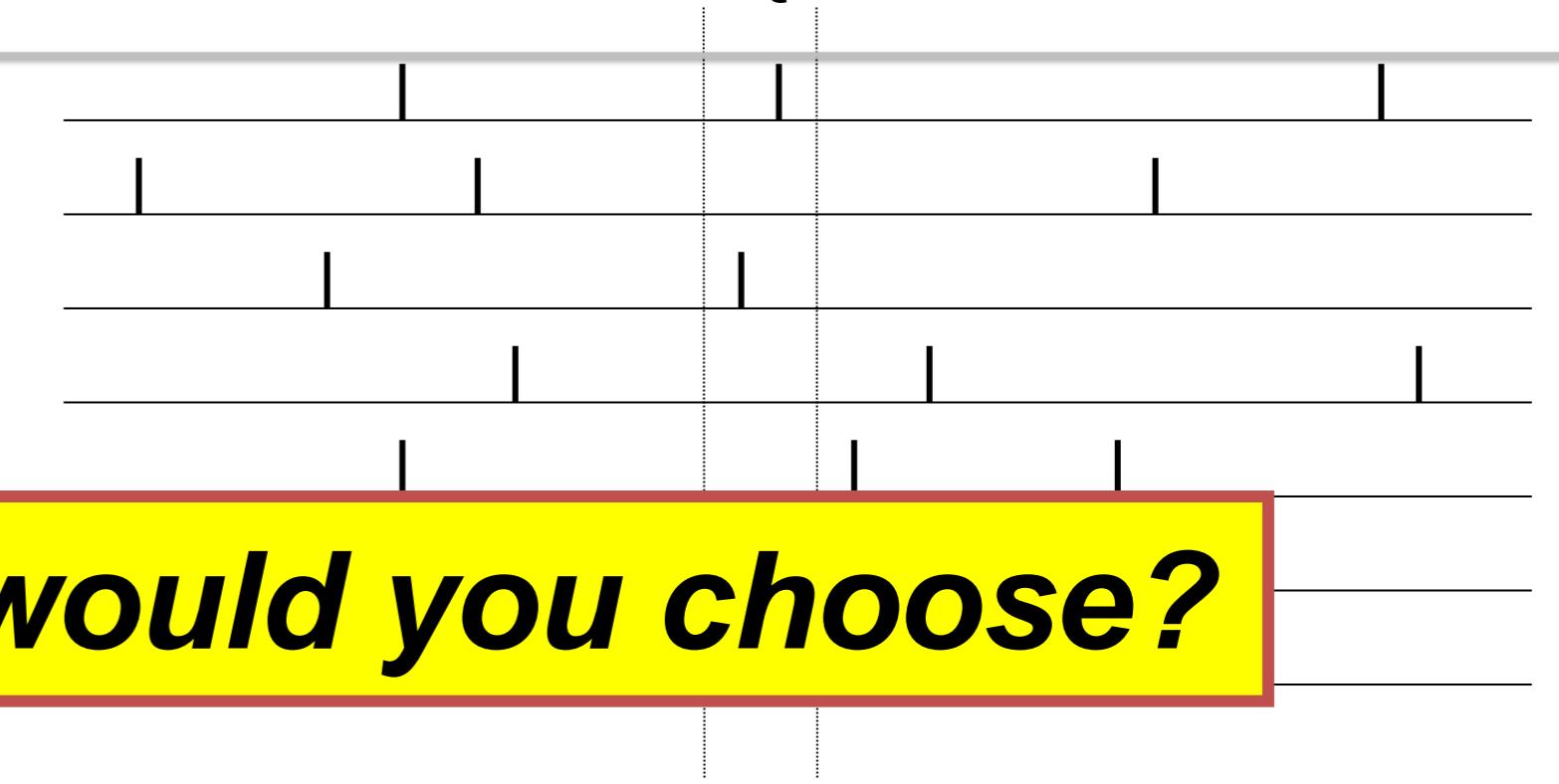
$$A(t) = F(\underline{h(t)}) = F\left(\int \kappa(s) I(t - s) ds\right)$$

$$A(t) = g(\underline{I(t)})$$

## 2. Transients in a population of **uncoupled** neurons



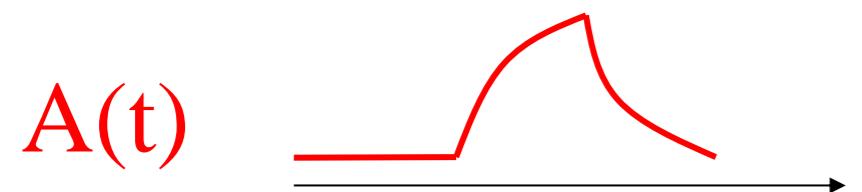
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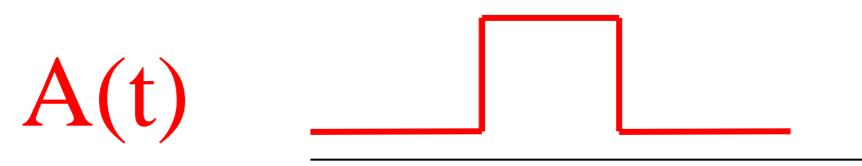
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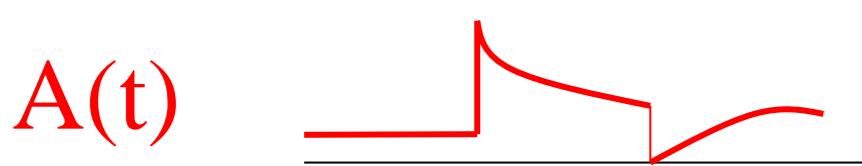
$$\tau \frac{d}{dt} A(t) = -A(t) + F(h(t))$$



$$A(t) = F(\underline{h(t)}) = F\left(\int \kappa(s) I(t-s) ds\right)$$

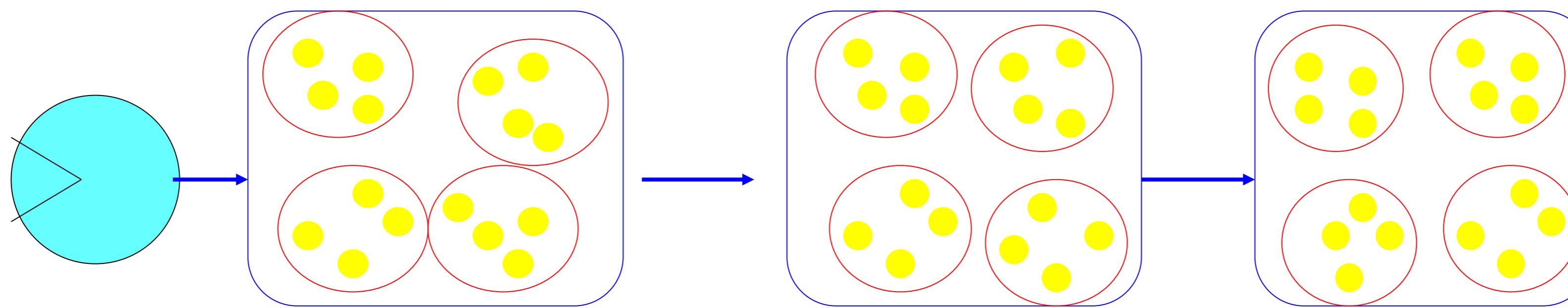


$$A(t) = g(\underline{I(t)})$$

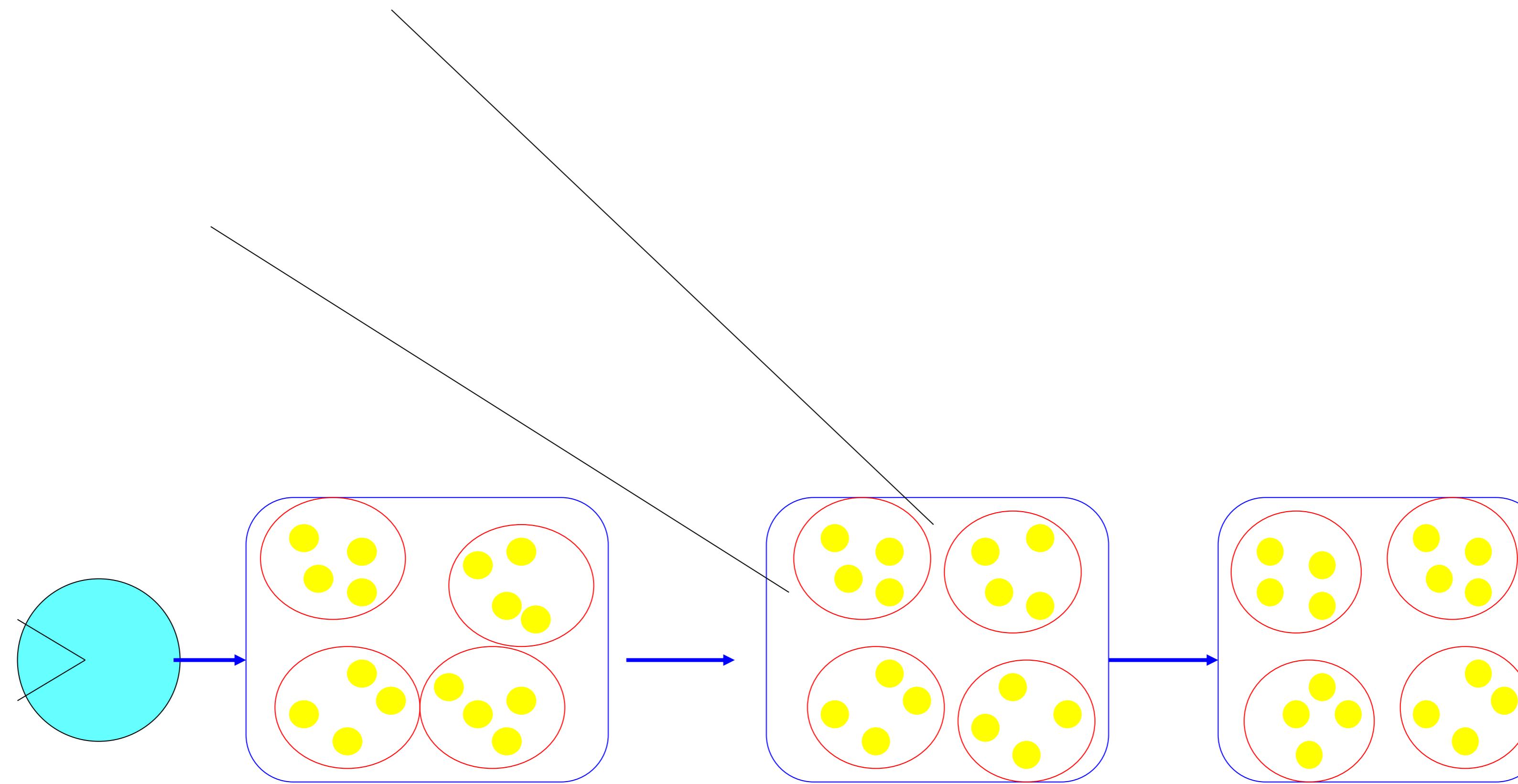


$$A(t) = g(I(t), I'(t))$$

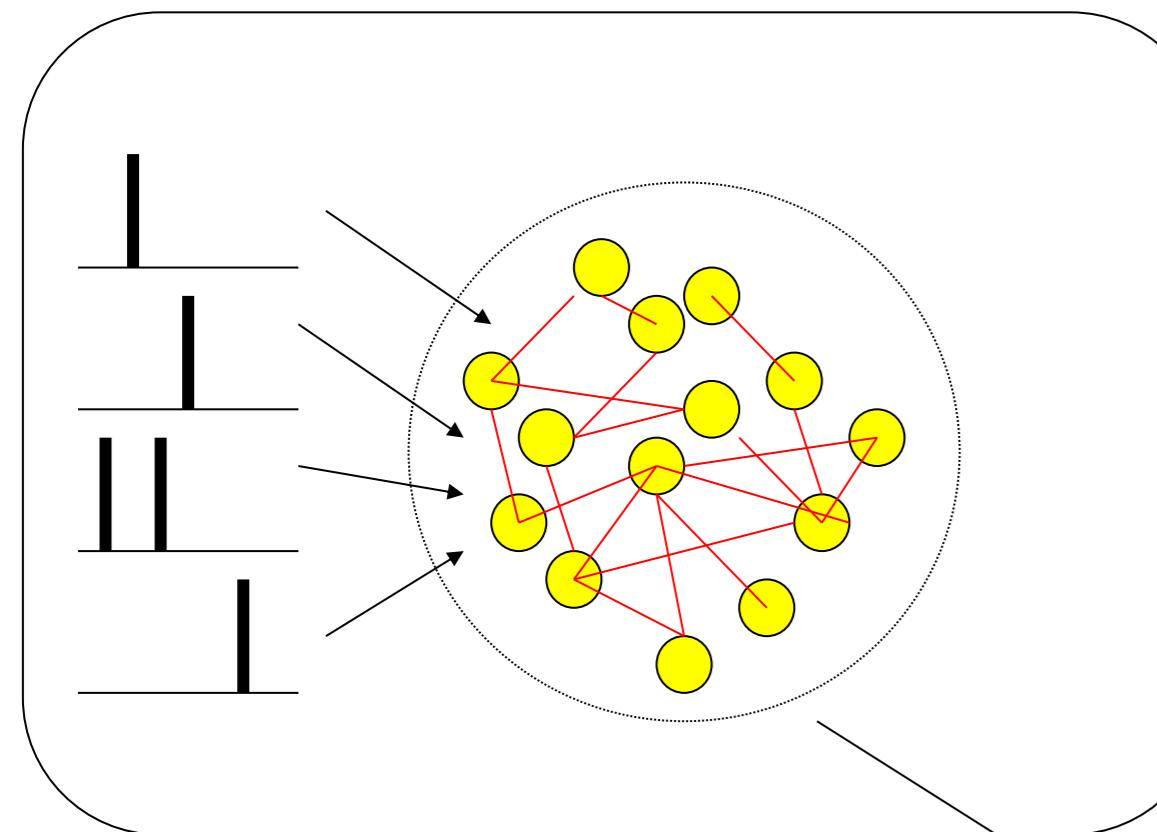
## 2. Transients in a population of neurons: simulations



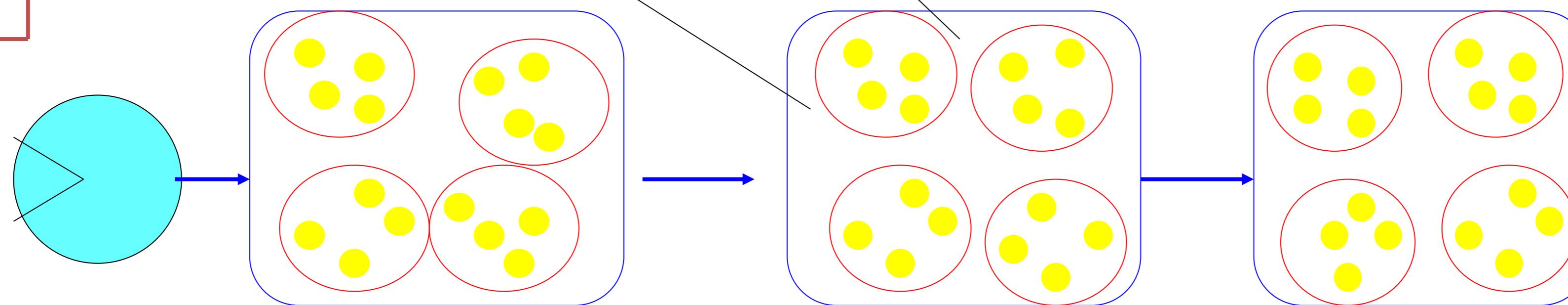
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## 2. Transients in a population of neurons: simulations



input {  
low rate  
-high rate



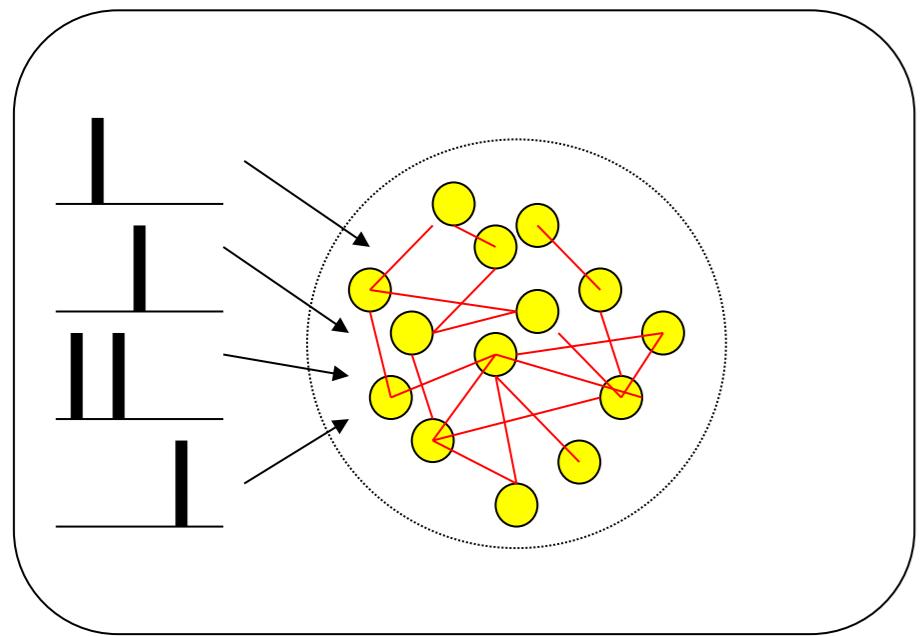
### Population

- 50 000 neurons
- 20 percent inhibitory
- randomly connected

### Connections

- 4000 external
- 4000 within excitatory
- 1000 within inhibitory

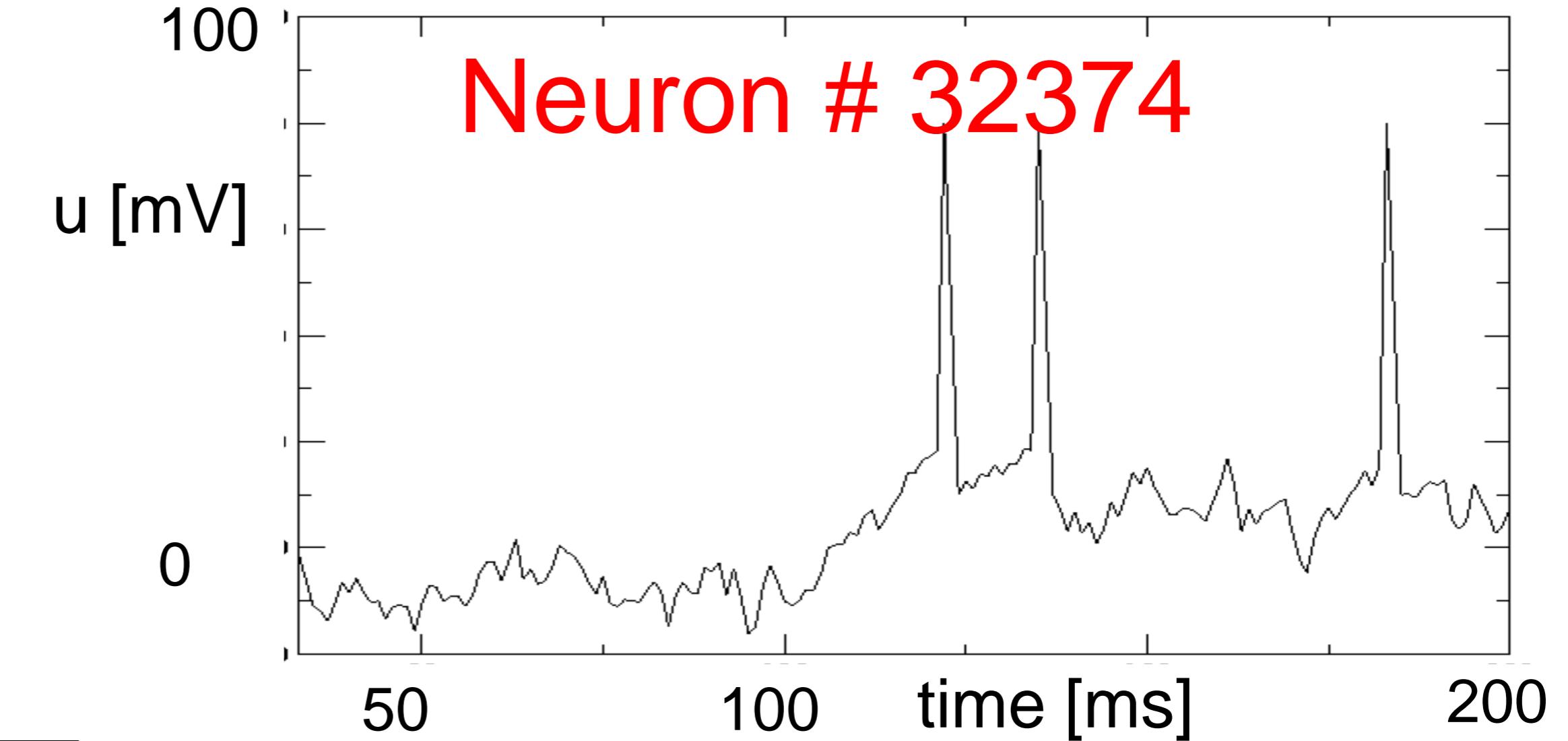
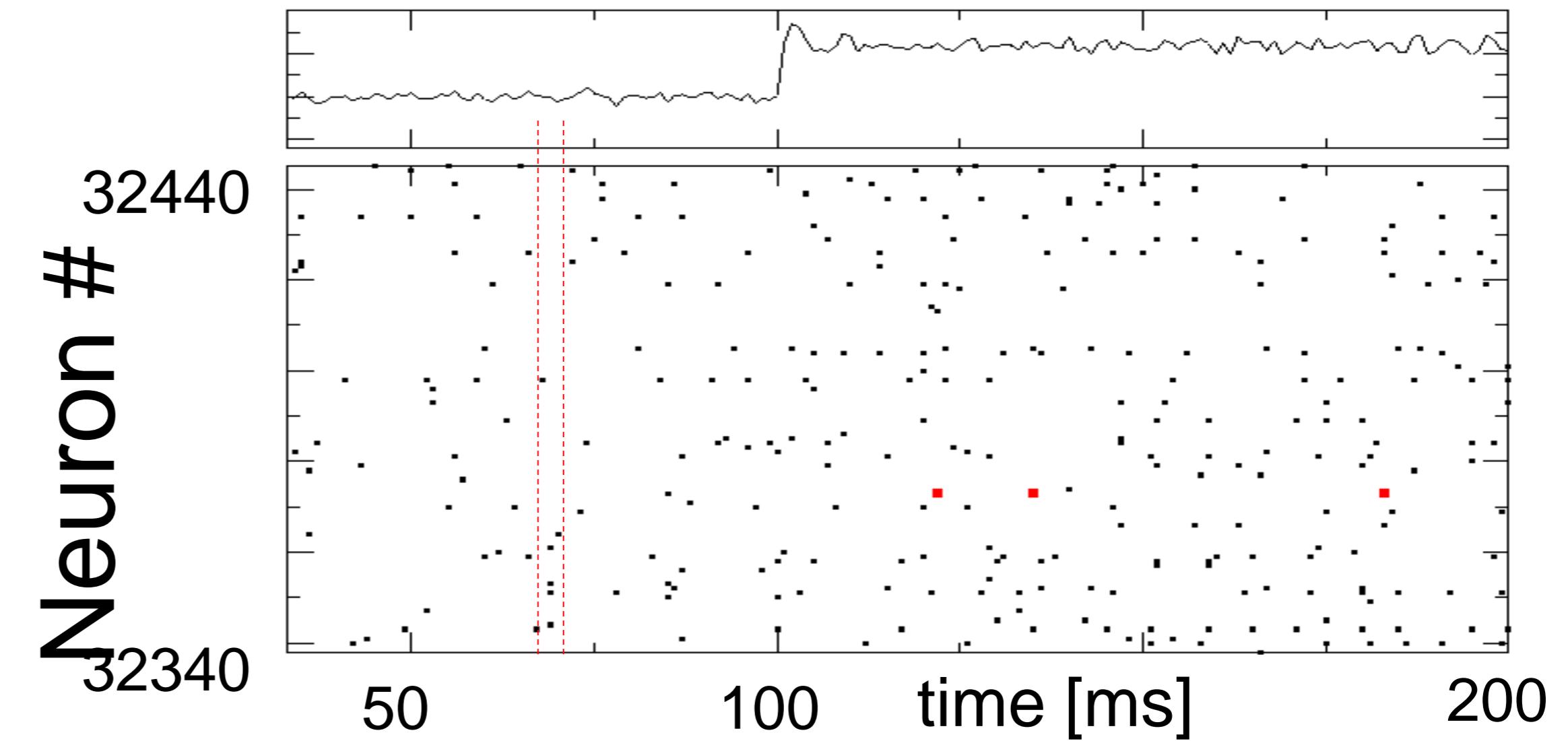
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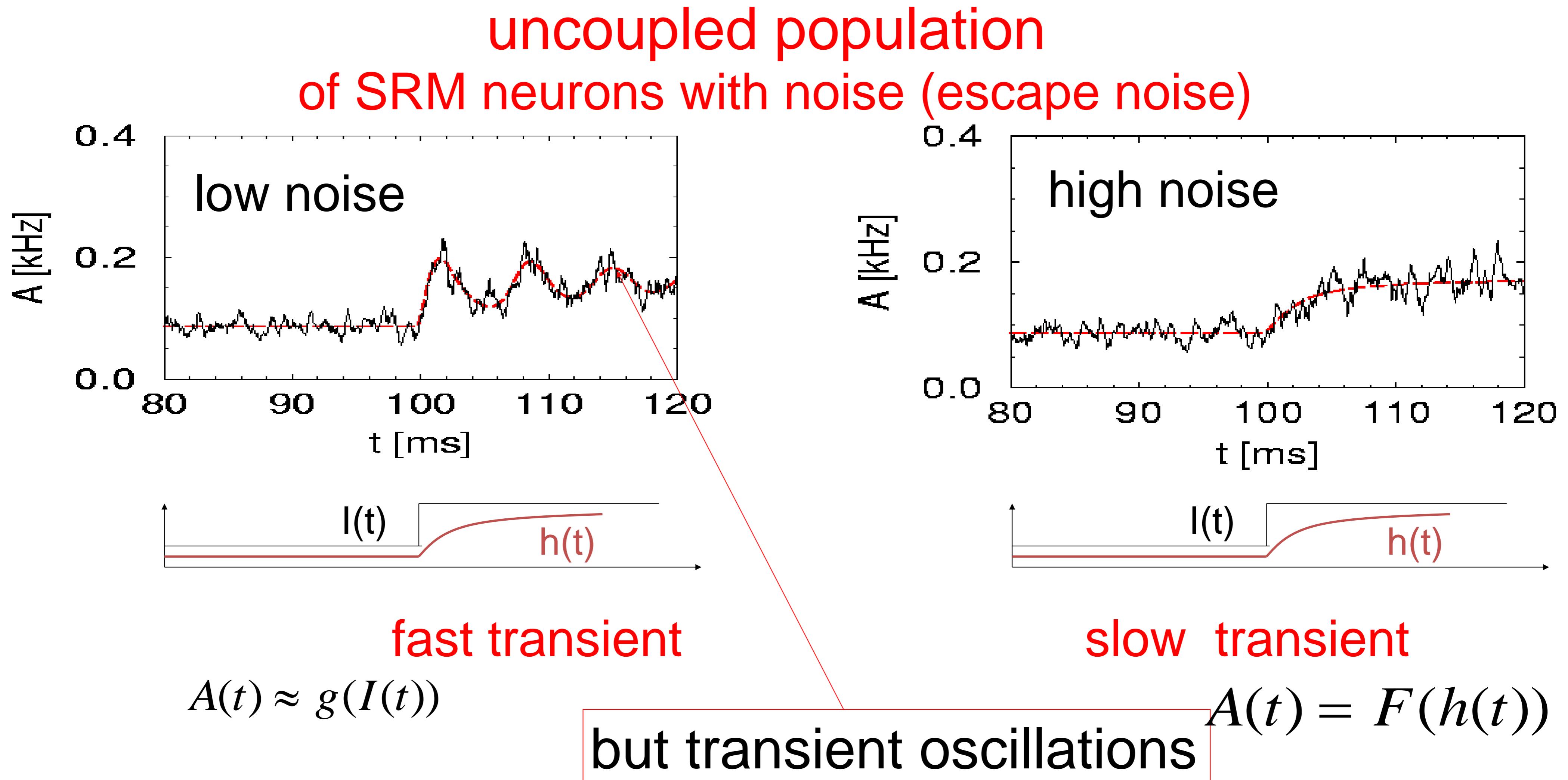
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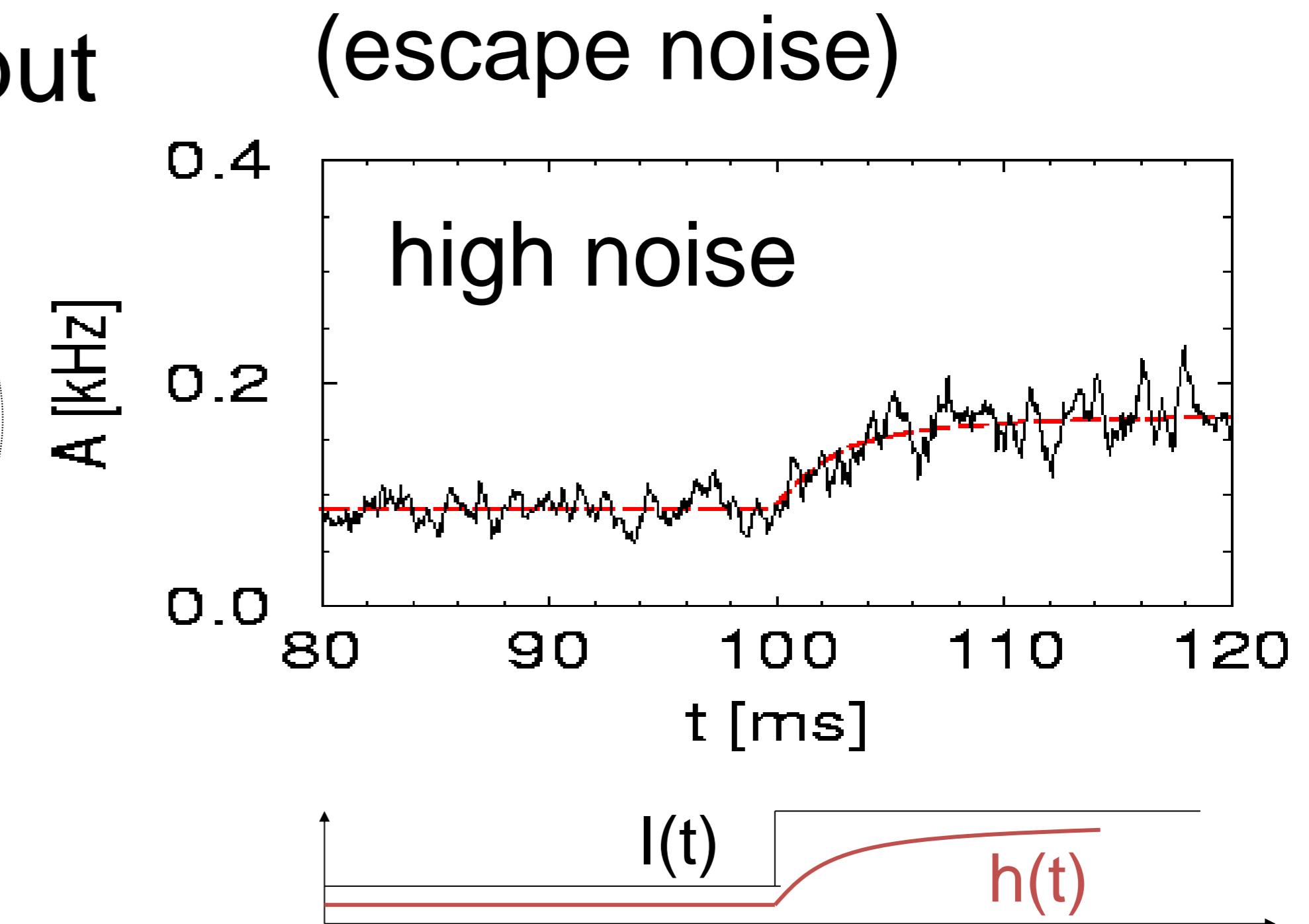
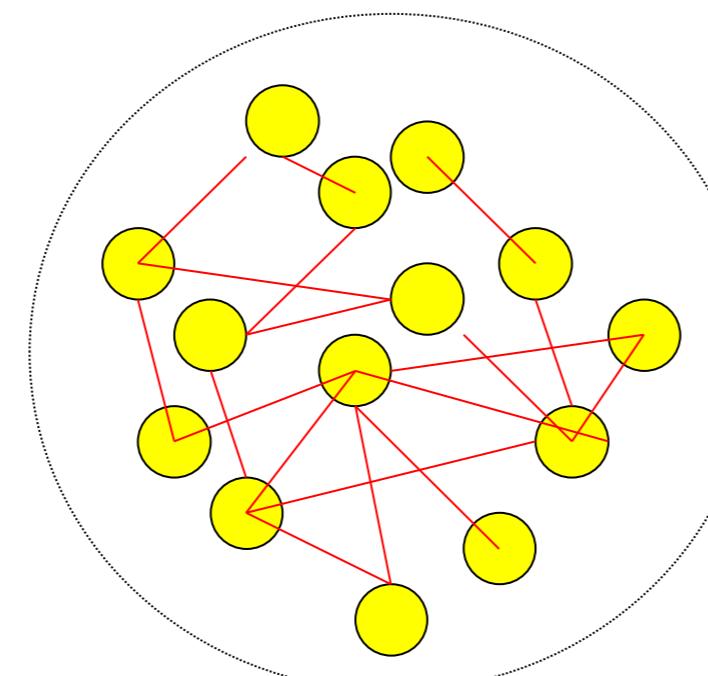
## 2. Transients for populations of noisy neurons



