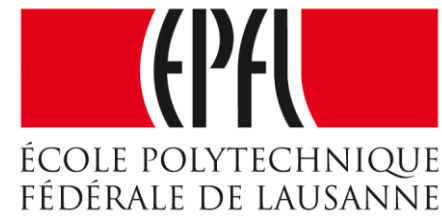


# Computational Neuroscience: Neuronal Dynamics of Cognition



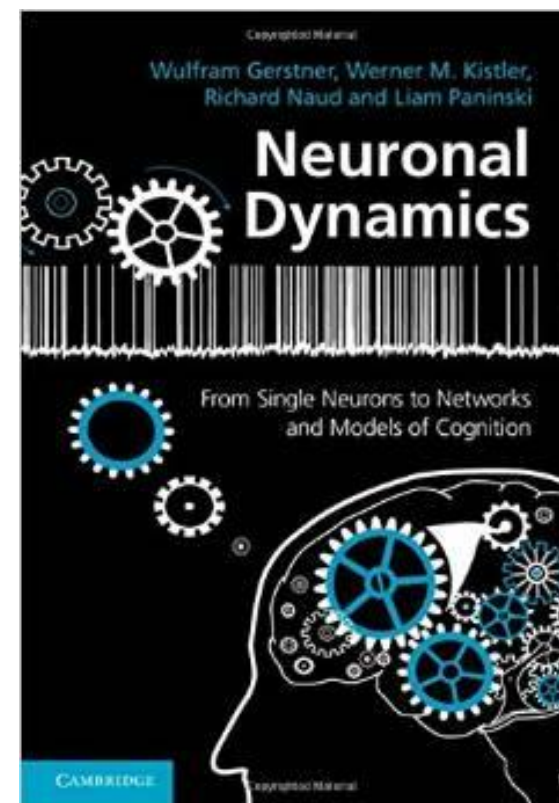
## Continuum models: Cortical fields and perception

Wulfram Gerstner

EPFL, Lausanne, Switzerland

*Reading:*  
**NEURONAL DYNAMICS**  
Ch. 18 +  
+Ch. 12.3.7+Ch 15.1-15.2.3

Cambridge Univ. Press



### 1. Aims and challenges

- review: mean-field arguments

### 2. Transients

- generalized integrate-and-fire model
- transients can be sharp or slow

### 3. Spatial continuum (cortex)

- orientation columns

### 4. Spatial continuum (model)

- field equations

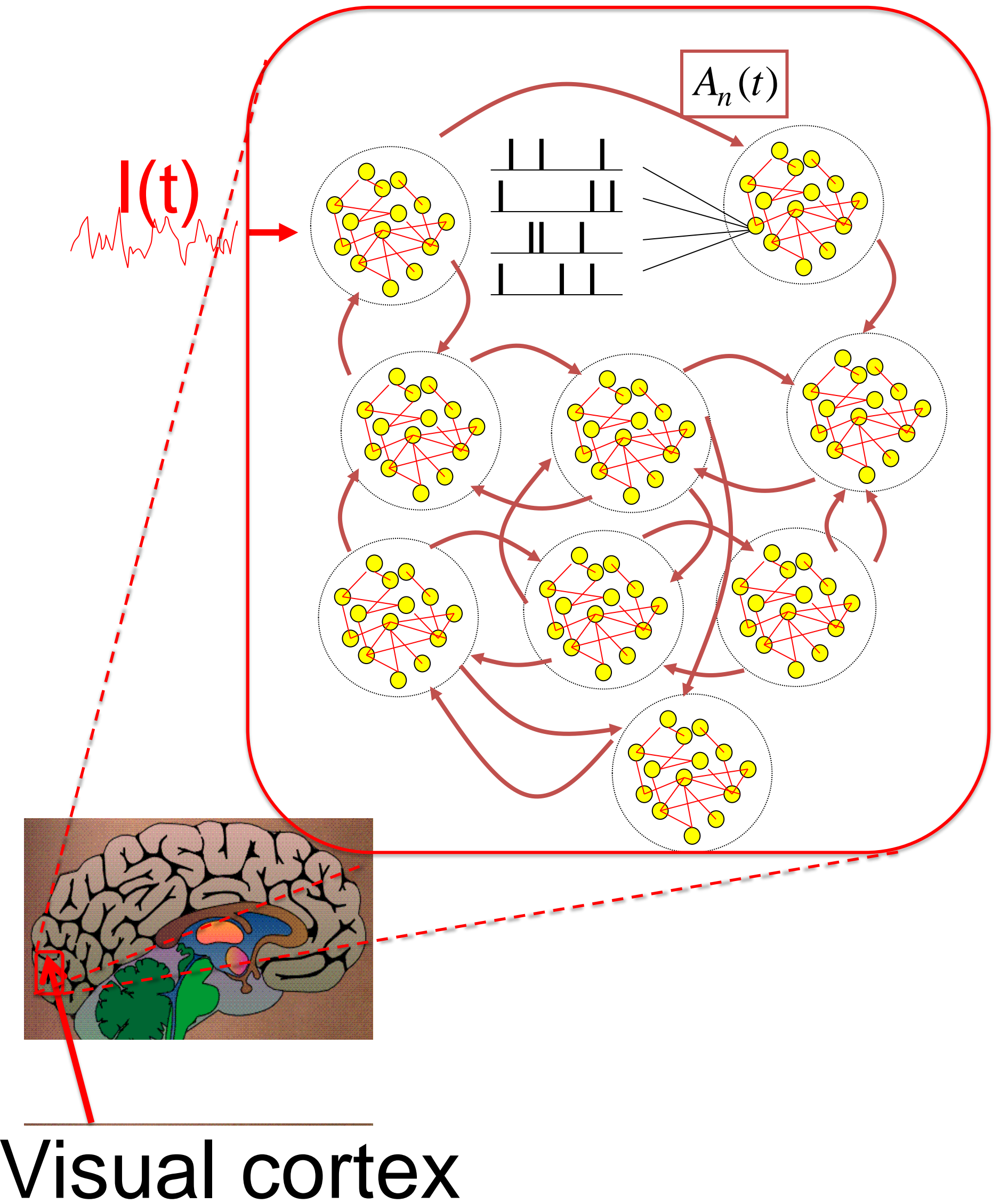
### 5. Solution types

- uniform solution
- bump solution

### 6. Perception

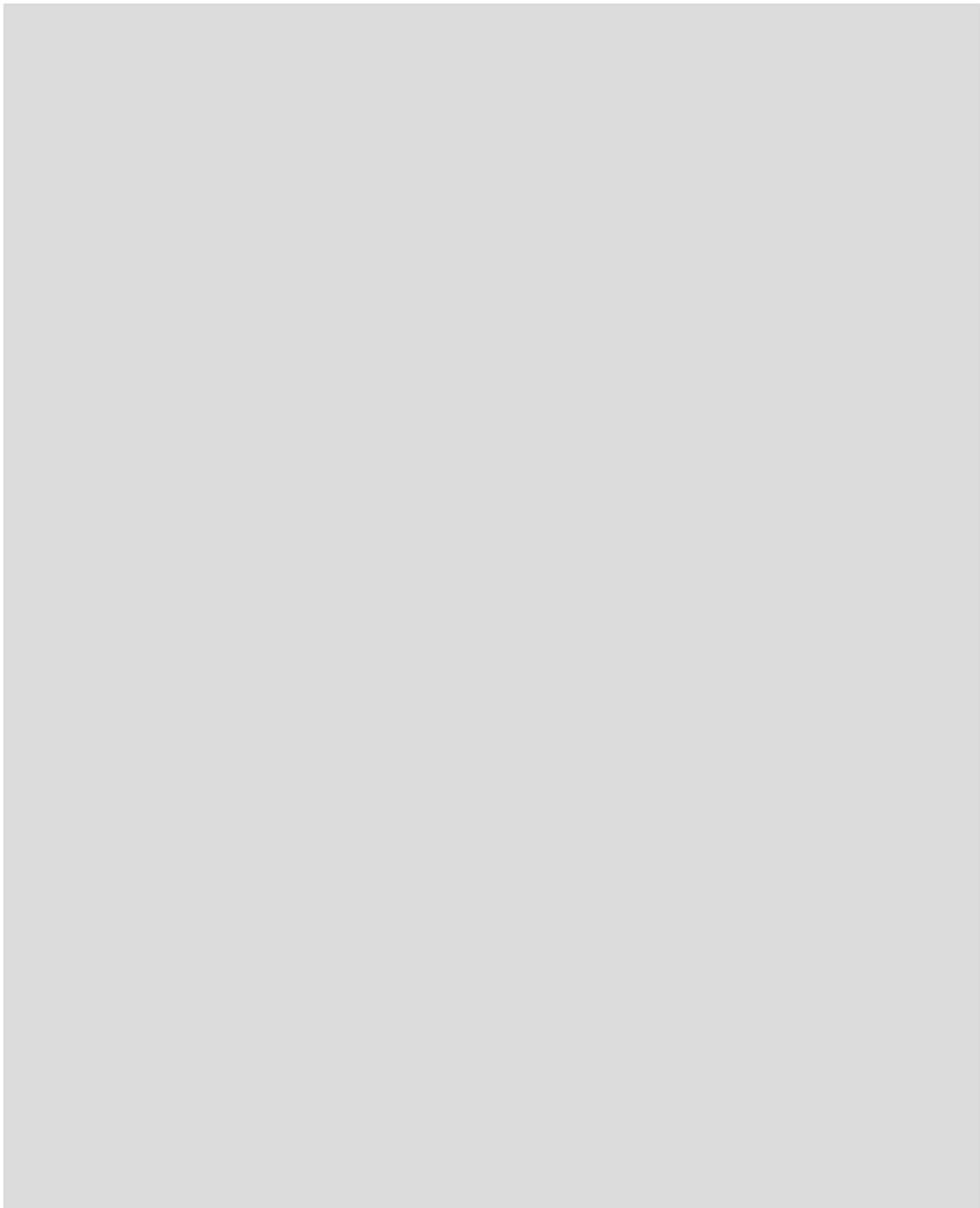
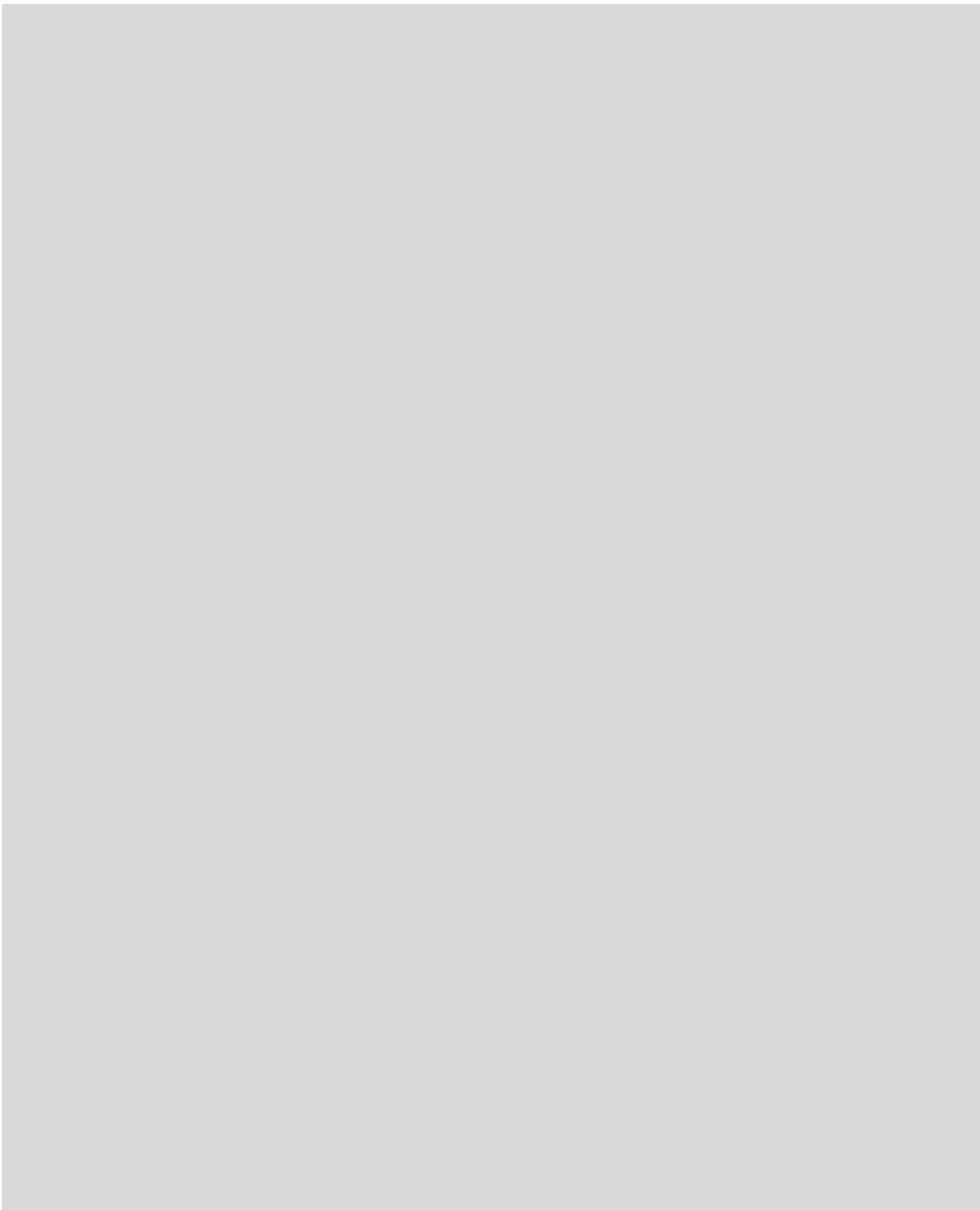
### 7. Head direction cells

# 1. Aims and Challenges



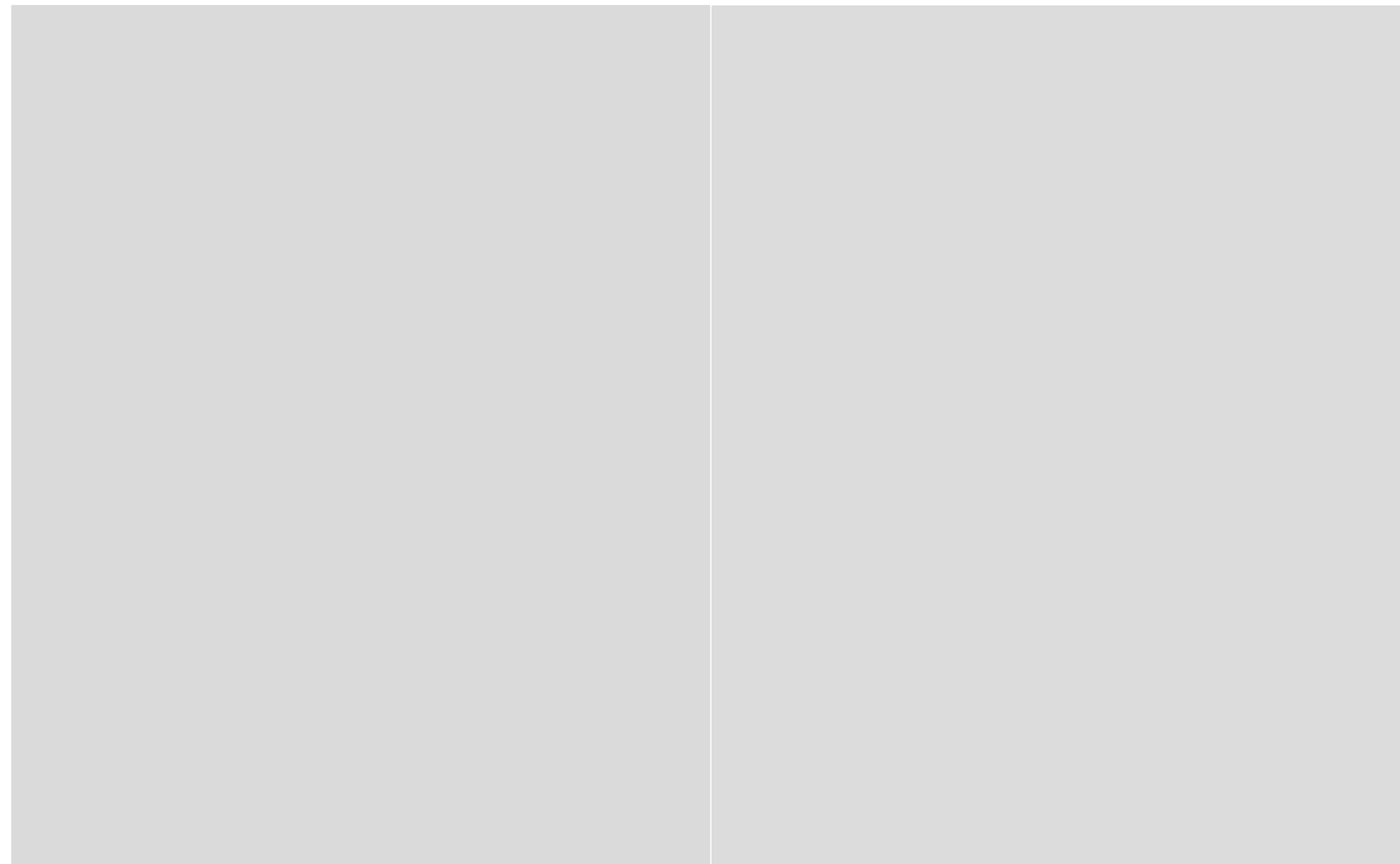
# 1. Aims and Challenges: Visual Perception

---



# 1. Aims and Challenges: Visual Perception

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Visual Perception  
→ weak contrasts  
→ world is continuous

# 1. Aims and Challenges: sense of direction

Sense of direction

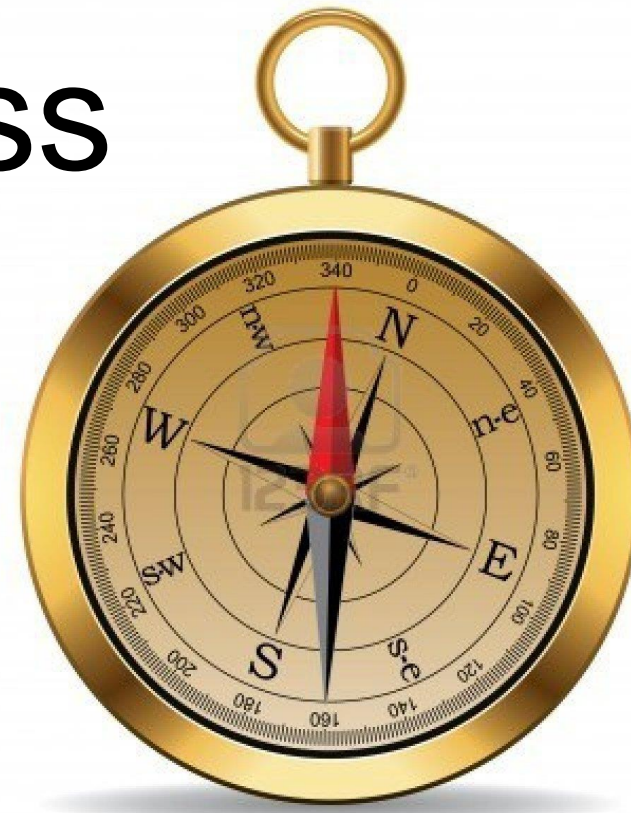
→ internal compass



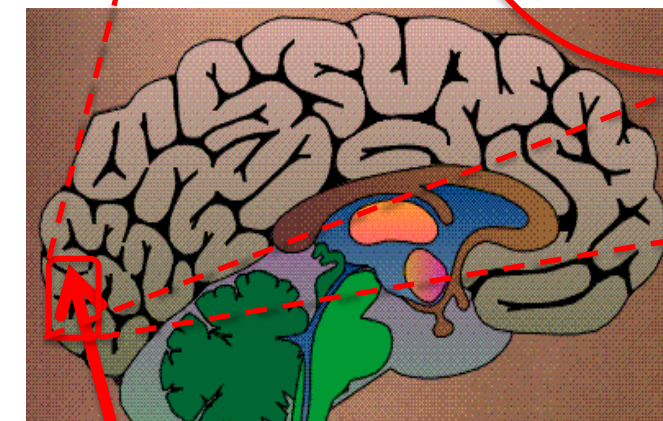
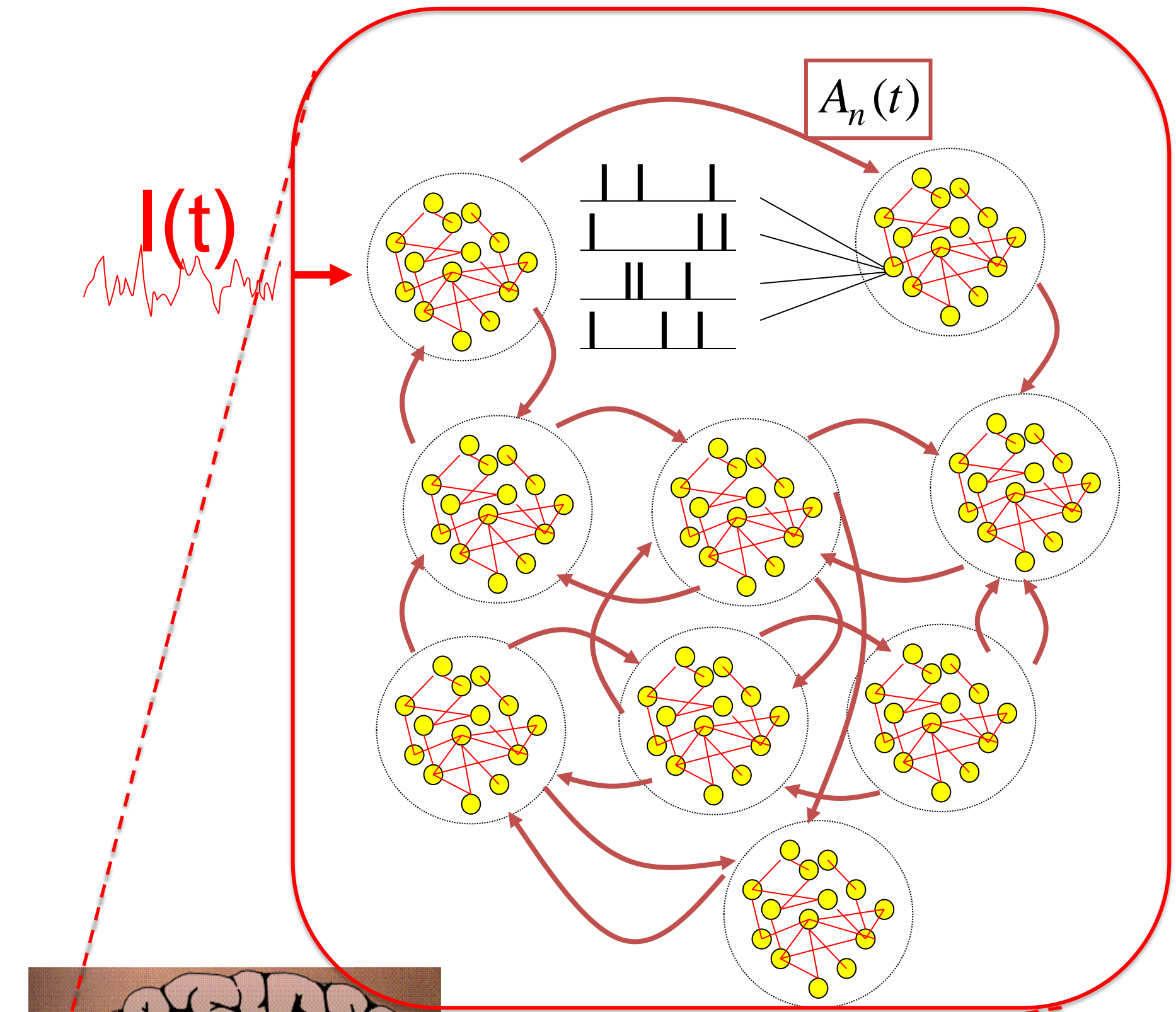