## 2. High-noise activity equation

Membrane potential caused by input

$$\tau \frac{d}{dt} h(t) = -h(t) + R I(t)$$



slow transient

$$A(t) = F(h(t))$$

In the limit of high noise,

## 2. High-noise activity equation

Population activity

$$A(t) = F(h(t))$$

Membrane potential caused by input

$$\tau \frac{d}{dt} h(t) = -h(t) + R I(t)$$

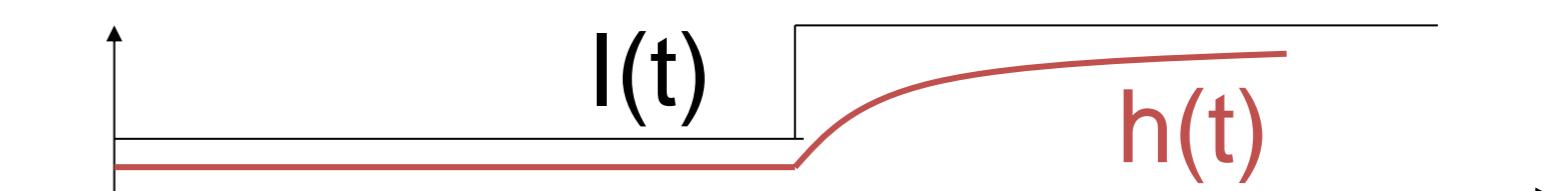
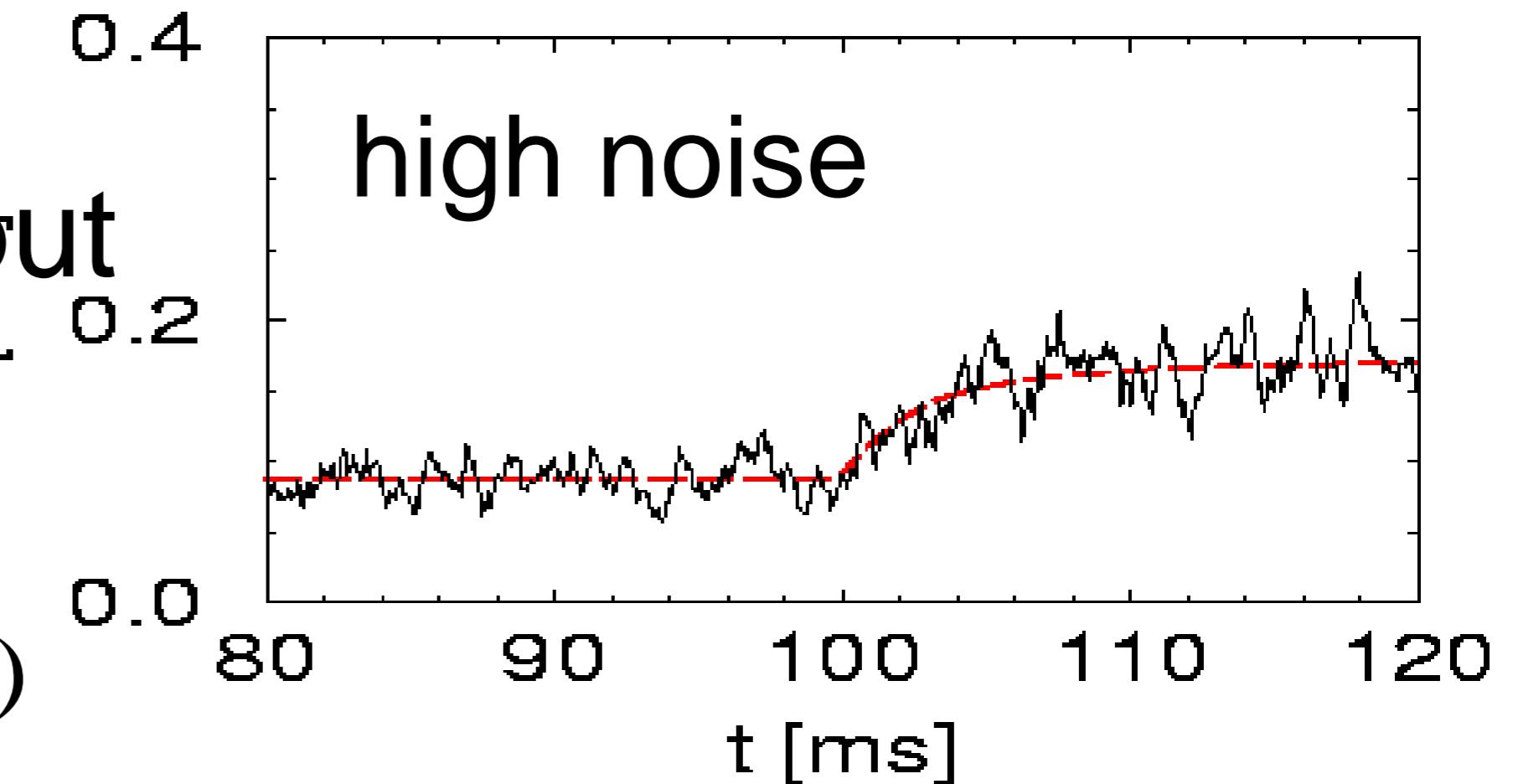
$$I(t) = I^{ext}(t) + I^{netw}(t)$$

$$I(t) = I^{ext}(t) + J_0 q A(t)$$

$$I(t) = I^{ext}(t) + J_0 q F(h(t))$$

$$\boxed{\tau \frac{d}{dt} h(t) = -h(t) + R I^{ext}(t) + \gamma F(h(t))}$$

(escape noise)



slow transient

$$\boxed{A(t) = F(h(t))}$$

1 population = 1 differential equation

## 2. Summary: Transients and population equations

$$\tau \frac{d}{dt} h(t) = -h(t) + R I^{ext}(t) + \gamma F(h(t))$$

- population activity equation
- smooth transient
- input potential determines activity

$$A = F(h(t))$$

- valid in high-noise regime
- misses sharp transients
- misses transient oscillations

## Quiz 1, now

### Population equations

A single homogeneous population of neurons is driven by a step current causing a transient response of the population activity.

- [ ] A single cortical model population can exhibit transient oscillations
- [ ] Transients are always sharp
- [ ] Transients are always slow
- [ ] in a certain limit transients can be slow
- [ ] An escape noise model in the high-noise limit has transients which are always slow
- [ ] A single population described by a single first-order differential equation (no integrals/no delays) can exhibit transient oscillations

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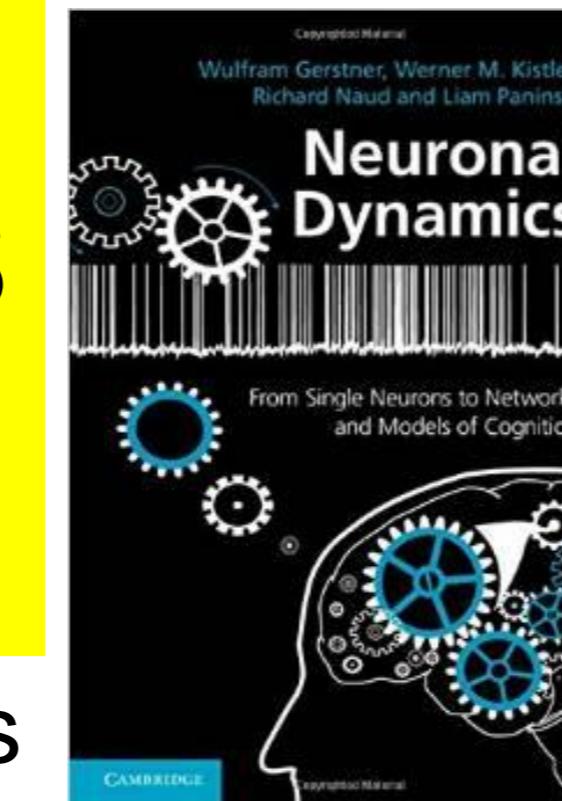
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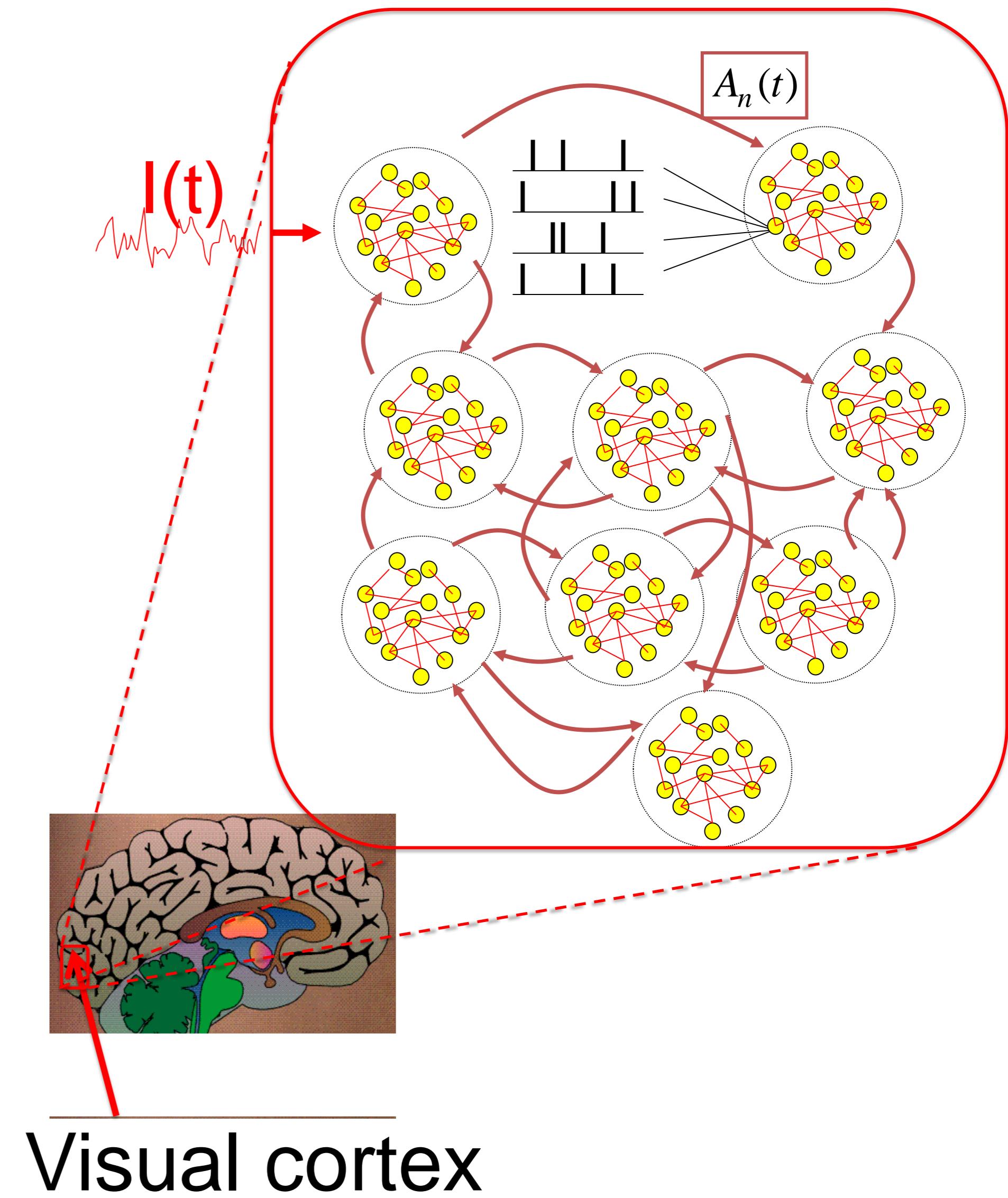
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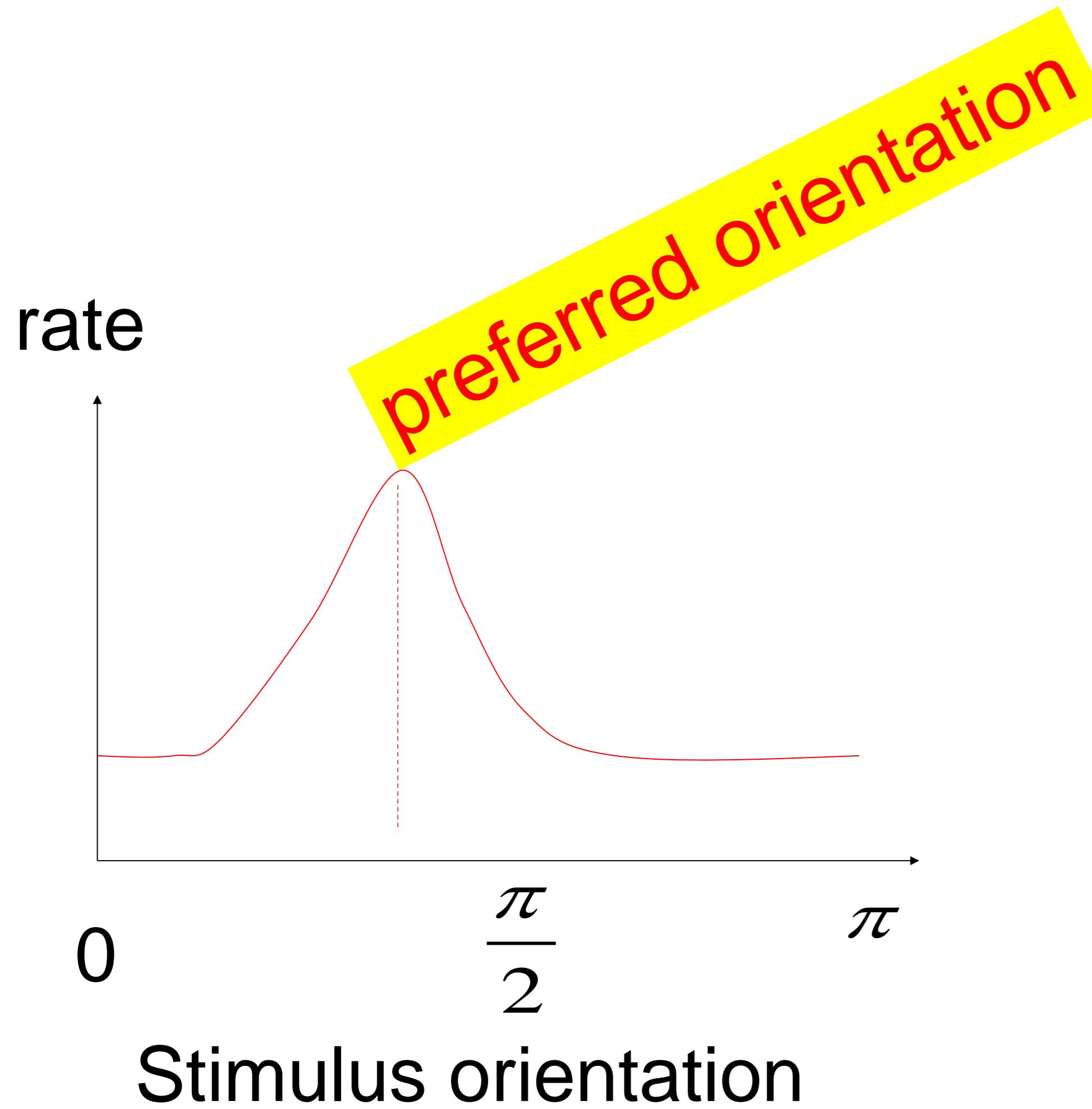


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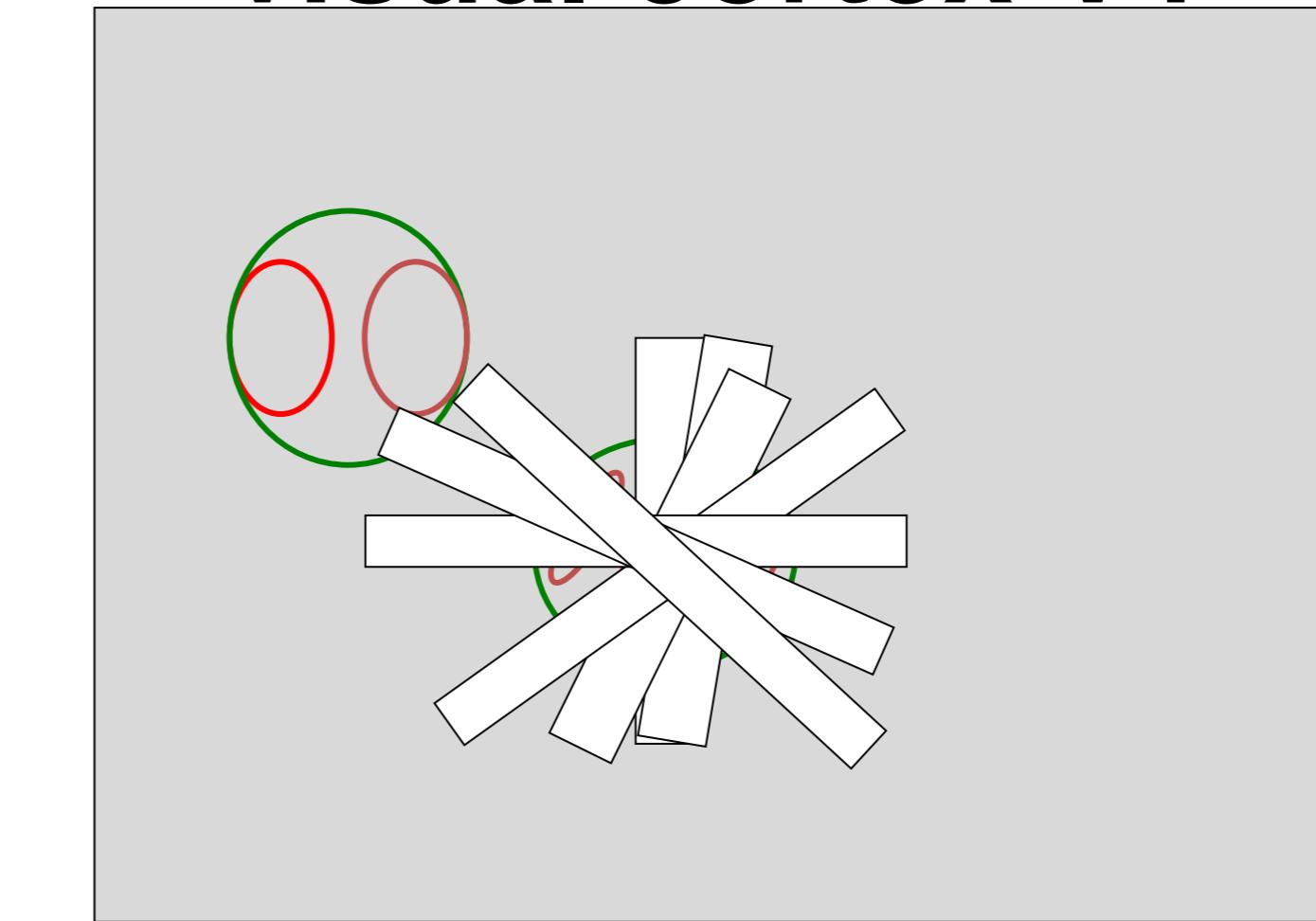
### 3. Interacting Populations: how many populations?



### 3. Review: Receptive fields with Orientation Tuning



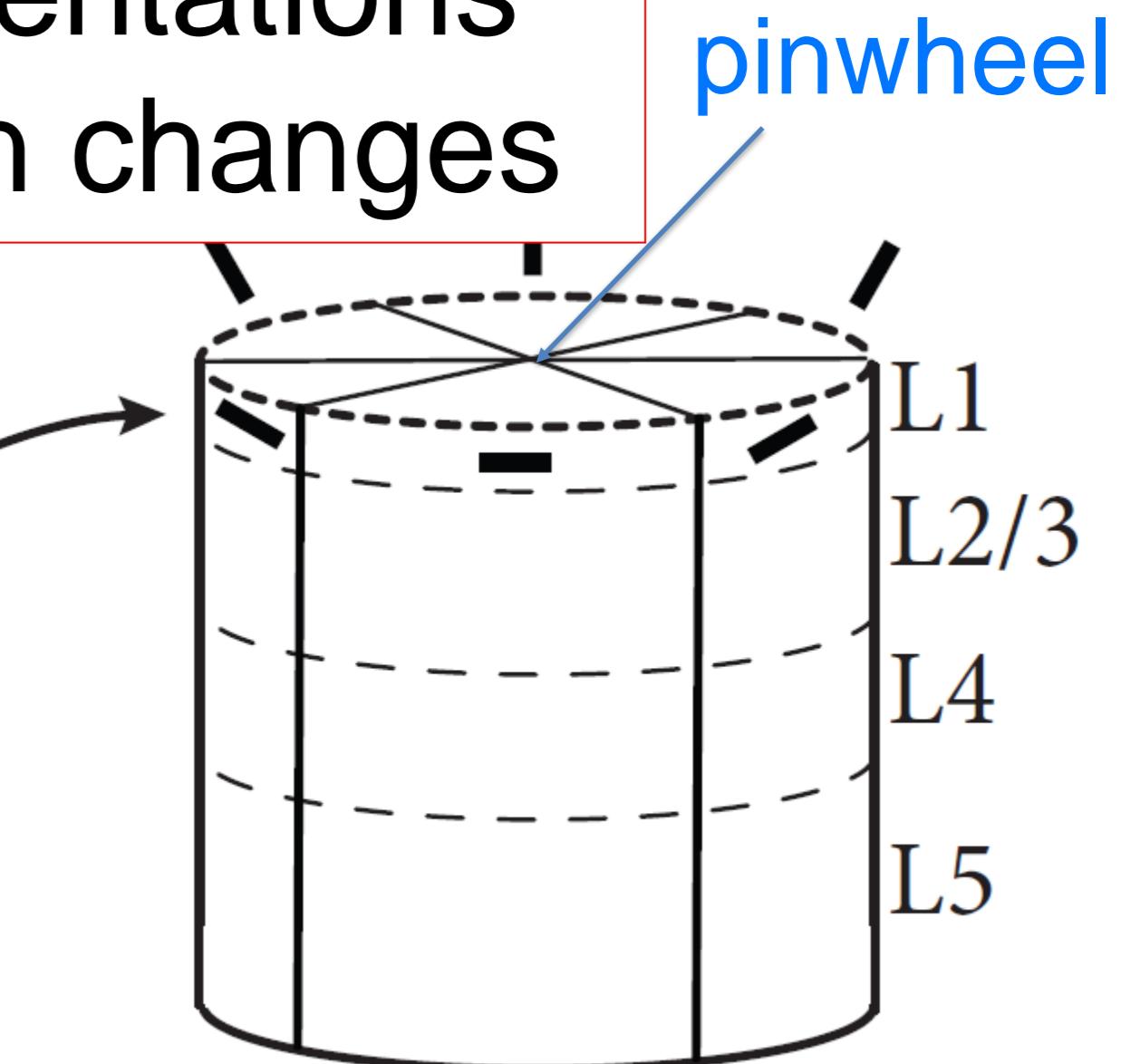
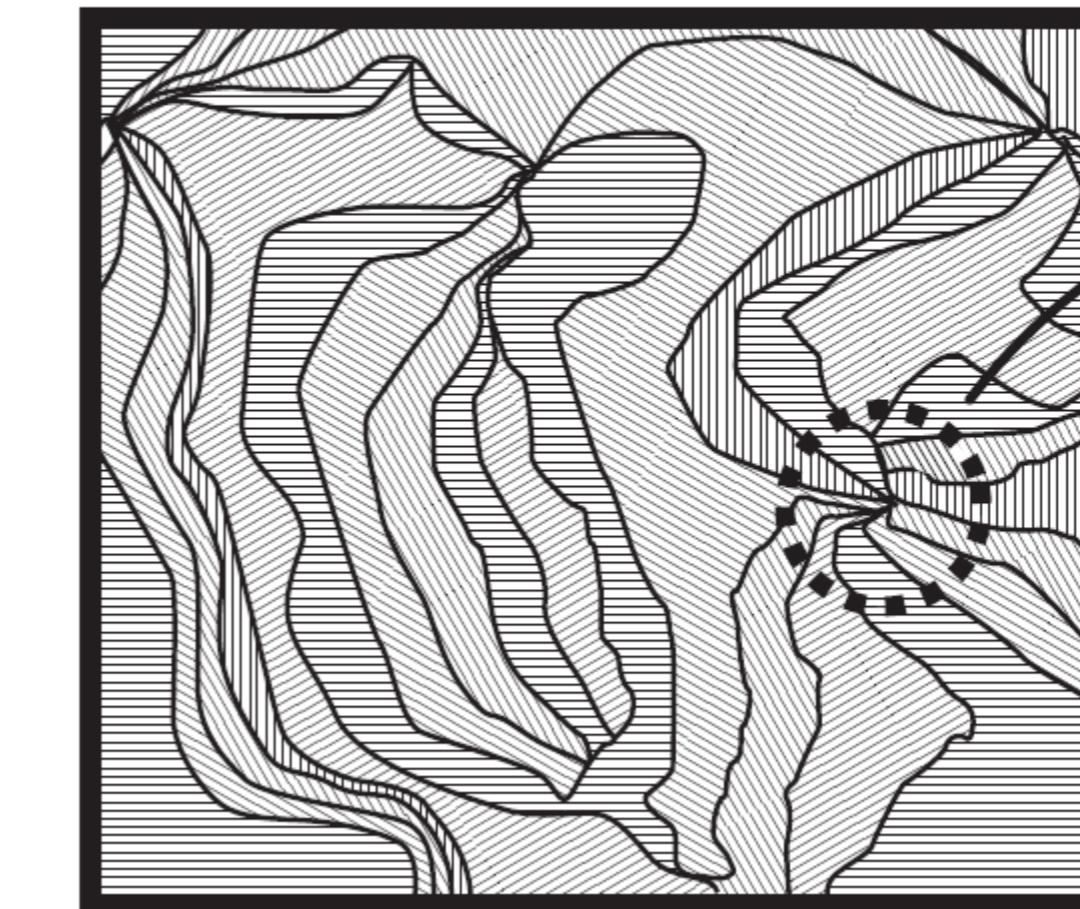
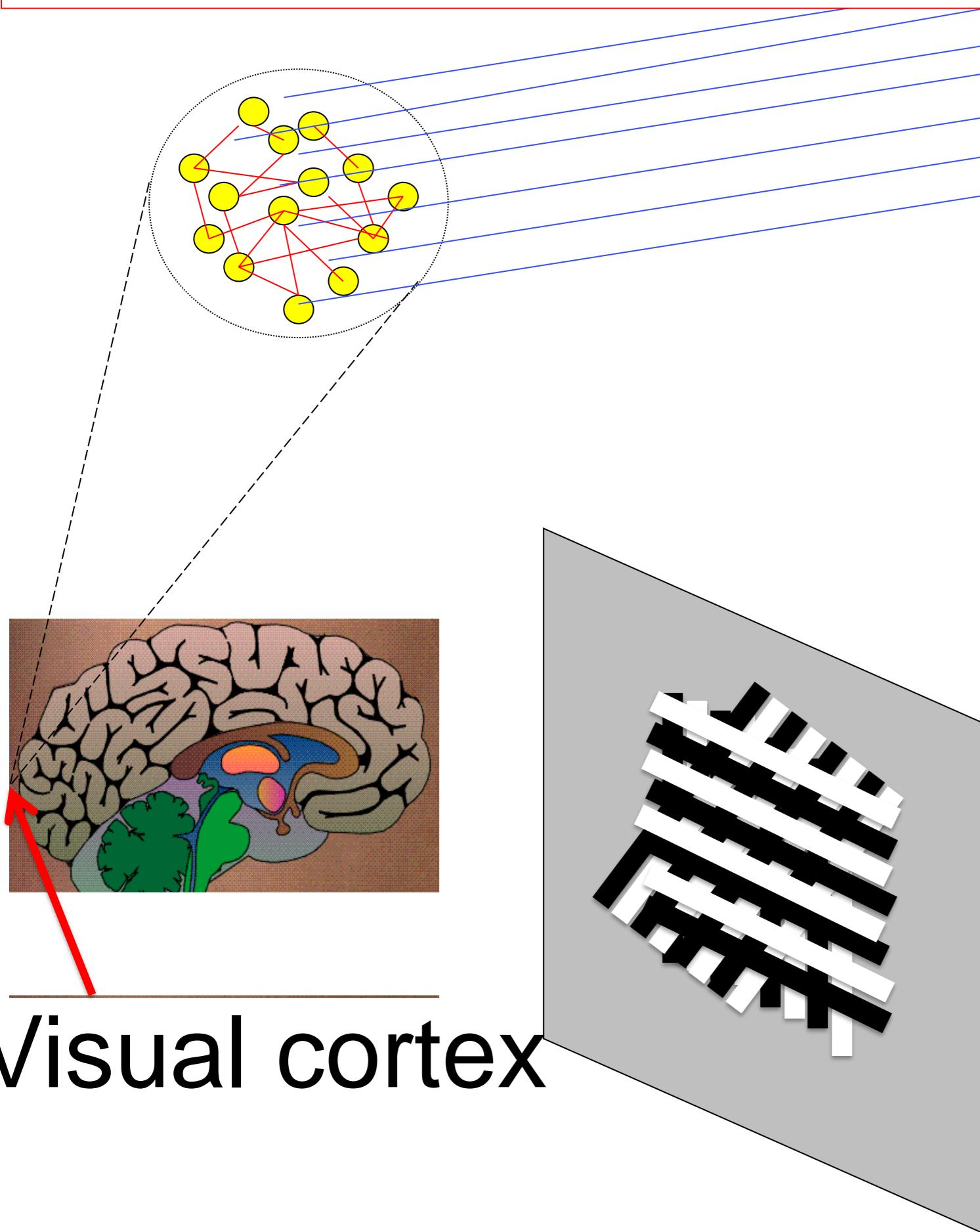
Receptive fields:  
**visual cortex V1**