# 1. Aims and Challenges: sense of direction

Sense of direction  
→ internal compass



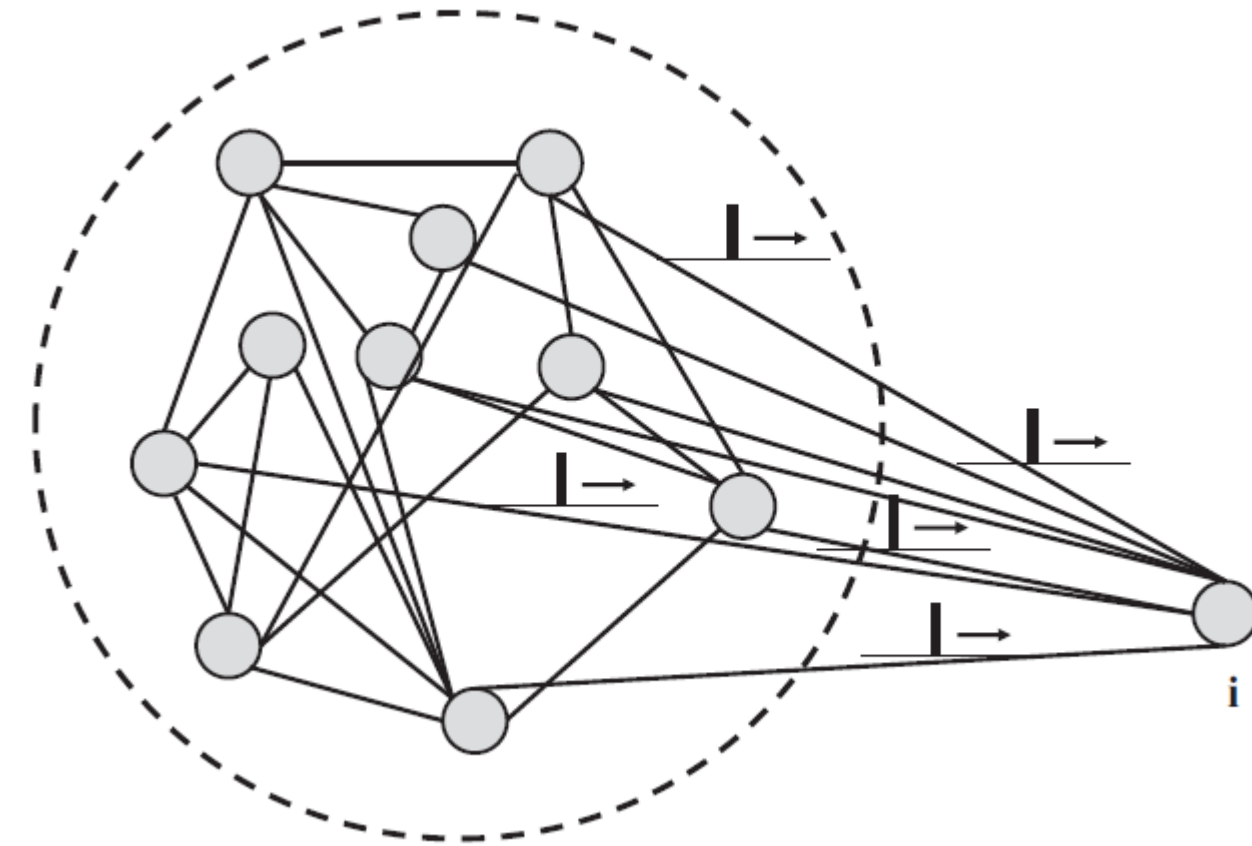
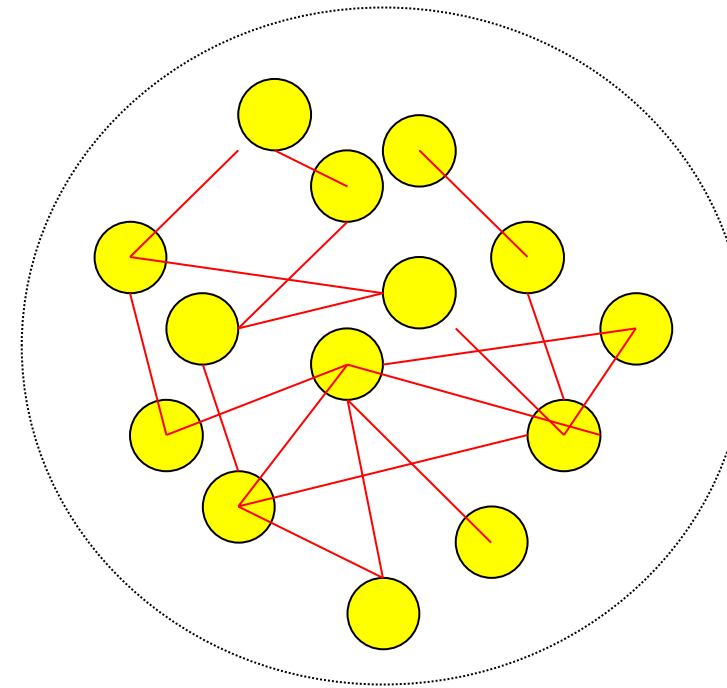
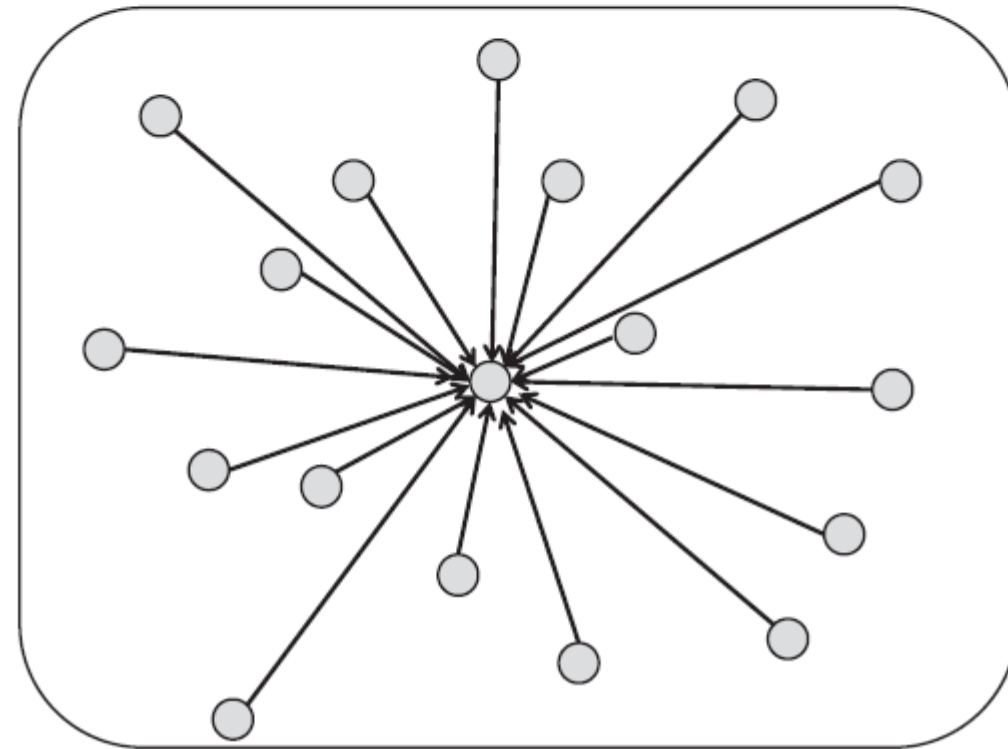
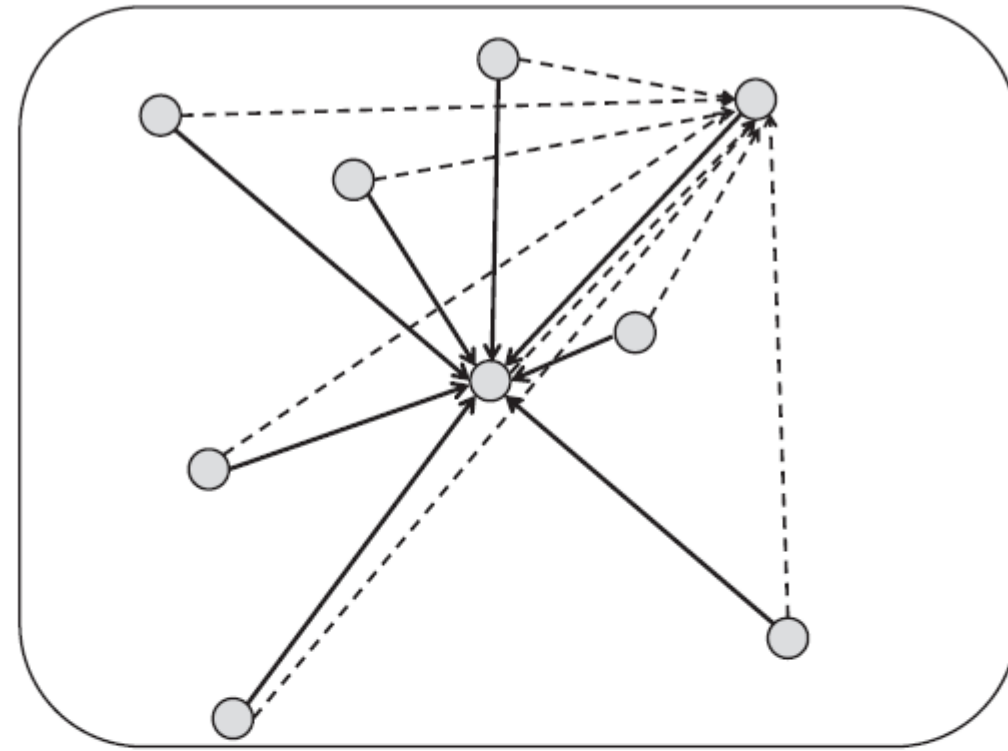
Visual Perception  
→ weak contrasts  
→ world is continuous



Visual cortex

# 1. review: mean-field arguments

## Single population full connectivity



All neurons receive the same  
total input current ('mean field')

# 1. Review: mean-field arguments

All neurons receive the same total input current ('mean field')

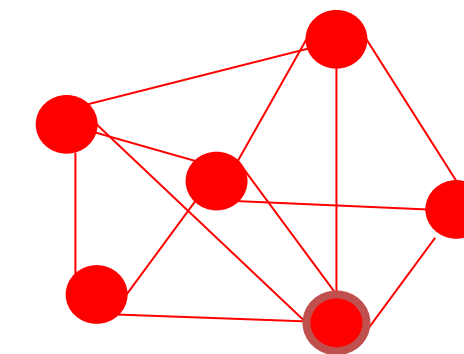
$$I(t) = J_0 q A(t) + I^{ext}(t)$$

Ultra-short current pulse

$$I_i(t) = J_0 \int \alpha(s) \underline{A(t-s)} ds + I^{ext}(t)$$

index  $i$  disappears

$$w_{ij} = \frac{J_0}{N}$$



fully  
connected

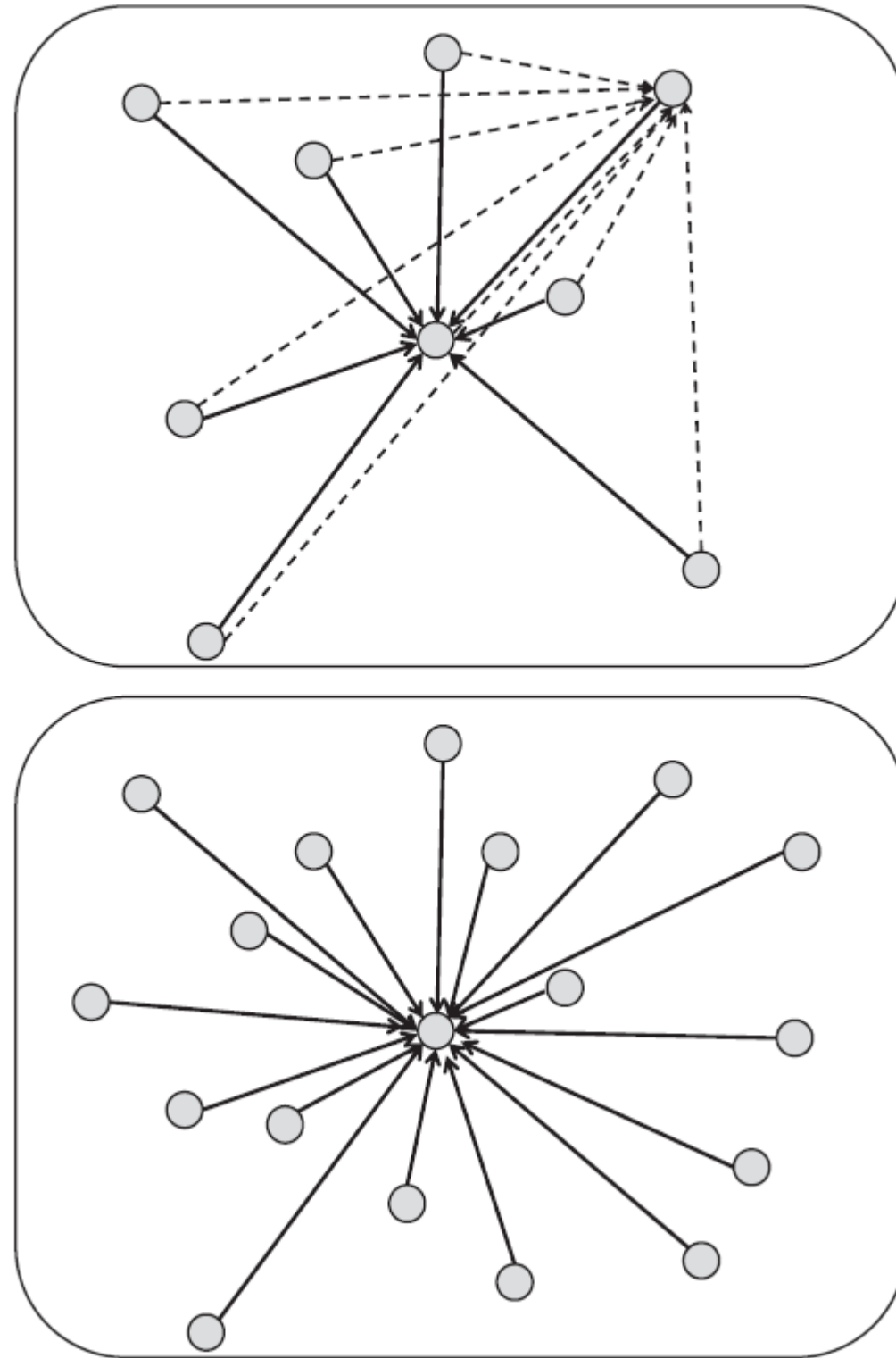
All spikes, all neurons

$$I_i^{net}(t) = \sum_j \sum_f w_{ij} \underline{\alpha(t - t_j^f)} + I^{ext}$$

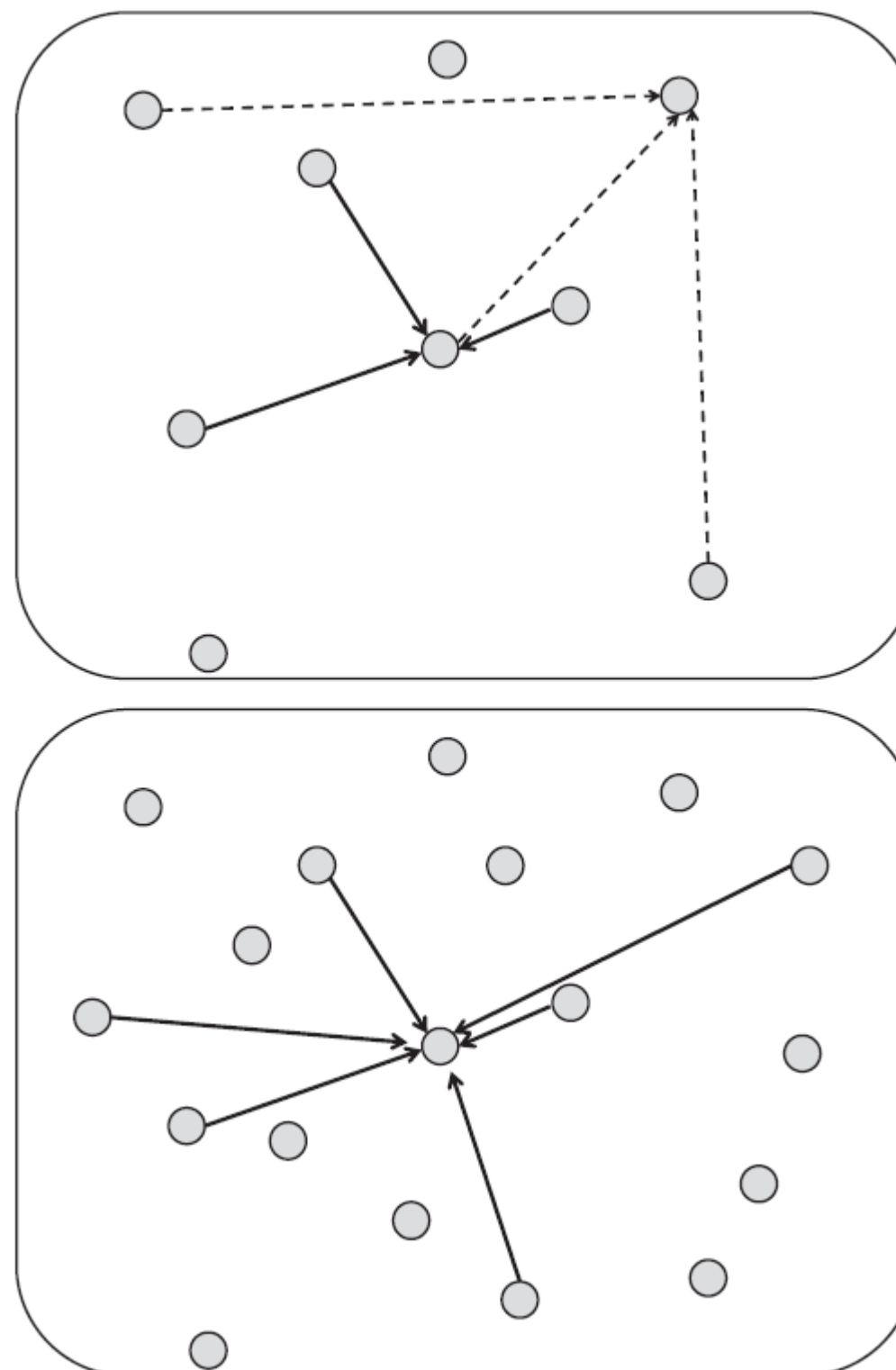


# 1. Review: mean-field also works for random coupling

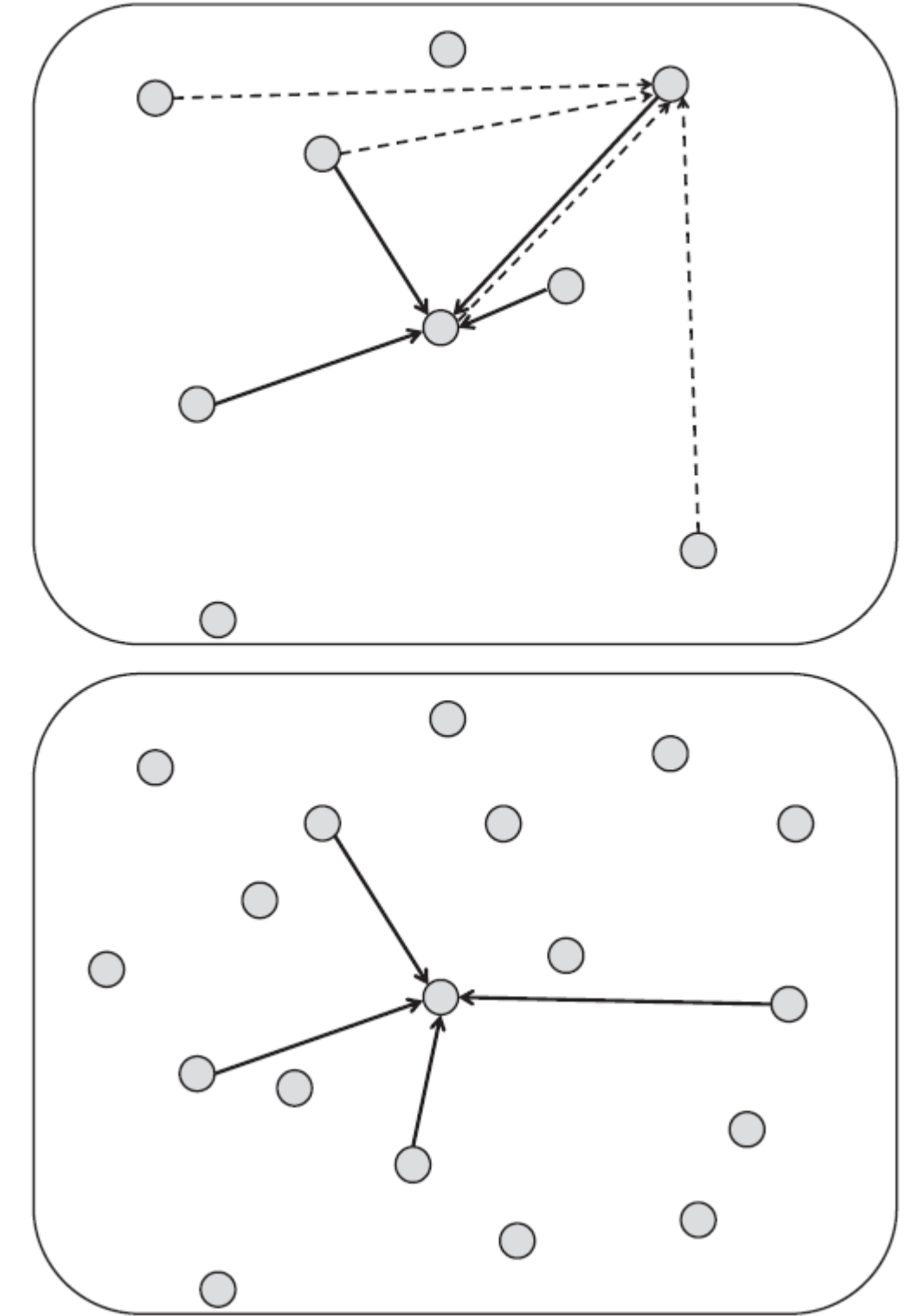
full connectivity



random: prob  $p$  fixed



random: number  $K$   
of inputs fixed



*Image: Gerstner et al.  
Neuronal Dynamics (2014)*

# 1. Review: stationary state/asynchronous activity

Homogeneous network

All neurons are identical,

**Single neuron rate = population rate**

$$v = g(I_0) = A_0$$

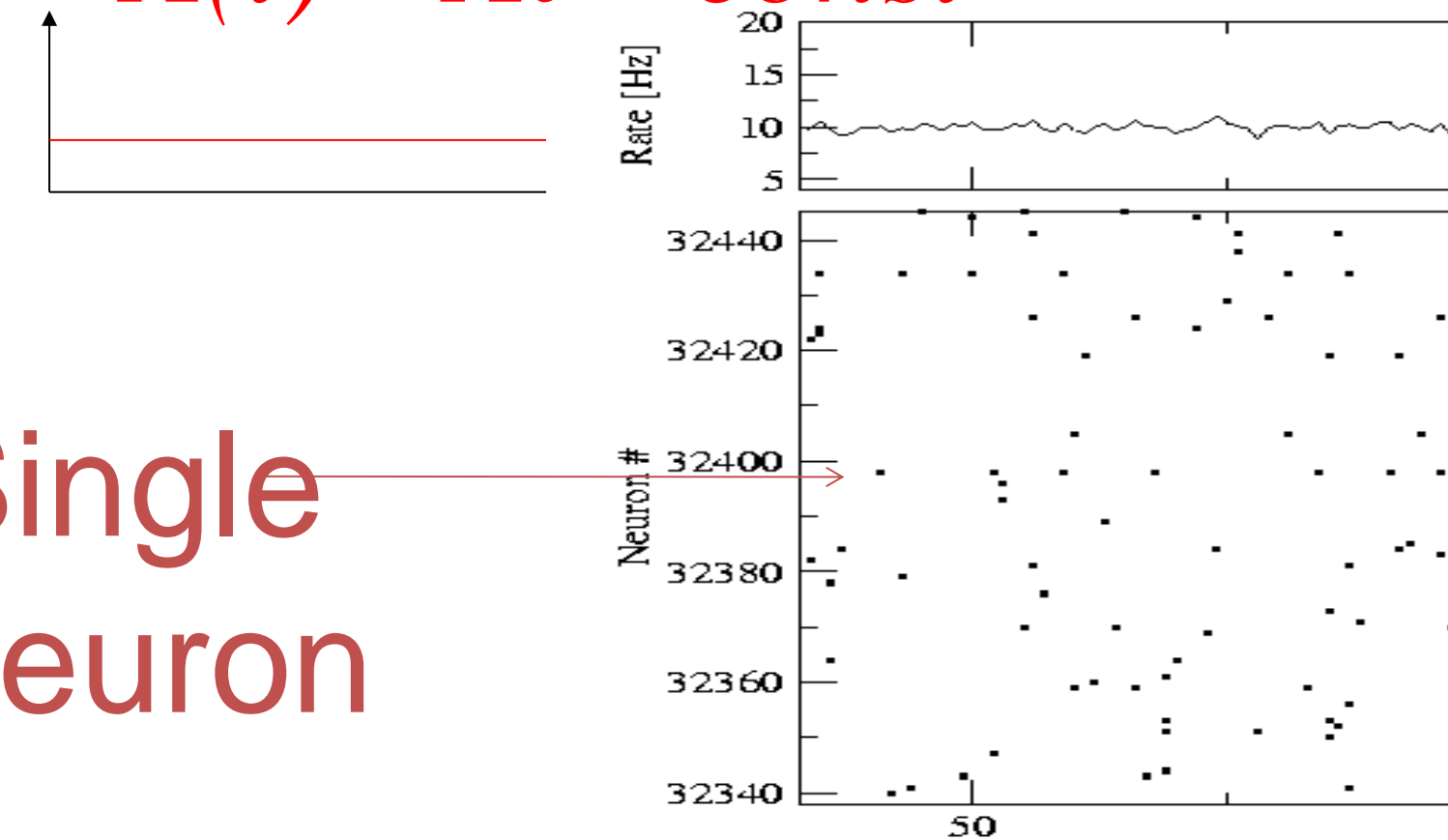
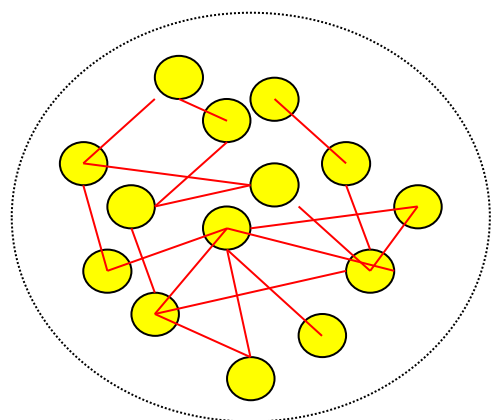
constant input

$$I_0 = c$$

Gain function at appropriate noise level

$$A(t) = A_0 = \text{const}$$

Single  
neuron



frequency (single neuron)  $v = 1/\langle s \rangle$       rate =  $1/(\text{meanInterval})$

# 1. Review : mean-field arguments for homogeneous population

- single neuron is driven by the 'population activity' of all others
- all neurons in populations receive the same input
- mean-field arguments work for fully connected and randomly connected populations
- in the **stationary** state, the single neuron firing rate is equal to the 'population activity' of a homogeneous population
- in the **stationary** state, 'population activity' can be predicted by
  - (i) single neuron gain function (f-I curve)
  - (ii) external input
  - (iii) intra-population coupling strength
- in the **stationary** state, choice of neuron model irrelevant  
(apart from gain function/f-I curve)

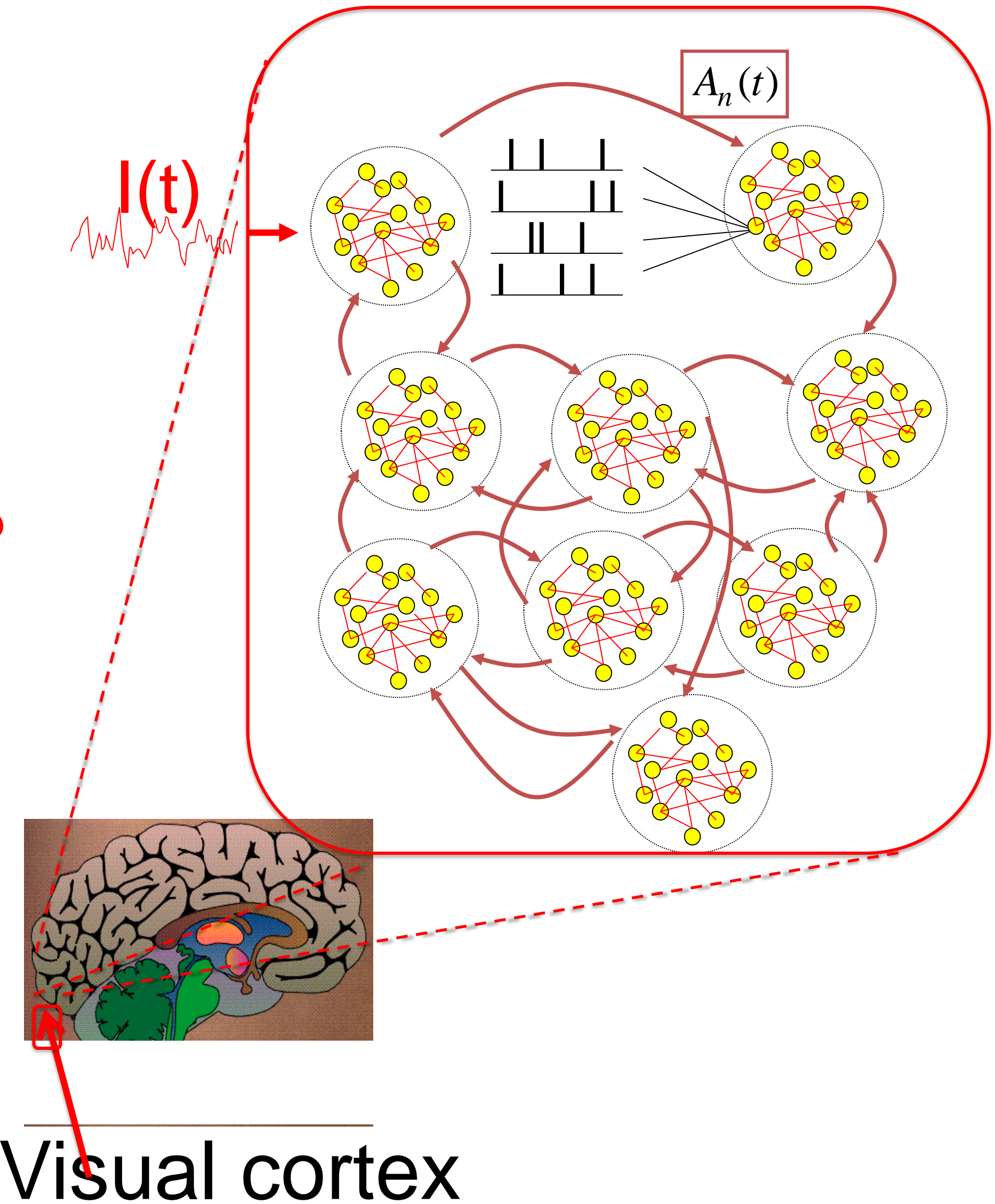
# 1. Aims and challenges

## Mathematical aims:

- beyond stationary states  
→ transients?
- more than one population  
→ how many? continuum?

## Cognitive Modeling aims:

- functional consequences  
→ visual perception?  
→ sense of direction?





# 1. Aims and challenges: compass and perception

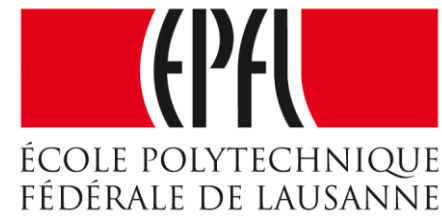
Sense of direction  
→ internal compass



Visual Perception  
→ weak contrasts  
→ world is continuous



# Computational Neuroscience: Neuronal Dynamics of Cognition



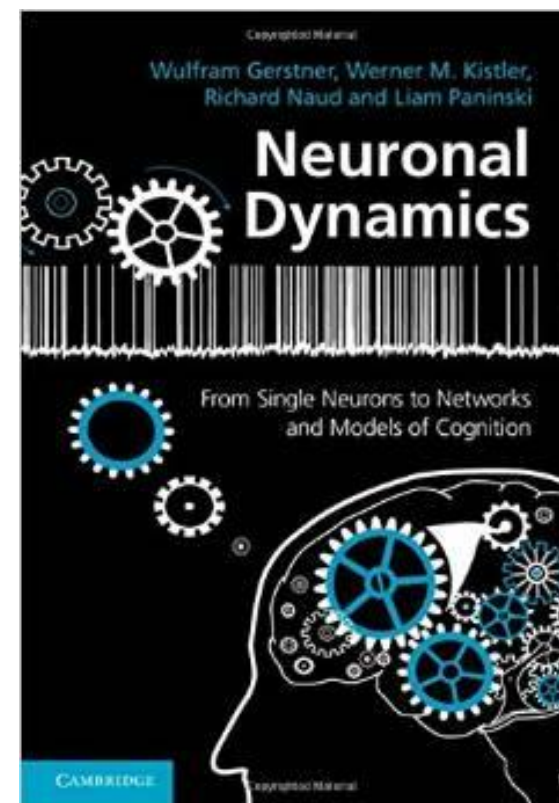
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### 1. Aims and challenges

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### 2. Transients

- generalized integrate-and-fire model
- transients can be sharp or slow

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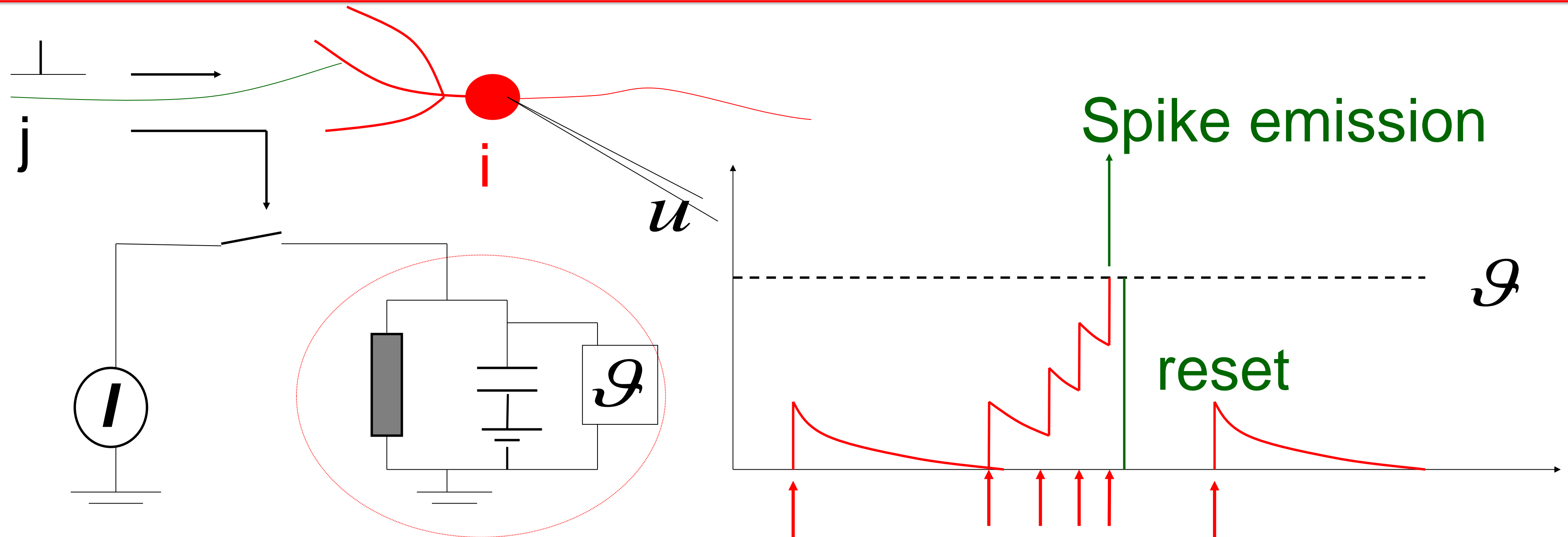
### 7. Head direction cells

## 2. Aims of this section: Transients

---

- beyond stationary states  
→ transients?
- but then neuron model matters!  
→ introduce **generalized integrate-and-fire models:**
  - Spike Response Model (SRM)
  - Generalized Linear Model (GLM)

## 2. Leaky Integrate-and-Fire Model



$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

linear

$$u(t) = \mathcal{G} \Rightarrow \text{Fire+reset} \quad u \rightarrow u_r$$

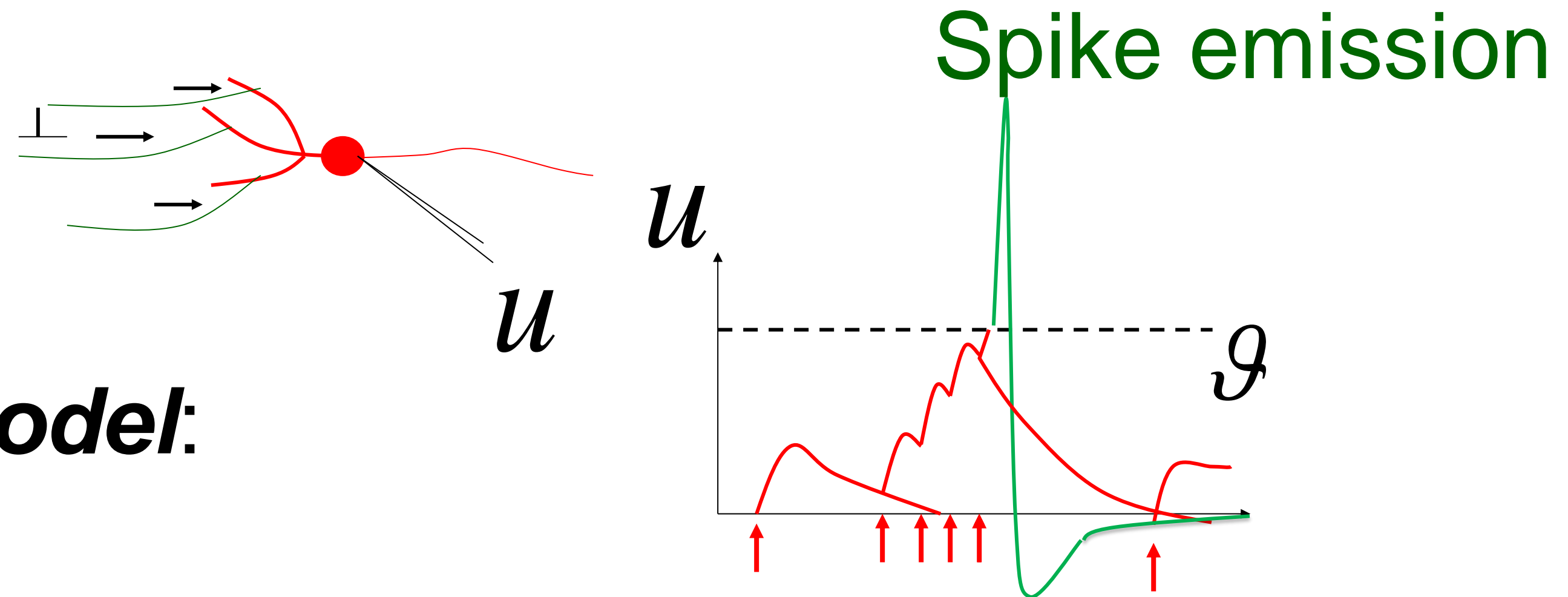
threshold

## 2. Generalized Integrate-and-Fire Model

### **Leaky Integrate-and-Fire Model:**

*passive membrane*  
+ *threshold*  
+ *reset*

equivalent  
description

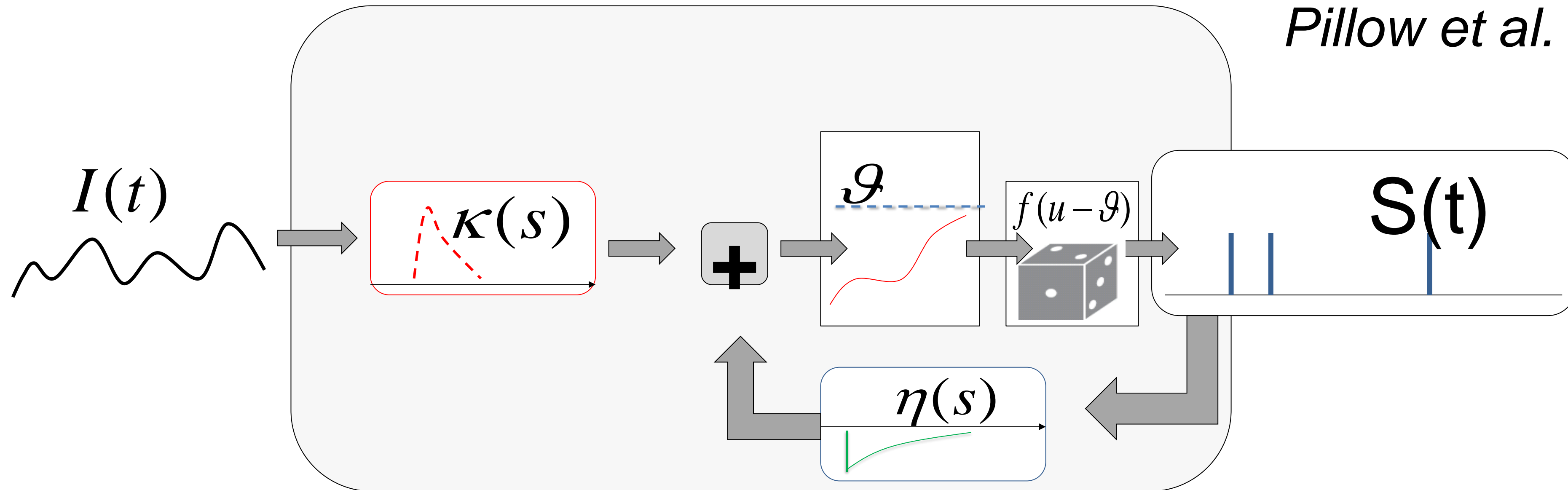


Input spike causes an EPSP  
= excitatory postsynaptic potential

- output spikes are events
- generated at threshold
- after spike: reset/refractoriness
- add  $\boxed{\eta(s)}$  (spike afterpotential)

# Spike Response Model (SRM) Generalized Linear Model (GLM)

*Gerstner et al.,  
1992, 2000  
Truccolo et al., 2005  
Pillow et al. 2008*



**potential**  $u(t) = \int \underline{\eta(s)} S(t-s) ds + \int_0^\infty \underline{\kappa(s)} I(t-s) ds + u_{rest}$

**firing intensity**  $\rho(t) = f(u(t) - g)$

(escape noise)

e.g.  $\rho(t) = \rho_0 \exp\left[\frac{u(t) - g}{\Delta}\right]$



## 2. Leaky Integrate-and-Fire Model: input potential

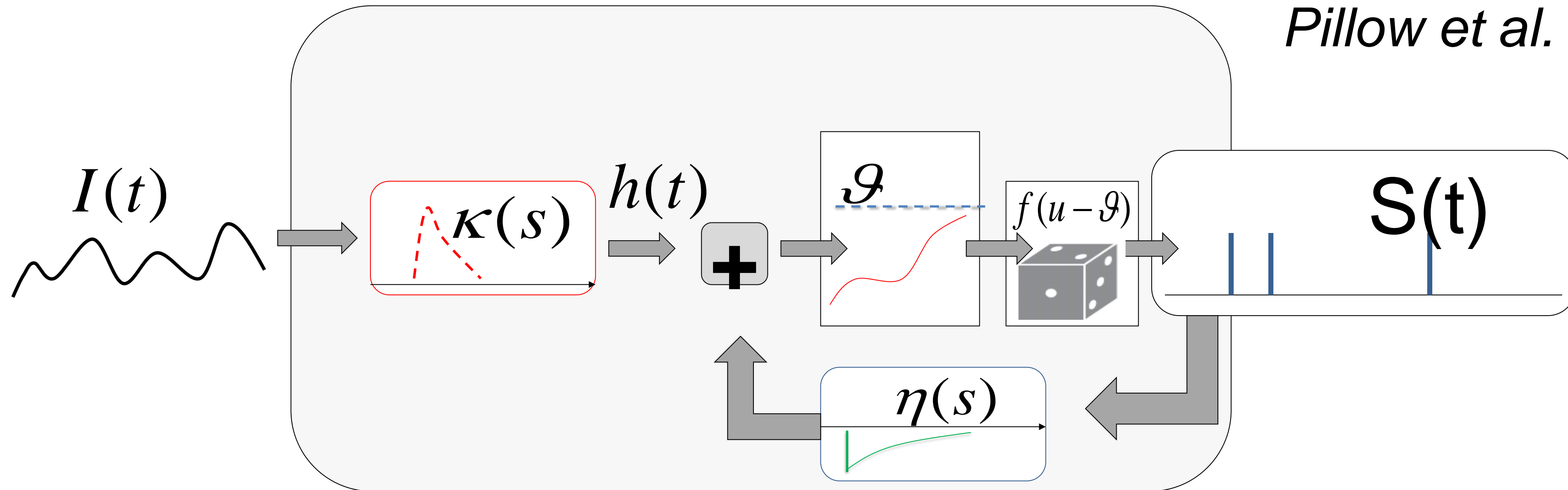
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$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

$$u(t) = u_{rest} + \textit{input potential} + \textit{reset potential}$$

# Spike Response Model (SRM) Generalized Linear Model (GLM)

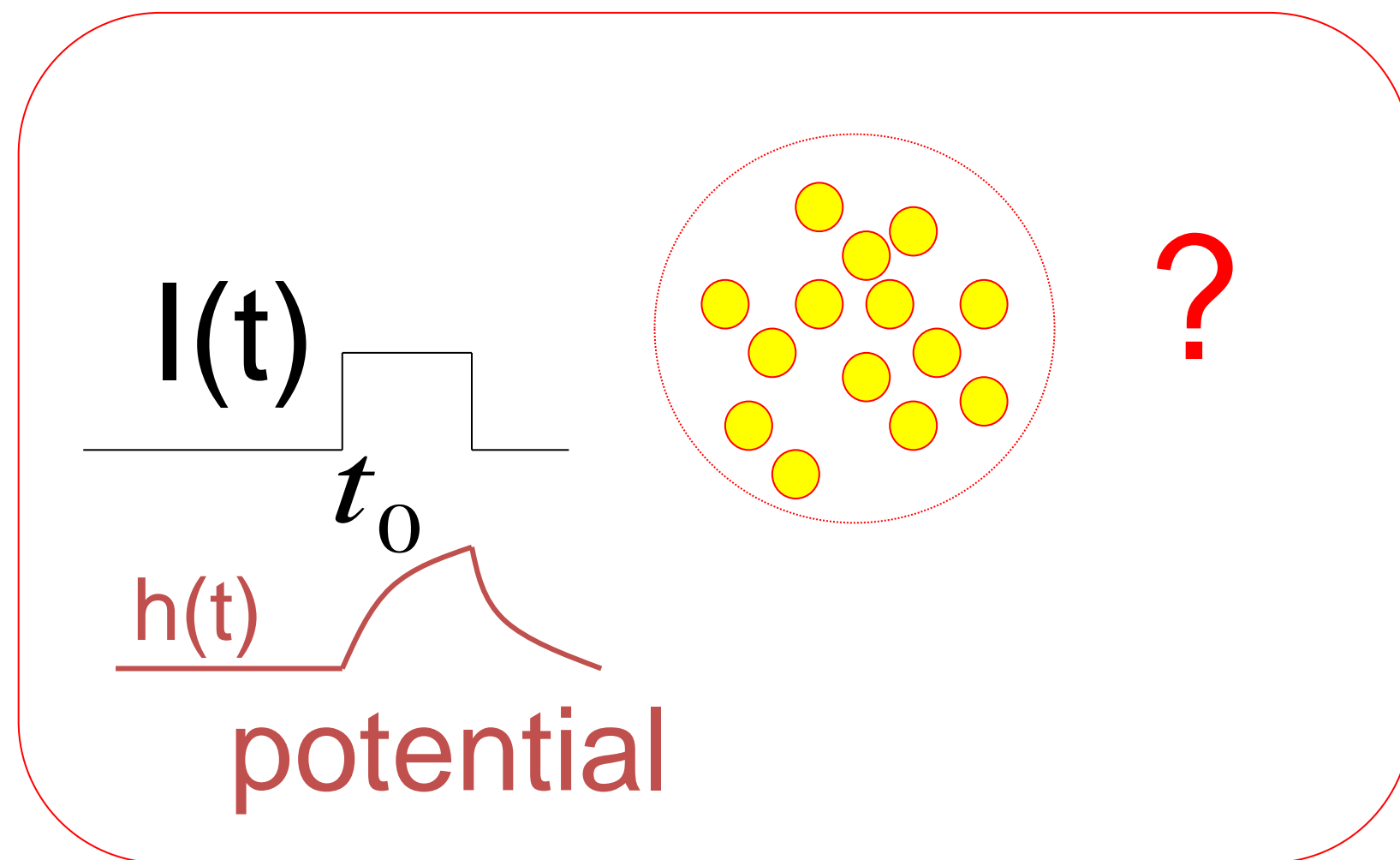
*Gerstner et al.,  
1992, 2000  
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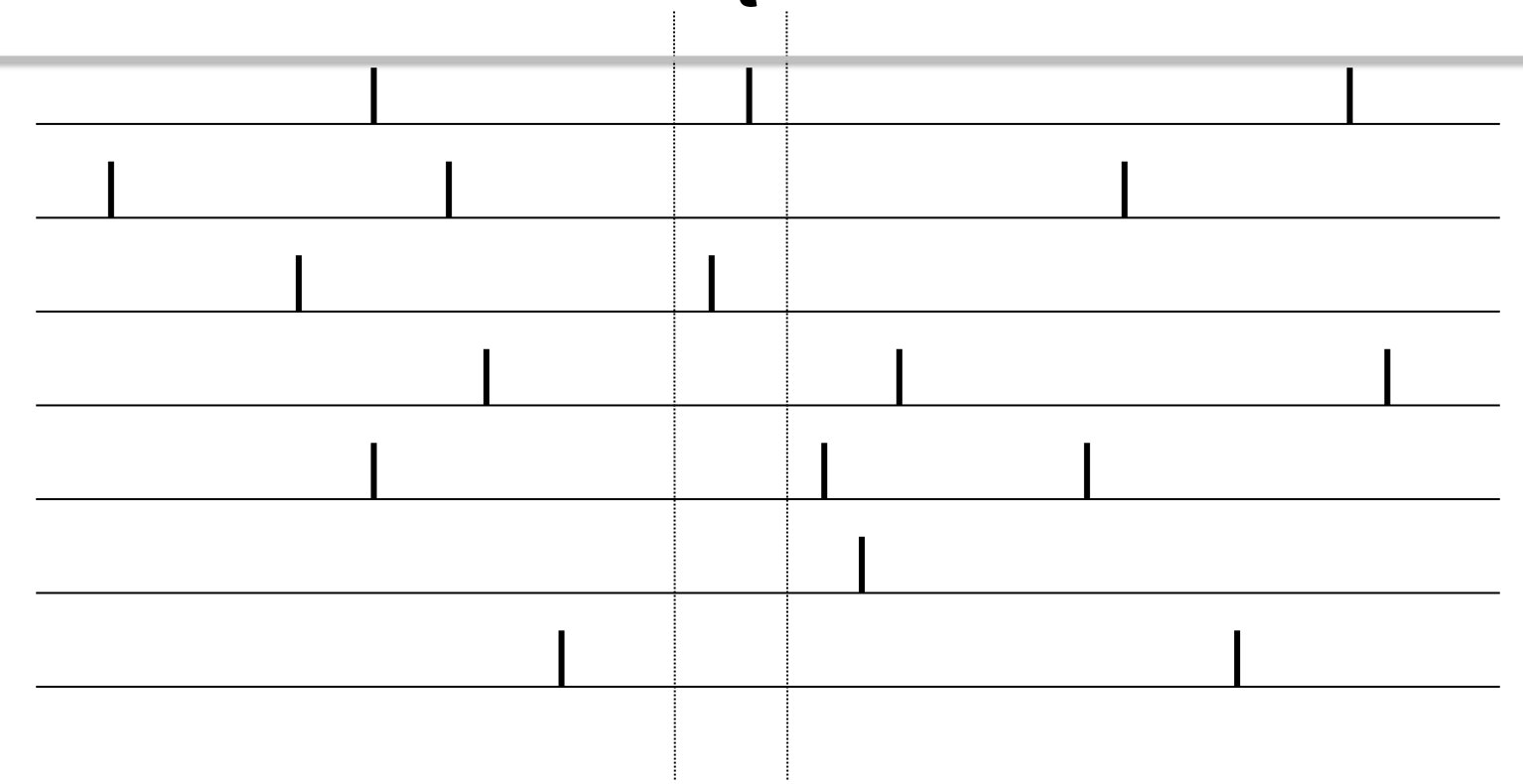
**potential**  $u(t) = u_{rest} + \underbrace{\int_0^\infty \kappa(s) I(t-s) ds}_{h(t)} + \int \eta(s) S(t-s) ds$

$$u(t) = u_{rest} + \text{input potential} + \text{reset potential}$$

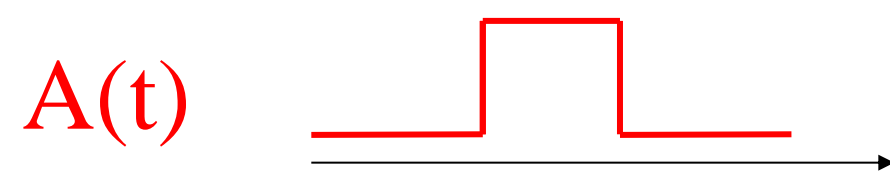
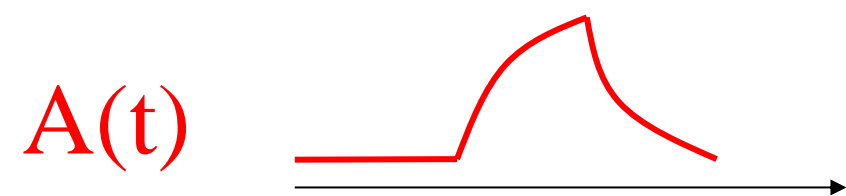
## 2. Transients in a population of **uncoupled** neurons