Orientation selective

### 3. Orientation Map

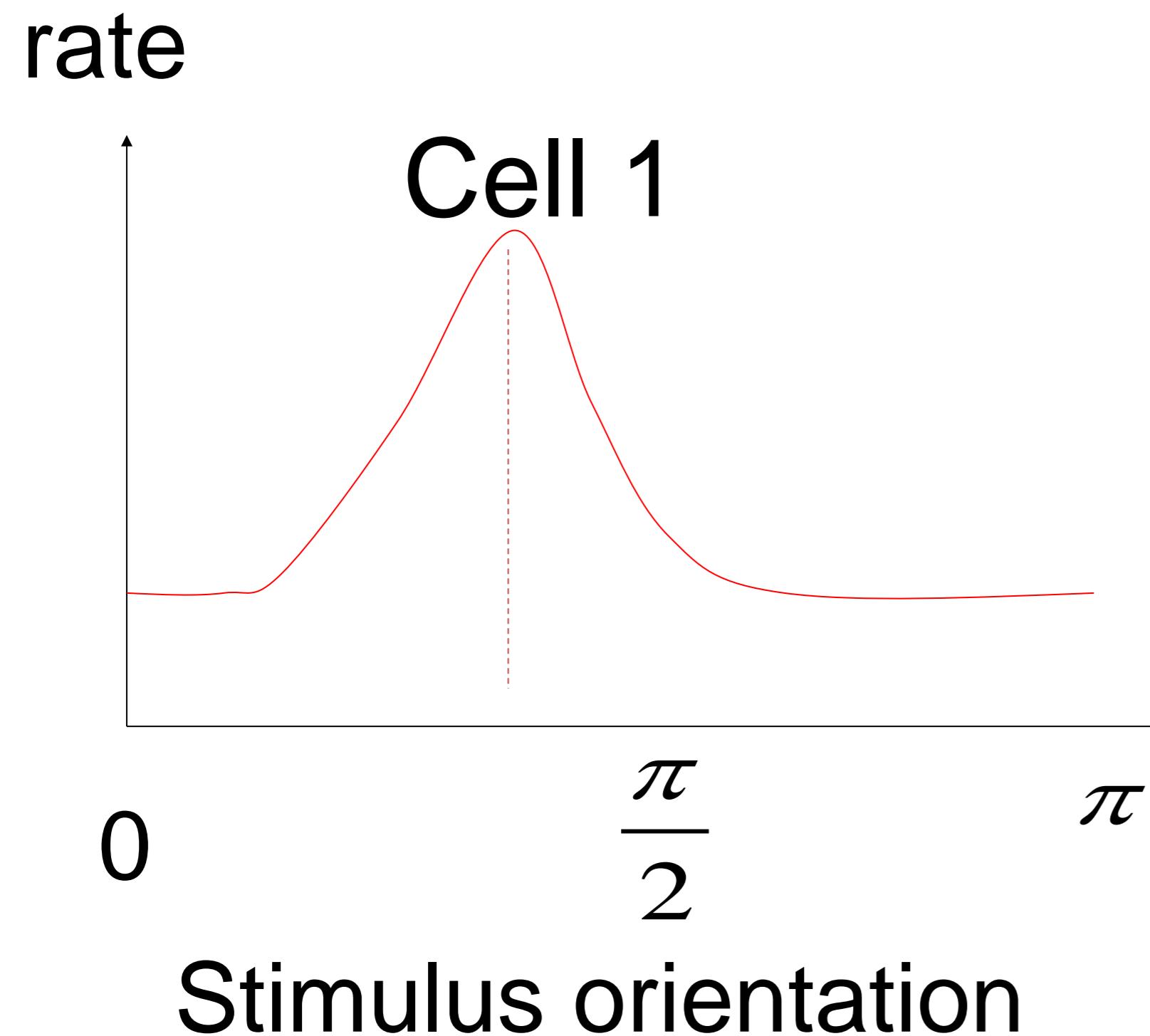
population of neighboring neurons: similar orientations  
as we move along cortical surface: orientation changes



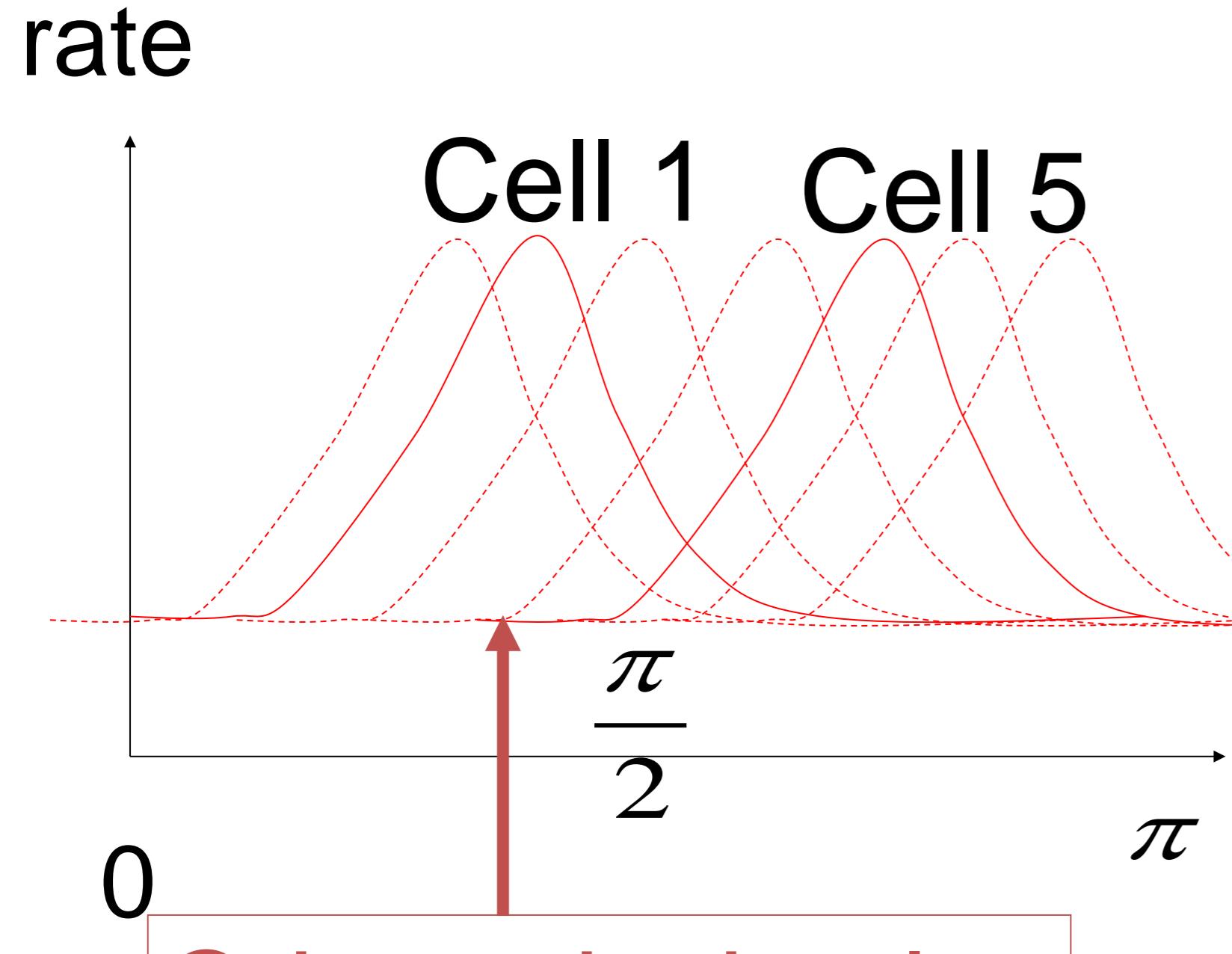
*Image: Gerstner et al.  
Neuronal Dynamics (2014)*

*Bonhoeffer&Grinvald, 1991;  
Bressloff&Cowan, 2002;  
Kaschube et al. 2010*

### 3. Do Orientation Columns exist? Do identical cells exist?



### 3. Do Orientation columns exist? Do identical cells exist?

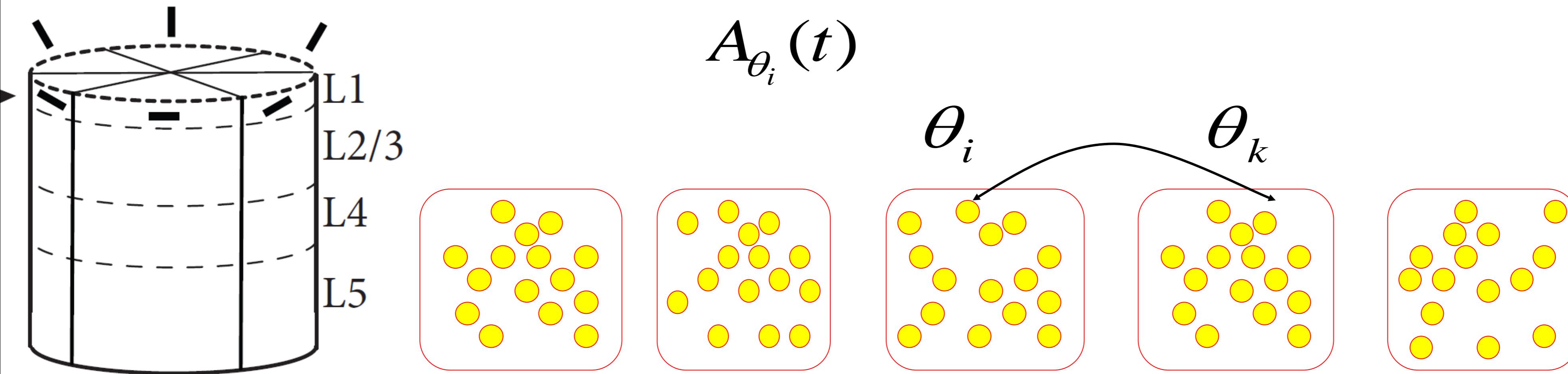


#### Coarse coding

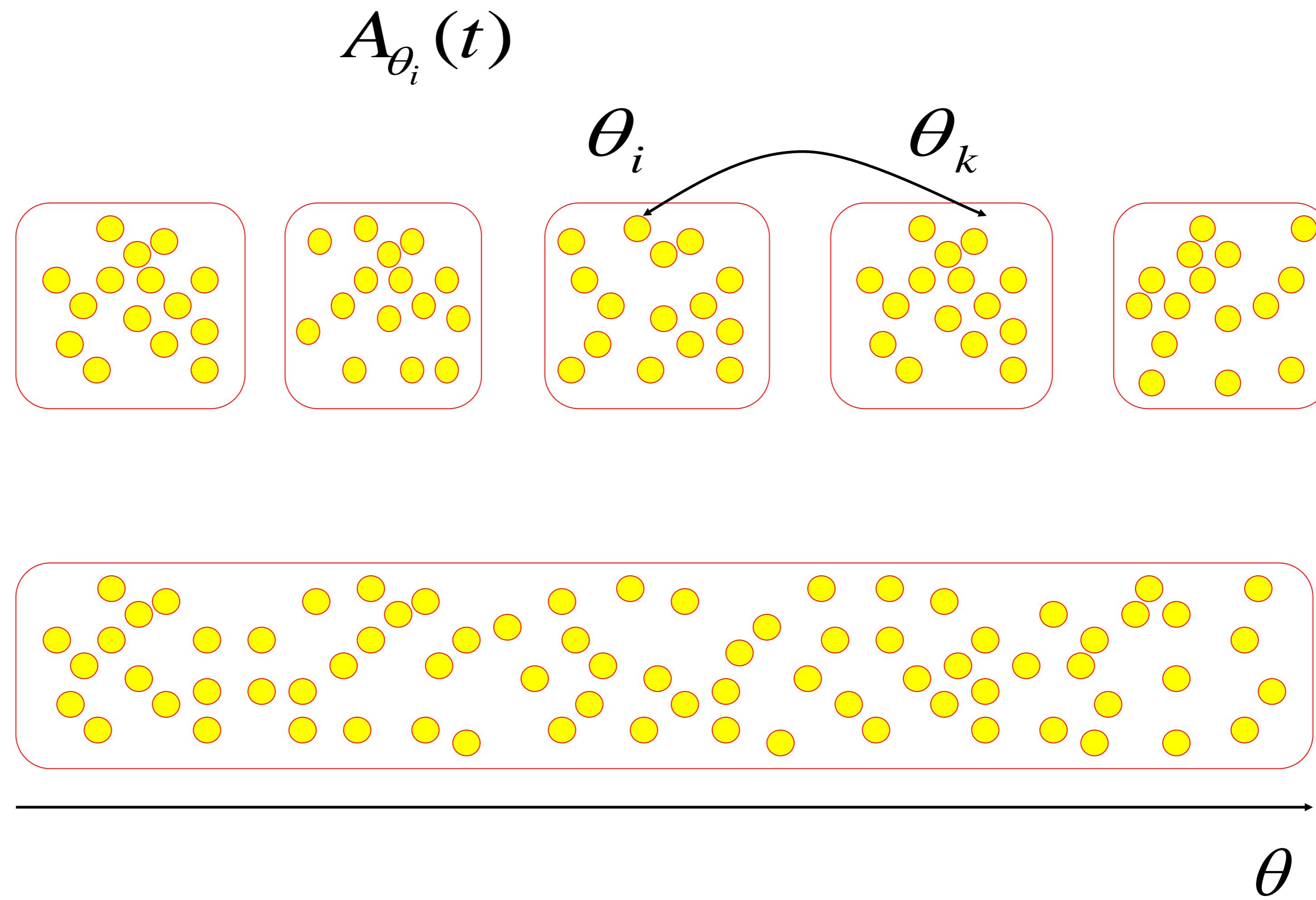
Many cells  
(from different columns)  
respond to a single  
stimulus with different rate

→ no discrete columns

### 3. multiple populations → continuum



### 3. multiple populations $\rightarrow$ continuum



# Computational Neuroscience: Neuronal Dynamics of Cognition



## Continuum models: Cortical fields and perception

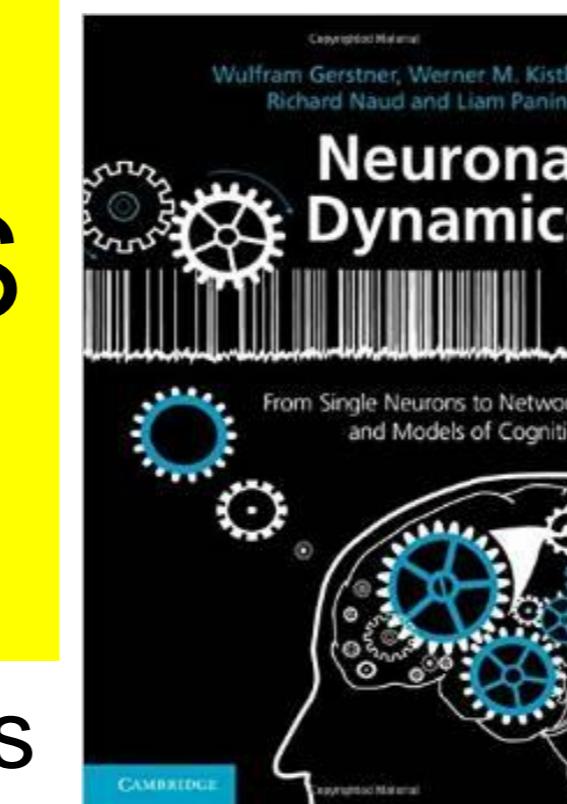
Wulfram Gerstner

EPFL, Lausanne, Switzerland

*Reading:*  
**NEURONAL DYNAMICS**

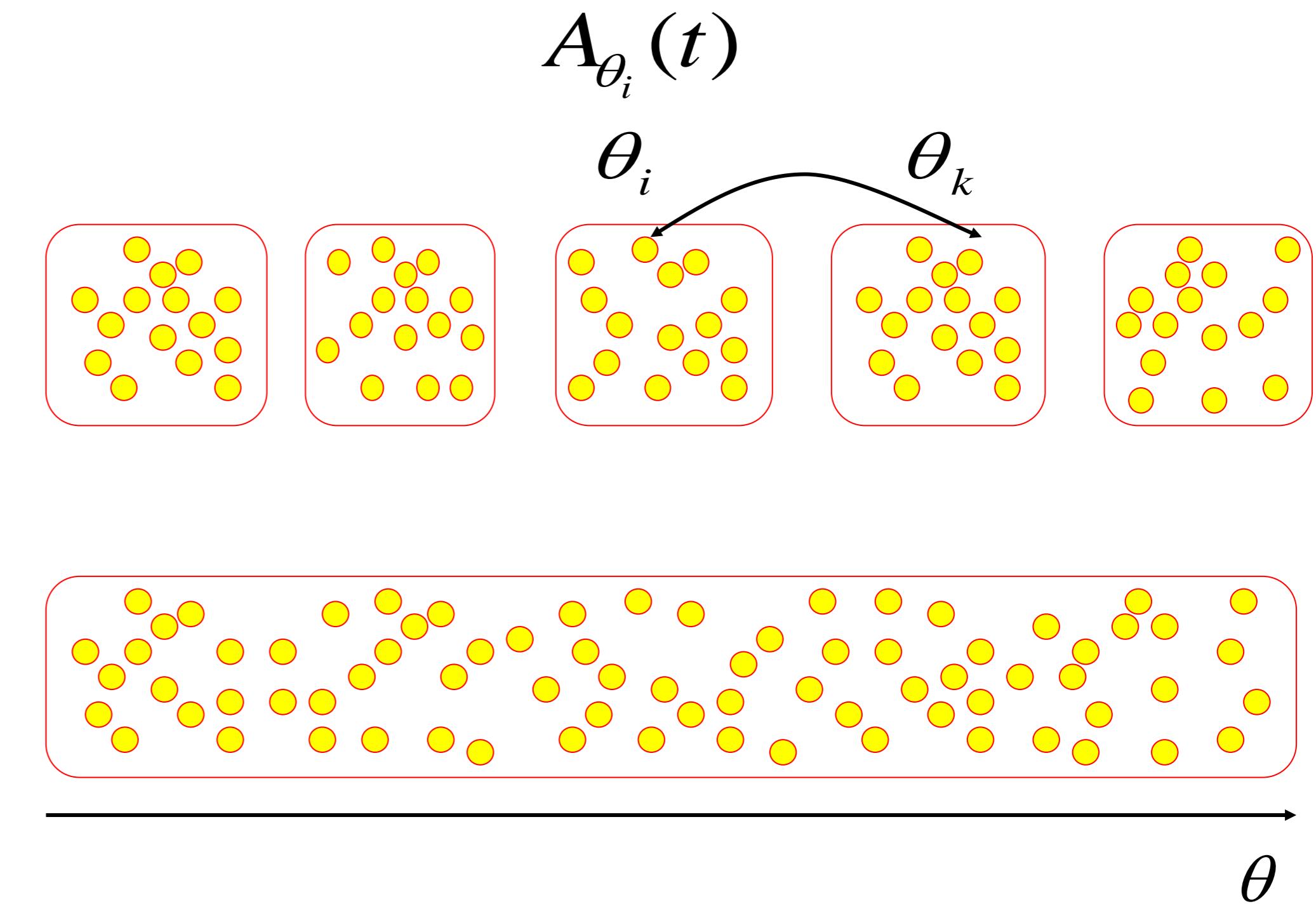
Ch. 18 +  
+Ch. 12.3.7+Ch 15.1-15.2.3

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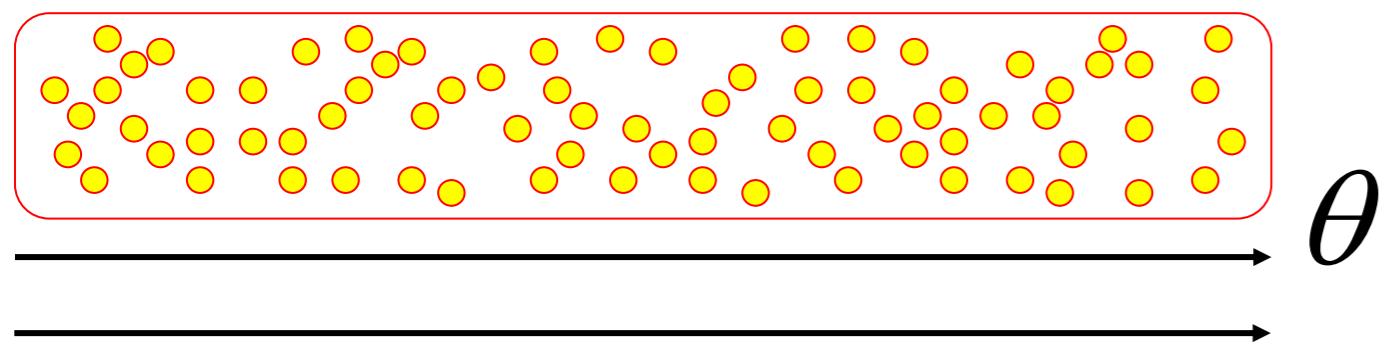
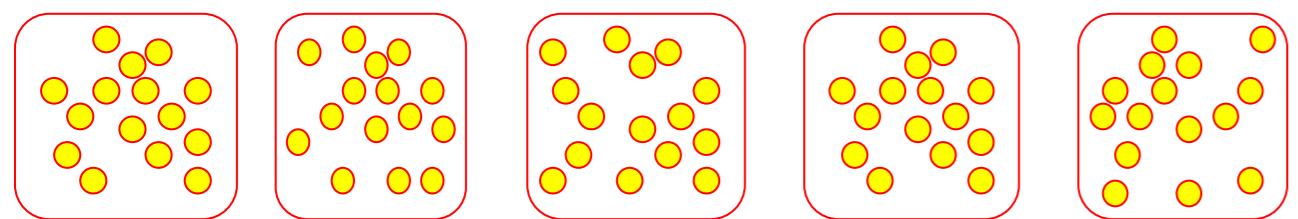
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4. Spatial continuum (model)
  - field equations
5. Solution types
  - uniform solution
  - bump solution
6. Perception
7. Head direction cells