population  
activity



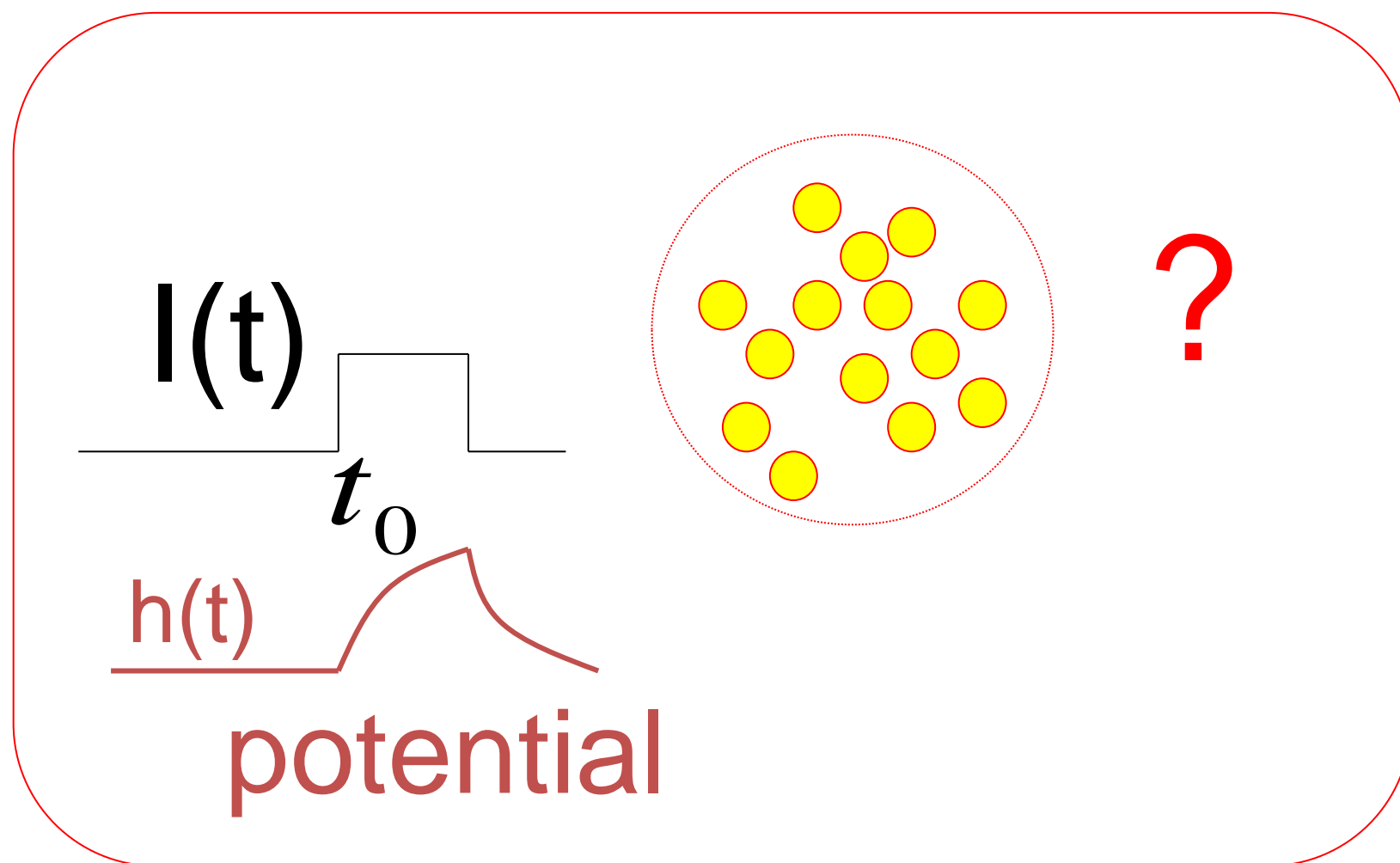
$$A(t) = \frac{n(t - \frac{\Delta t}{2}; t + \frac{\Delta t}{2})}{N \Delta t}$$



$$A(t) = F(\underline{h(t)}) = F\left(\int \kappa(s) I(t-s) ds\right)$$

$$A(t) = g(\underline{I(t)})$$

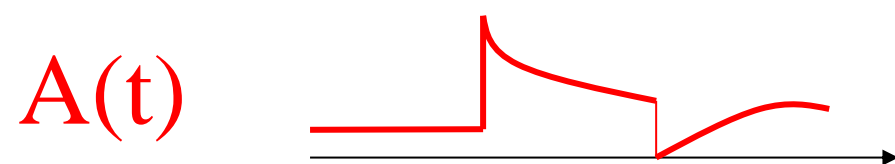
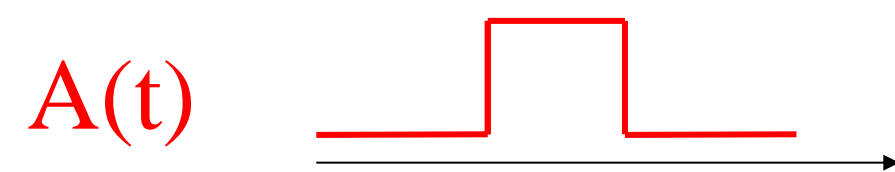
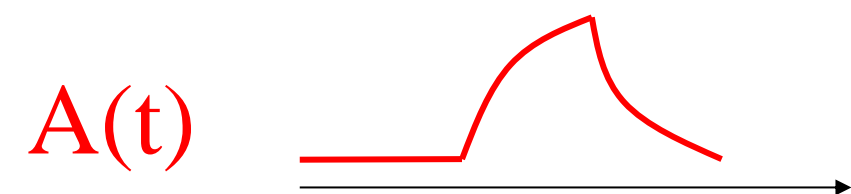
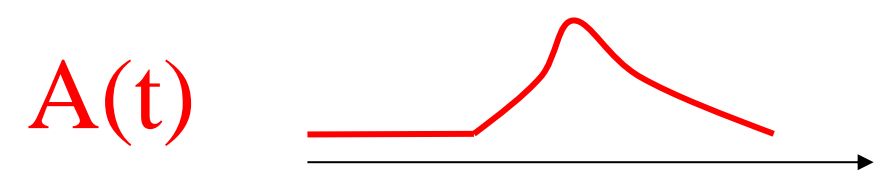
## 2. Transients in a population of **uncoupled** neurons



***Which would you choose?***

$$A(t) = \frac{n(t - \frac{\Delta t}{2}; t + \frac{\Delta t}{2})}{N \Delta t}$$

population  
activity



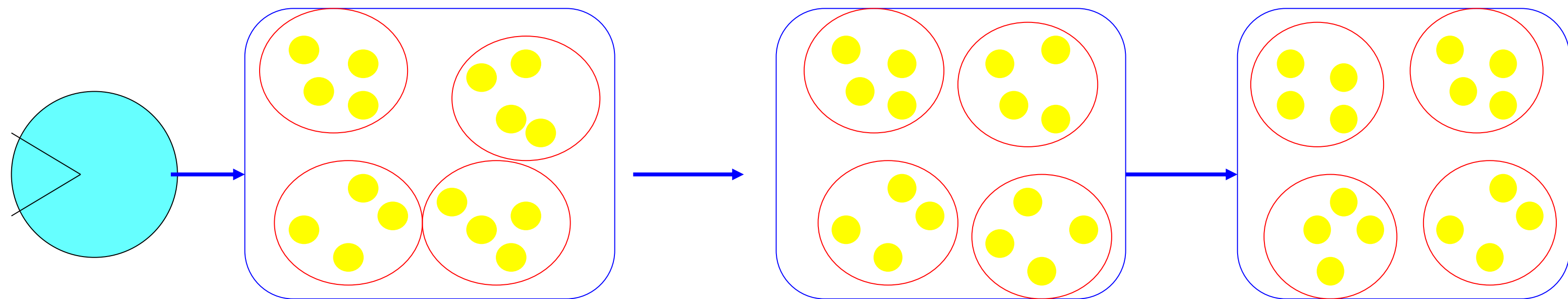
$$\tau \frac{d}{dt} A(t) = -A(t) + F(h(t))$$

$$A(t) = F(\underline{h(t)}) = F\left(\int \kappa(s) I(t-s) ds\right)$$

$$A(t) = g(\underline{I(t)})$$

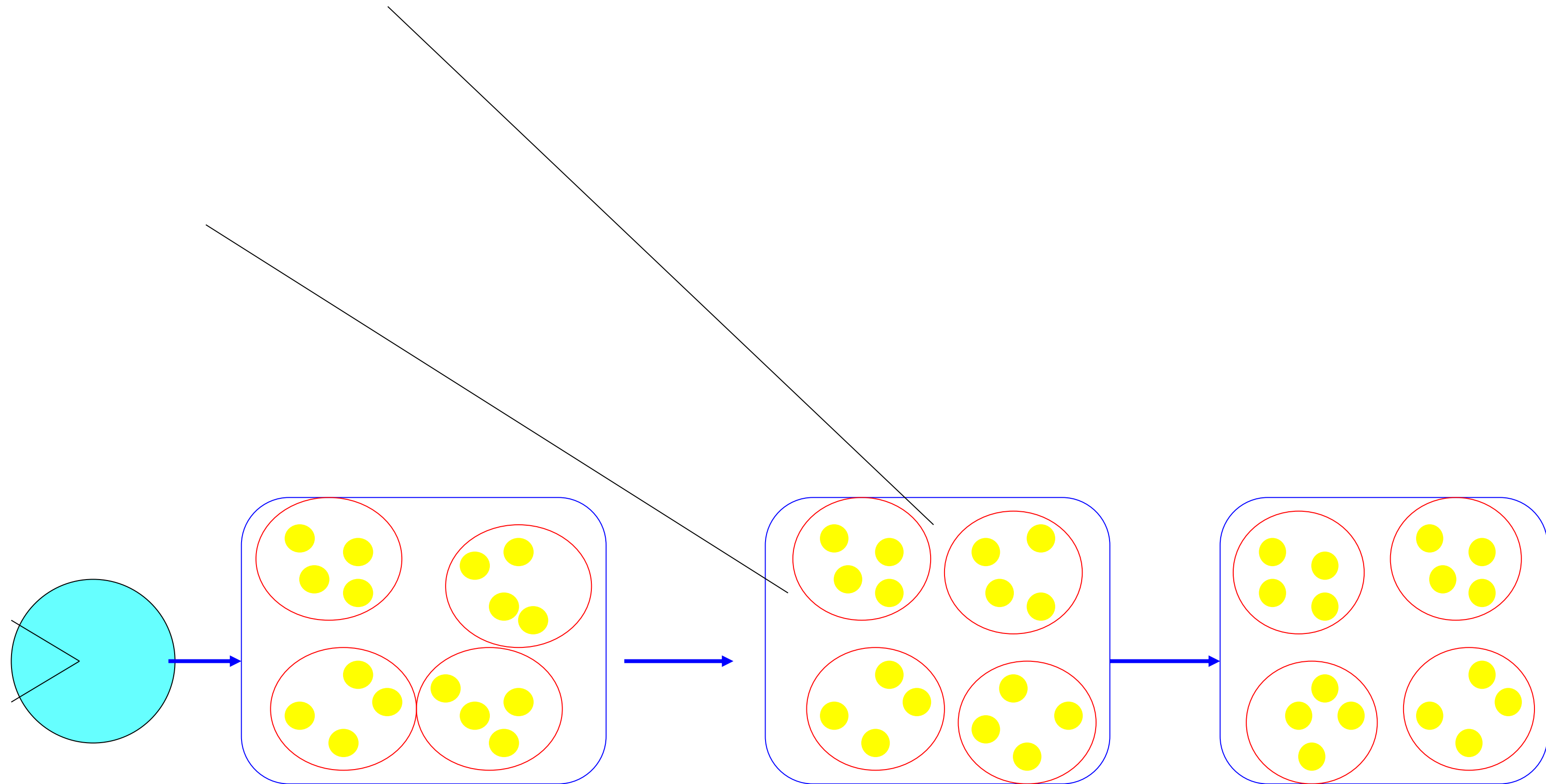
$$A(t) = g(I(t), I'(t))$$

## 2. Transients in a population of neurons: simulations

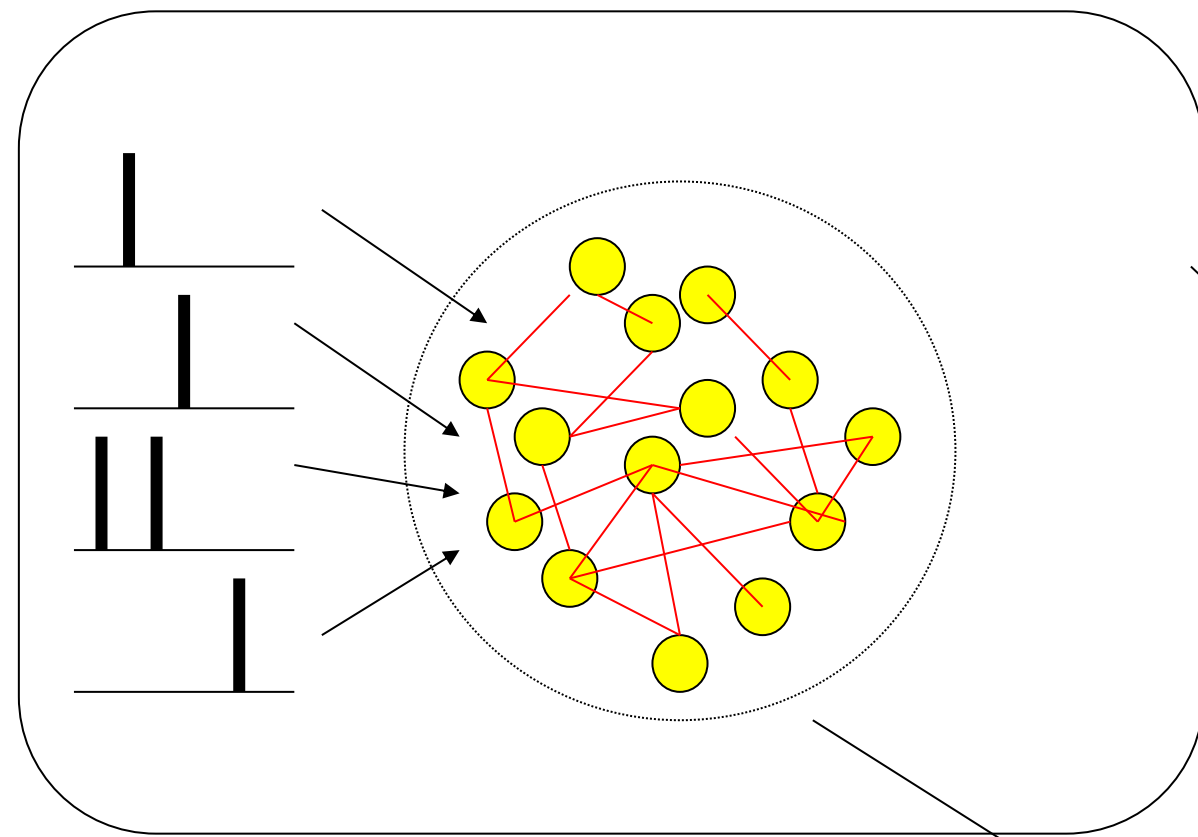




## 2. Transients in a population of neurons



## 2. Transients in a population of neurons: simulations



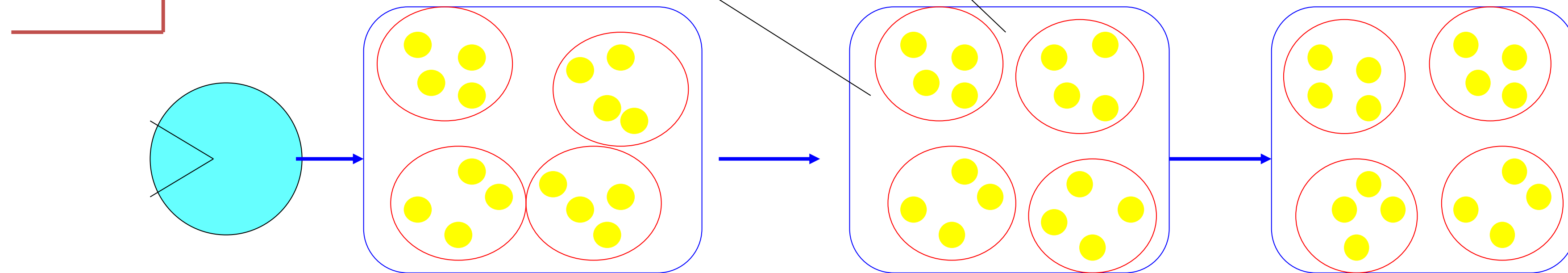
### Population

- 50 000 neurons
- 20 percent inhibitory
- randomly connected

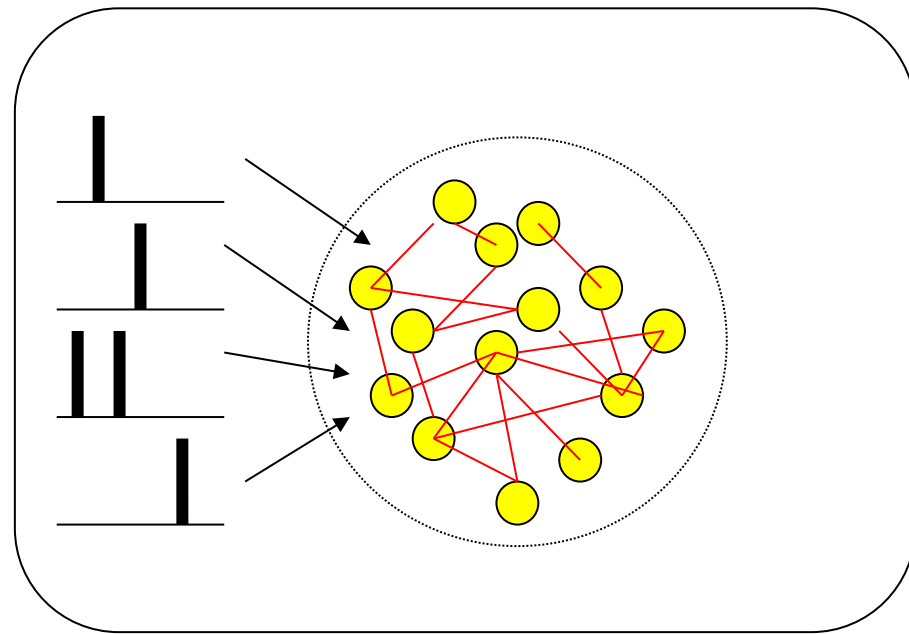
### Connections

4000 external  
4000 within excitatory  
1000 within inhibitory

**input** { low rate  
          - high rate



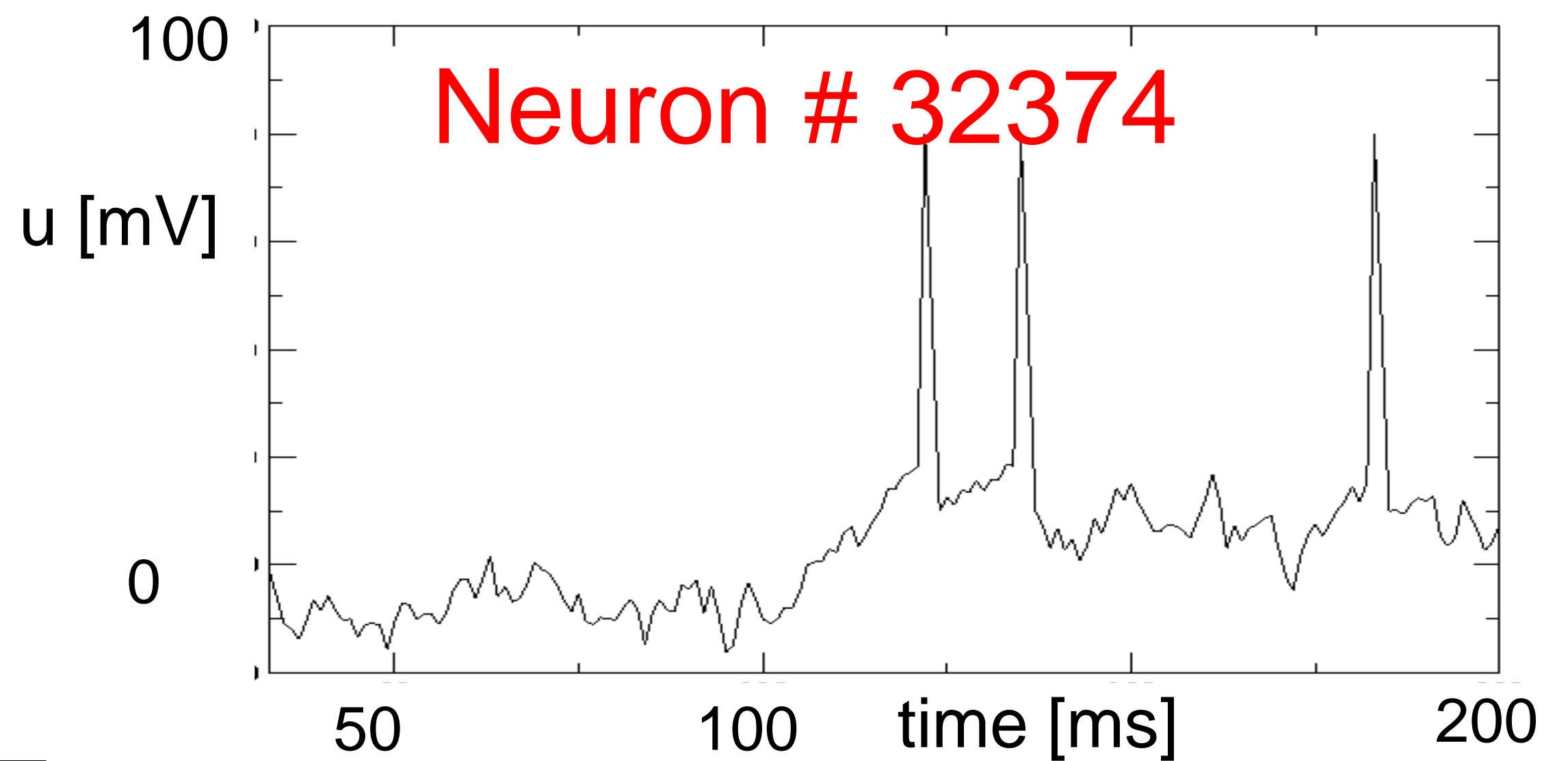
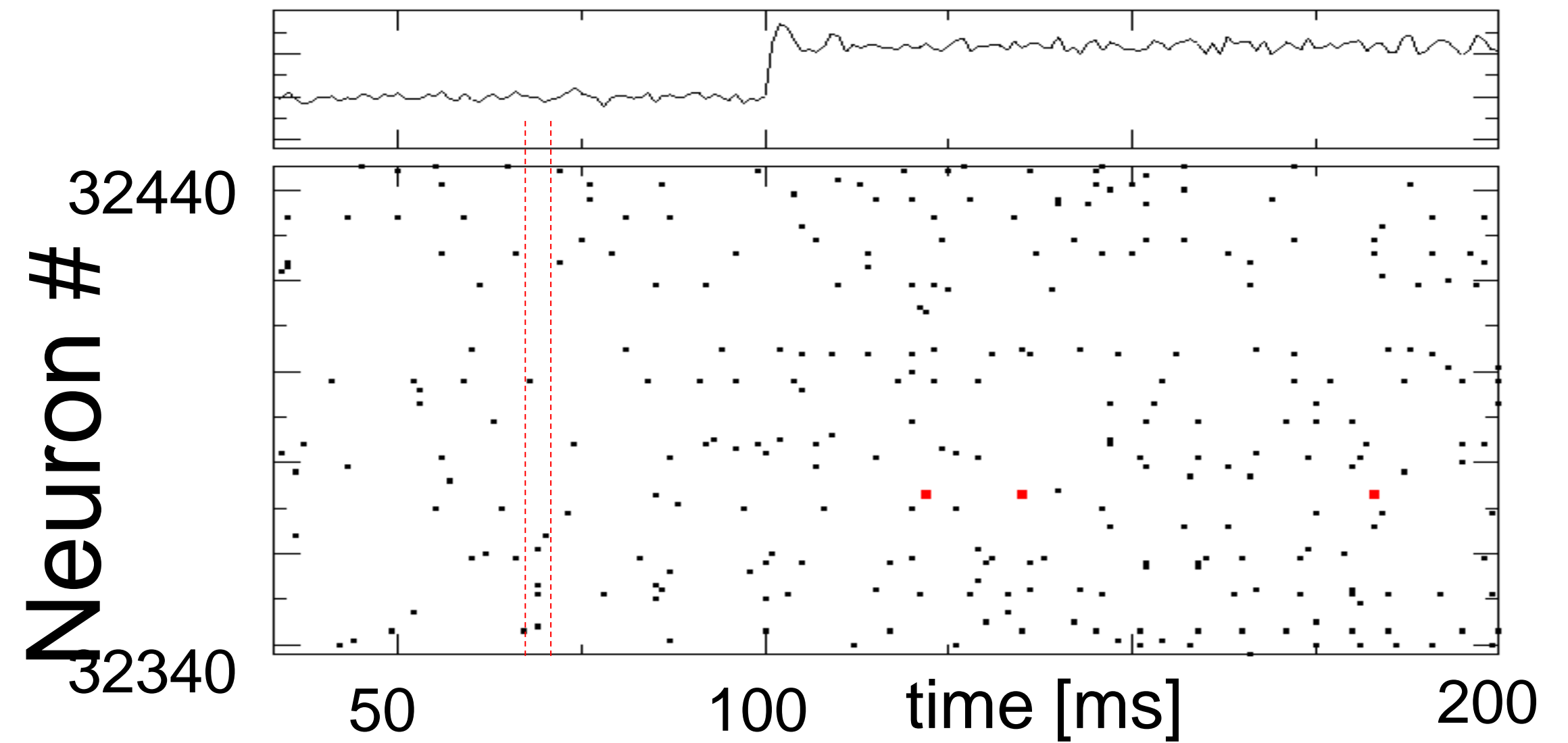
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input { low rate  
high rate

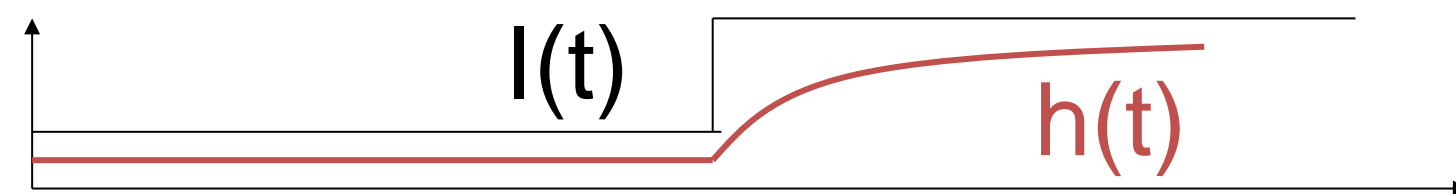
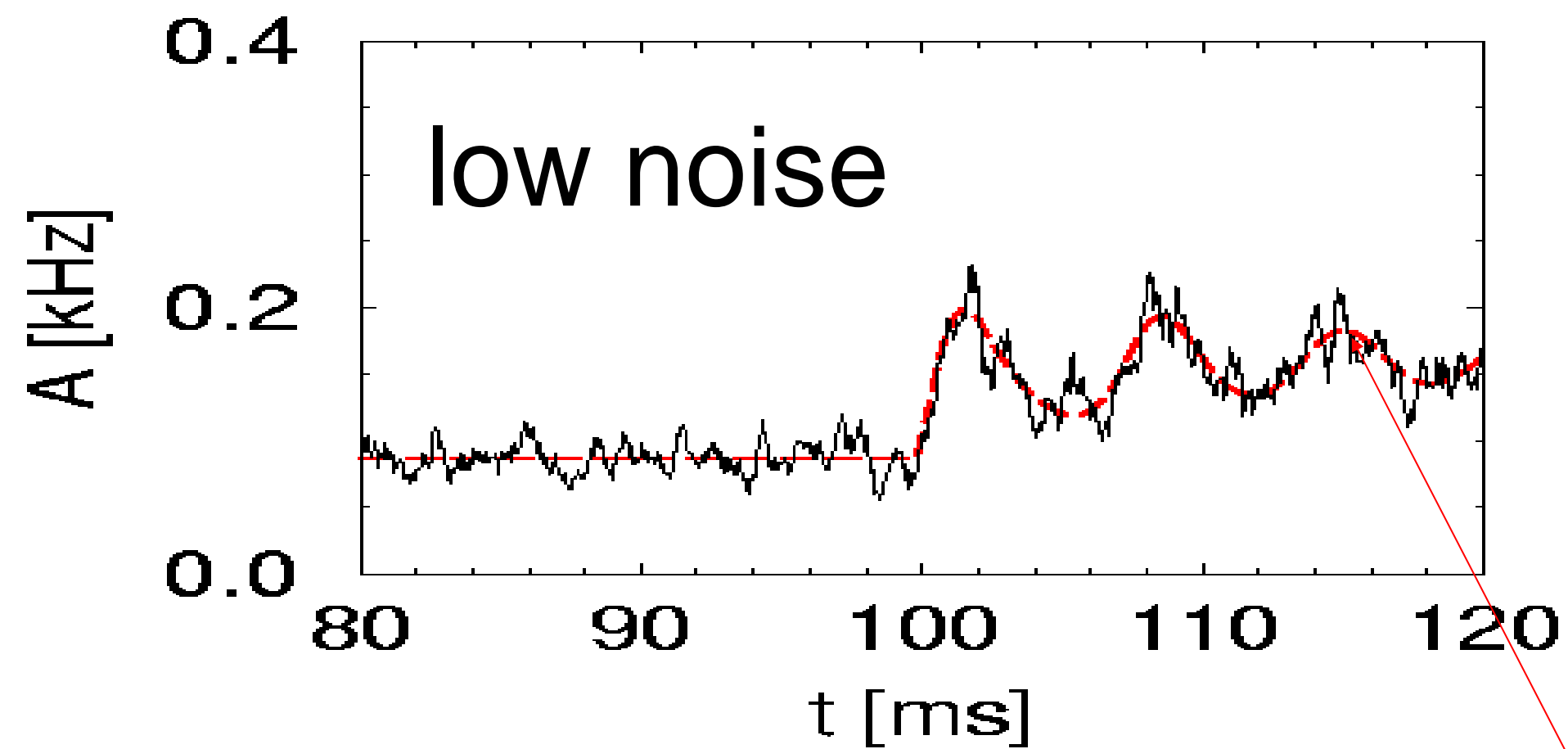
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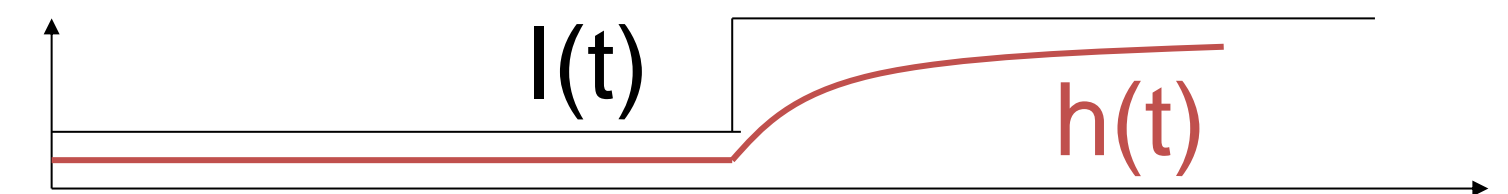
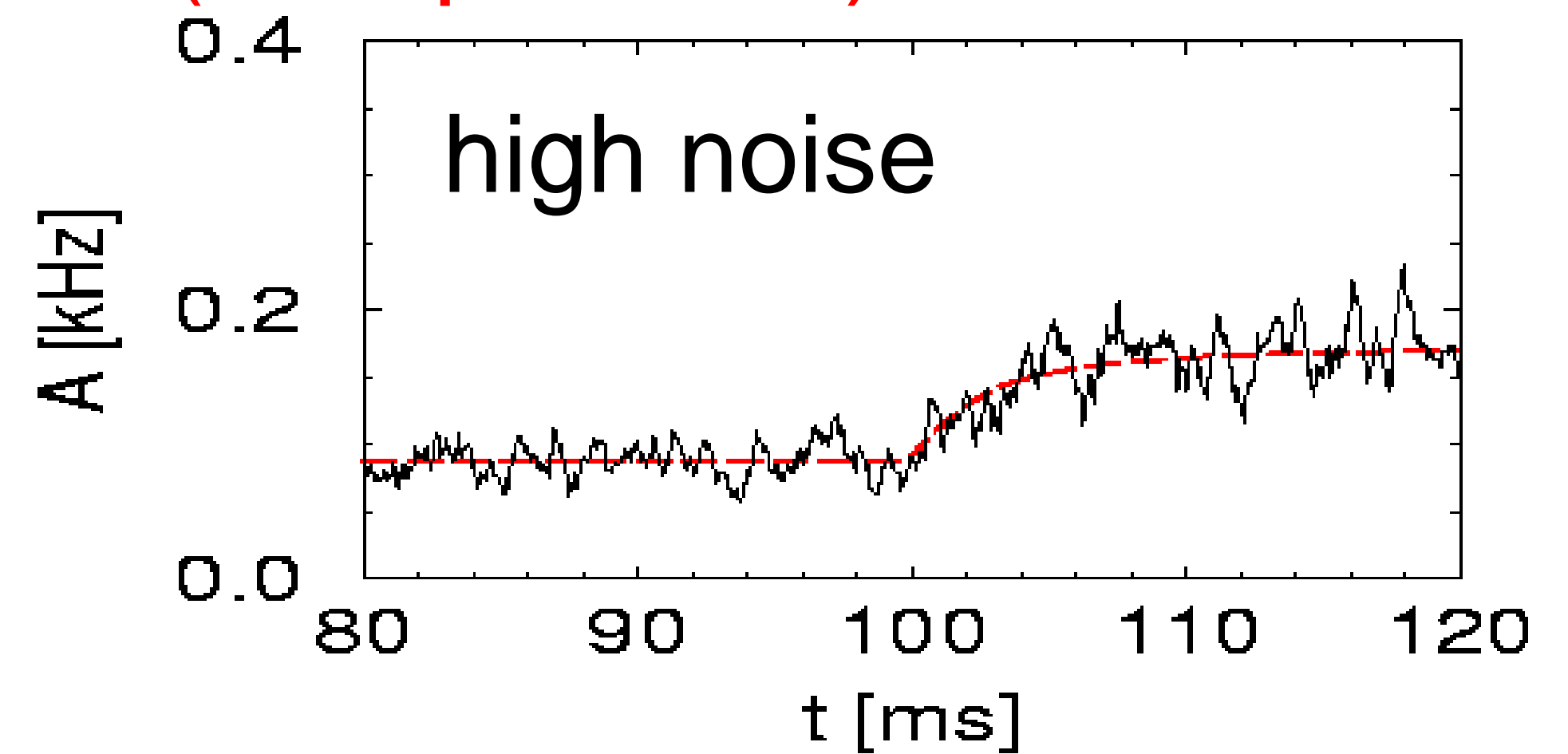
## 2. Transients for populations of noisy neurons

uncoupled population  
of SRM neurons with noise (escape noise)



fast transient

$$A(t) \approx g(I(t))$$



slow transient

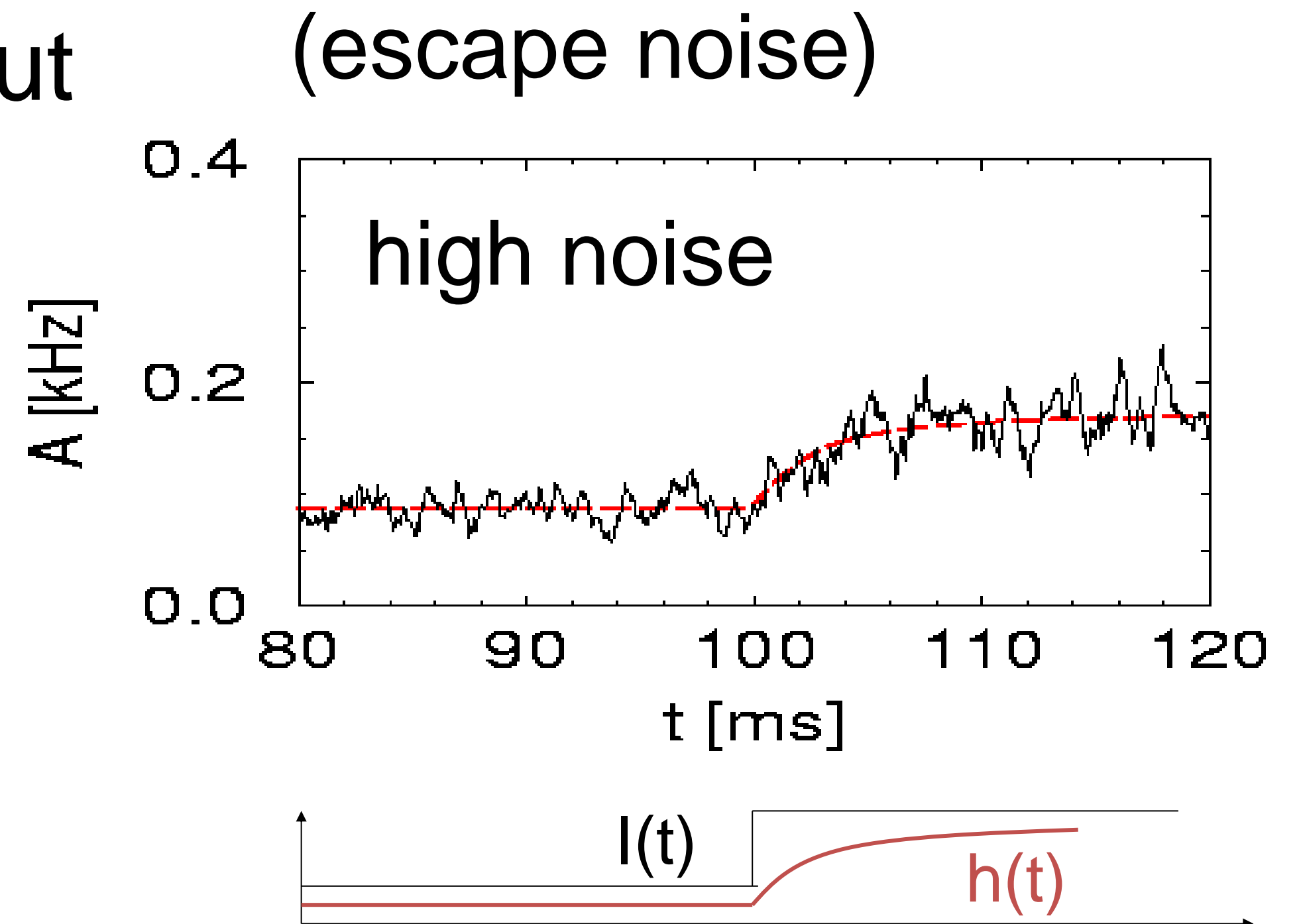
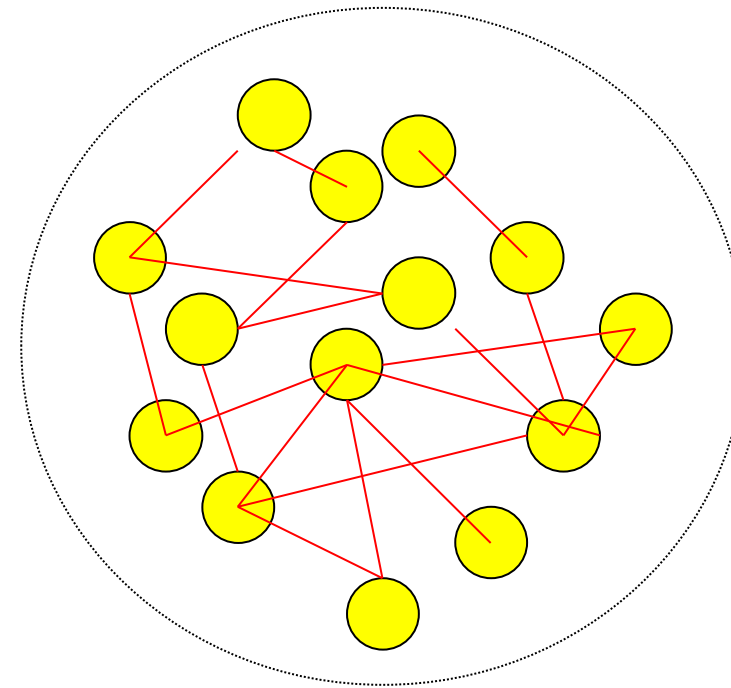
$$A(t) = F(h(t))$$

but transient oscillations

## 2. High-noise activity equation

Membrane potential caused by input

$$\tau \frac{d}{dt} h(t) = -h(t) + R I(t)$$



slow transient

$$A(t) = F(h(t))$$

In the limit of **high noise**,



## 2. High-noise activity equation

Population activity

$$A(t) = F(h(t))$$

Membrane potential caused by input

$$\tau \frac{d}{dt} h(t) = -h(t) + R I(t)$$

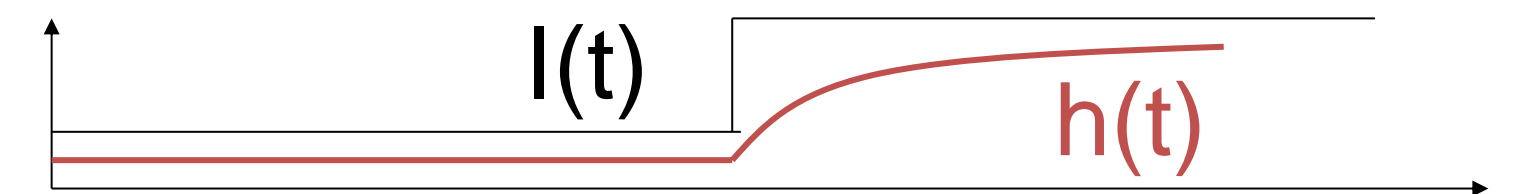
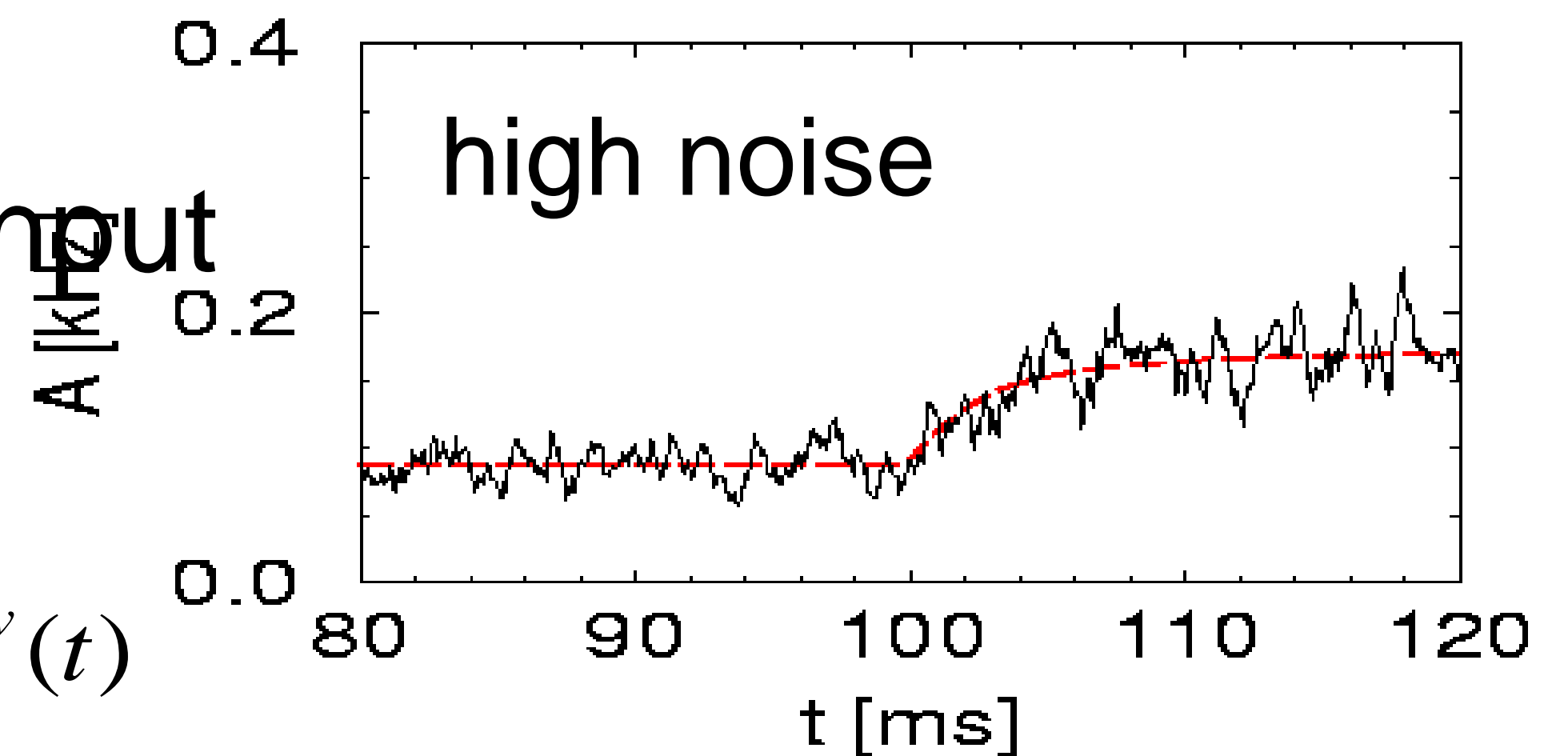
$$I(t) = I^{ext}(t) + I^{netw}(t)$$

$$I(t) = I^{ext}(t) + J_0 q A(t)$$

$$I(t) = I^{ext}(t) + J_0 q F(h(t))$$

$$\tau \frac{d}{dt} h(t) = -h(t) + R I^{ext}(t) + \gamma F(h(t))$$

(escape noise)



slow transient

$$A(t) = F(h(t))$$

1 population = 1 differential equation

## 2. Summary: Transients and population equations

$$\tau \frac{d}{dt} h(t) = -h(t) + R I^{ext}(t) + \gamma F(h(t))$$

- population activity equation
- smooth transient
- input potential determines activity

$$A = F(h(t))$$

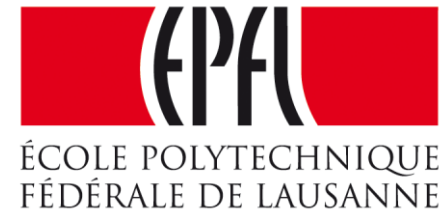
- valid in high-noise regime
- misses sharp transients
- misses transient oscillations

## Population equations

A single homogeneous population of neurons is driven by a step current causing a transient response of the population activity.

- ☐ A single cortical model population can exhibit transient oscillations
- ☐ Transients are always sharp
- ☐ Transients are always slow
- ☐ in a certain limit transients can be slow
- ☐ An escape noise model in the high-noise limit has transients which are always slow
- ☐ A single population described by a single first-order differential equation (no integrals/no delays) can exhibit transient oscillations

# Computational Neuroscience: Neuronal Dynamics of Cognition



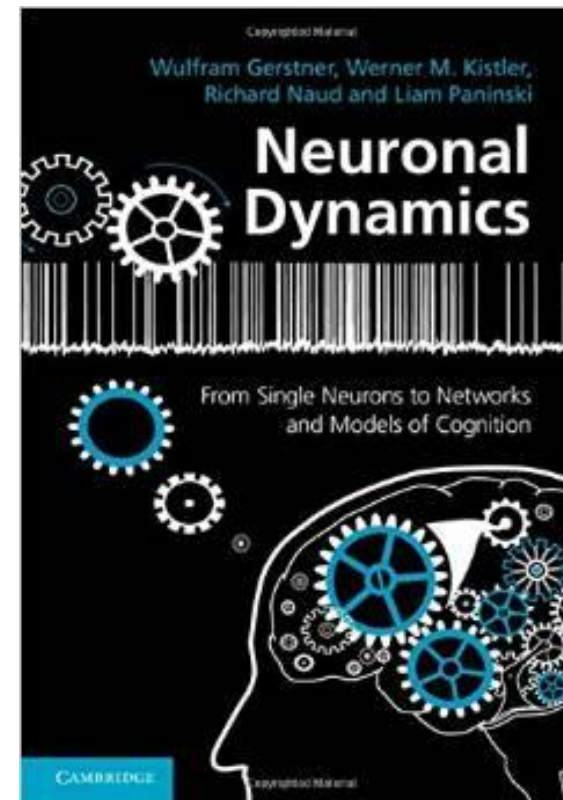
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### 1. Aims and challenges

- review: mean-field arguments

### 2. Transients

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- transients can be sharp or slow

### 3. Spatial continuum (cortex)

- orientation columns

### 4. Spatial continuum (model)

- field equations

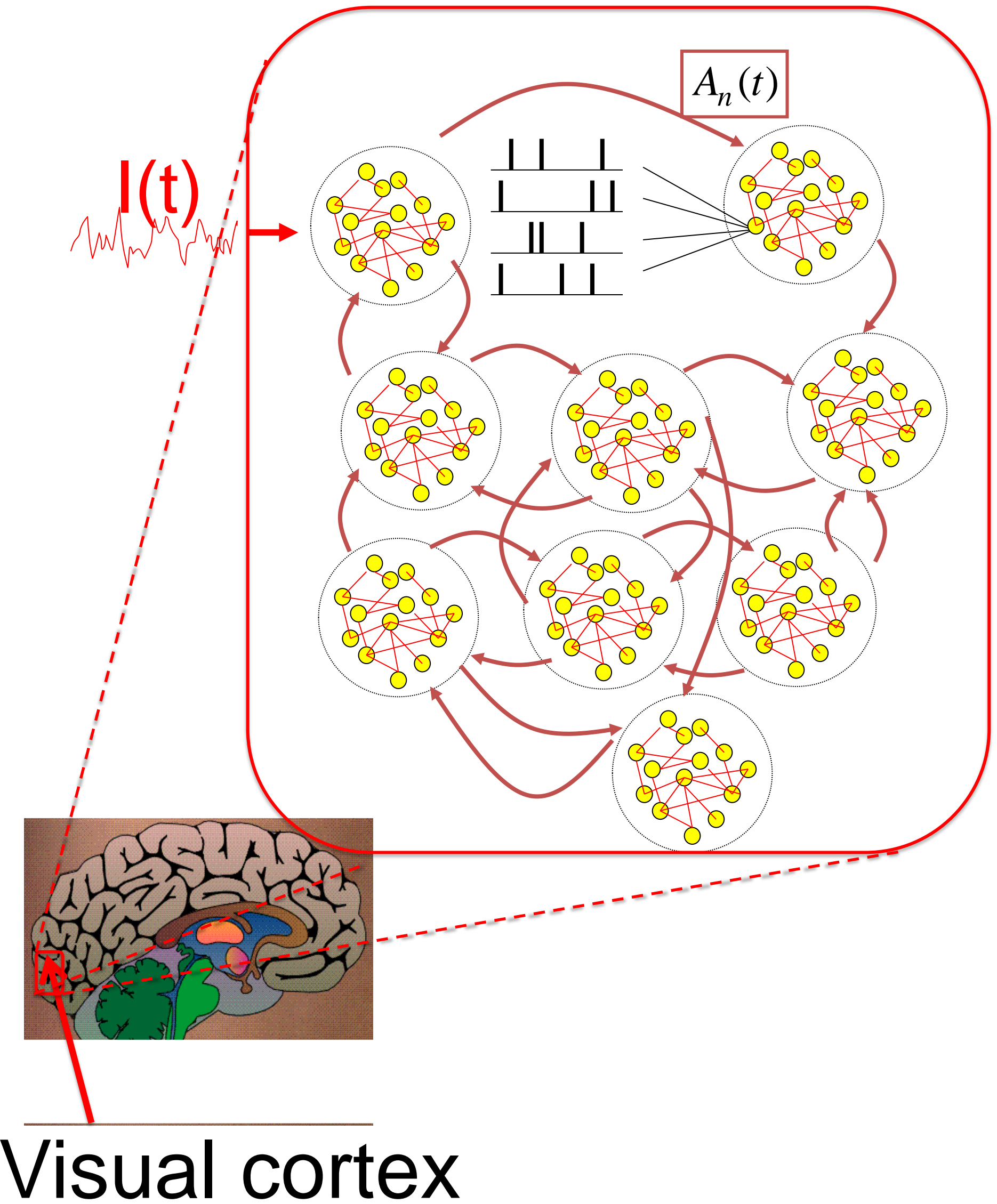
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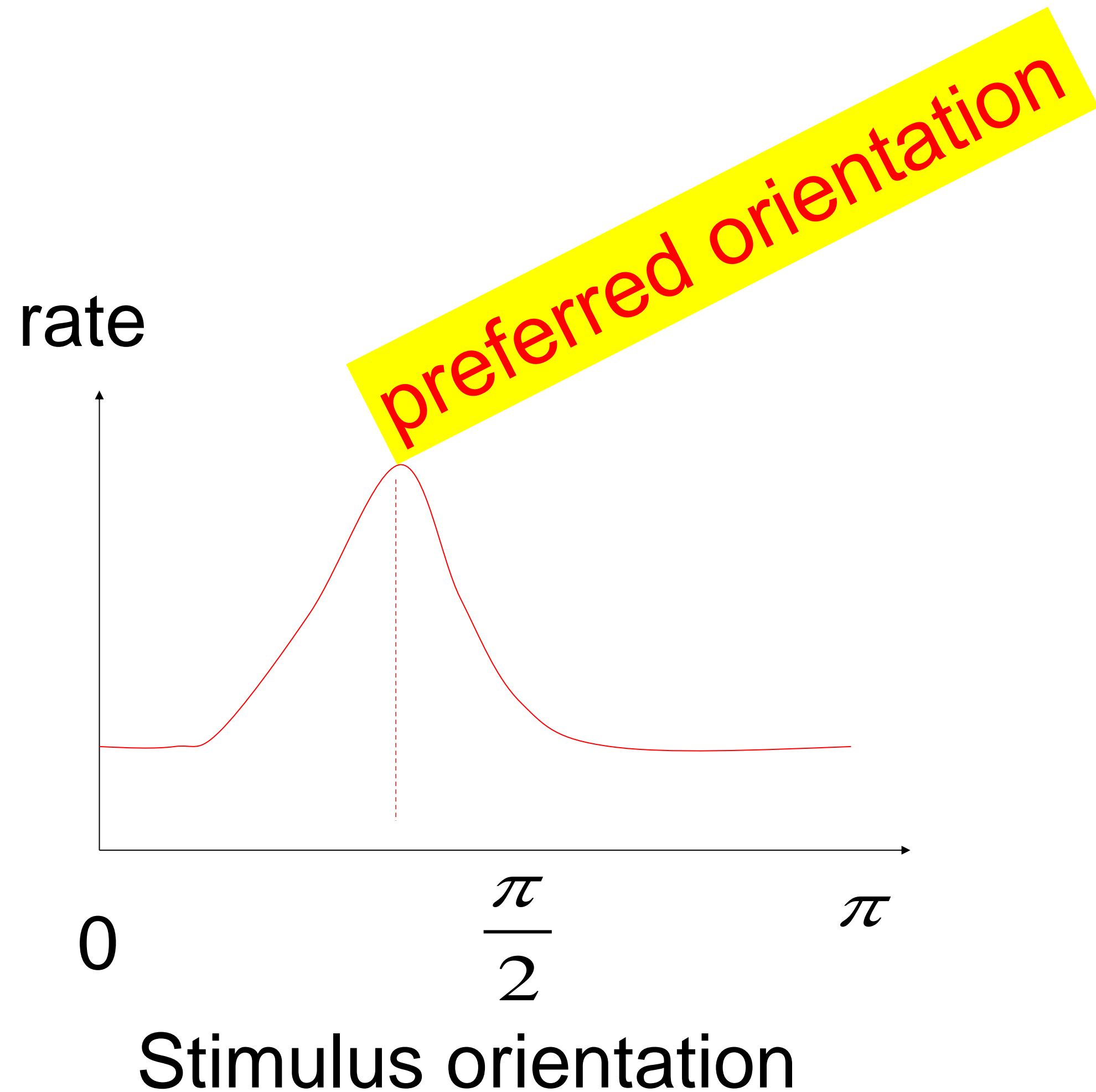
### 7. Head direction cells