## 4. multiple populations $\rightarrow$ continuum



**Mathematical aim:**  
perform continuum limit

## 4. multiple populations → continuum



## 4. Field equation (continuum model)

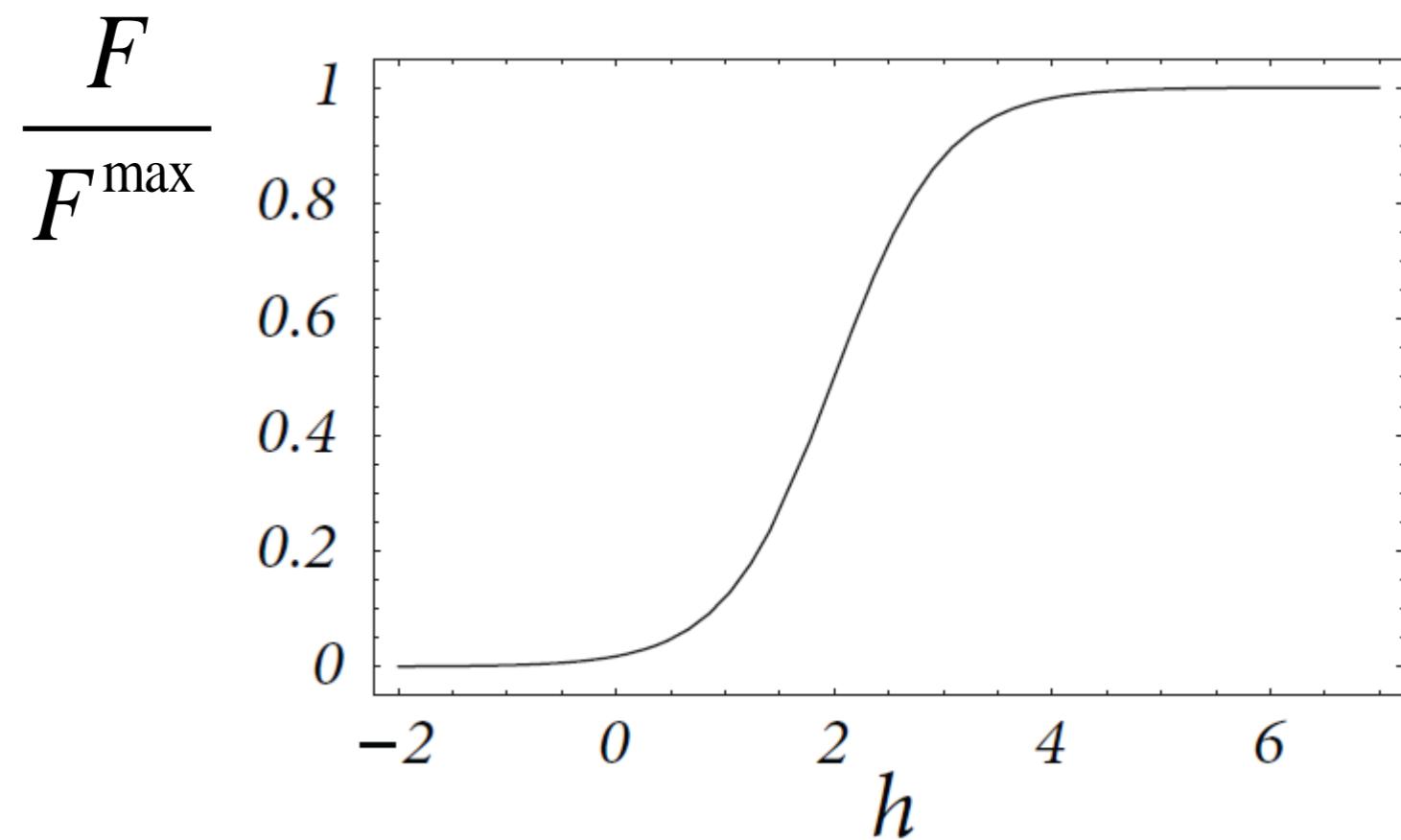
Wilson and Cowan, 1973

Population activity

$$A(x, t) = F(h(x, t))$$

Membrane potential caused by input

$$\tau \frac{d}{dt} h(x, t) = -h(x, t) + R I(x, t)$$



## 4. Field equation (continuum model)

Wilson and Cowan, 1973

Population activity

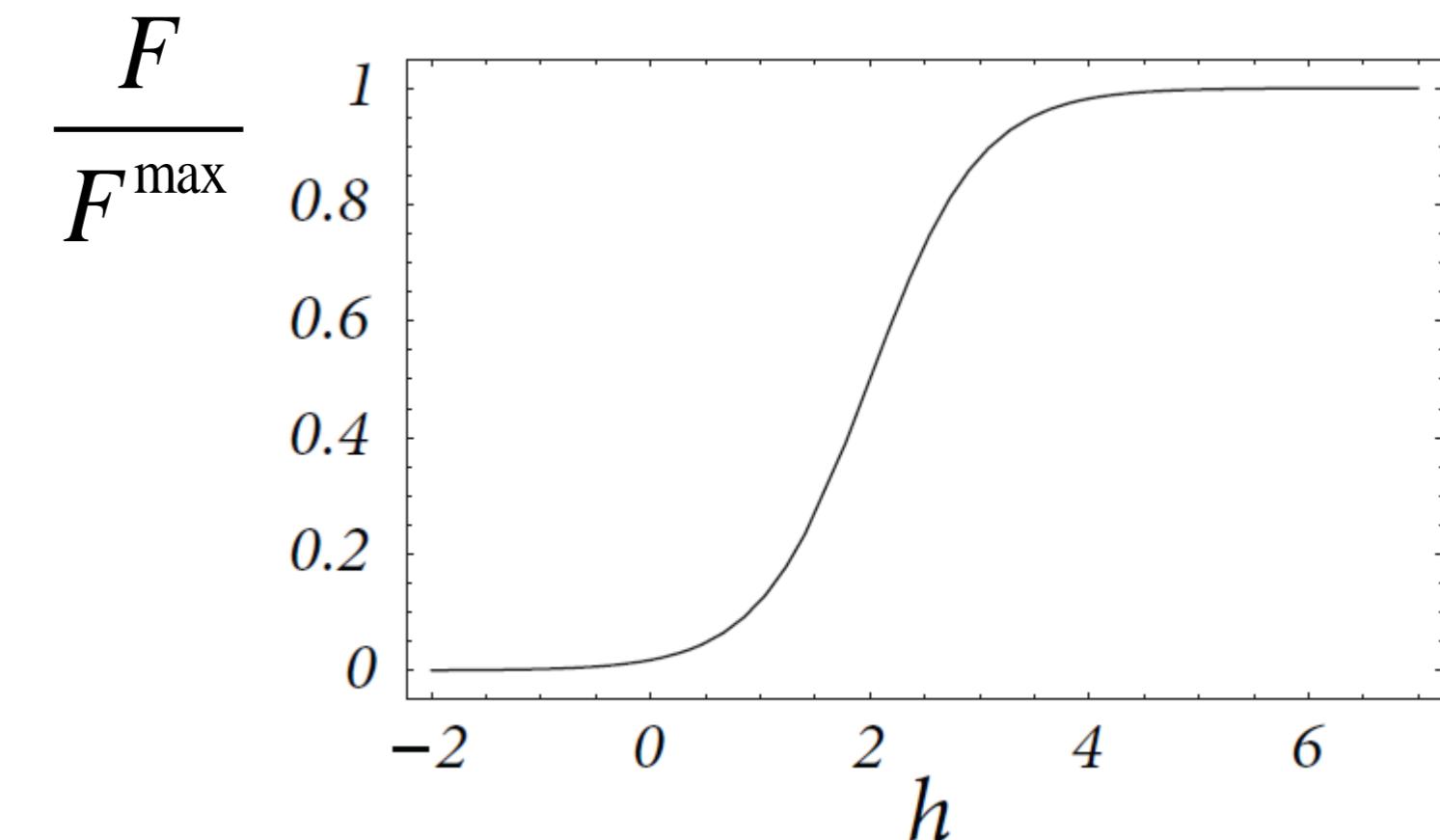
$$A(x, t) = F(h(x, t))$$

Membrane potential caused by input

$$\tau \frac{d}{dt} h(x, t) = -h(x, t) + R I(x, t)$$

$$I(x, t) = I^{ext}(x, t) + I^{netw}(x, t)$$

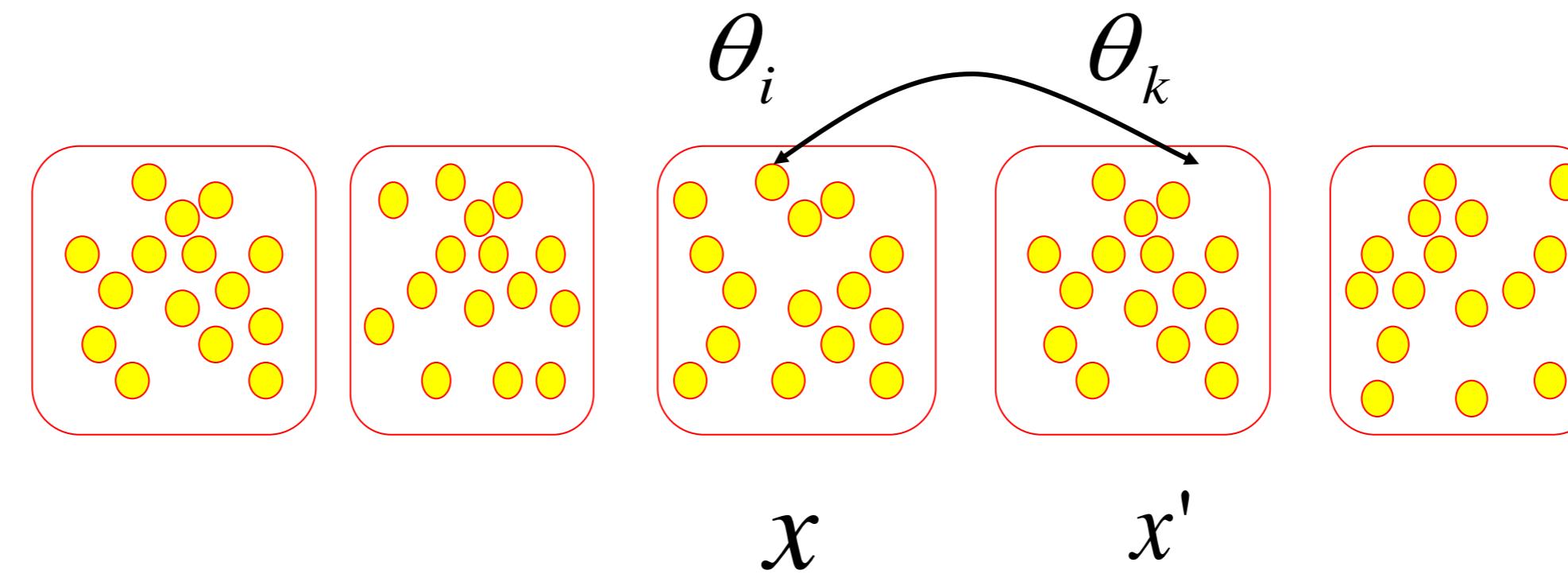
$$I^{netw}(x, t) = d \int w(x - x', t) A(x', t) dx'$$



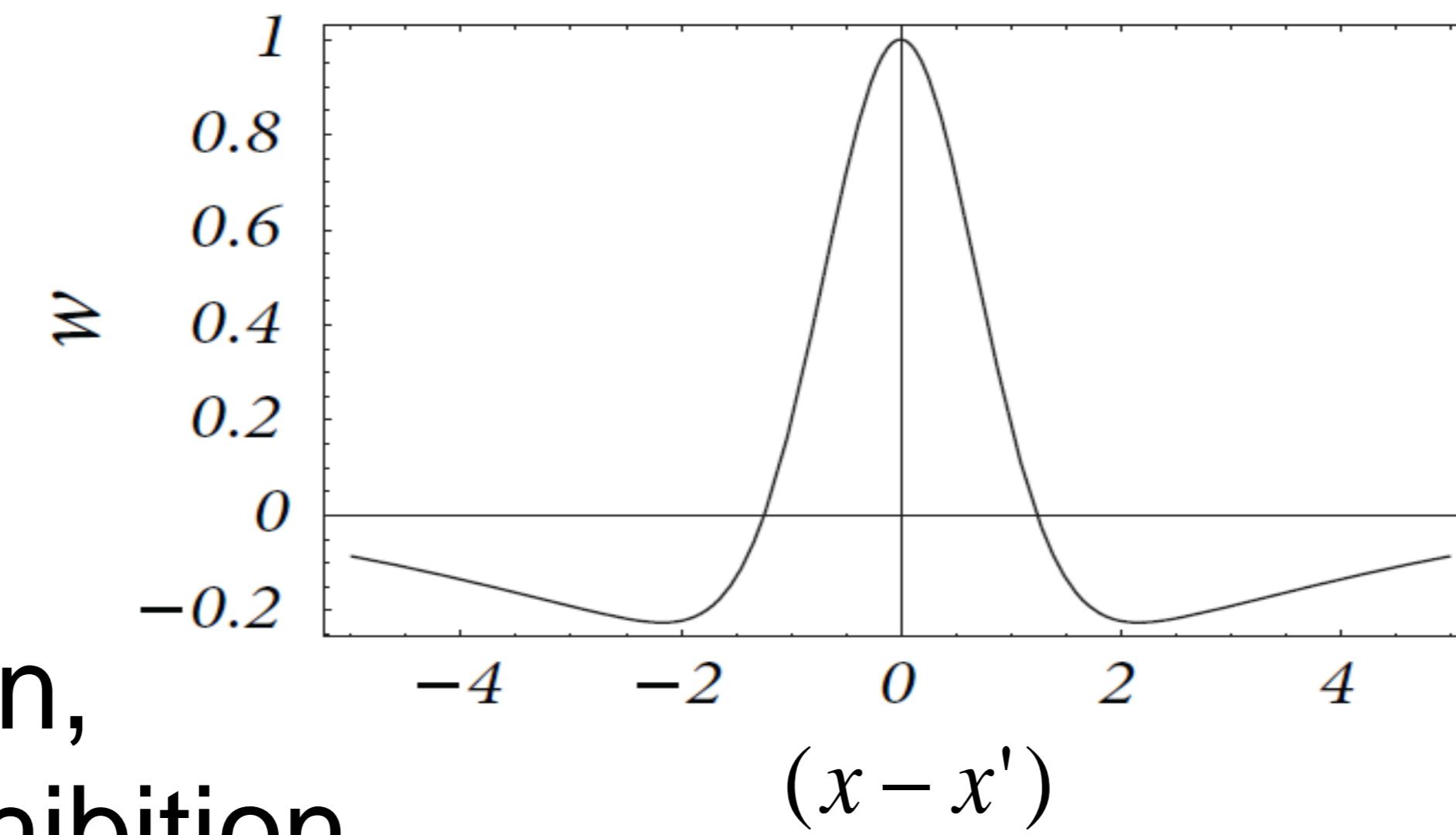
$$\tau \frac{d}{dt} h(x, t) = -h(x, t) + R I^{ext}(x, t) + d \int w(x - x') F(h(x', t)) dx'$$

1 field = 1 integro-differential equation

## 4. coupling across continuum: Mexican hat



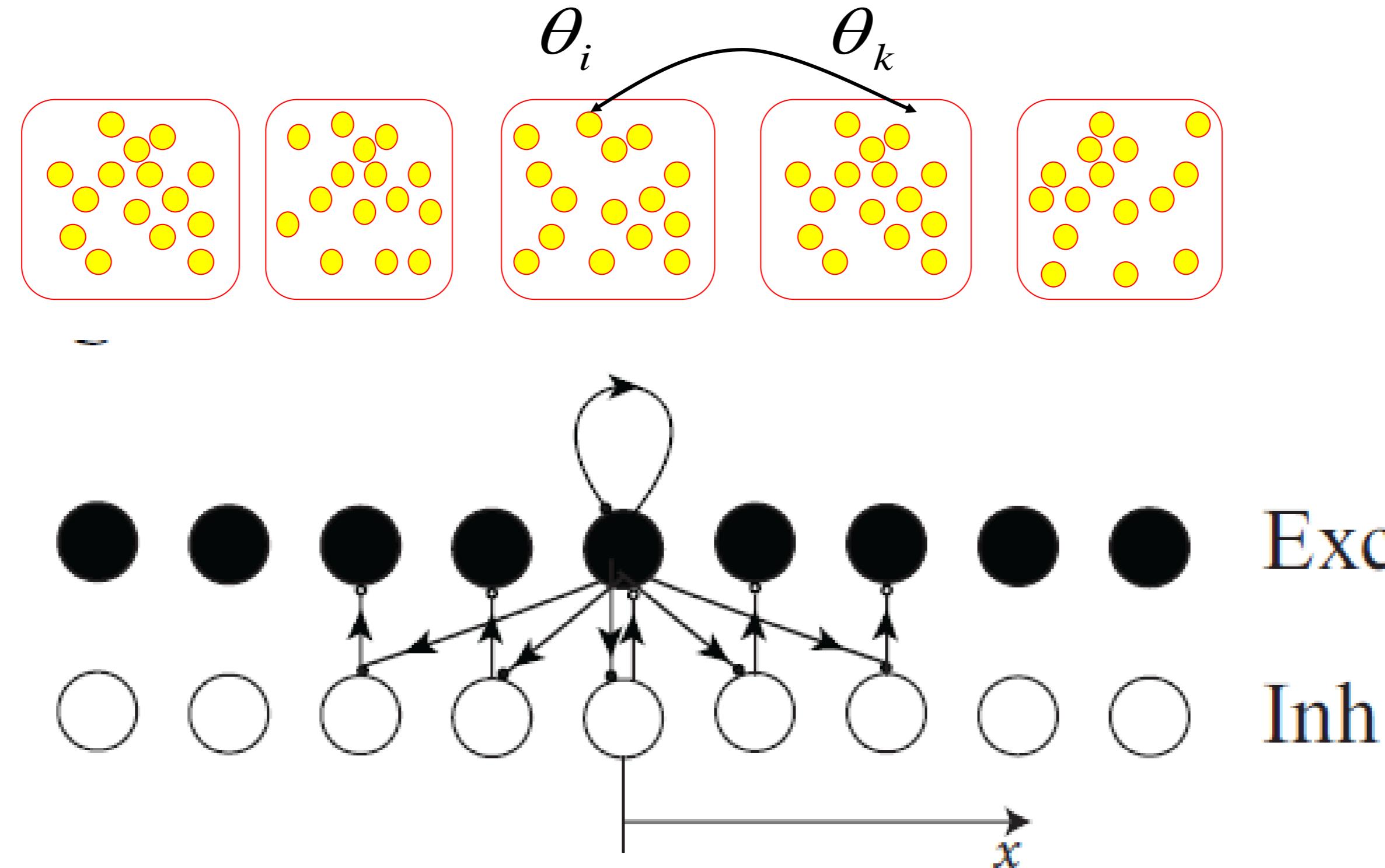
**Mexican hat**



local excitation,  
long-range inhibition

$$w(x, x') = w(|x - x'|)$$

## 4. more realistic cortical coupling



Effective long-range negative interaction with local inhibition

## 4. Summary: Field equations and coupling

$$\tau \frac{d}{dt} h(x, t) = -h(x, t) + RI^{ext}(x, t) + d \int w(x - x') F(h(x', t)) dx'$$

- field equations = population activity models in the spatial continuum
- coupling often distance-dependent  
 $w(x, x') = w(|x - x'|)$
- activity  $A = F(h(t))$
- effective long-range inhibition instead of local inhibitory neurons
- variable  $x$  can represent space or abstract quantity (e.g., orientation)

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## Continuum models: Cortical fields and perception

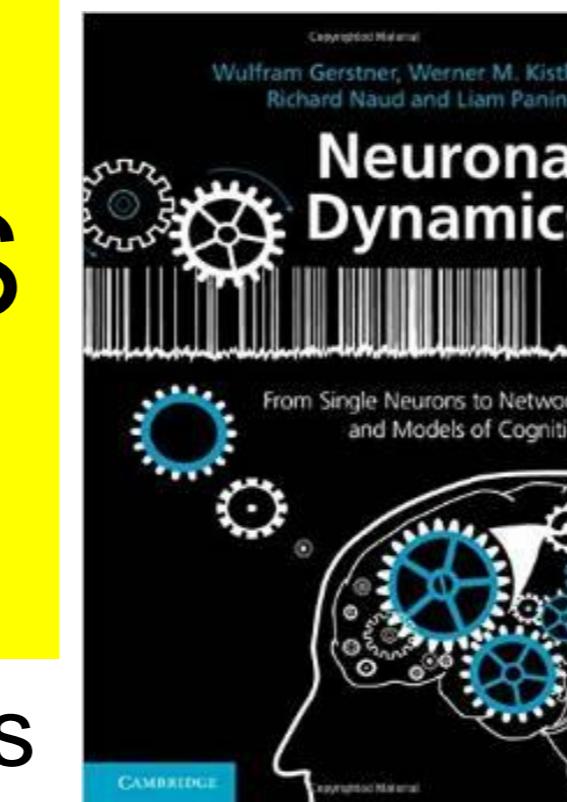
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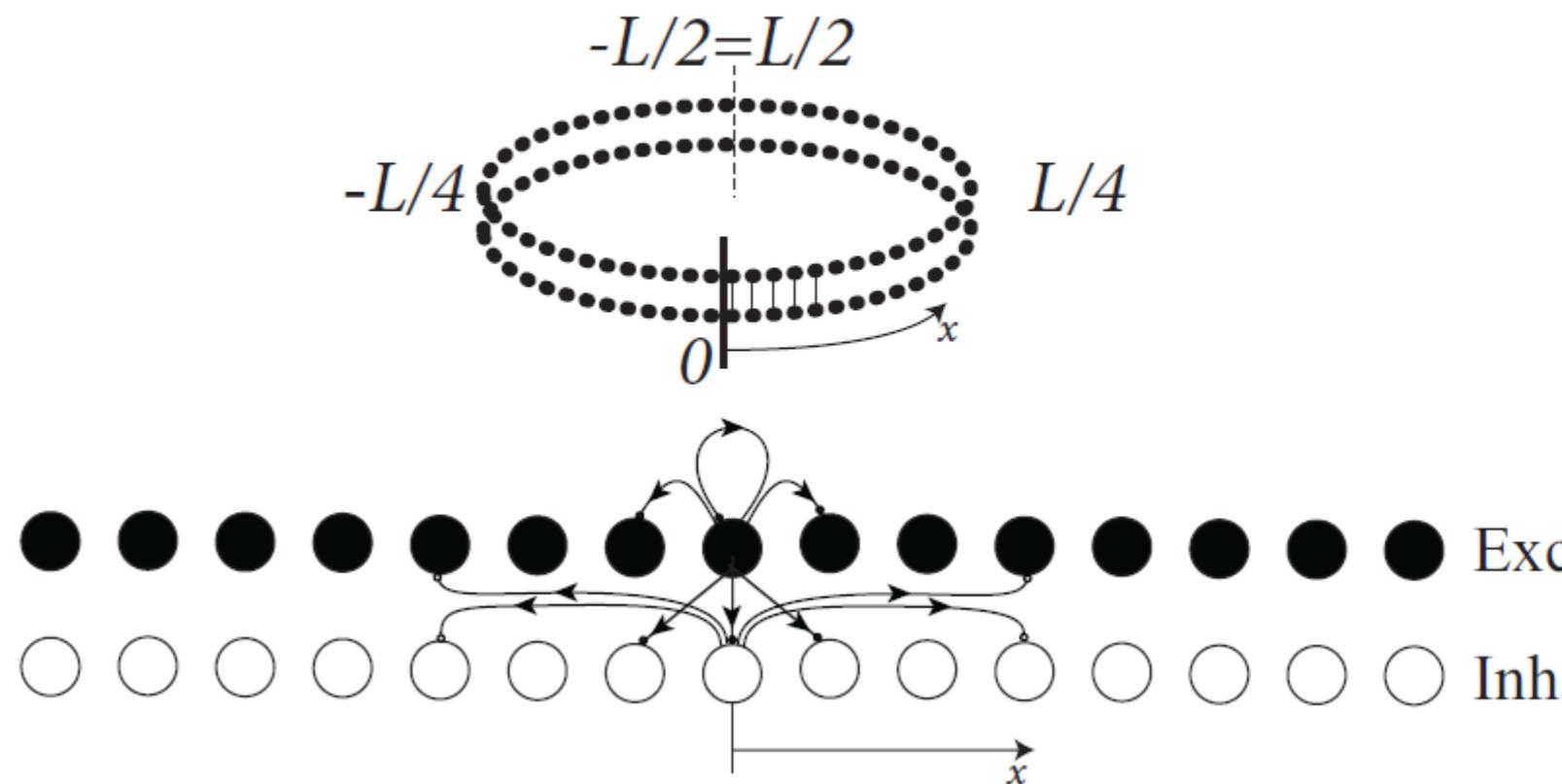


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  - field equations
- 5. Solution types**
  - uniform solution
  - bump solution
- 6. Perception**
- 7. Head direction cells**

## 5. Two Solution Types (ring model)

Coupling:

A



Input-driven regime

B

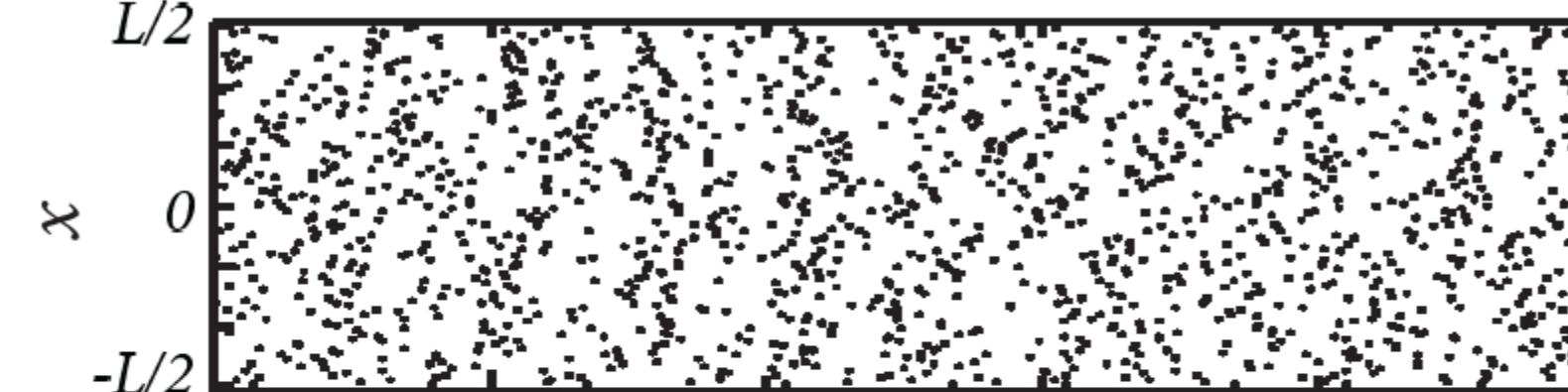
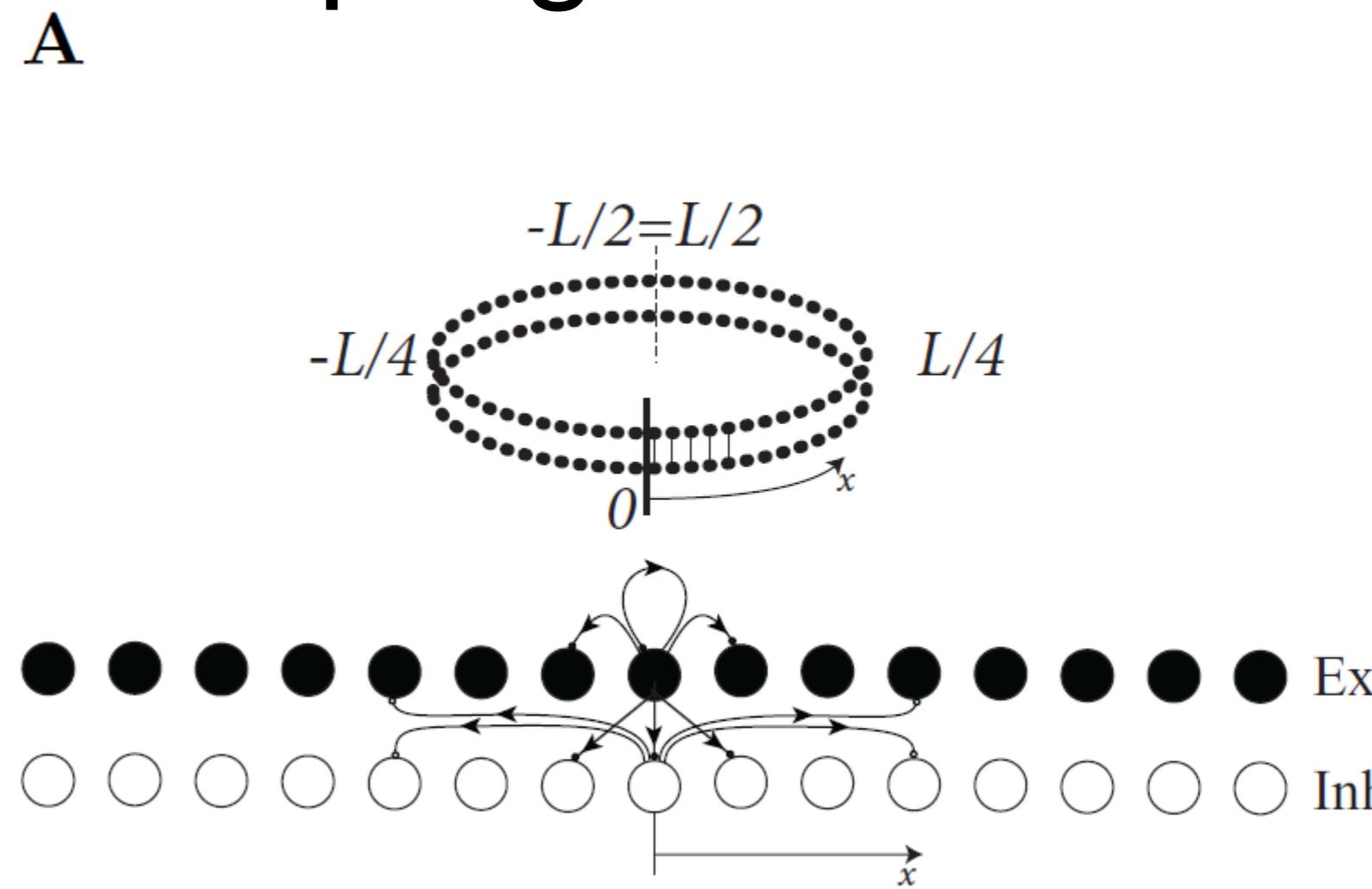