### 3. Interacting Populations: how many populations?

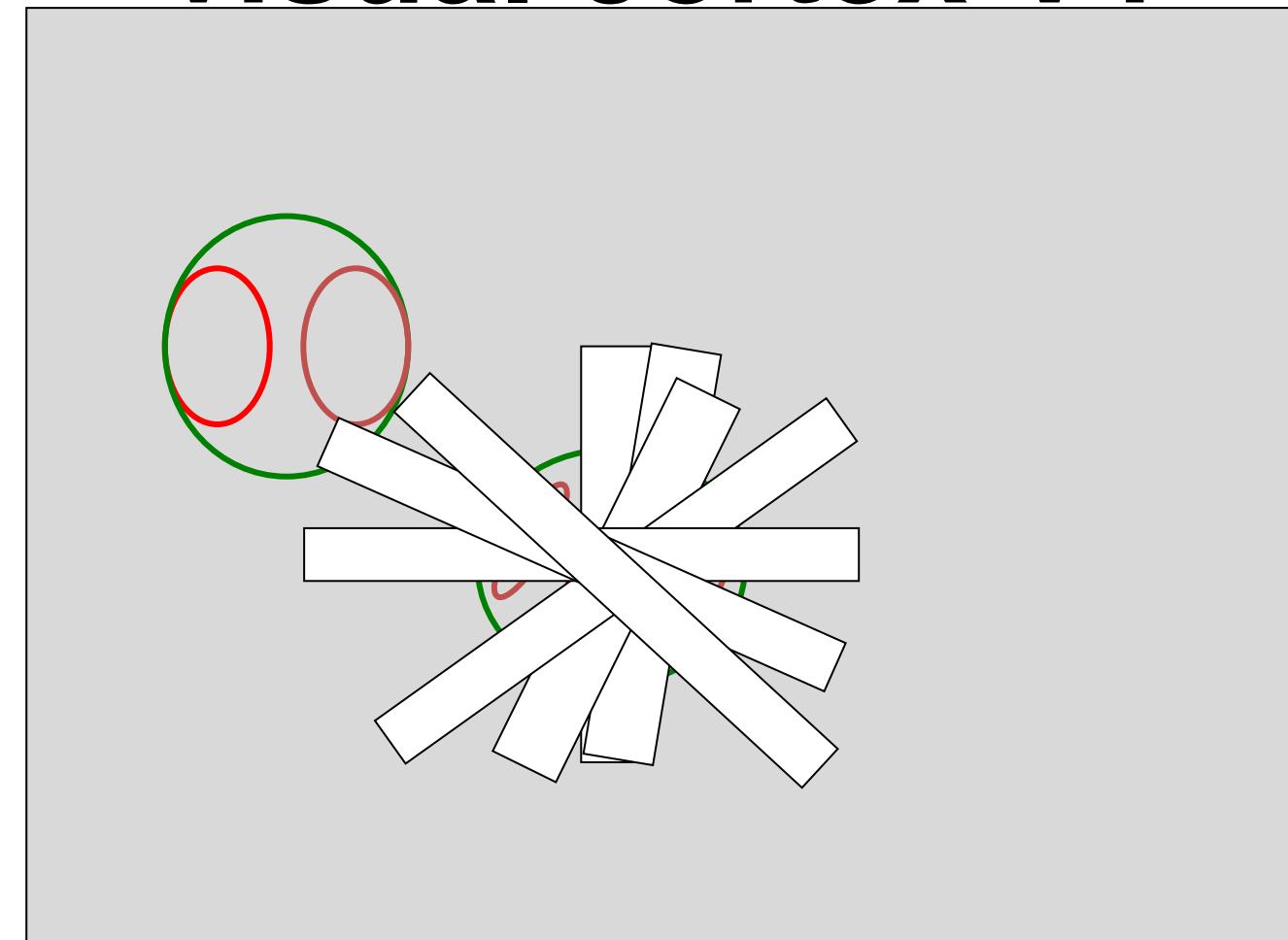




### 3. Review: Receptive fields with Orientation Tuning



Receptive fields:  
**visual cortex V1**



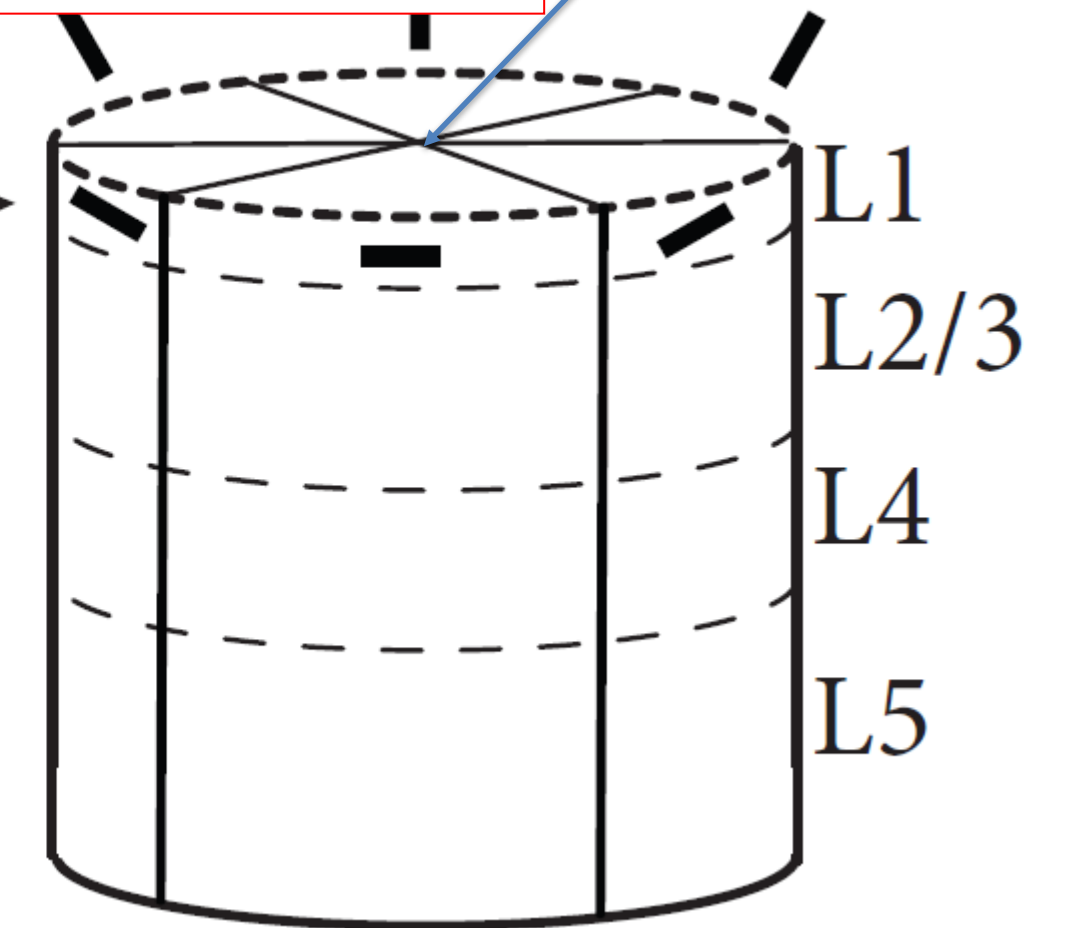
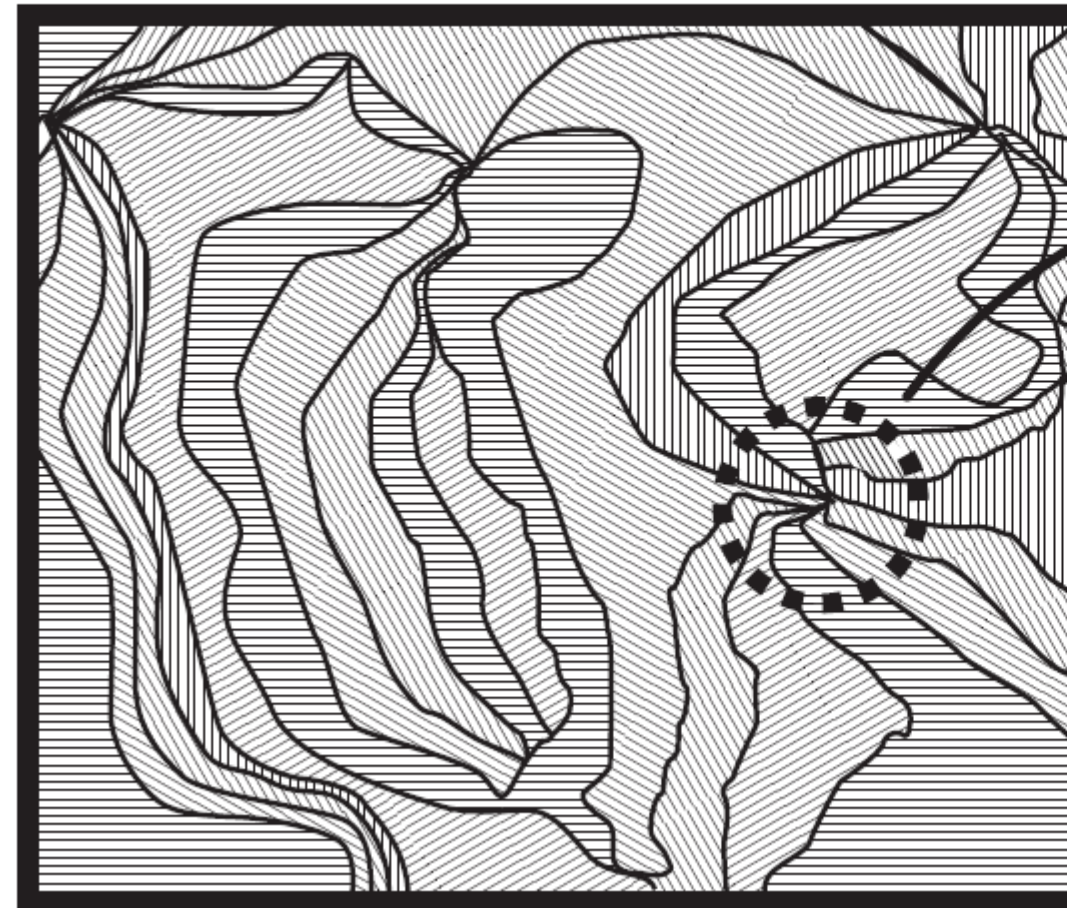
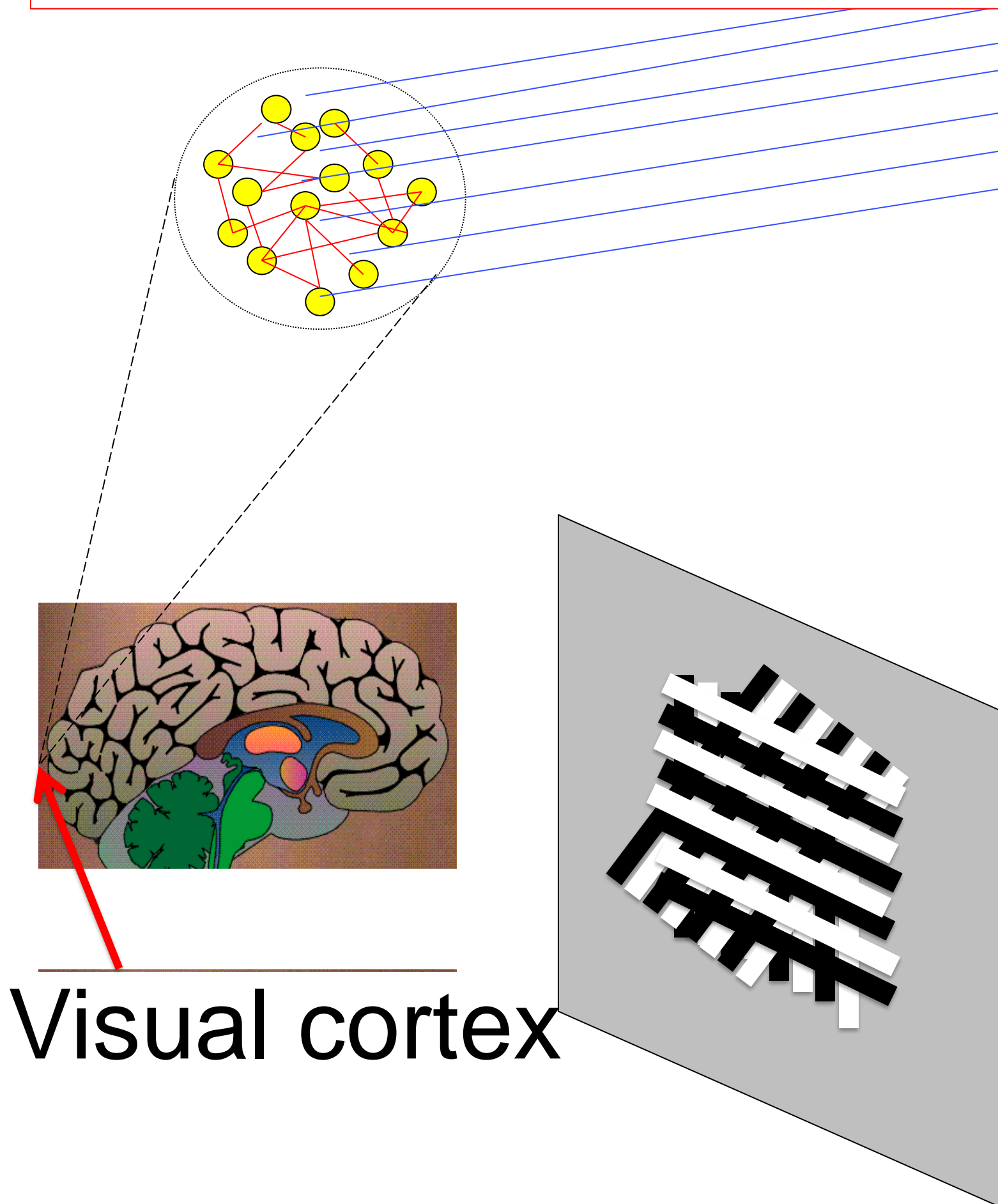
Orientation selective



### 3. Orientation Map

population of neighboring neurons: similar orientations  
as we move along cortical surface: orientation changes

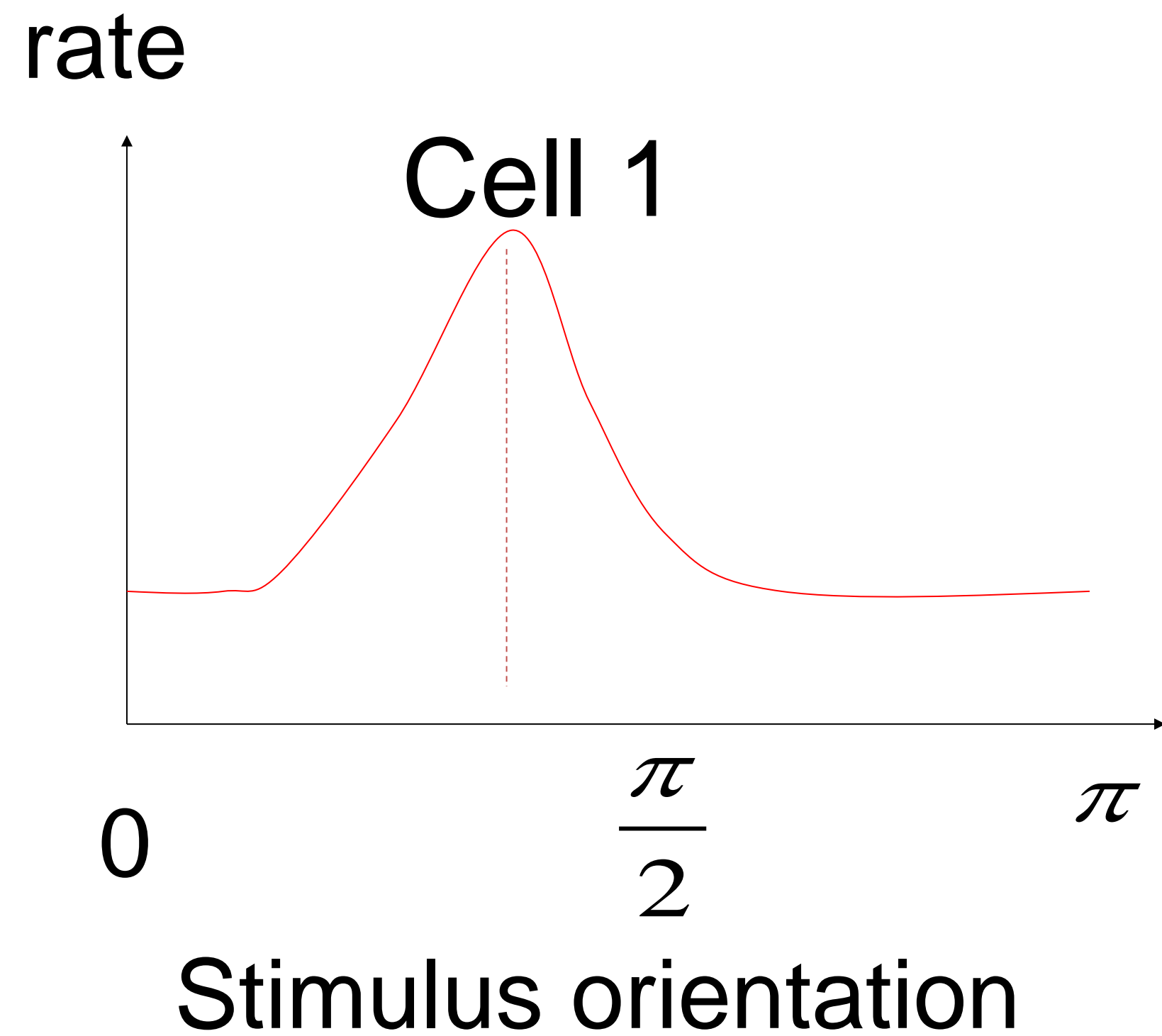
pinwheel



*Image: Gerstner et al.  
Neuronal Dynamics (2014)*

*Bonhoeffer&Grinvald, 1991;  
Bressloff&Cowan, 2002;  
Kaschube et al. 2010*

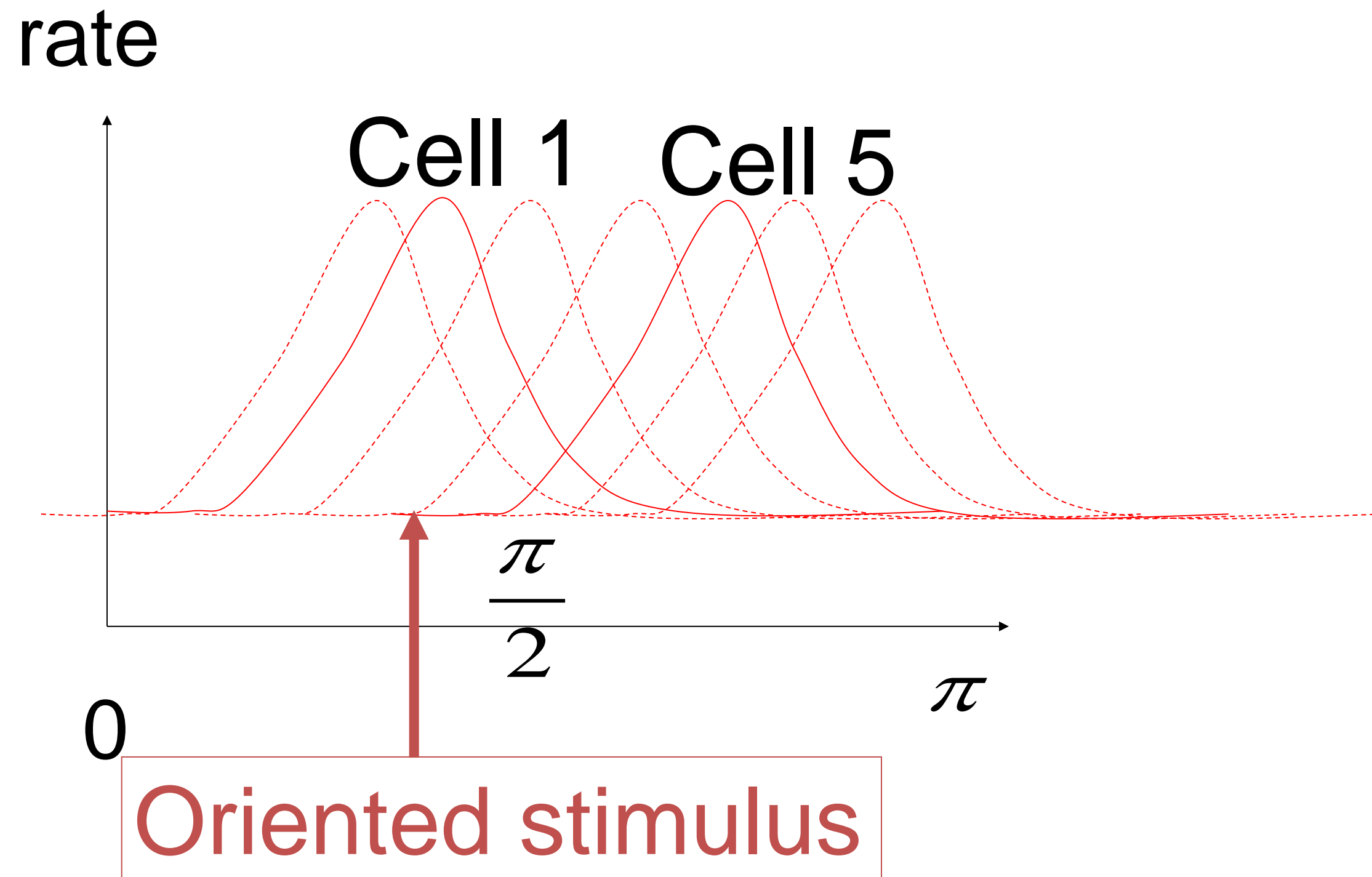
### 3. Do Orientation Columns exist? Do identical cells exist?