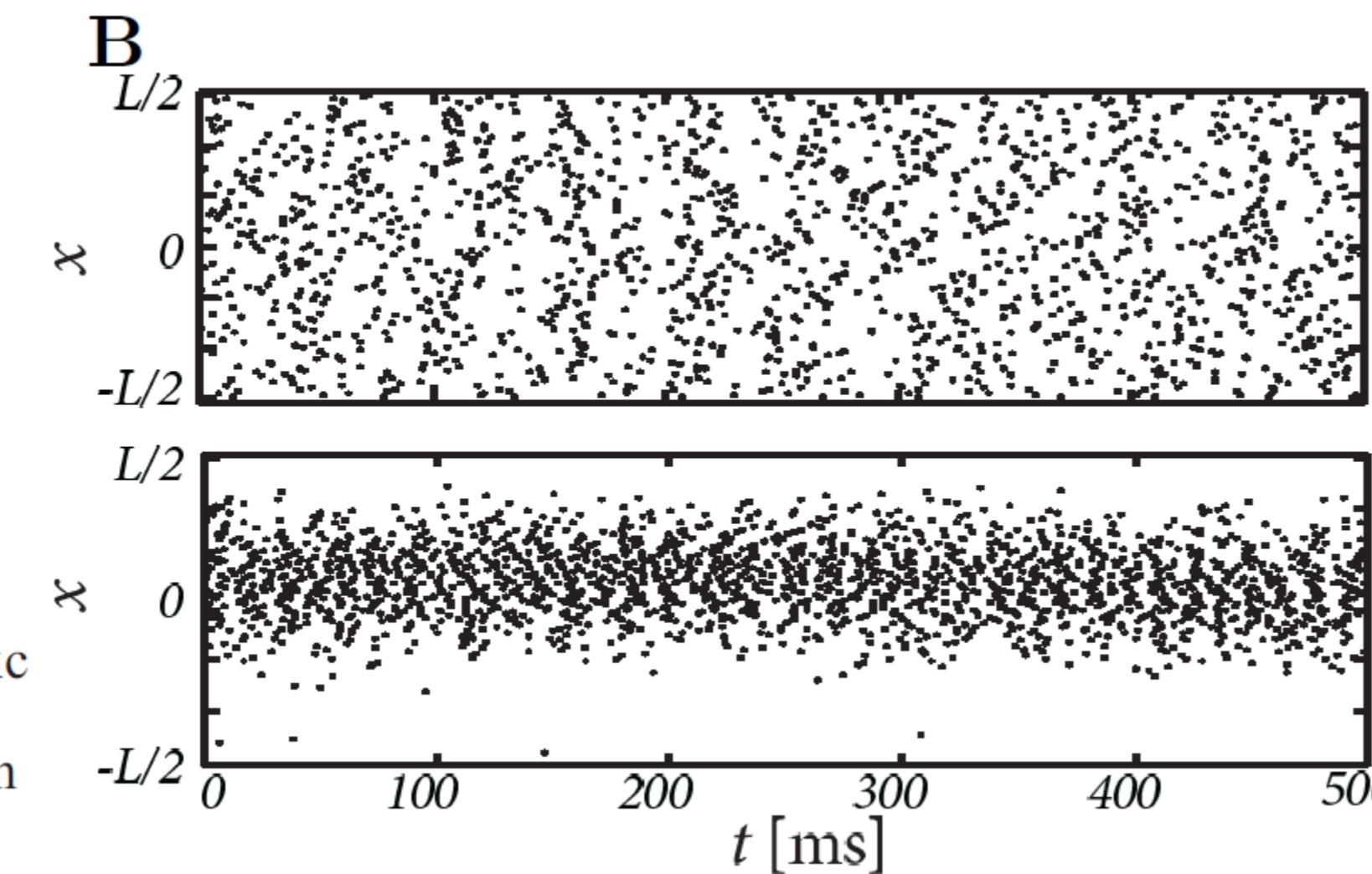
Image: *Neuronal Dynamics*,  
Gerstner et al.,  
Cambridge Univ. Press (2014),

## 5. Two Solution Types (ring model)

Coupling:



Input-driven regime



Bump attractor regime

## 5. Solution type A: homogeneous solution=input driven regime

Field Equations:  
*Wilson and Cowan, 1973*

### Edge enhancement



Weak lateral connectivity

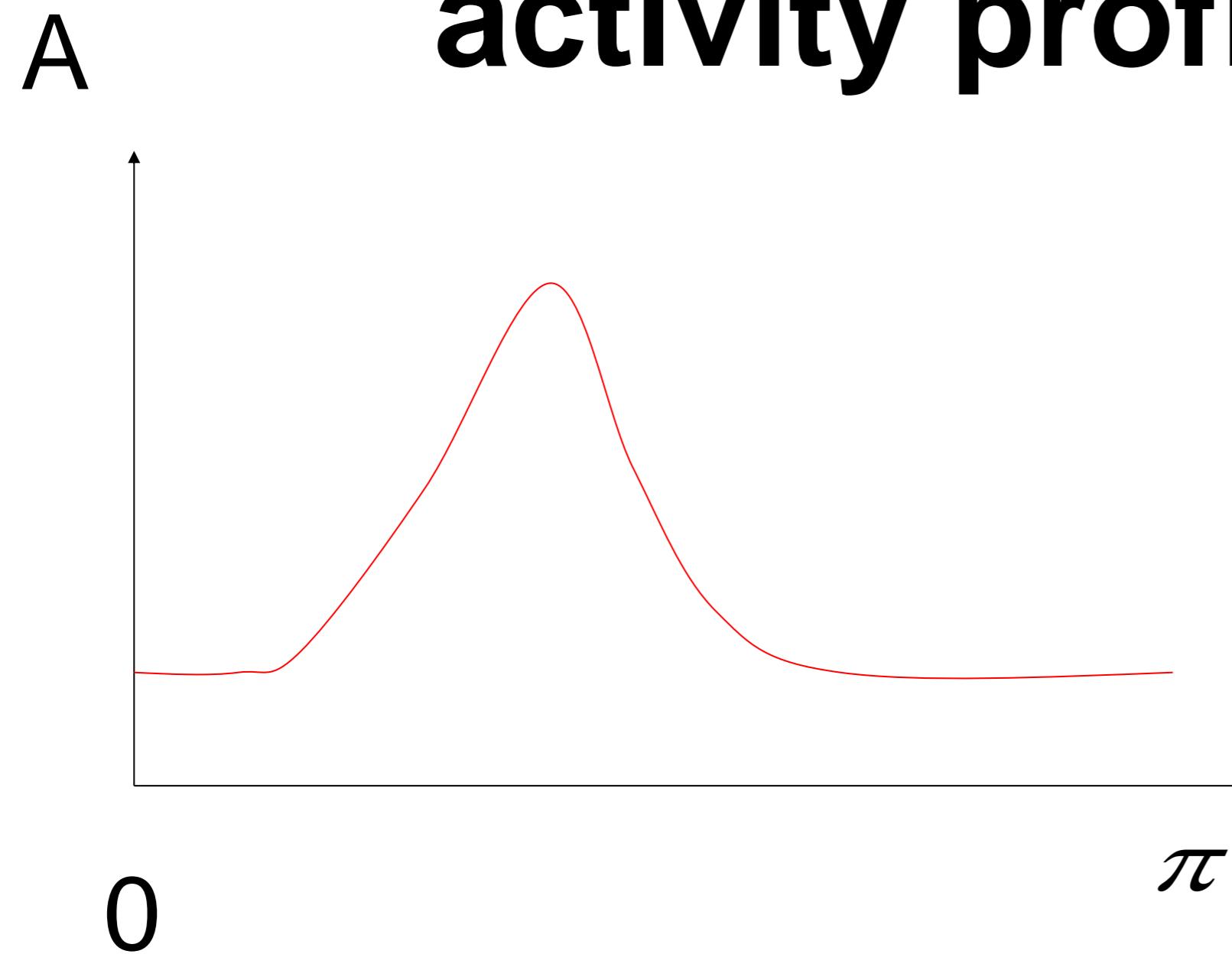
Possible application  
visual cortex cells:

(see next part)

## 5. Solution type B: bump solution

Field Equations:  
*Wilson and Cowan, 1973*

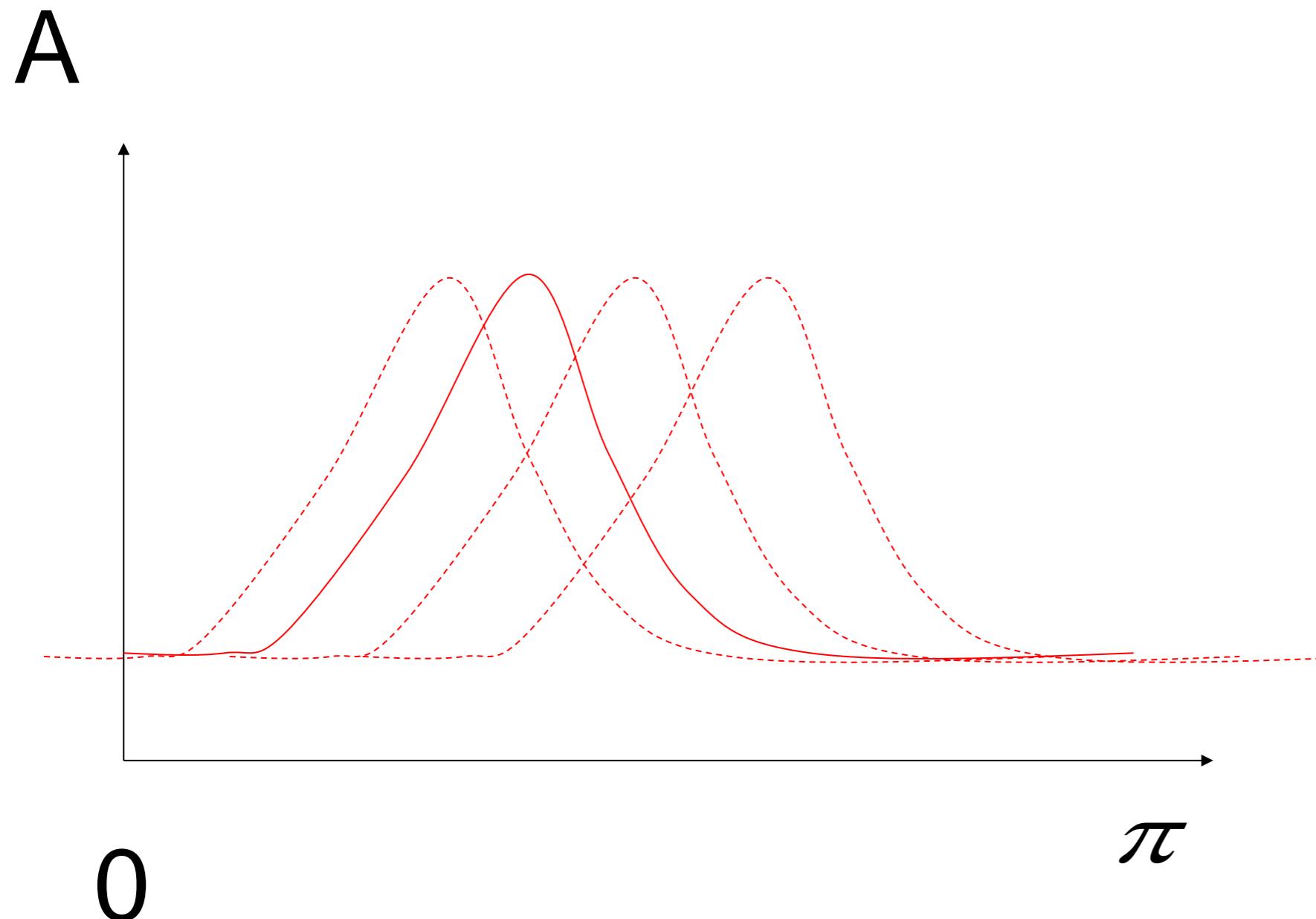
**Bump formation:  
activity profile in the absence of input**



strong lateral connectivity

## 5. Solution type B: bump solution

**Bump formation:  
activity profile in the absence of input**



Field Equations:  
*Wilson and Cowan, 1973*

- strong lateral connectivity;
- long-range inhibition

**Possible application**

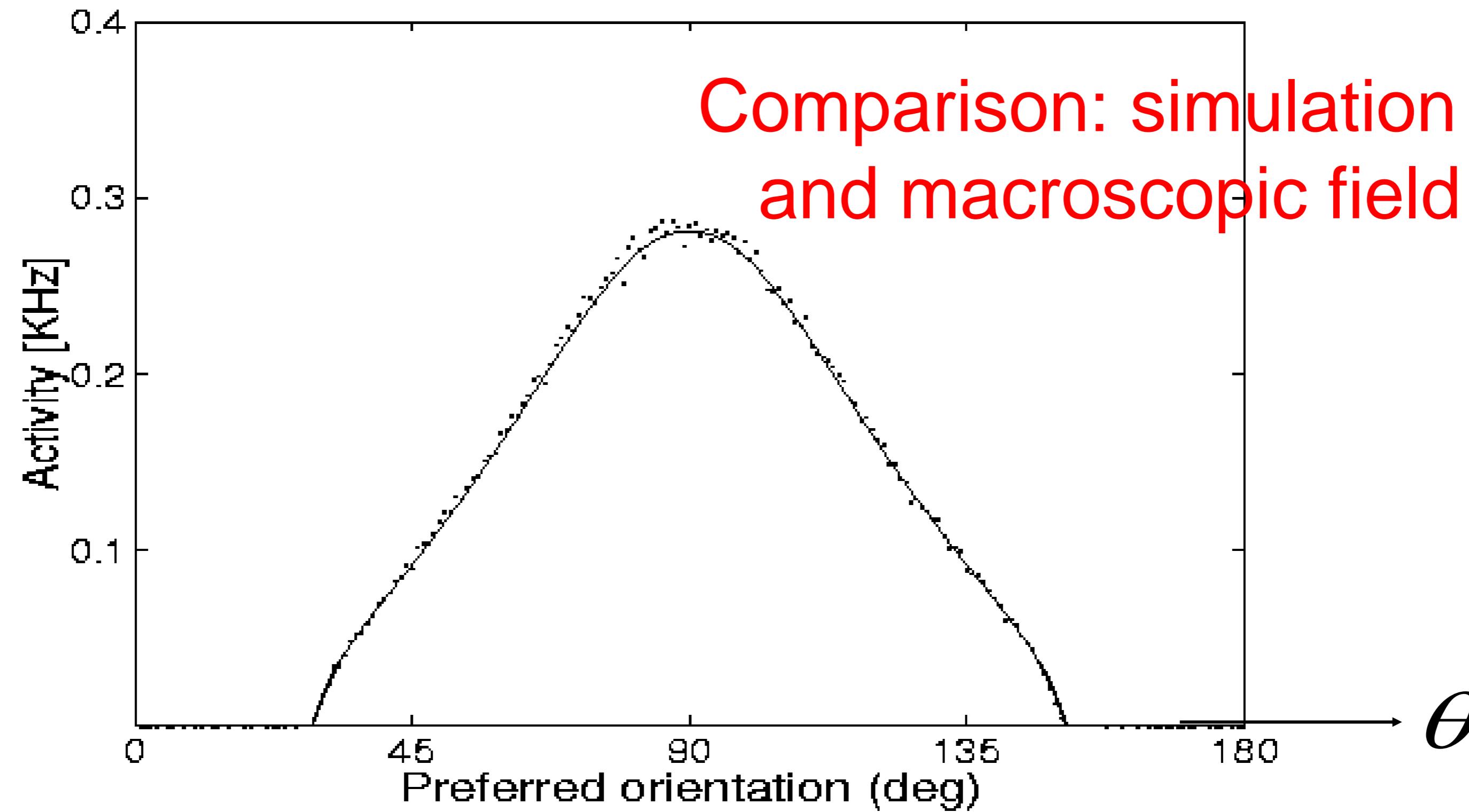
- head direction cells  
→ (see part 7)
- spatial working memory

A Compte, N Brunel, PS Goldman-Rakic, XJ Wang (2000) Synaptic mechanisms and network dynamics underlying spatial working memory, *Cerebral Cortex* 10 (9), 910-923

## 5. Solution type B: bump solution

$$A(\theta, t) = A(\theta)$$

Spiridon&Gerstner



Continuum: stationary profile

Image: *Neuronal Dynamics*,  
Gerstner et al.,  
Cambridge Univ. Press (2014),

# 5. Solution types: multiple bump solutions with local interaction

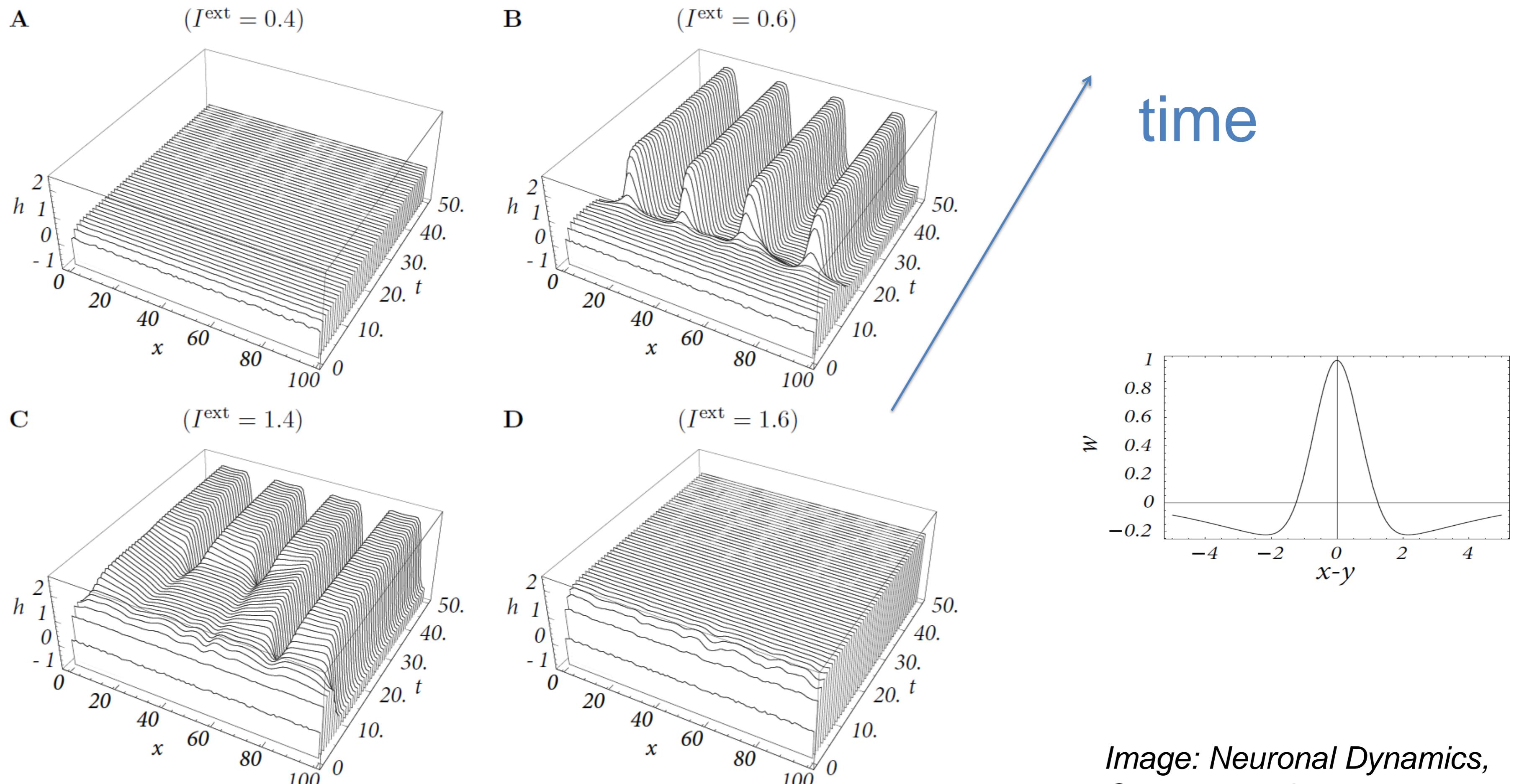


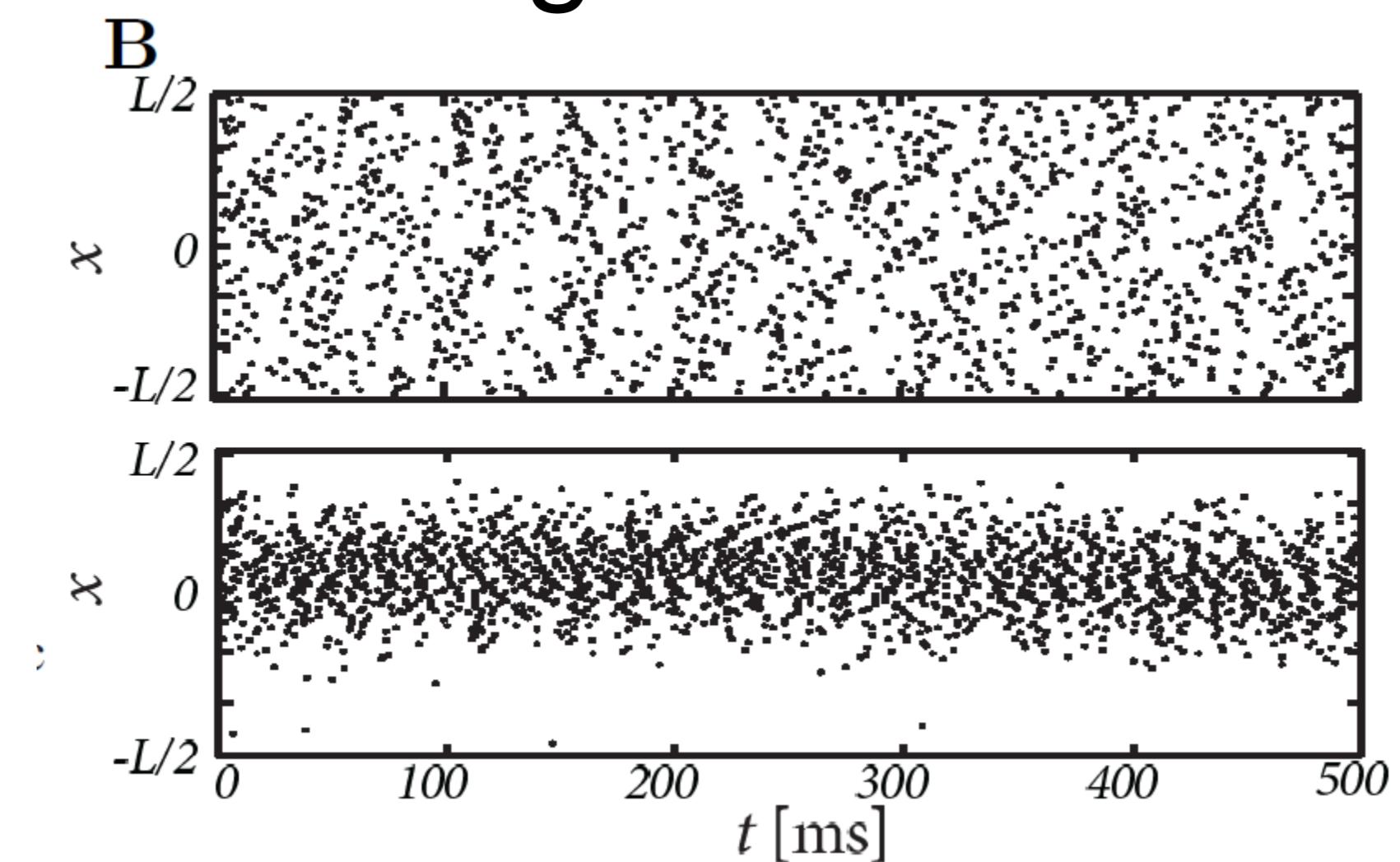
Image: *Neuronal Dynamics*,  
Gerstner et al.,  
Cambridge Univ. Press (2014),

## 5. Two Solution Types (ring model)

### Two stationary solution types:

- homogeneous for flat input
  - responds to input
- bump attractor for flat input
  - moves to location of input

Input-driven regime:  
homogeneous solution



Bump attractor regime

## **Solution of Field equations (1-dimensional ring model)**

- [ ] If a solution exists with a single bump localized around  $x_0$ , there are also bump solutions at other locations.
- [ ] If the interaction is Mexican hat, a stationary solution can have at most a single bump
- [ ] A homogeneous solution (constant in time and space) always exists
- [ ] A homogeneous solution (constant in time and space) is always stable
- [ ] If I increase in a model the spatial scale of inhibition, the activity profile of an existing bump solution becomes broader
- [ ] If I increase in a model the amplitude of excitation and the spatial scale of inhibition, a bump solution is more likely to exist

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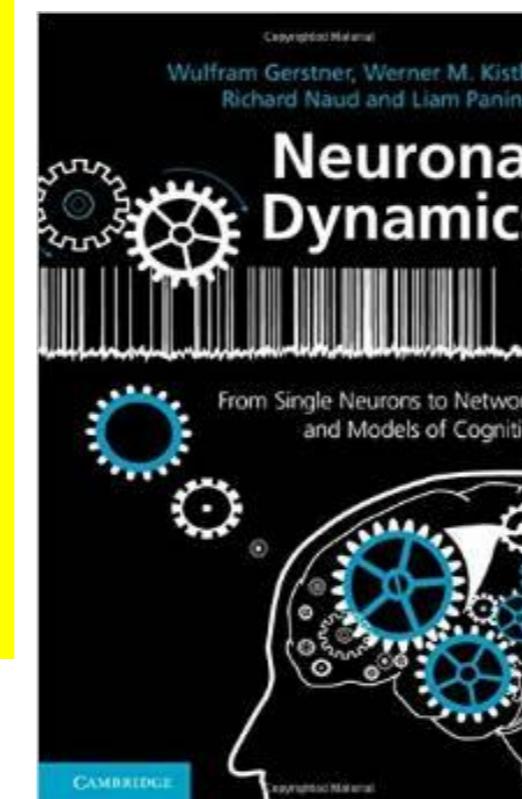
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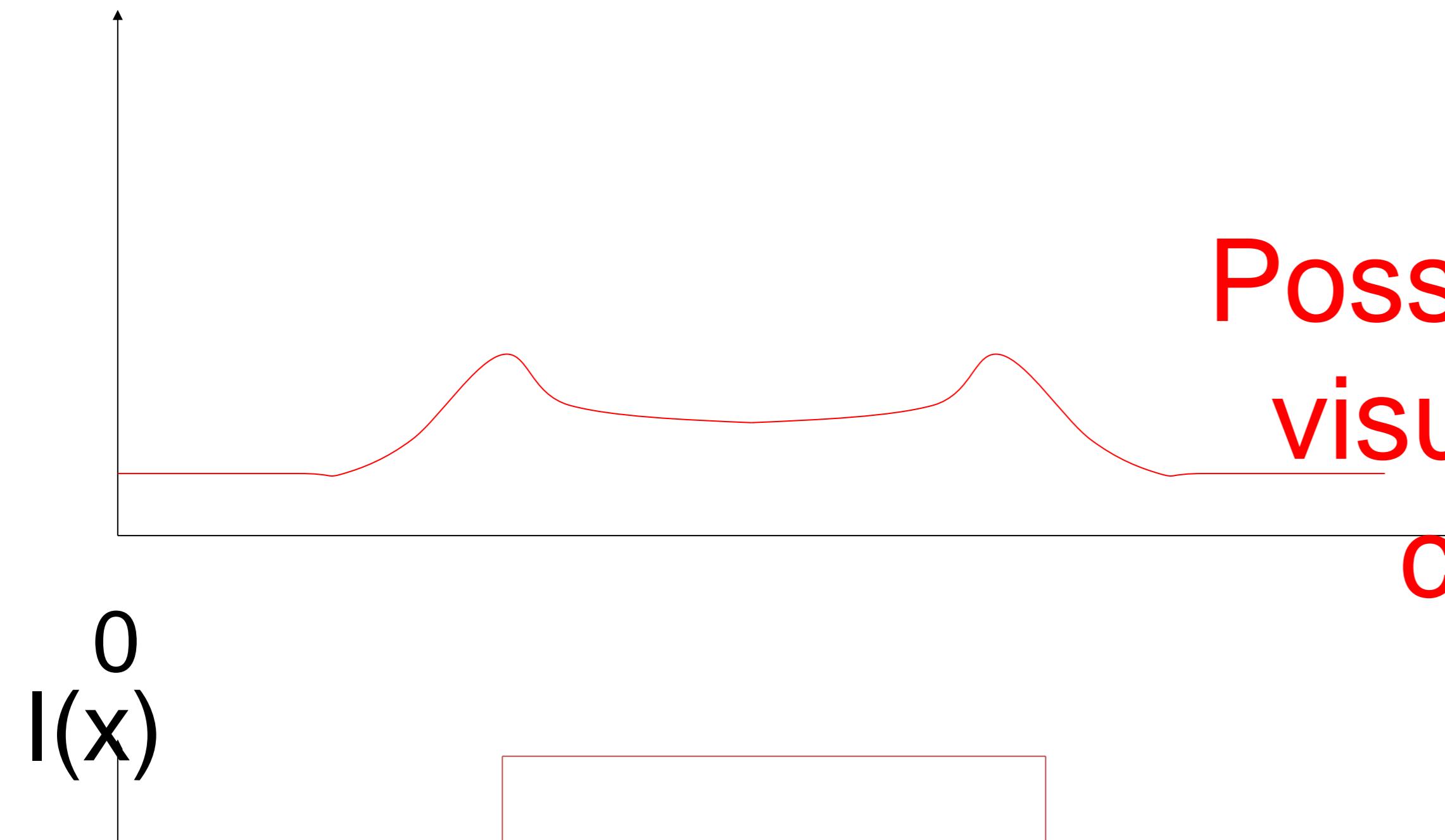


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## 6. homogeneous/input driven solution