### 3. Do Orientation columns exist? Do identical cells exist?

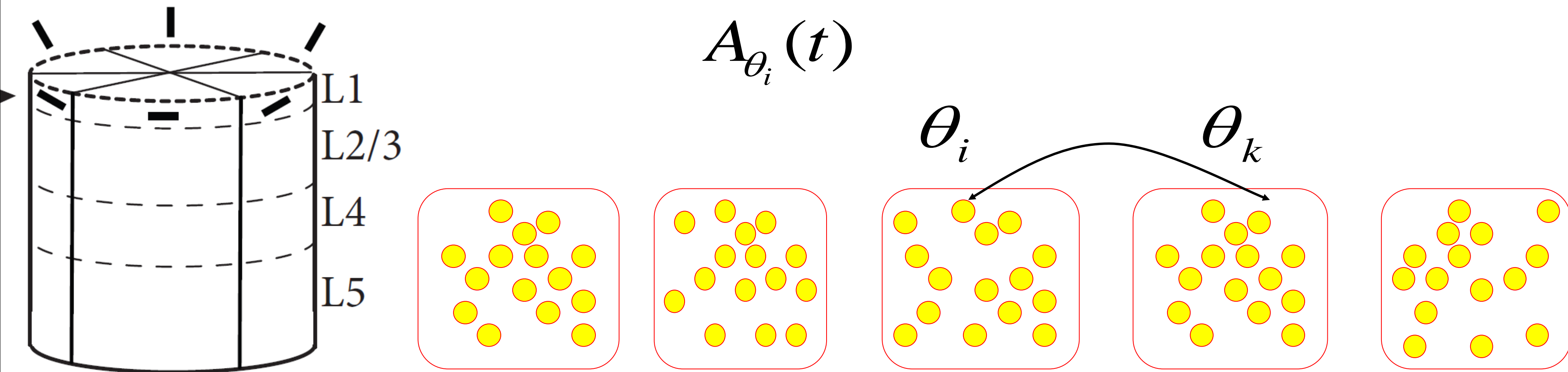
## Coarse coding

Many cells  
(from different columns)  
respond to a single  
stimulus with different rate

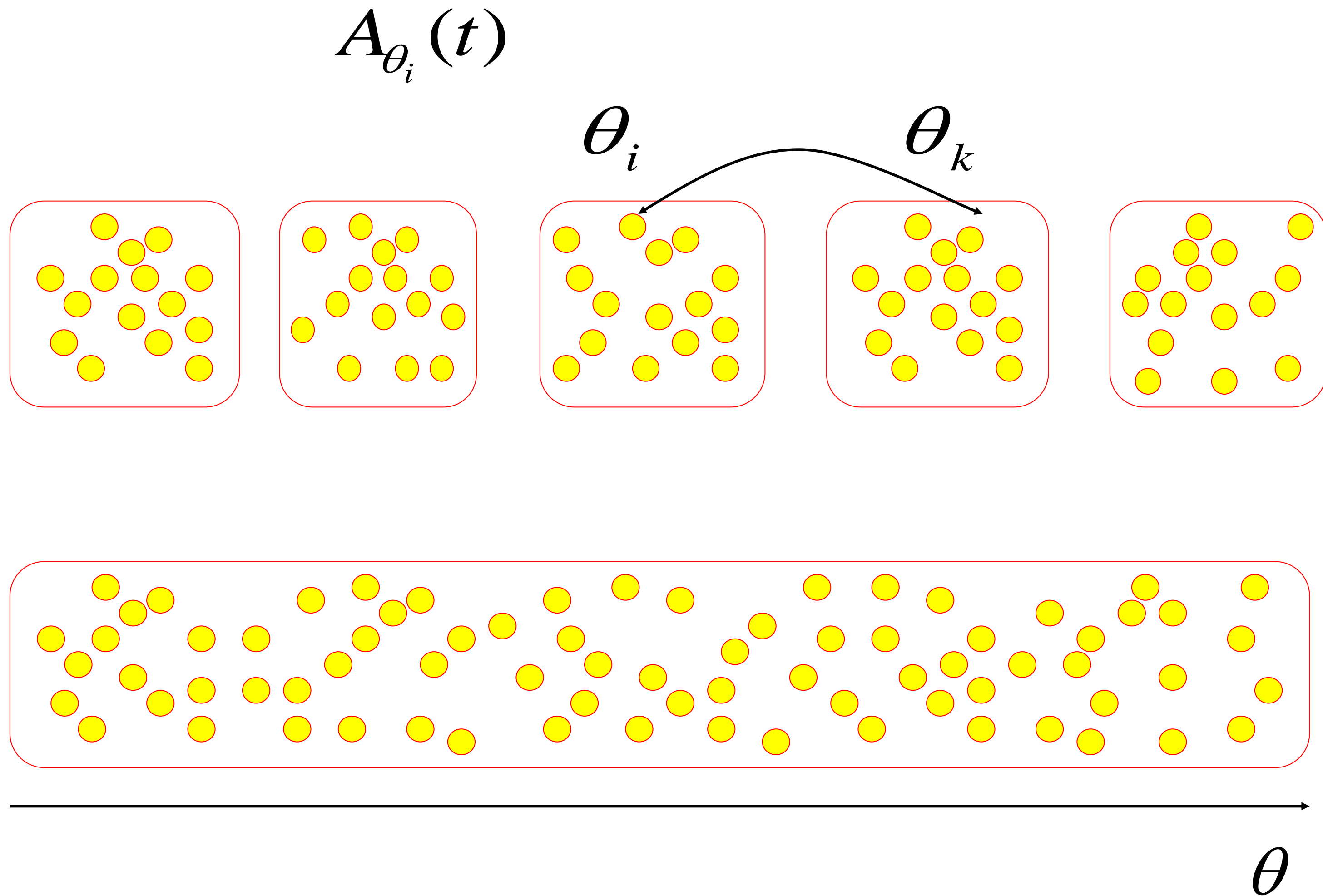


→ no discrete columns

### 3. multiple populations → continuum

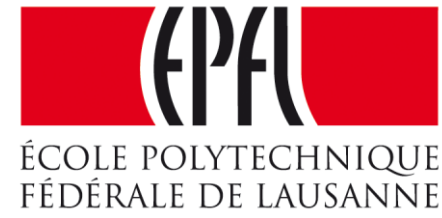


### 3. multiple populations $\rightarrow$ continuum





# Computational Neuroscience: Neuronal Dynamics of Cognition



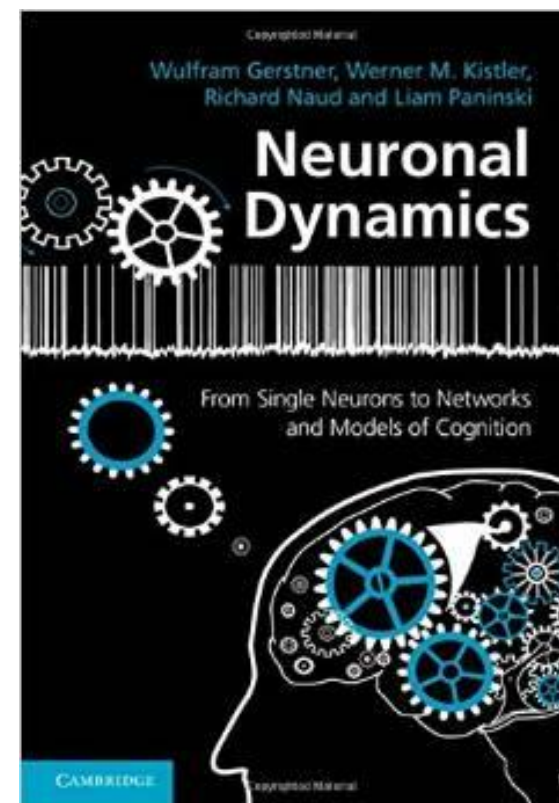
## Continuum models: Cortical fields and perception

Wulfram Gerstner

EPFL, Lausanne, Switzerland

*Reading:*  
**NEURONAL DYNAMICS**  
Ch. 18 +  
+Ch. 12.3.7+Ch 15.1-15.2.3

Cambridge Univ. Press



### 1. Aims and challenges

- review: mean-field arguments

### 2. Transients

- generalized integrate-and-fire model
- transients can be sharp or slow

### 3. Spatial continuum (cortex)

- orientation columns

### 4. Spatial continuum (model)

- field equations

### 5. Solution types

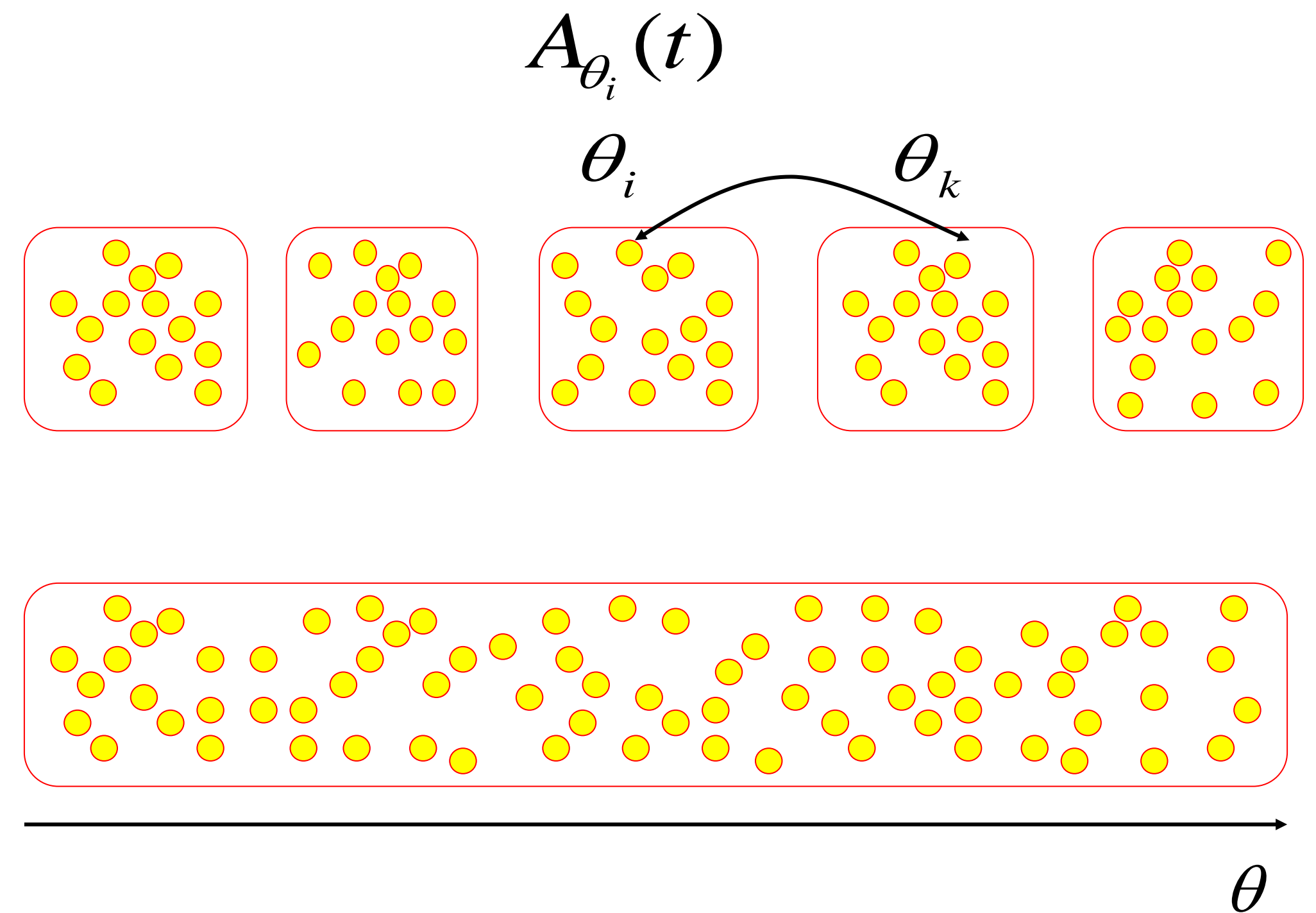
- uniform solution
- bump solution

### 6. Perception

### 7. Head direction cells

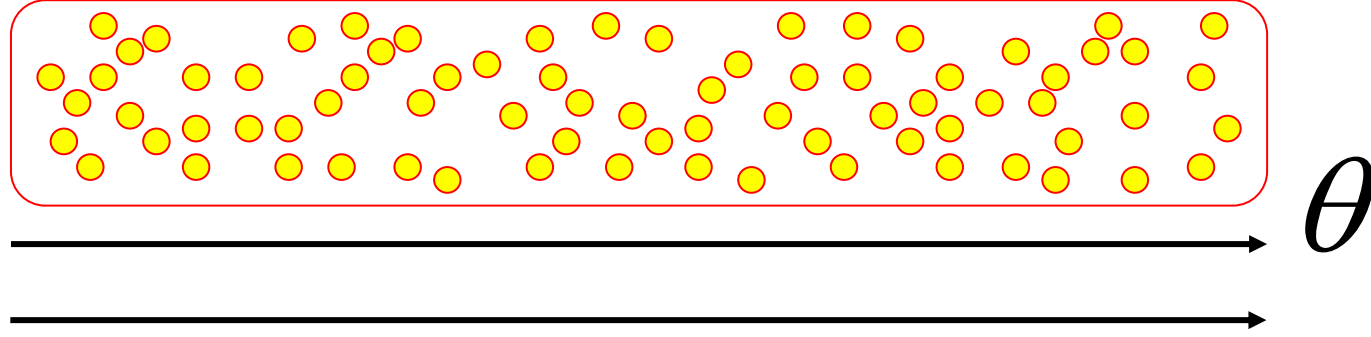
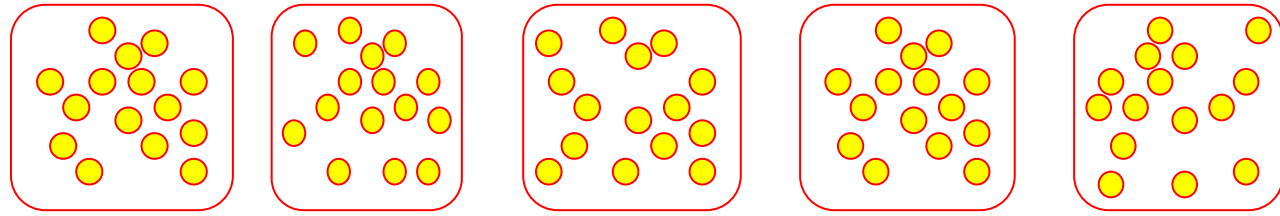


## 4. multiple populations $\rightarrow$ continuum



**Mathematical aim:  
perform continuum limit**

## 4. multiple populations → continuum



## 4. Field equation (continuum model)

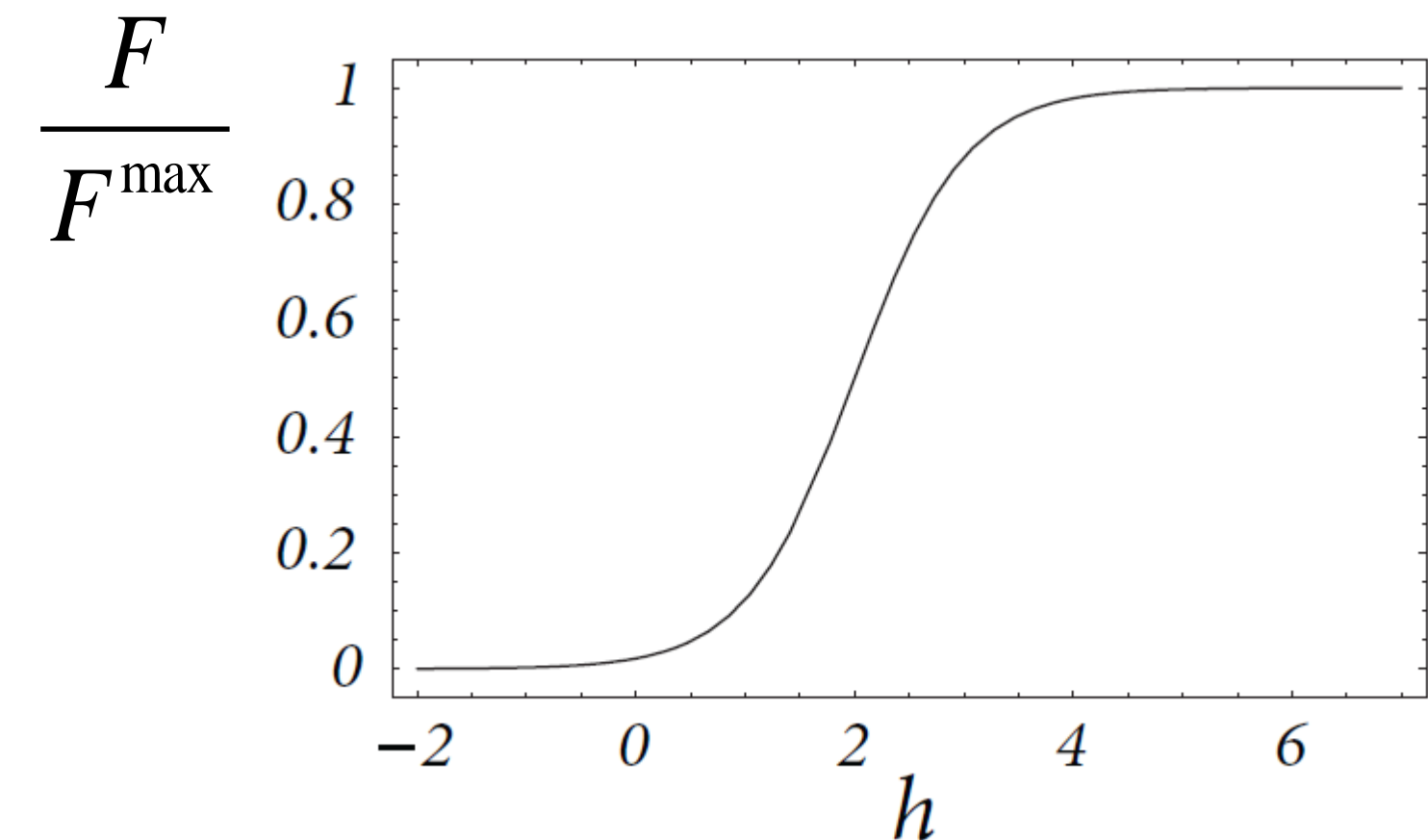
*Wilson and Cowan, 1973*

Population activity

$$A(x, t) = F(h(x, t))$$

Membrane potential caused by input

$$\tau \frac{d}{dt} h(x, t) = -h(x, t) + R I(x, t)$$



## 4. Field equation (continuum model)

*Wilson and Cowan, 1973*

Population activity

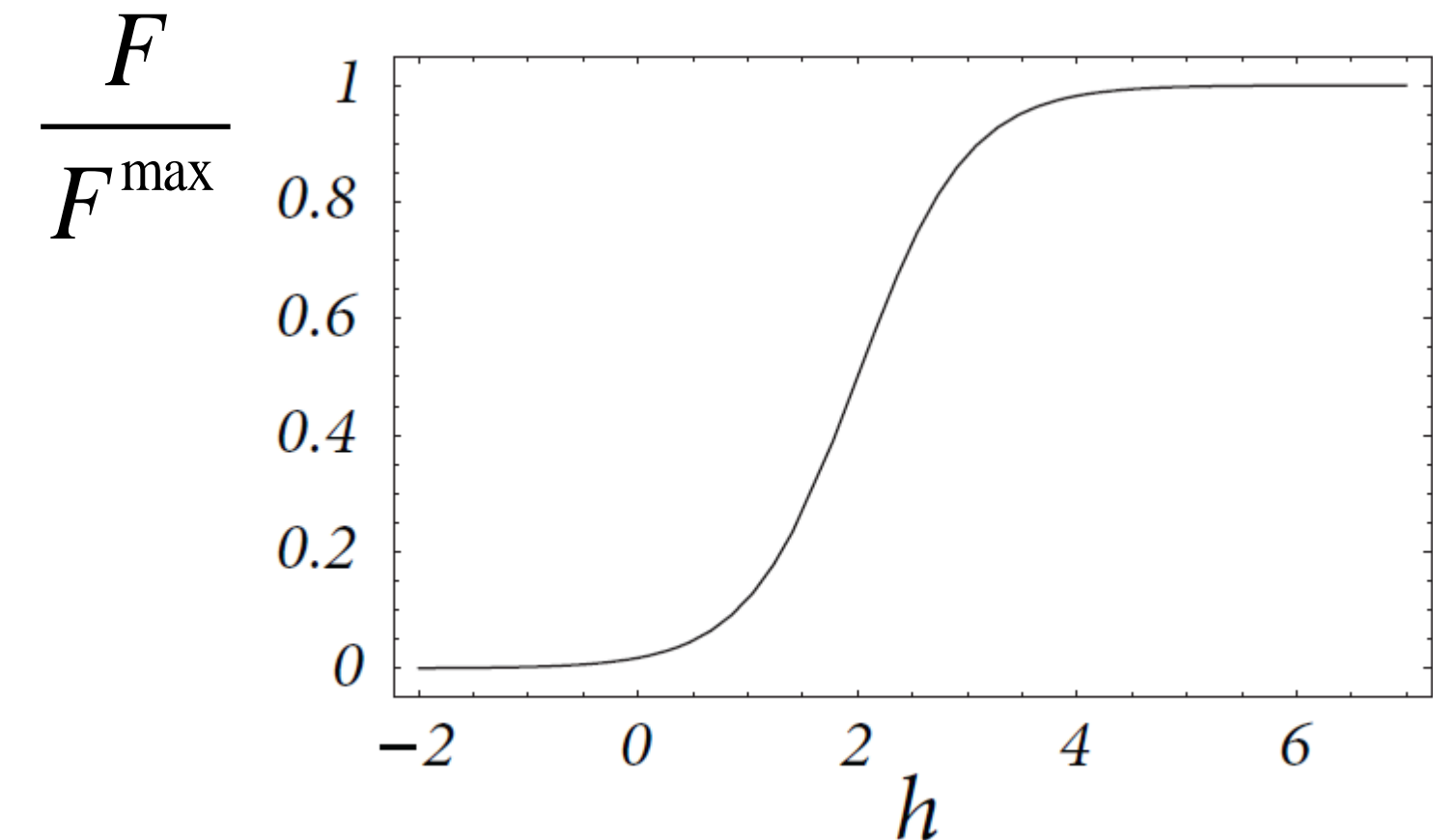
$$A(x, t) = F(h(x, t))$$

Membrane potential caused by input

$$\tau \frac{d}{dt} h(x, t) = -h(x, t) + R I(x, t)$$

$$I(x, t) = I^{ext}(x, t) + I^{netw}(x, t)$$

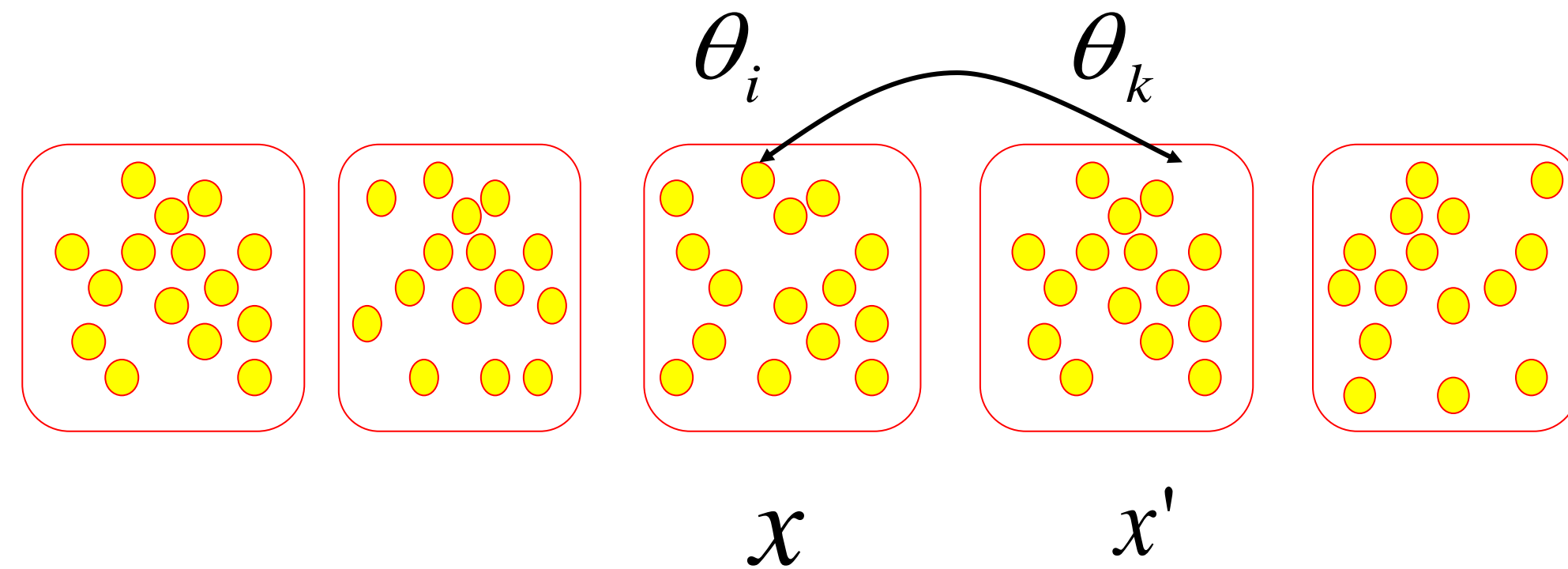
$$I^{netw}(x, t) = d \int w(x - x', t) A(x', t) dx'$$



$$\tau \frac{d}{dt} h(x, t) = -h(x, t) + R I^{ext}(x, t) + d \int w(x - x') F(h(x', t)) dx'$$

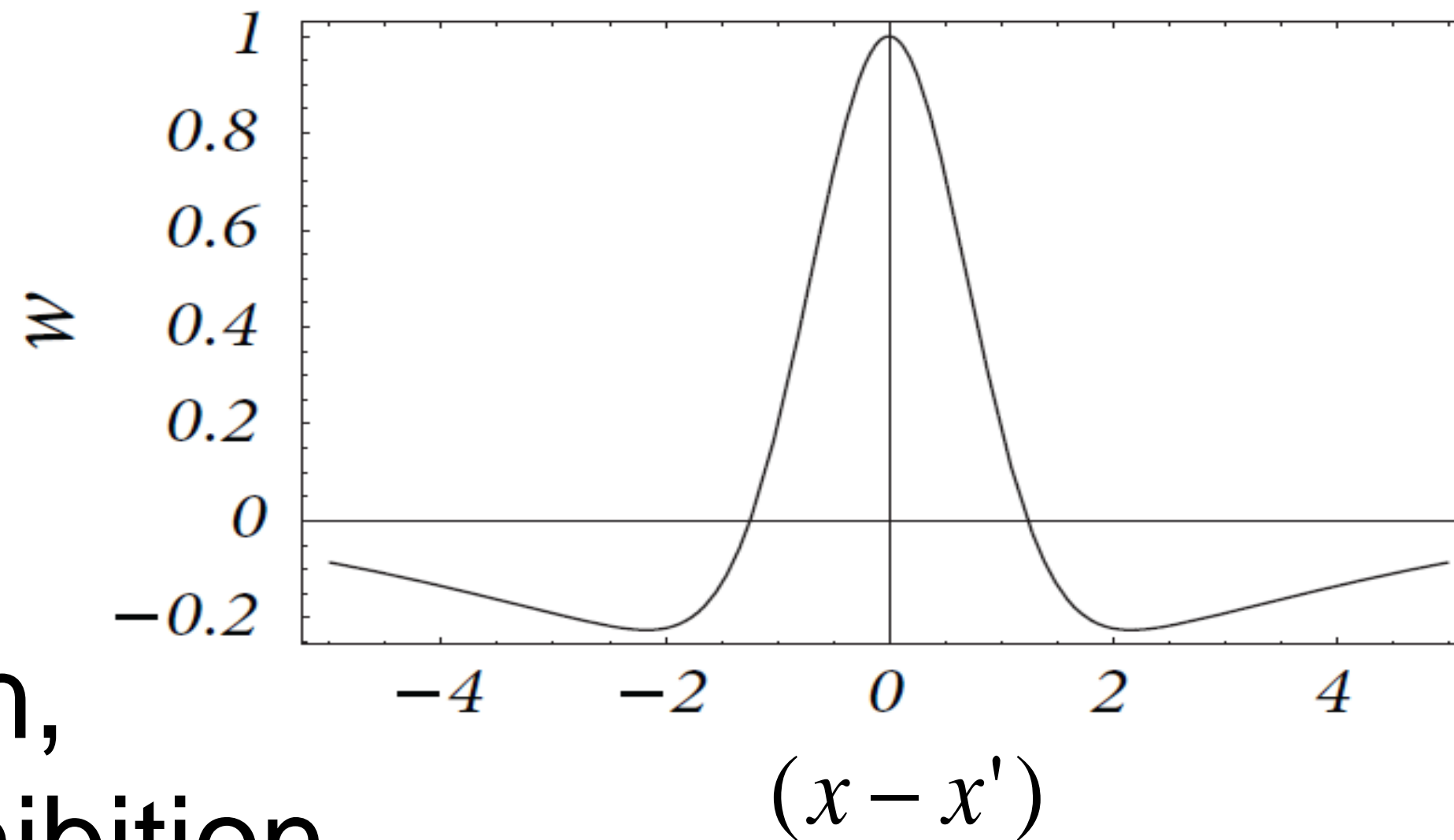
1 field = 1 integro-differential equation