### Edge enhancement

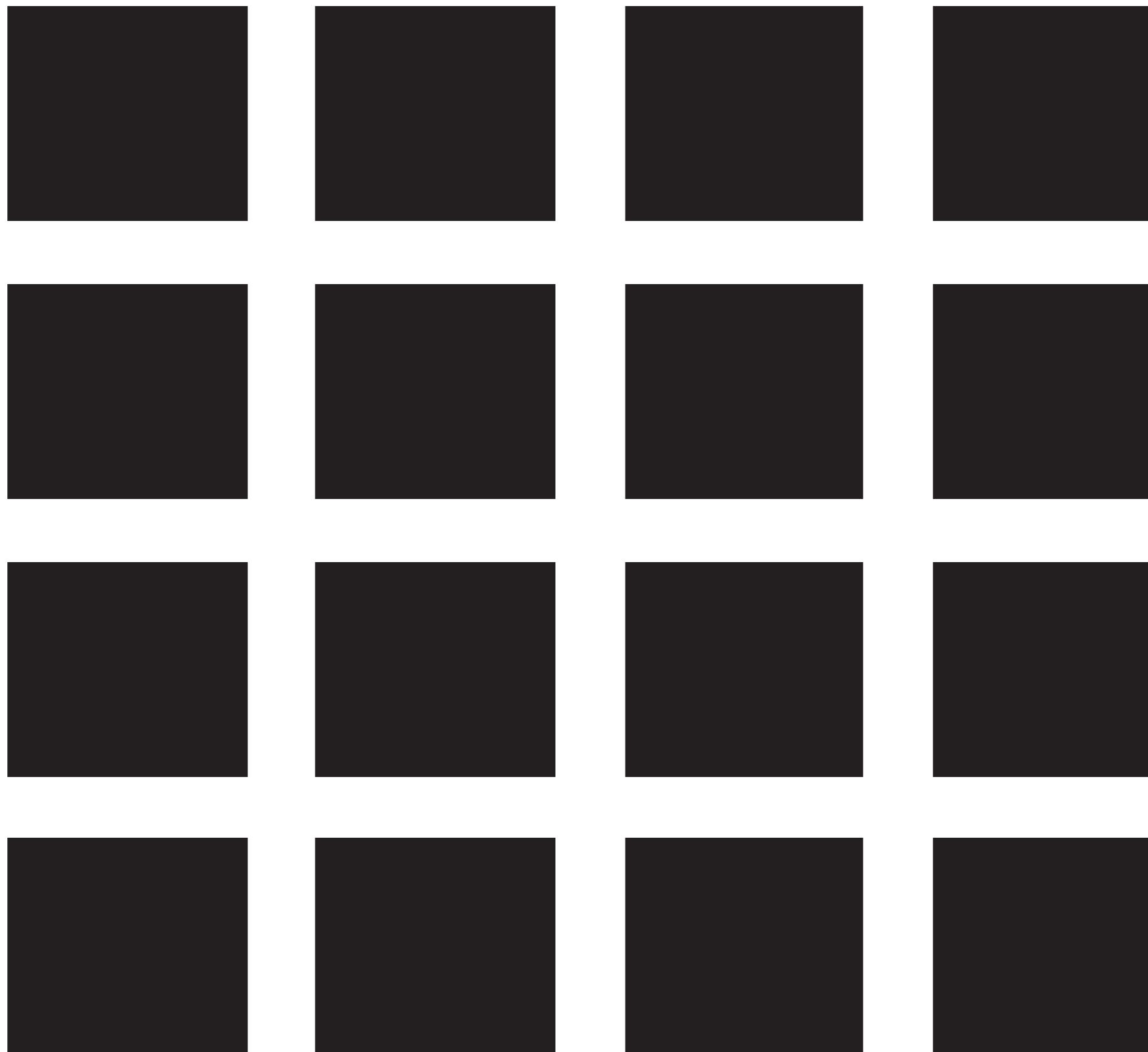
$A(x)$  (Weak lateral connectivity)



Field Equations  
for edge enhancement  
*Wilson and Cowan, 1973*  
*Grossberg, 1973*

Possible application to  
visual cortex cells:  
contrast enhancement in  
- orientation  
- location

# 6. Perception - grid illusion



*Image: Neuronal Dynamics,  
Gerstner et al.,  
Cambridge Univ. Press (2014),*

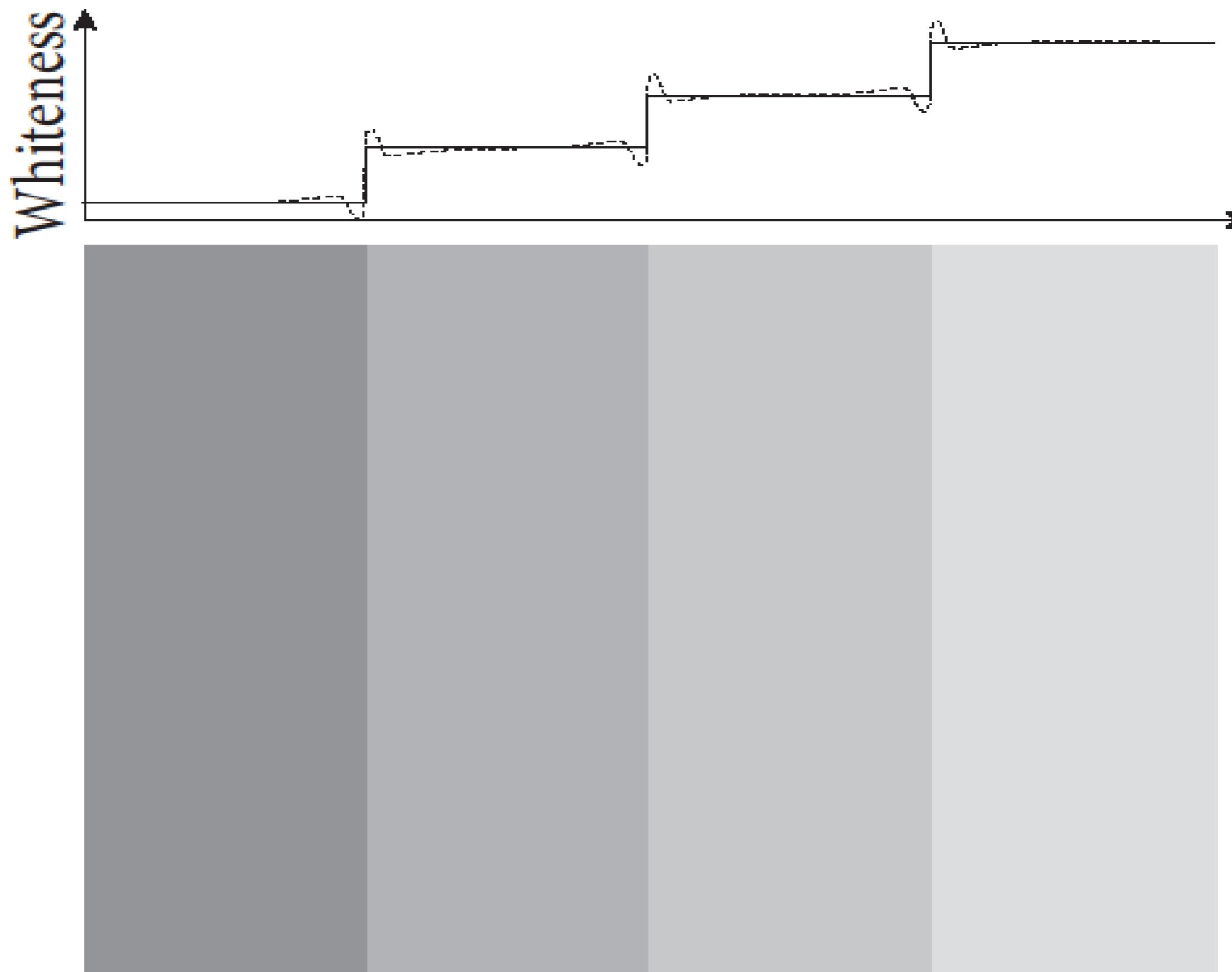
# 6. Perception - grid illusion



*Image: Neuronal Dynamics,  
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# 6. Perception – Mach bands

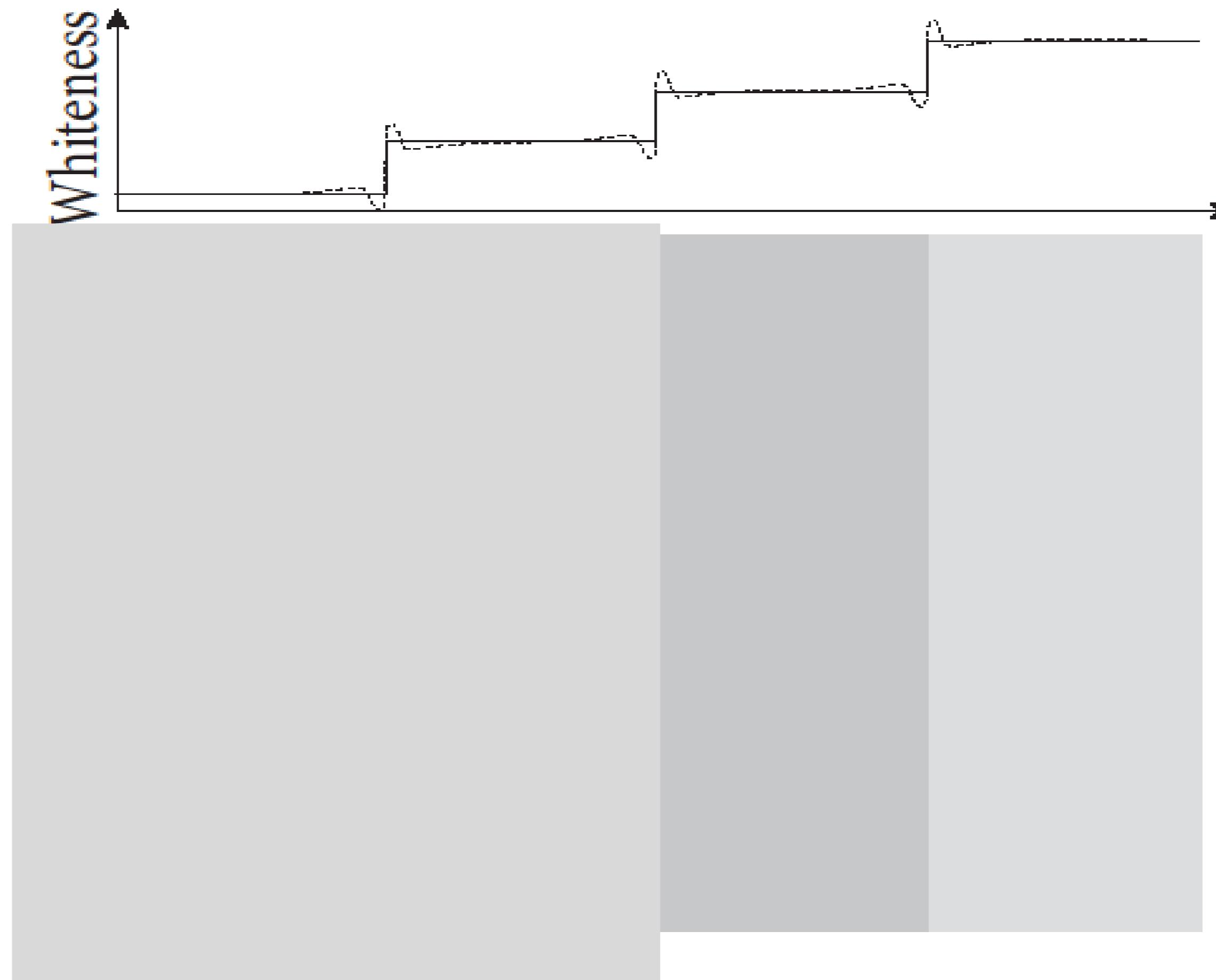
Mach, 1865, 1906



*Image: Neuronal Dynamics,  
Gerstner et al.,  
Cambridge Univ. Press (2014),*

# 6. Perception – Mach bands

Mach, 1865, 1906



*Image: Neuronal Dynamics,  
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# 6. Mach bands in a continuum model

Mexican-hat coupling

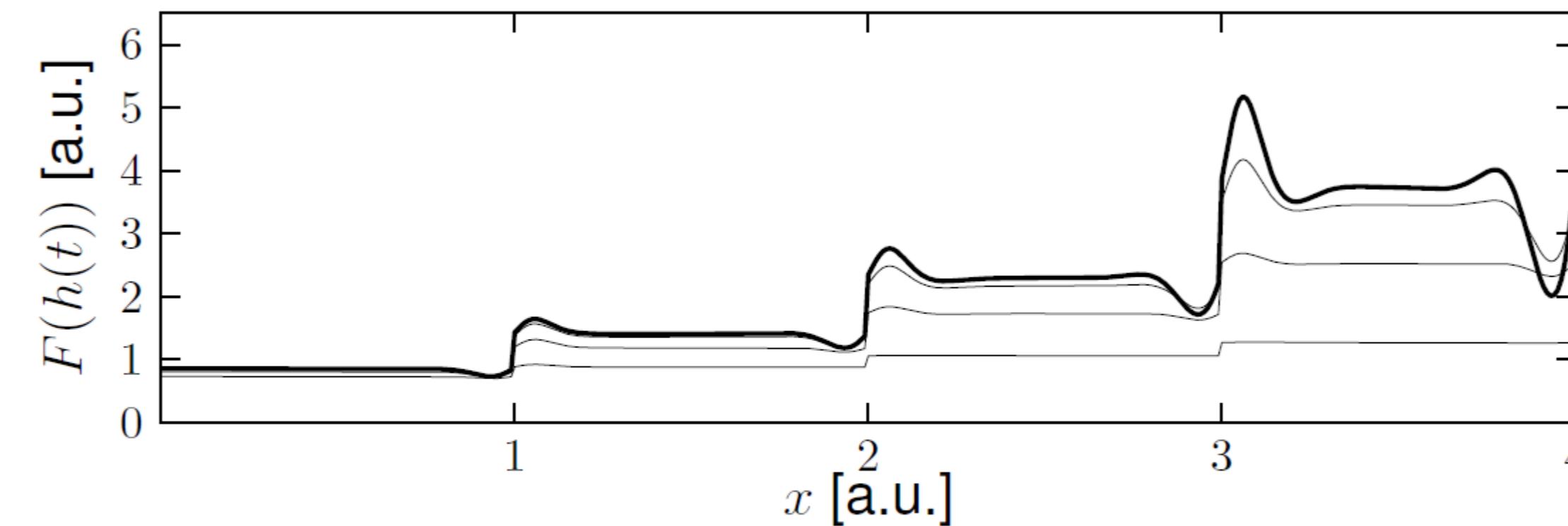
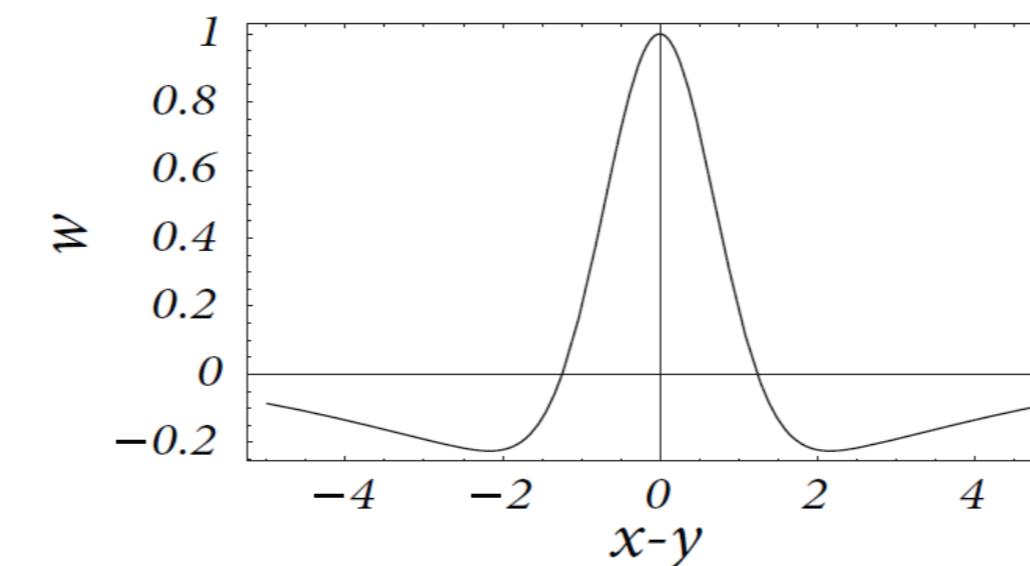
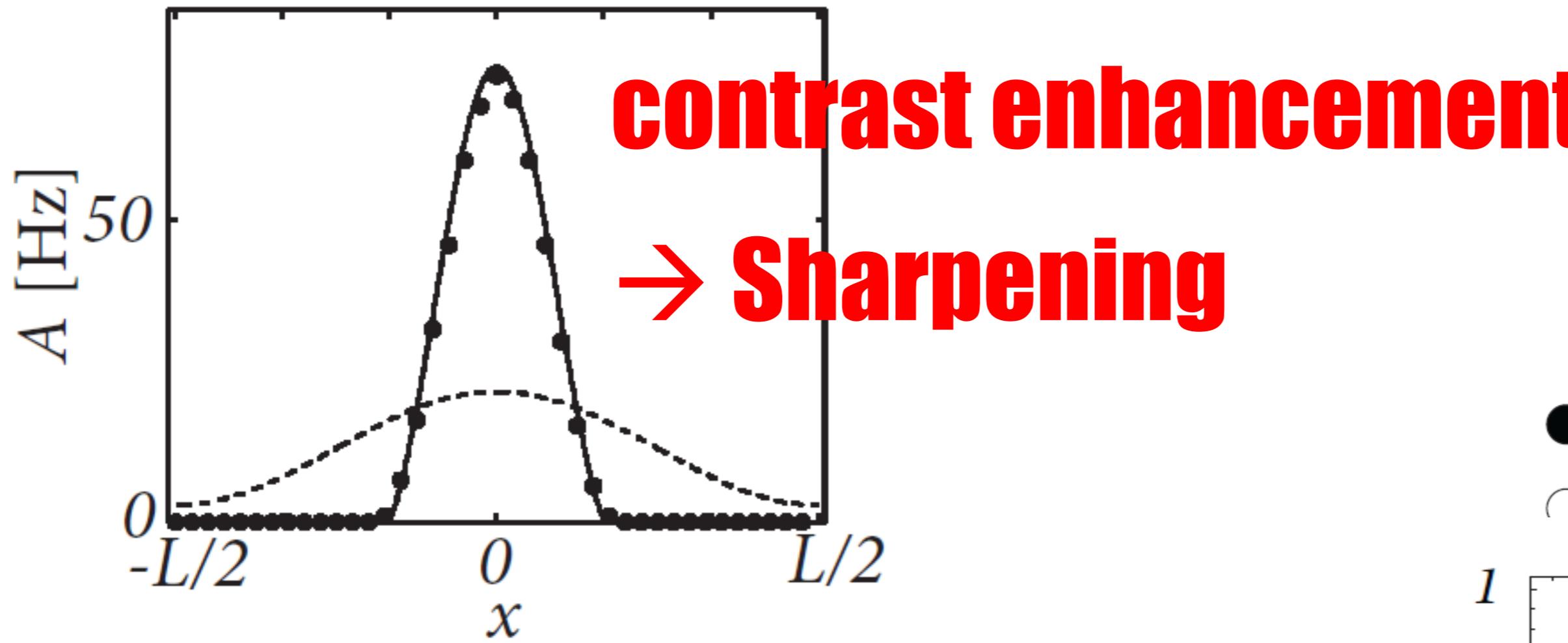


Fig. 18.9: A. Mach bands in a field model with mexican hat

*Image: Neuronal Dynamics,  
Gerstner et al.,  
Cambridge Univ. Press (2014),*

# 6. Field models and Perception: contrast enhancement

B



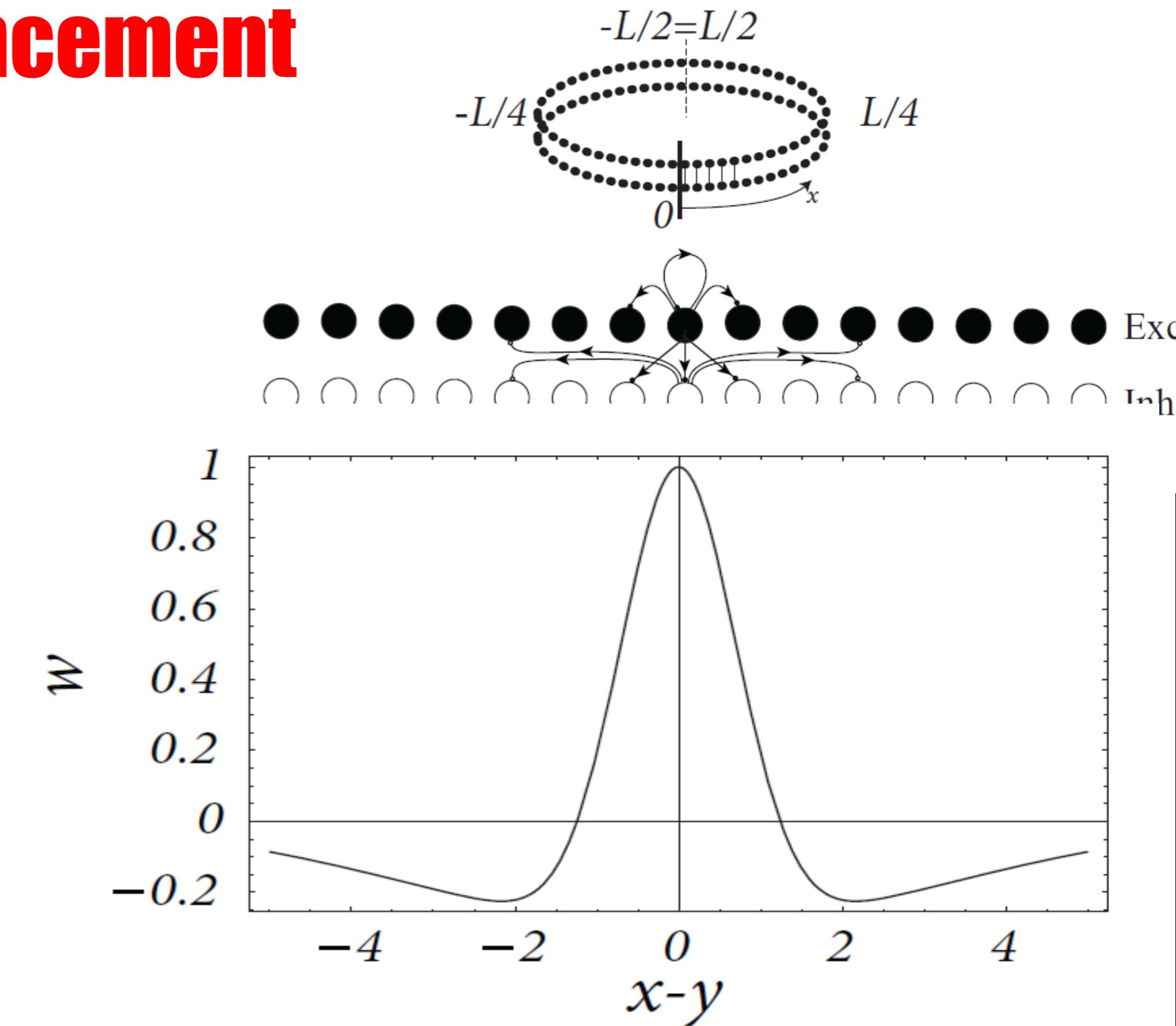
Shriki et al. (2003):

Ring model in input-driven regime,  
driven by broad input (dashed line),  
causes sharp activity bump;

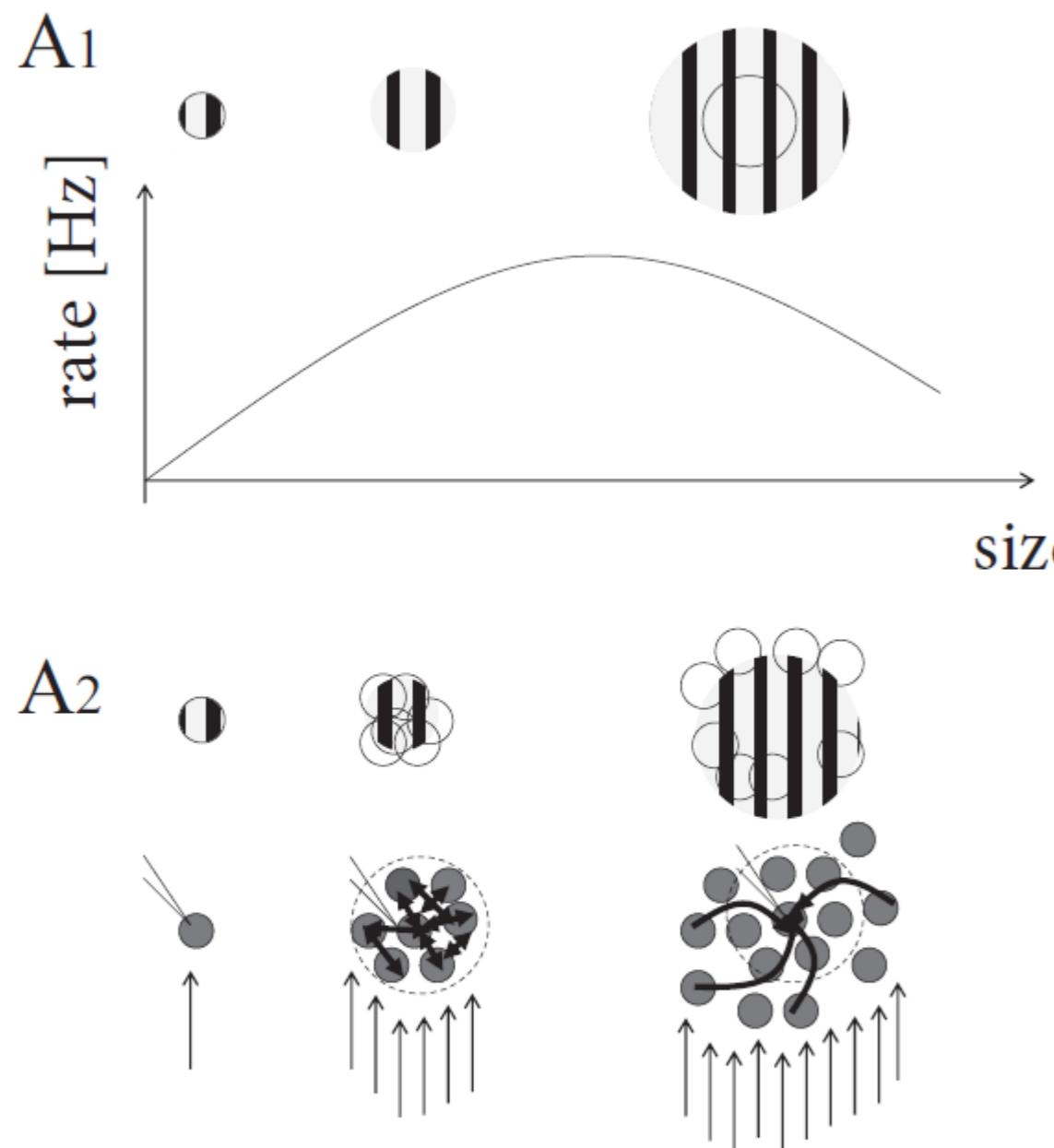
See also: Ben-Yishai et al. 1995;

Hansel and Sompolinsky, 1998

A

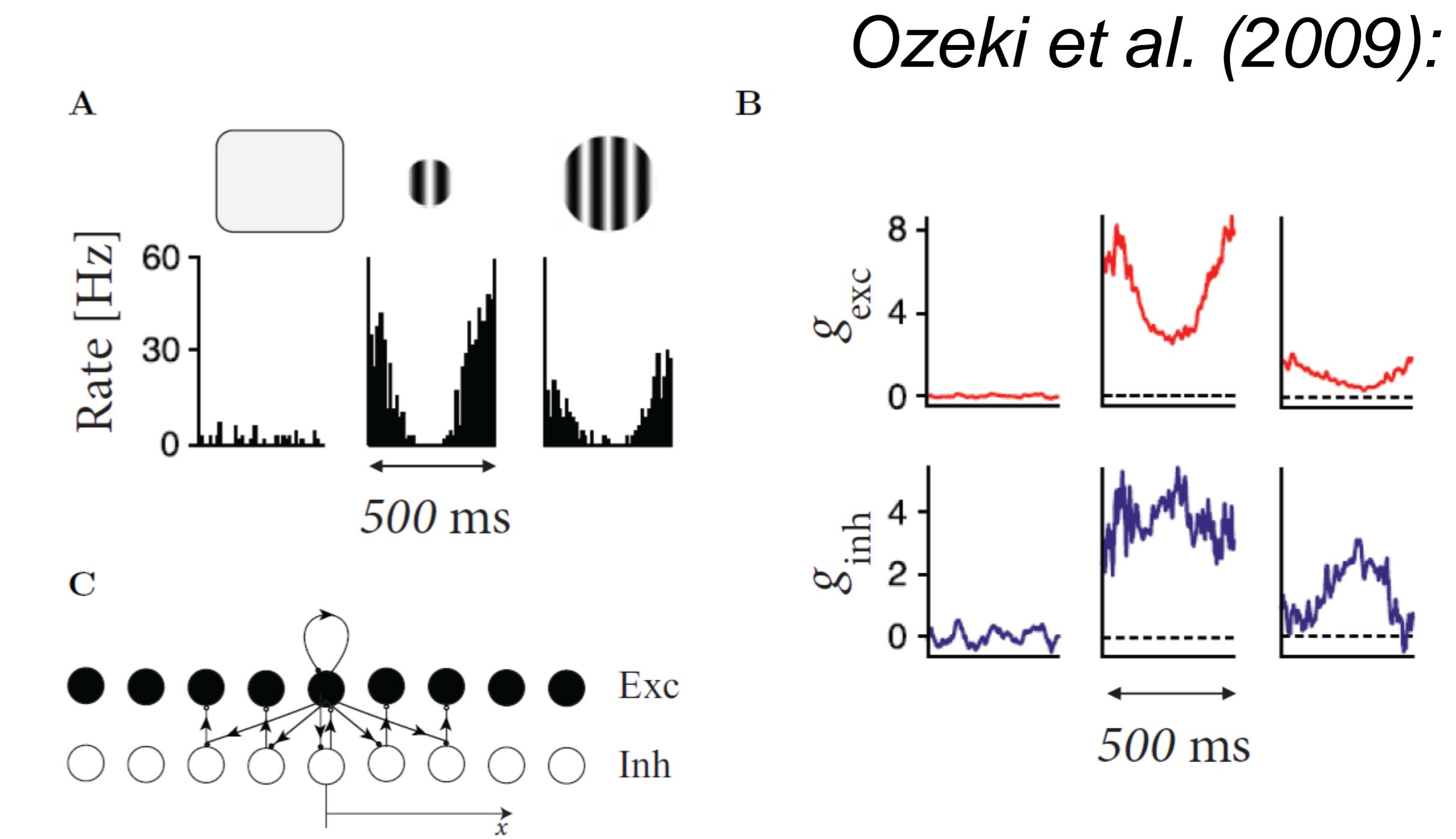


# 6: Field models and Perception: surround suppression



**Fig. 18.12:** Surround suppression.

*Image: Neuronal Dynamics,  
Gerstner et al.,  
Cambridge Univ. Press (2014),*



**Fig. 18.13:** Network stabilized by local inhibition. The schematic model could potentially explain why larger gratings lead not only to less excitatory input  $g_{\text{exc}}$ , but also to less inhibitory input  $g_{\text{inh}}$ . **A.** The firing rate as a function of the phase of the moving grating for the three stimulus conditions (blank screen, small and large grating). **B.** Top: Excitatory input into the cell. Bottom: Inhibitory input into the same cell. As in A, left, middle and right correspond to a blank screen, a small grating and or a large grating. Note that the larger grating leads to a reduction of both excitation and inhibition; adapted from (Ozeki et al., 2009). **C.** Network model with long range excitation and local inhibition. Excitatory neurons within a local population excite themselves (feedback arrow), and also send excitatory input to inhibitory cells (downward arrows). Inhibitory neurons project to local excitatory neurons.

# 6. Field models and Perception

## Psychophysics:

- contrast enhancement is a stable psychophysical phenomenon
- Mach bands are but one example

## Neuronal:

- the activity of V1 cell first increases and then decreases with size of stimulus
- both excitatory and inhibitory input into a cell show similar changes

## Modeling

- continuum model with Mexican-hat interaction in the input-driven regime for Mach bands
- Receptive Field tuning:  
contrast enhancement = 'sharpening'

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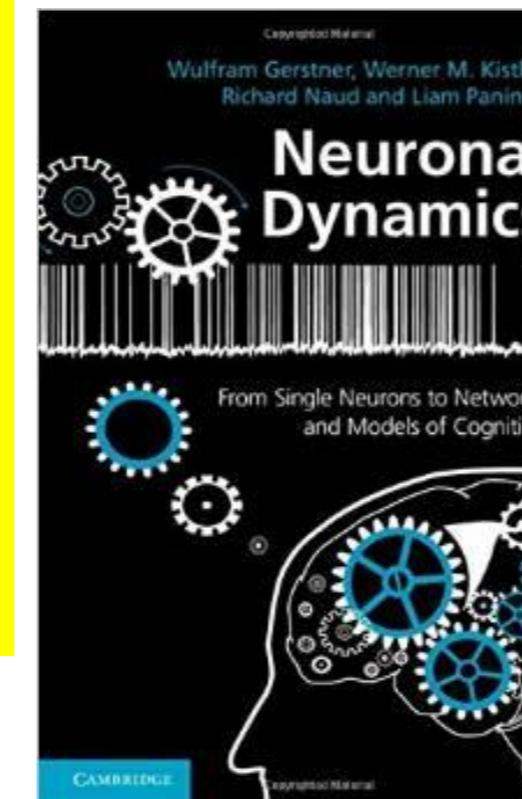
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## 7. Head direction cells: aims

sense of direction



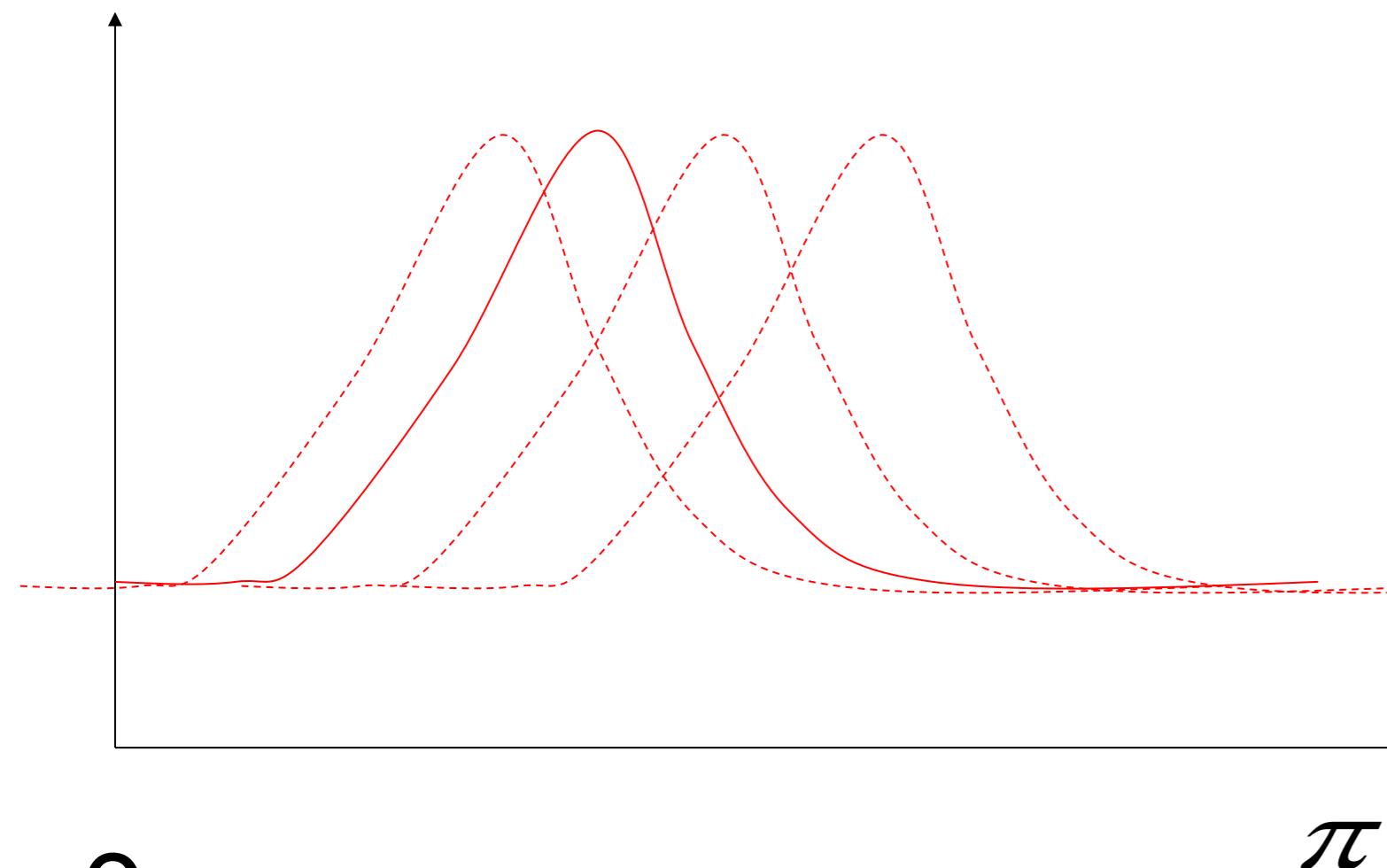
**Model**

memory of direction related  
to bump solution of ring model

# 7. Bump solution

## Basic phenomenology

A



0

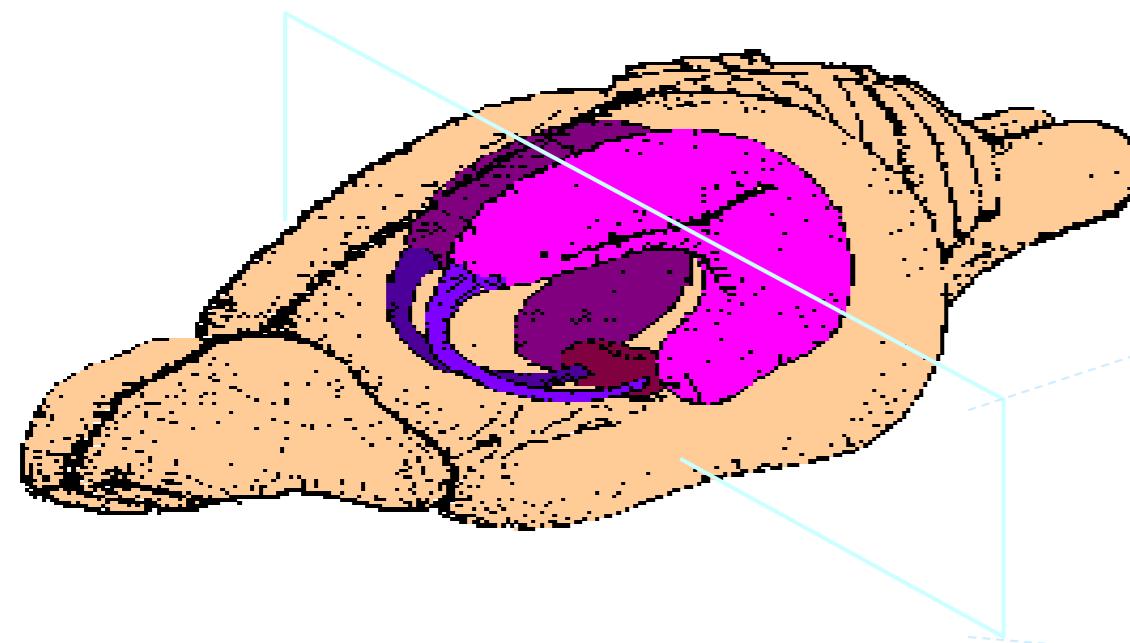
$\pi$

## Bump formation

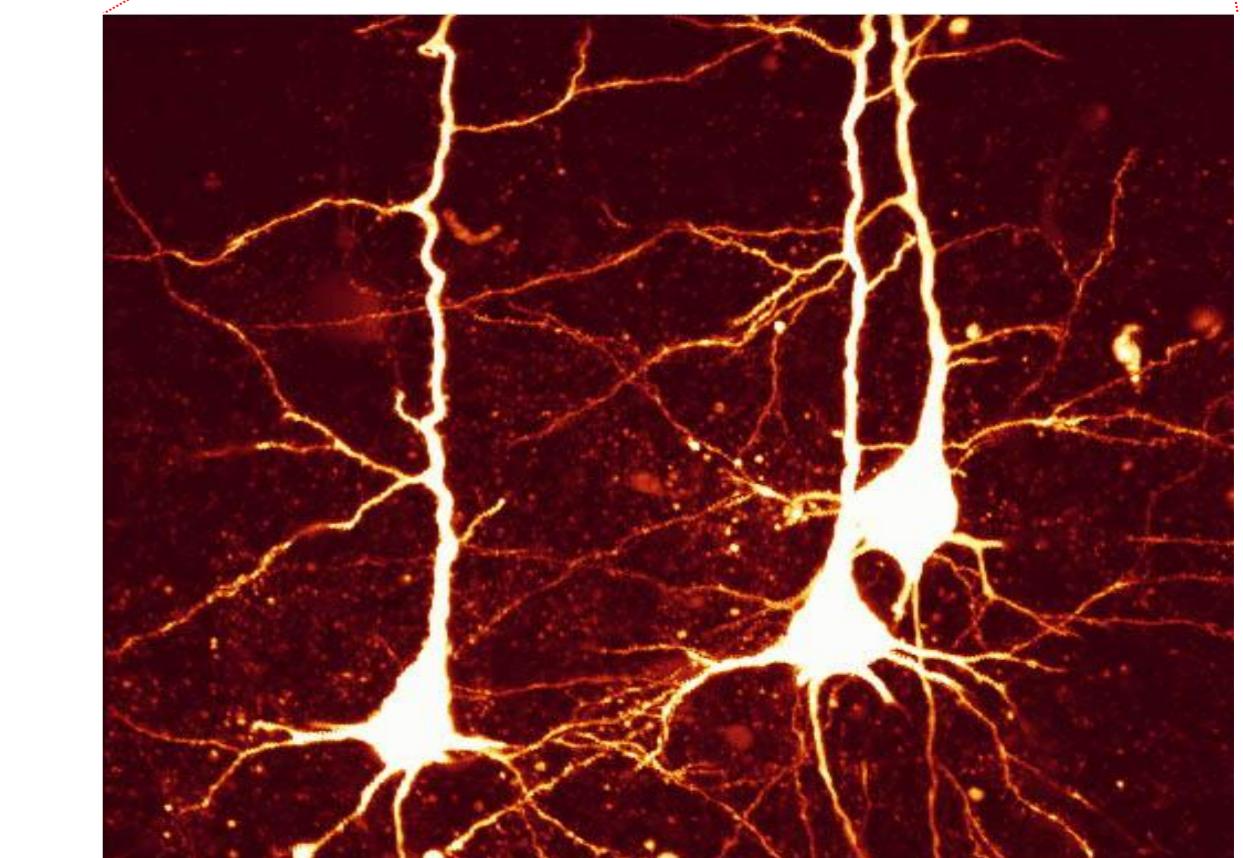
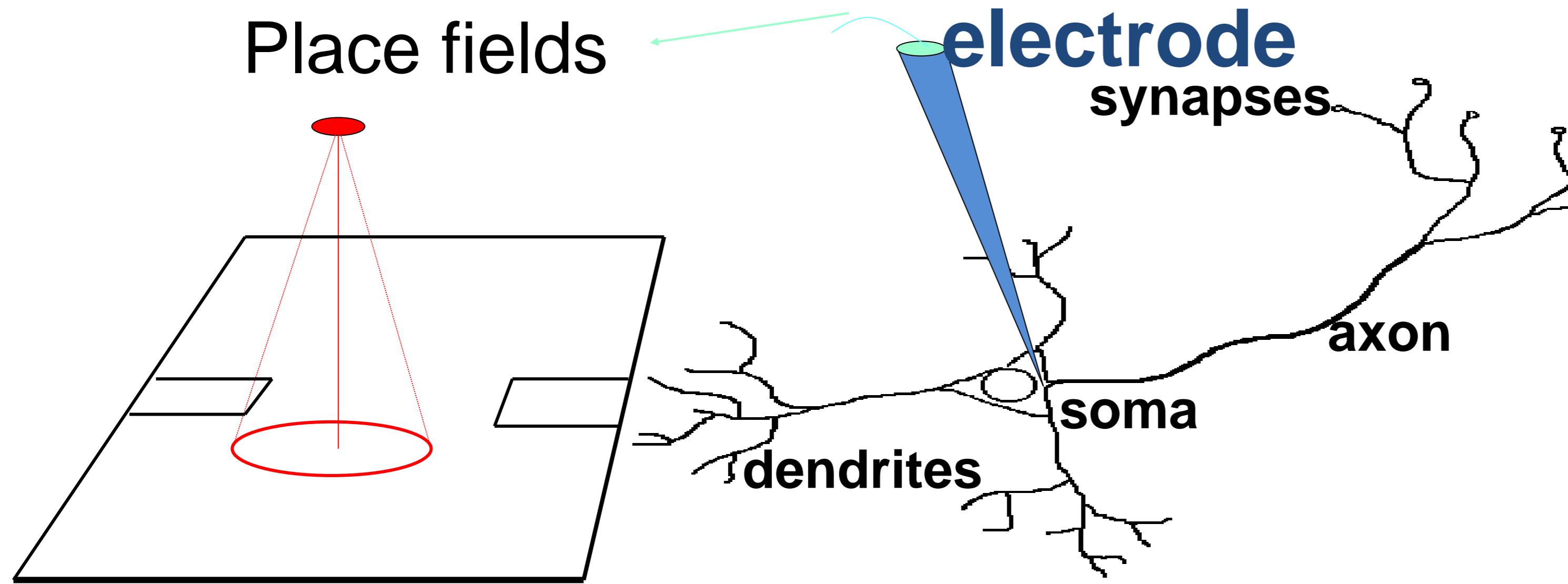
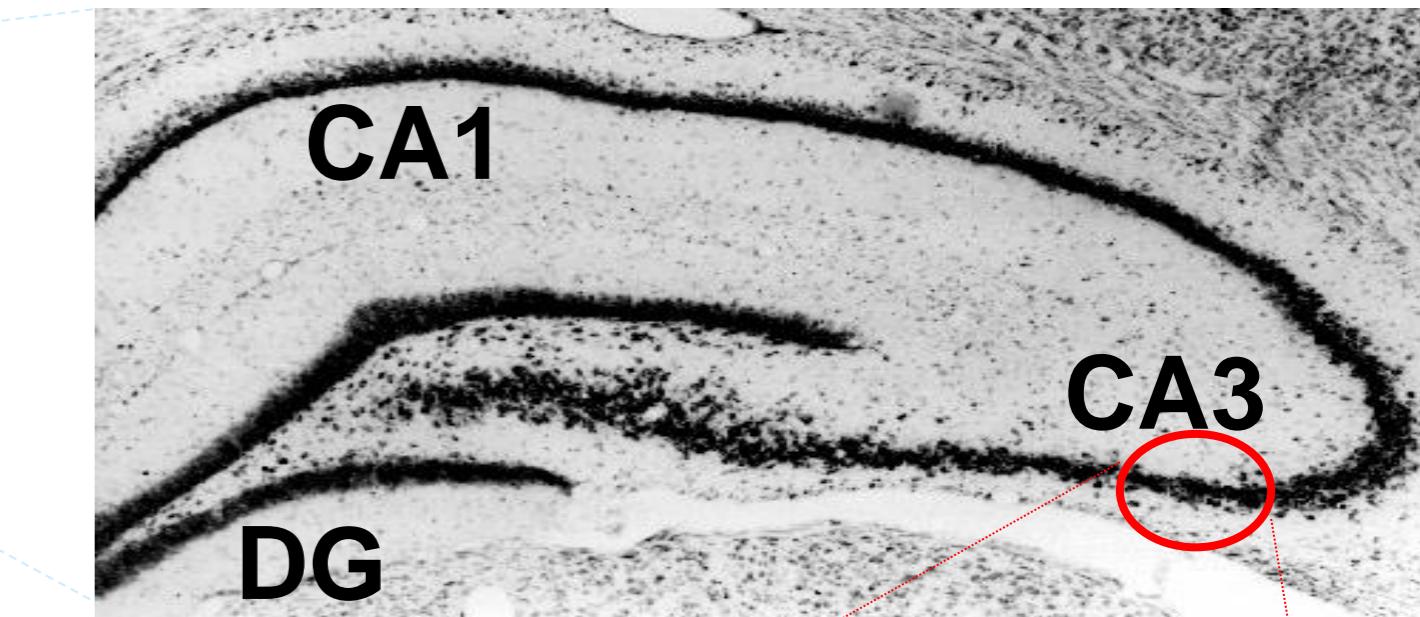
strong lateral connectivity

Possible application:  
head direction cells -  
bump of active cells  
→ indicate current orientation

# 7. Hippocampal place cells

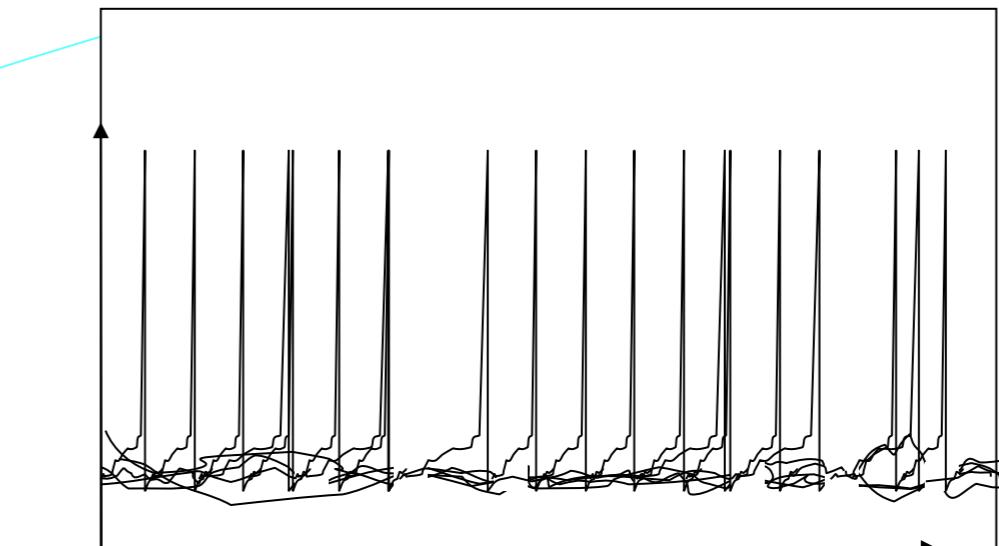
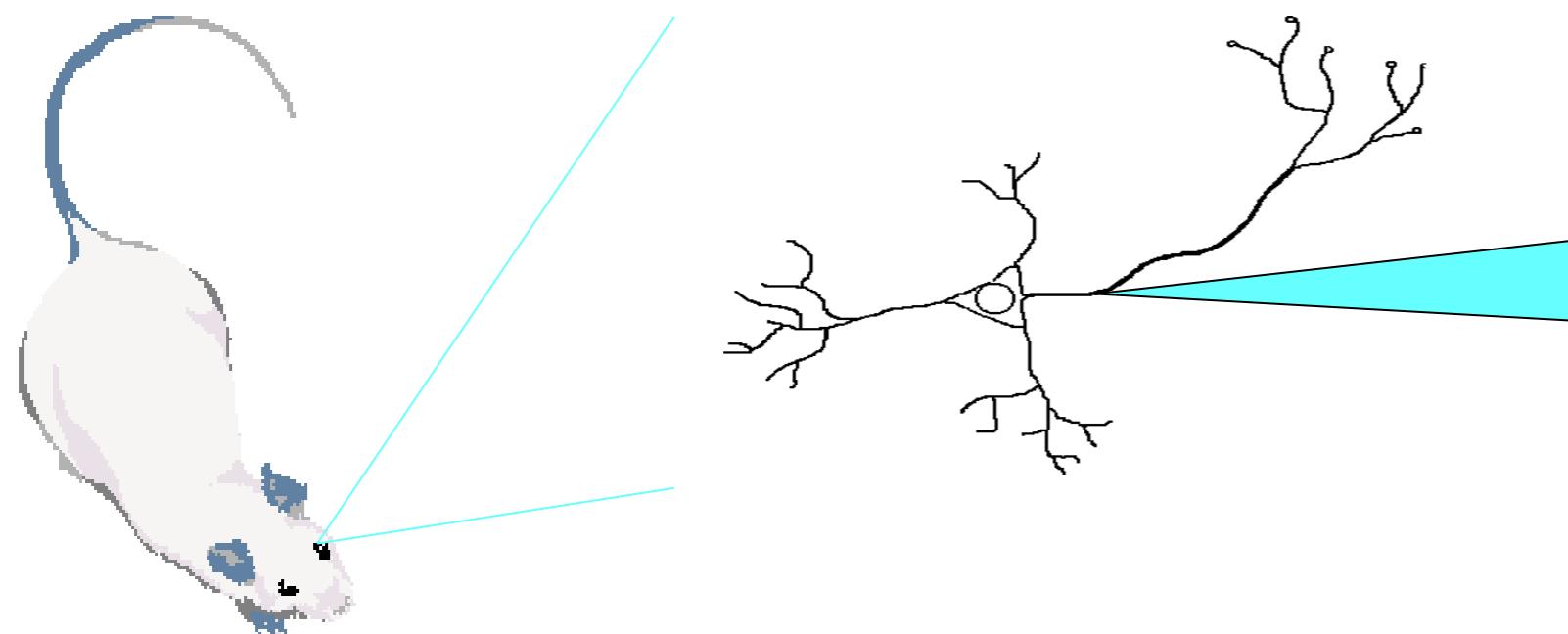
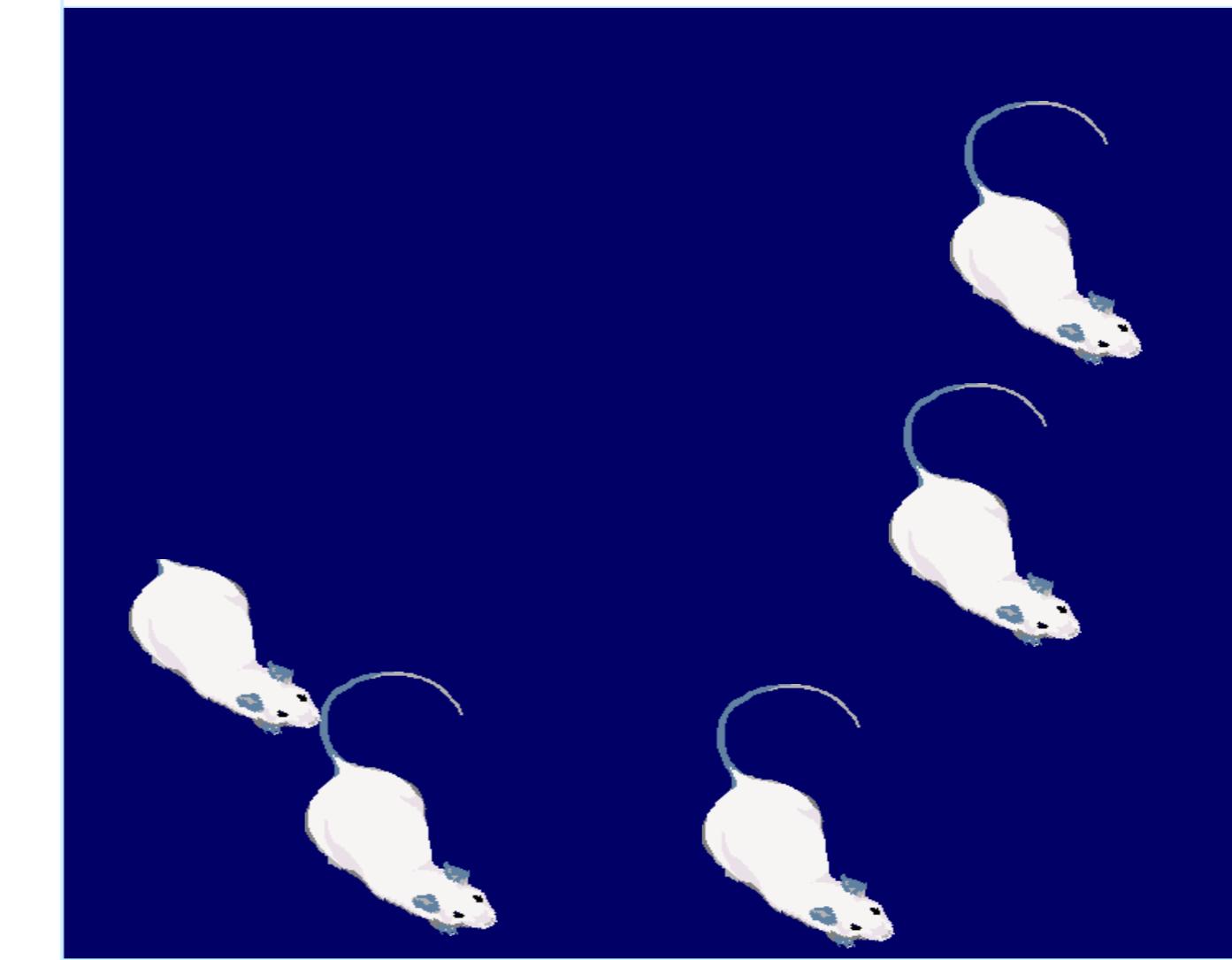
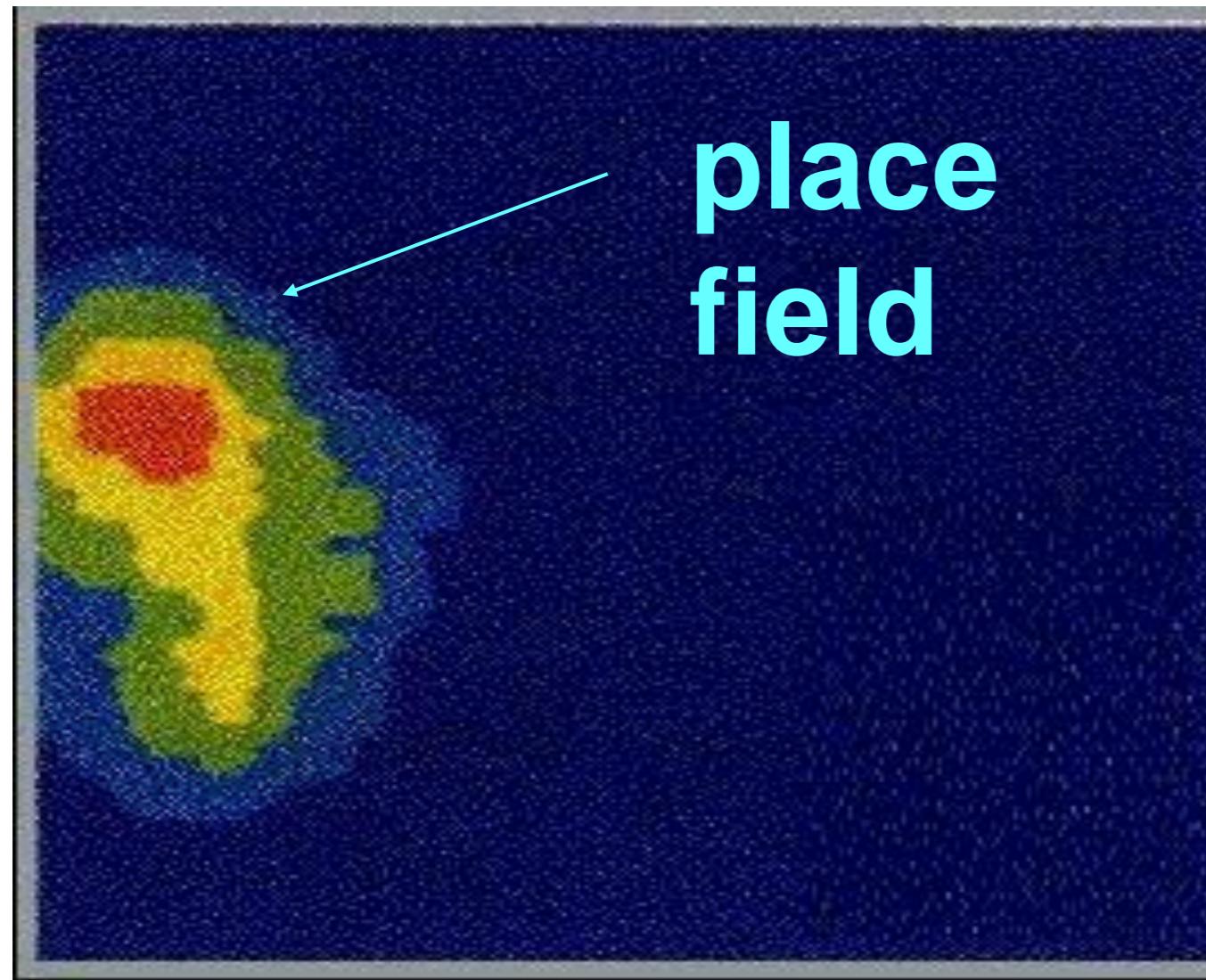


rat brain



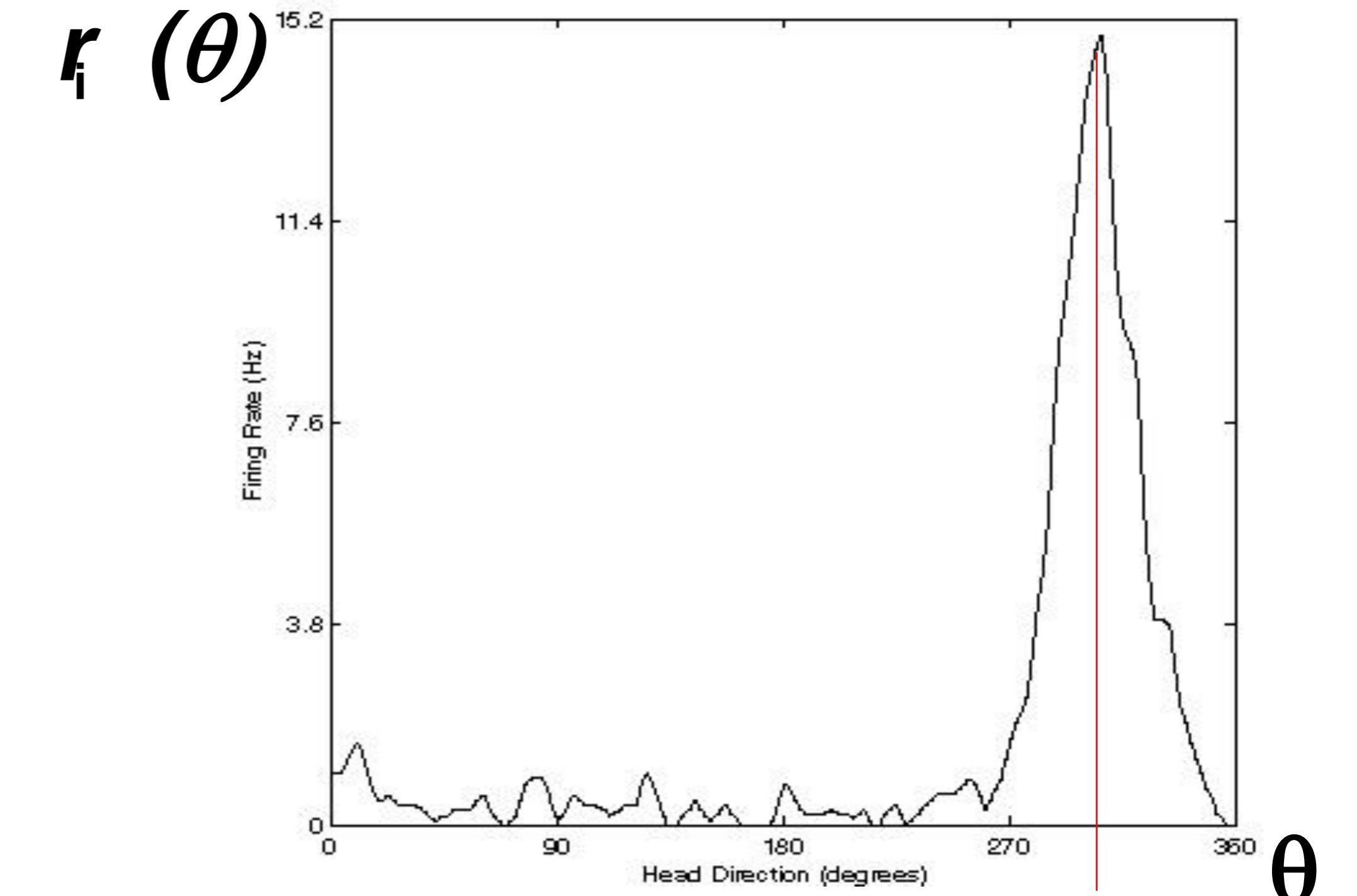
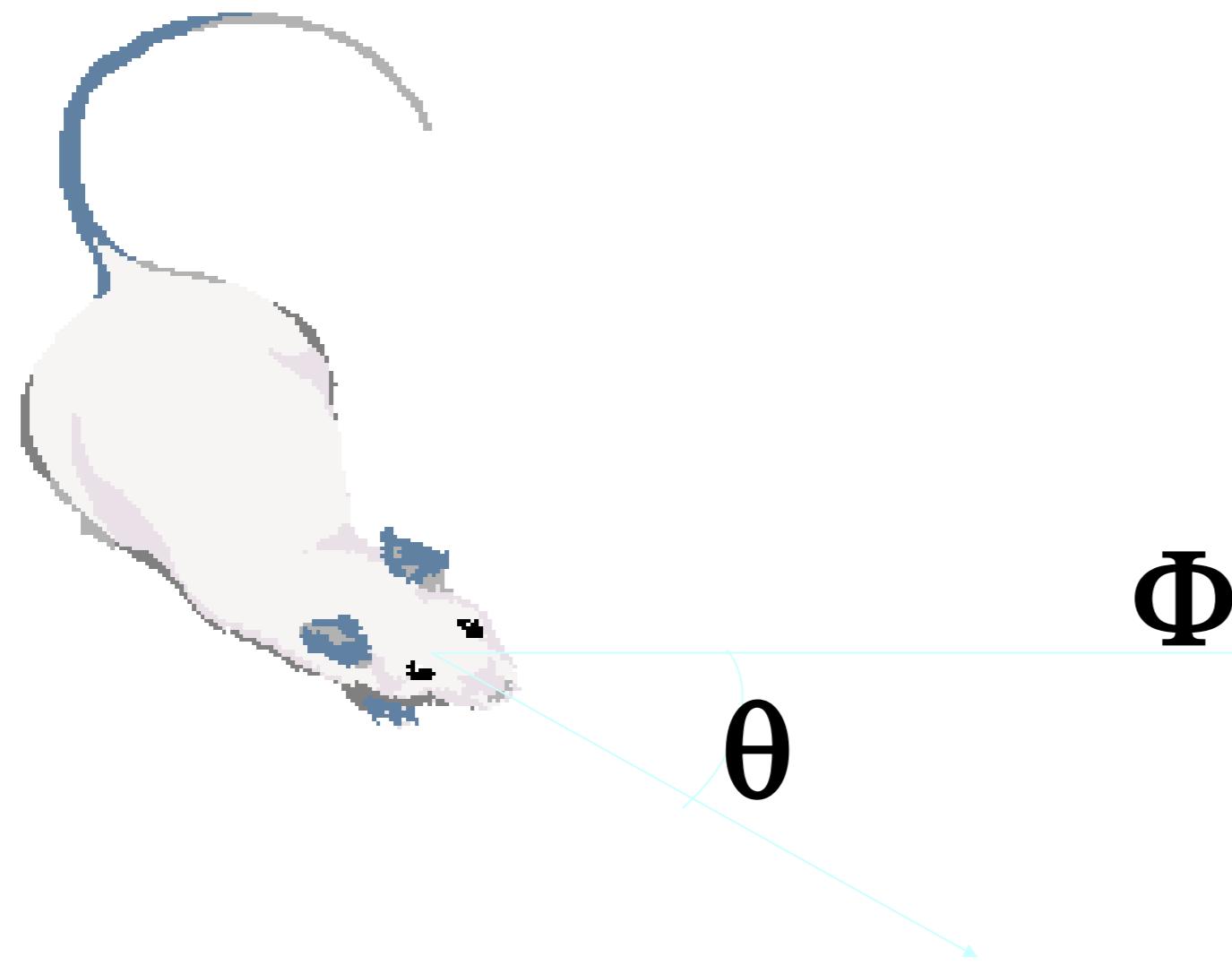
# .7. Hippocampal place cells

Main property: encoding the animal's location



# 7. Head direction cells

Main property: encoding the animal's heading

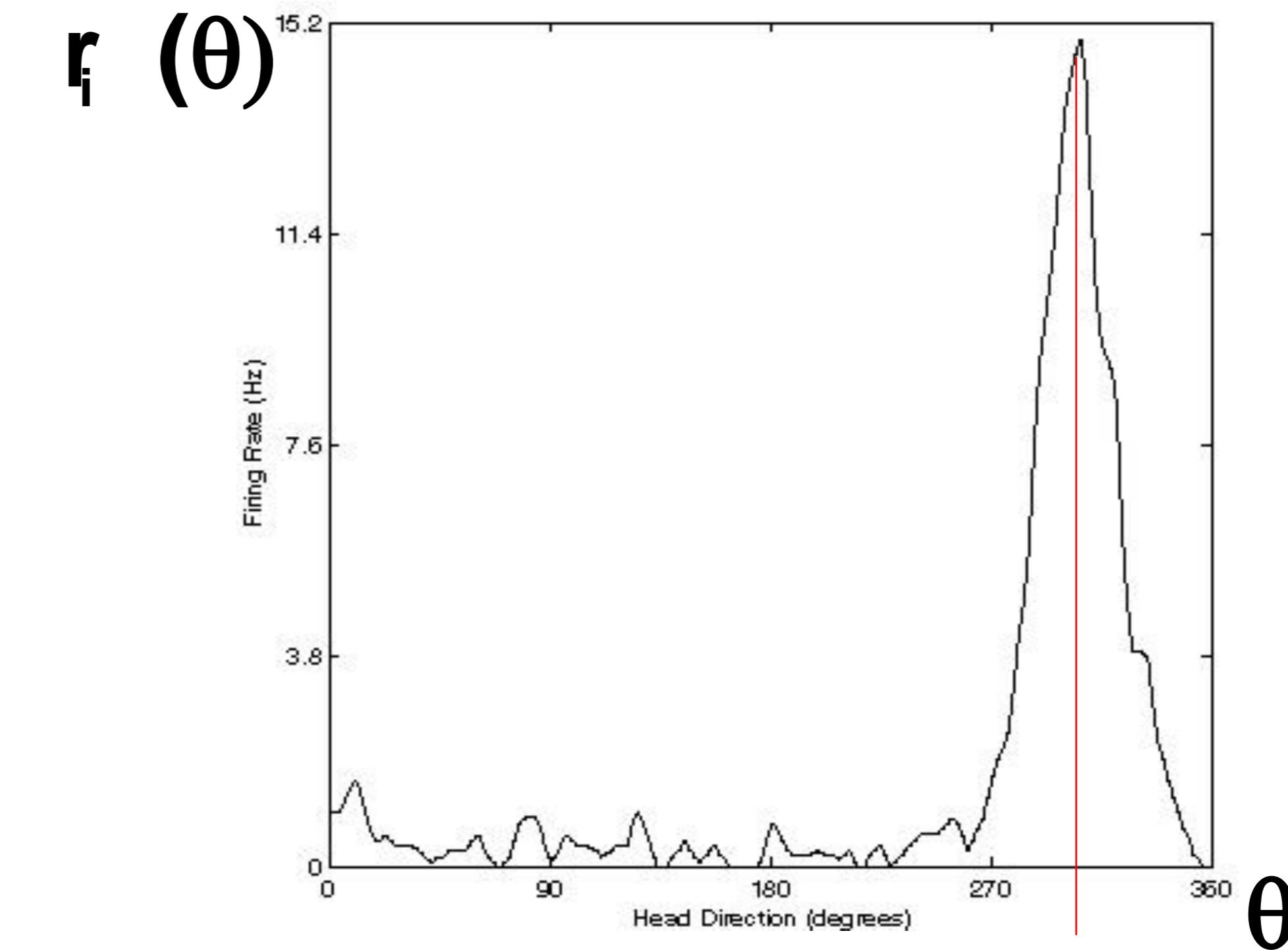
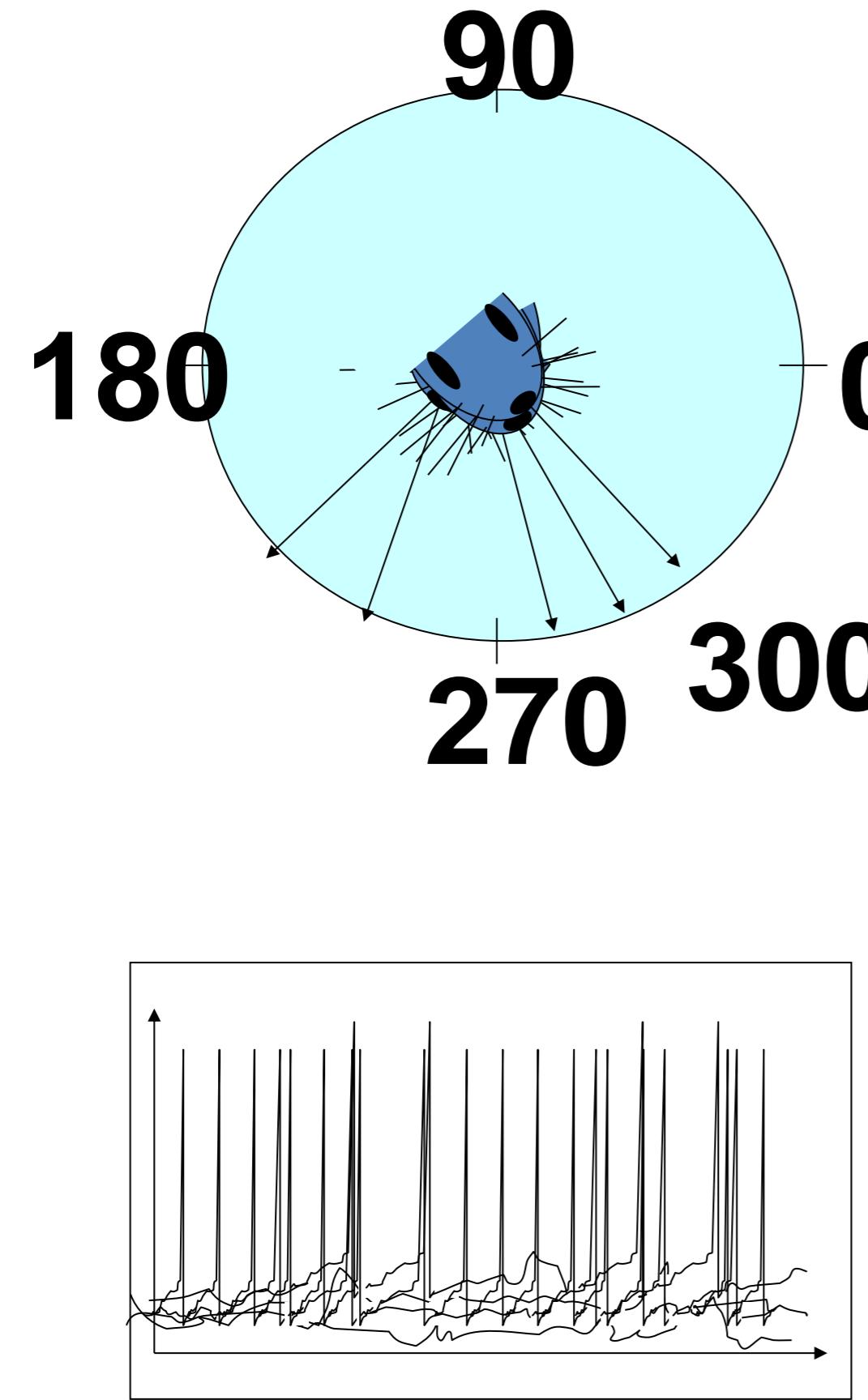


*Taube and Müller,  
Hippocampus 1998,*

$\theta_i$   
Preferred firing direction

# 7. Head direction cells

Main property: encoding the animal's allocentric heading



$\theta_i$   
Preferred firing direction

# 7. Head direction cells

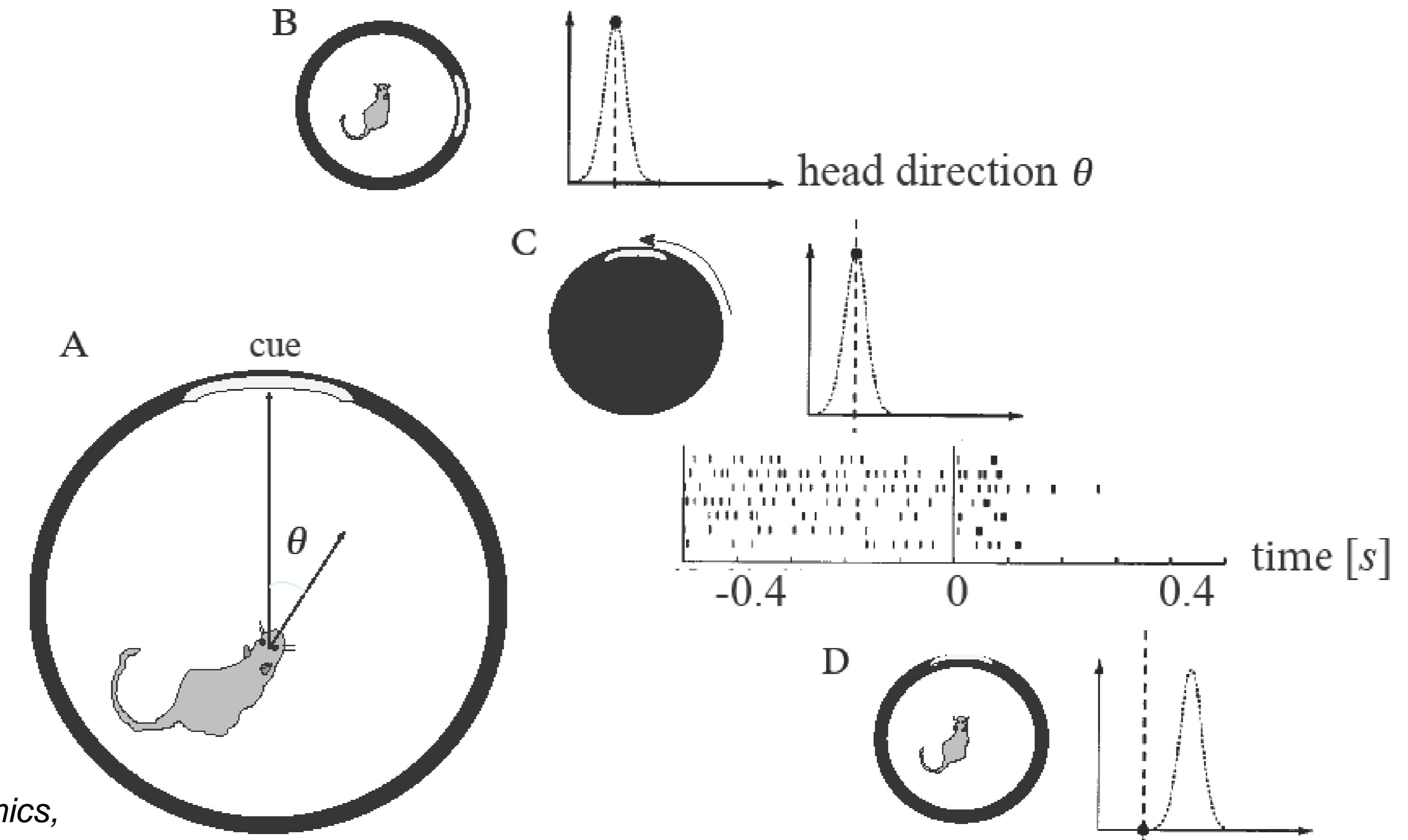
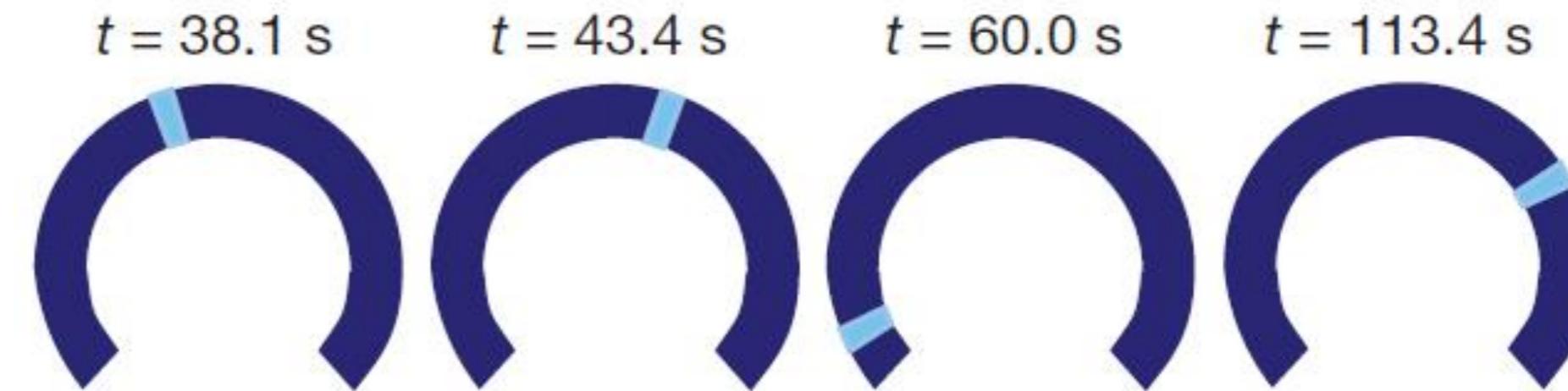


Image: *Neuronal Dynamics*,  
Gerstner et al.,  
Cambridge Univ. Press (2014),  
Adapted from Zugaro et al. (2003), *J. Neurosci.* 23:3478-3482

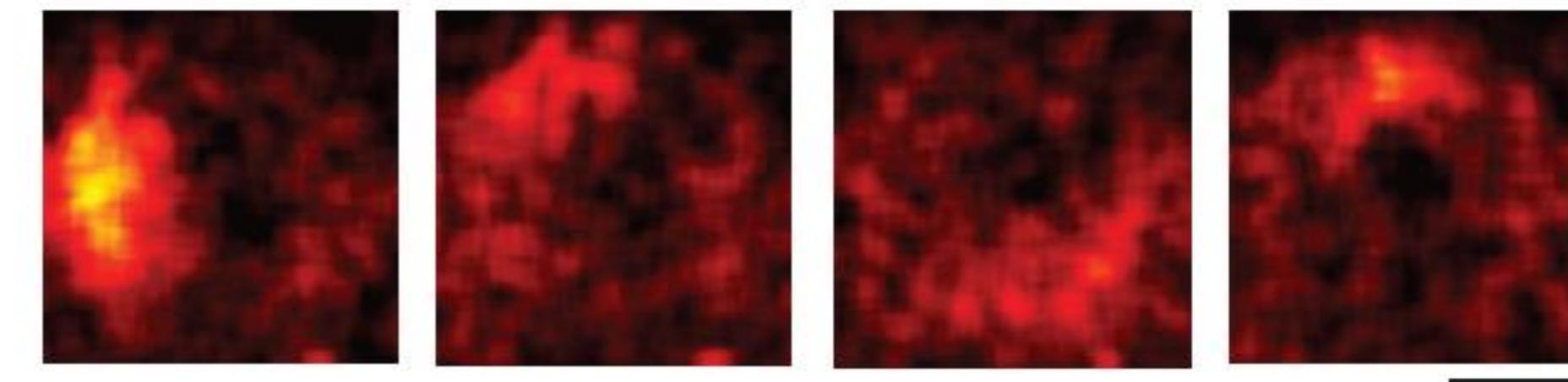
## 7. Head direction cells in the fly brain

Similar to the rat: head direction cells in fly brain (ellipsoid body)

stimulus on screen:



activity in  
ellipsoid body



- bump activity persists in the dark
- cue is landmark configuration

Seelig and Jarayaman, Nature, 2015,

*Neural dynamics for landmark orientation and angular path integration*

## 7. Head direction cells: summary

head direction cells

- are sensitive to direction of head with respect to visual cues
- keep their activity if light is switched off
- exist in rodents and in flies
- can be explained by bump solution in ring model



*Taube and Müller, Hippocampus 1998,*

*Zugaro et al., J. Neurosci. 2003*

*Seelig and Jarayaman, Nature, 2015*

*Redish et al., Network, 1996, Zhang, J. Neurosci. 1996*

## 7. Summary: field models

**Continuum model provides understanding for:**

- head direction cell  
    → bumps of activity
- spatial working memory  
    → bumps of activity
- place cells  
    → bumps of activity
- contrast enhancement and some visual illusions  
    → input driven regime
- receptive field properties  
    → input driven regime

# 7. Selected References: Field Models

## Field Models

H. R. Wilson and J. D. Cowan (1973) A mathematical theory of the functional dynamics of cortical and thalamic nervous tissue. *Kybernetik* 13, pp. 55–80

S. Grossberg (1973) Contour enhancement, short term memory and constancies in reverberating neural networks. *Studies in Applied Mathematics* 52:217-257.

## Mach bands and Field Models for Visual Cortex and contrast Enhancement

E. Mach (1865) Über die Wirkung der räumlichen vertheilung des Lichtreizes auf die Netzhaut. *Sitzungsberichte der mathematisch-naturwissenschaftlichen Classe kaiserl. Akademie der Wissenschaften* 52, pp. 303–322.

E. Mach (1906) *Die Analyse der Ampfindungen*. 5th edition, Gustav Fischer, Jena.

R. Ben-Yishai, R.L. Bar-Or and H. Sompolinsky (1995) Theory of orientation tuning in visual cortex. *Proc. Natl. Acad. Sci. USA* 92, pp. 3844–3848.

O. Shriki, D. Hansel and H. Sompolinsky (2003) Rate models for conductance-based cortical neuronal networks. *Neural Computation* 15, pp. 1809–1841

## Head Direction Cells and Field Models for Head direction and Spatial Working Memory

J. S. Taube and R. U. Muller (1998) Comparisons of head direction cell activity in the postsubiculum and anterior thalamus of freely moving rats. *Hippocampus* 8, pp. 87–108.

A.D. Redish, A.N. Elga and D.S. Touretzky (1996) A coupled attractor model of the rodent head direction system. *Network* 7, pp. 671–685.

K. Zhang (1996) Representaton of spatial orientation by the intrinsic dynamics of the head-direction ensemble: a theory. *J. Neurosci.* 16, pp. 2112–2126.

A Compte, N Brunel, PS Goldman-Rakic, XJ Wang (2000) Synaptic mechanisms and network dynamics underlying spatial working memory, *Cerebral Cortex* 10 (9), 910-923