## 4. coupling across continuum: Mexican hat



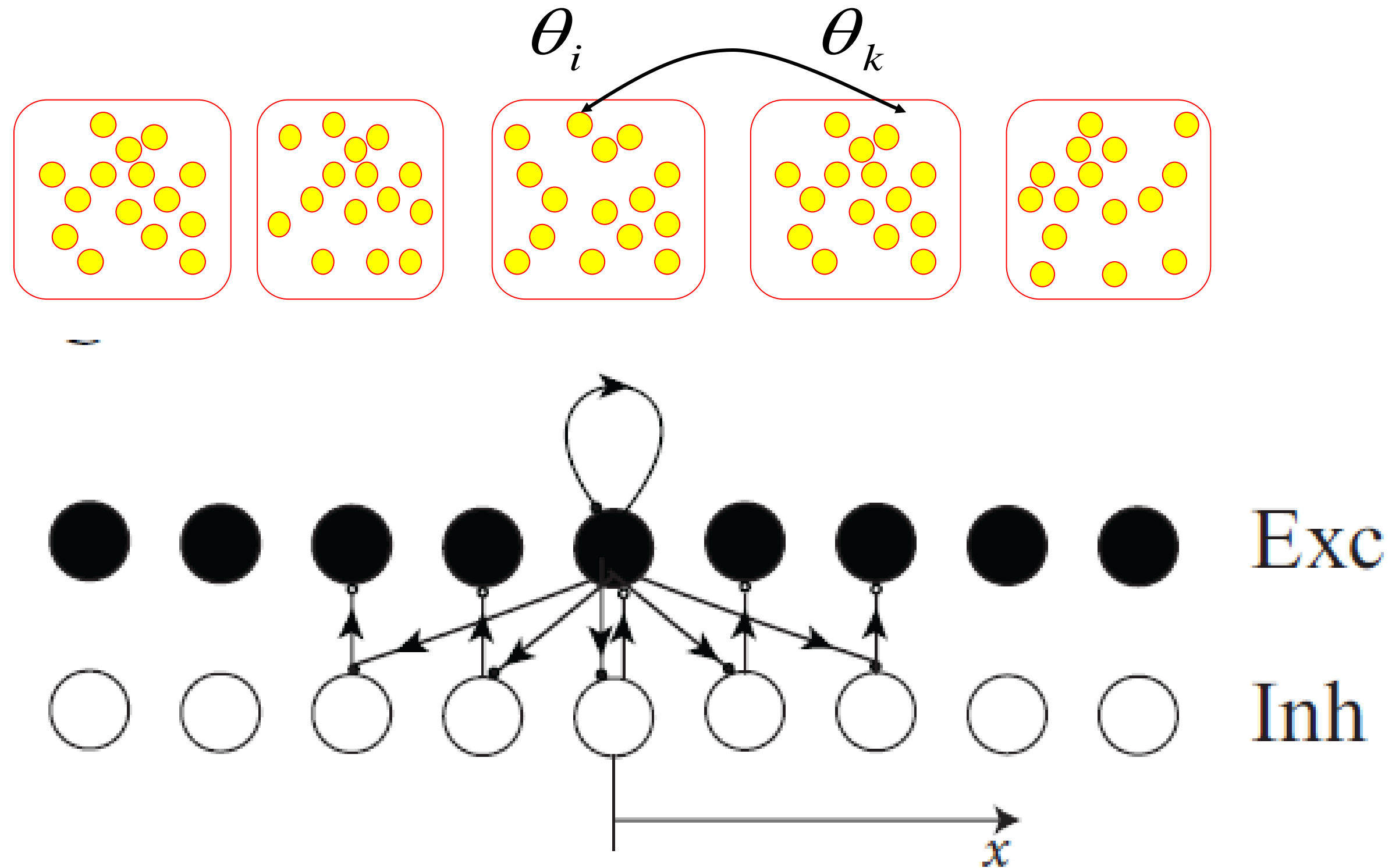
**Mexican hat**

local excitation,  
long-range inhibition



$$w(x, x') = w(|x - x'|)$$

## 4. more realistic cortical coupling



Effective long-range negative interaction with local inhibition

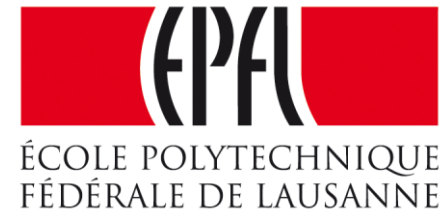


## 4. Summary: Field equations and coupling

$$\tau \frac{d}{dt} h(x, t) = -h(x, t) + R I^{ext}(x, t) + d \int w(x - x') F(h(x', t)) dx'$$

- field equations = population activity models in the spatial continuum
- coupling often distance-dependent
$$w(x, x') = w(|x - x'|)$$
- activity  $A = F(h(t))$
- effective long-range inhibition
  - instead of local inhibitory neurons
- variable  $x$  can represent space or abstract quantity (e.g., orientation)

# Computational Neuroscience: Neuronal Dynamics of Cognition



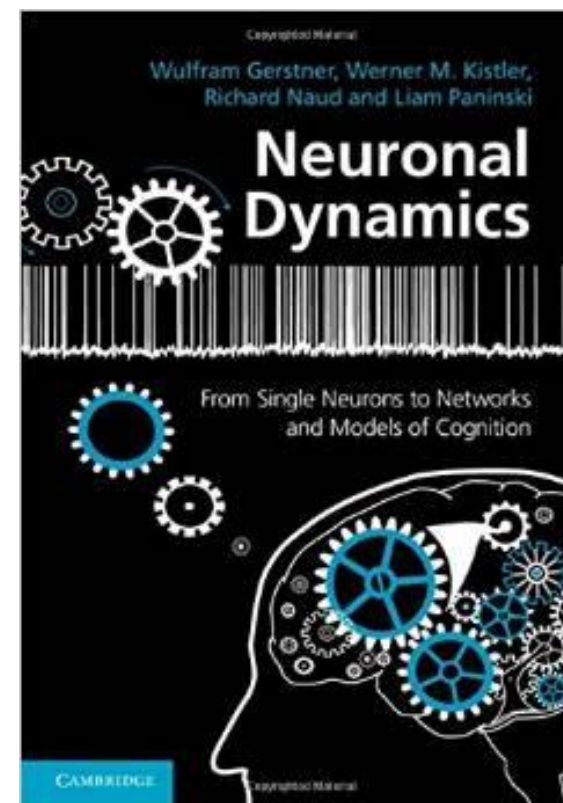
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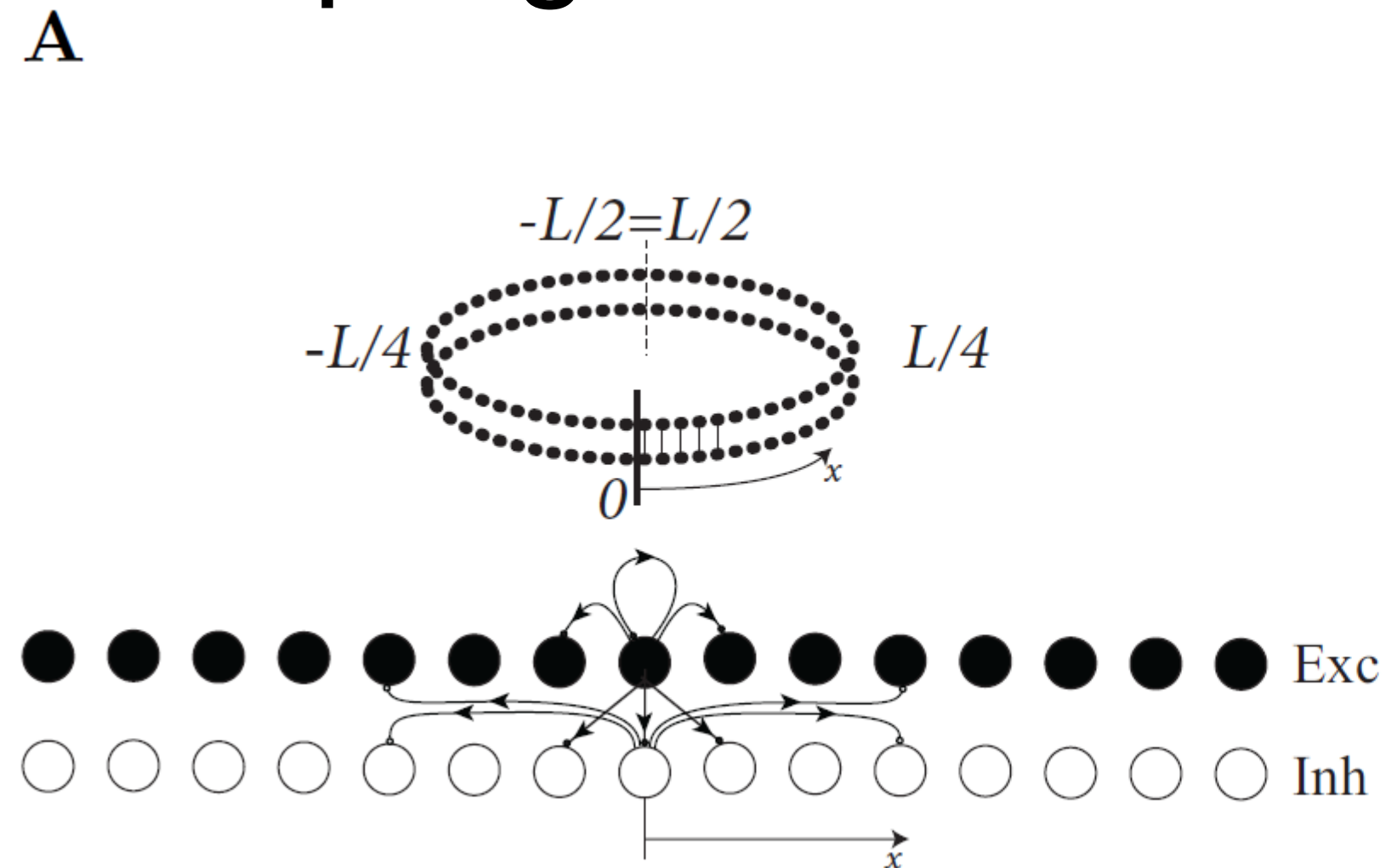
- uniform solution
- bump solution

### 6. Perception

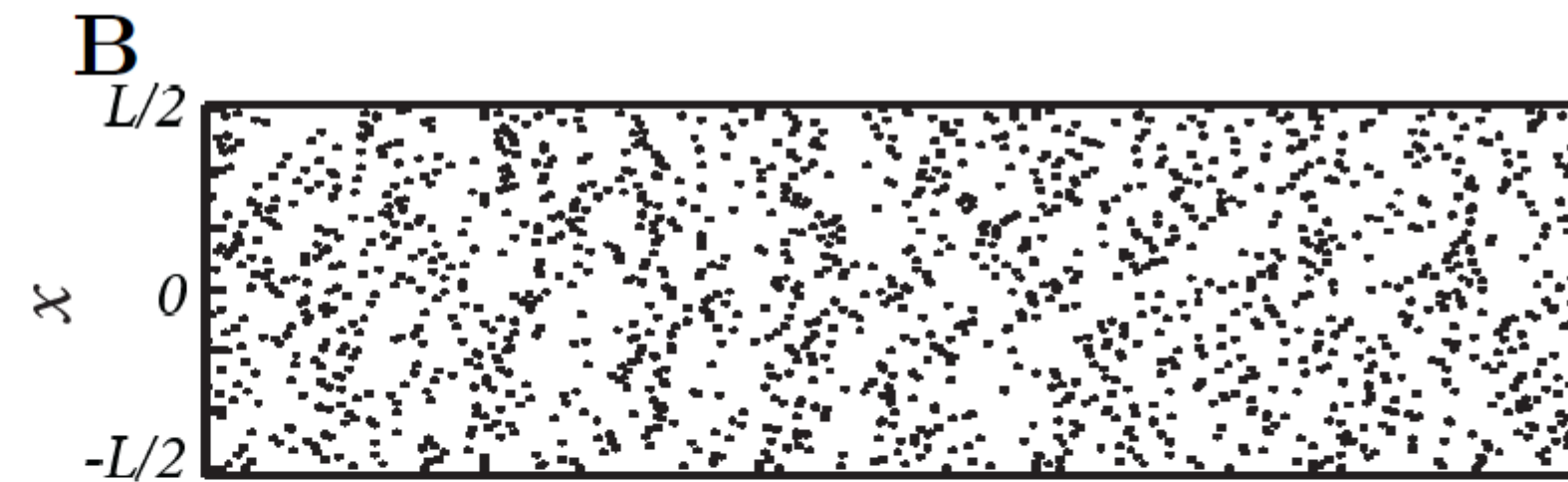
### 7. Head direction cells

# 5. Two Solution Types (ring model)

Coupling:



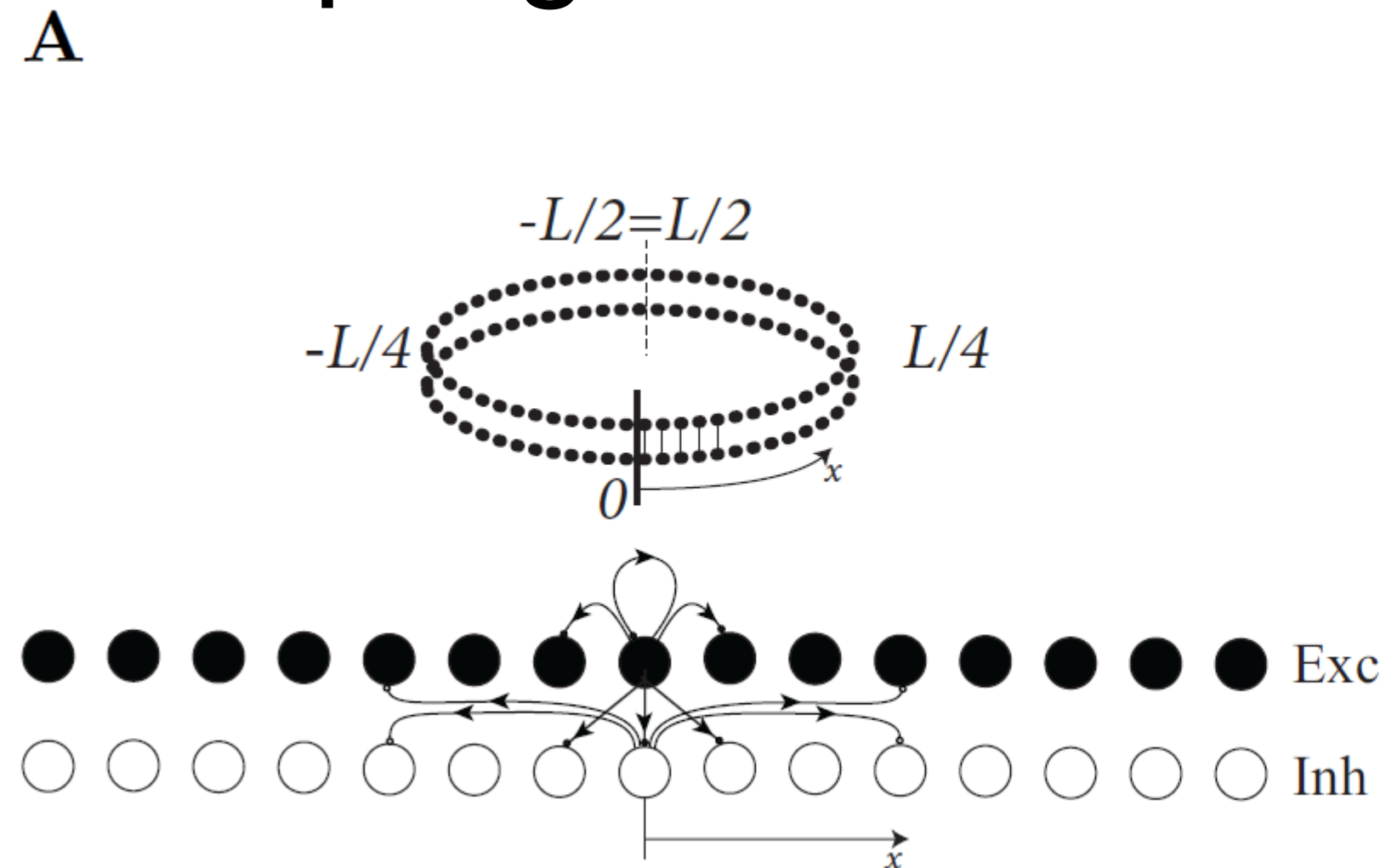
Input-driven regime



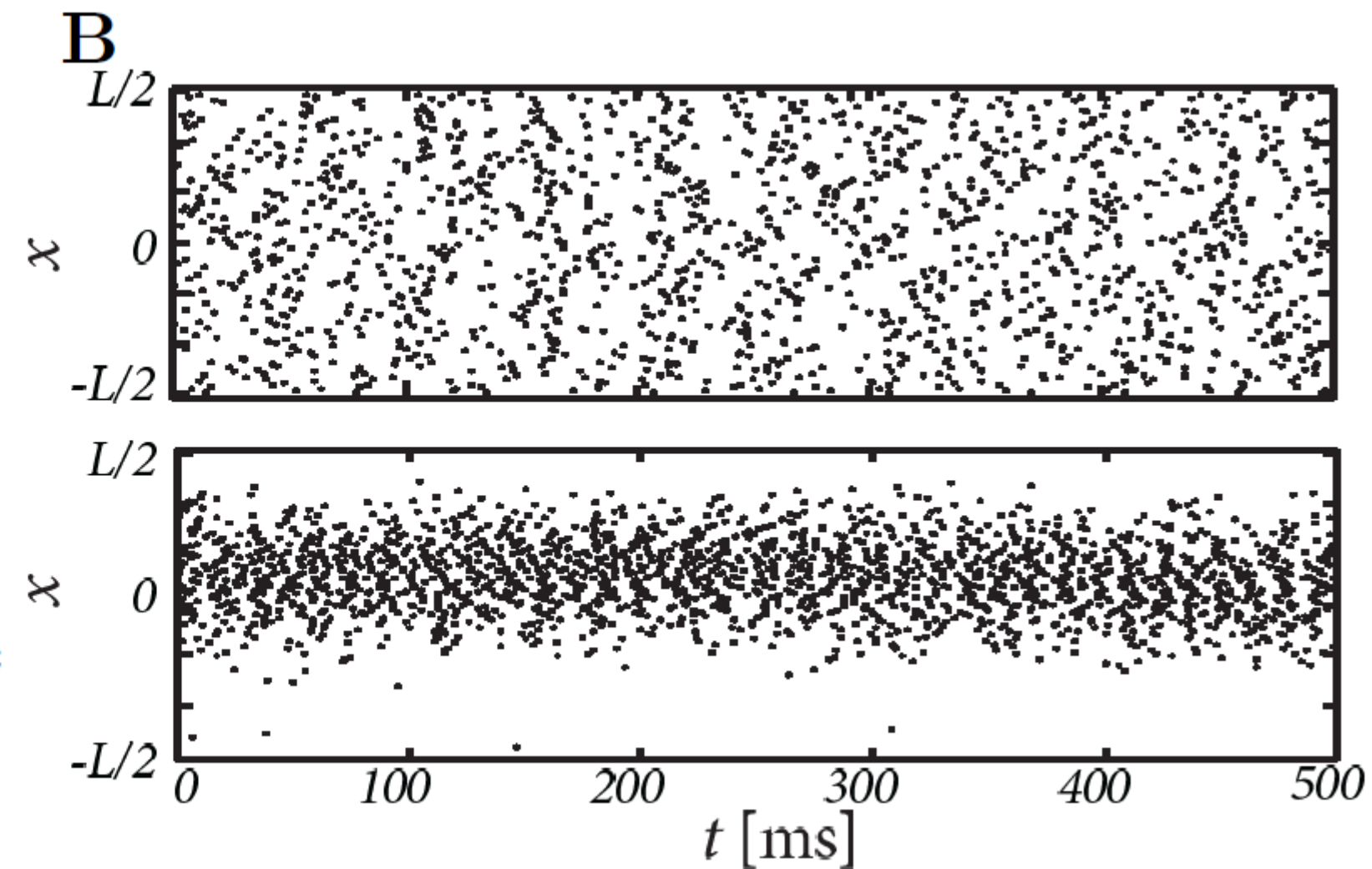
*Image: Neuronal Dynamics,  
Gerstner et al.,  
Cambridge Univ. Press (2014),*

# 5. Two Solution Types (ring model)

Coupling:



Input-driven regime



Bump attractor regime

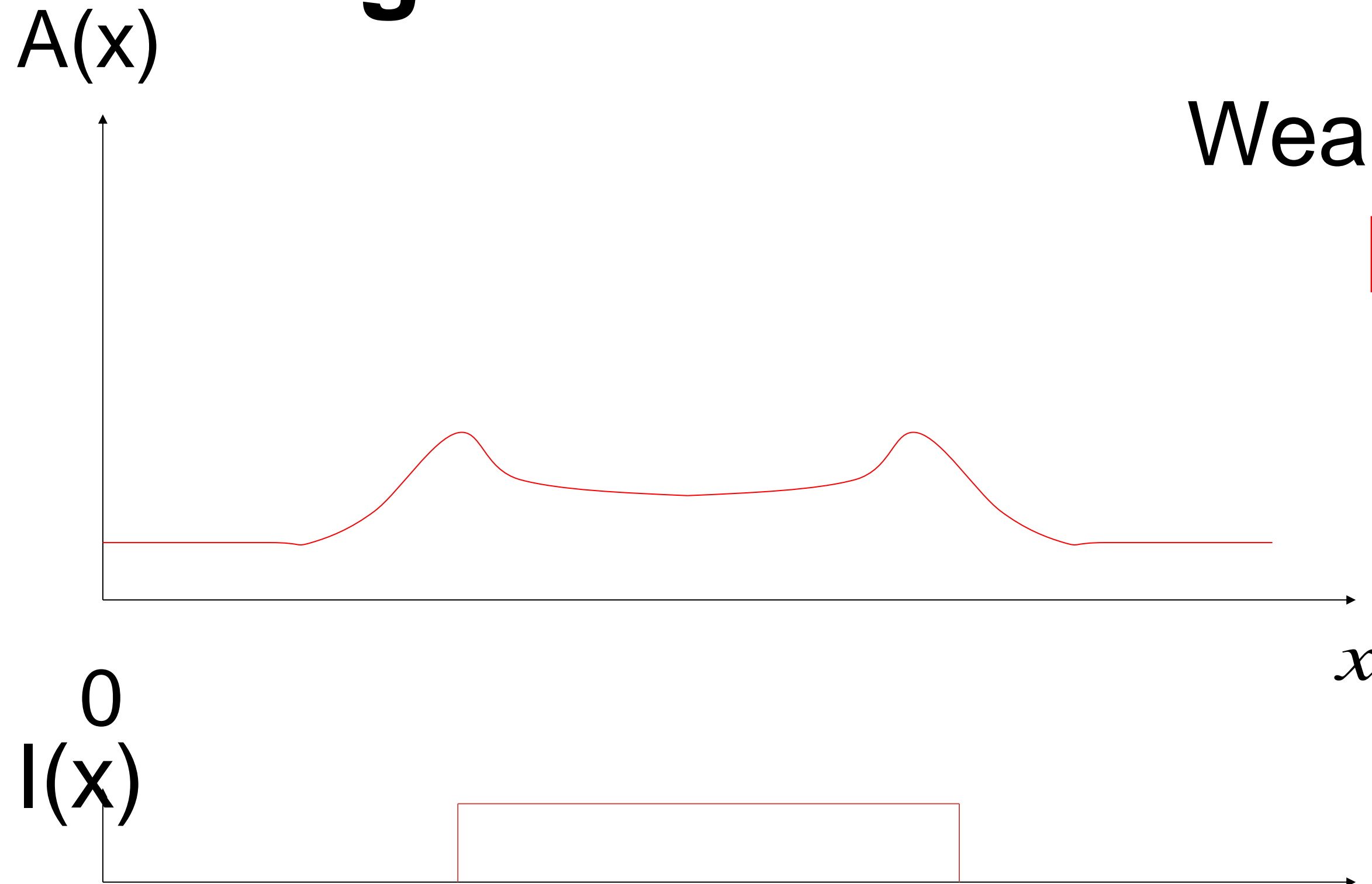
*Image: Neuronal Dynamics,  
Gerstner et al.,  
Cambridge Univ. Press (2014),*

## 5. Solution type A: homogeneous solution=input driven regime

Field Equations:

*Wilson and Cowan, 1973*

### Edge enhancement



Weak lateral connectivity

Possible application  
visual cortex cells:  
(see next part)

## 5. Solution type B: bump solution

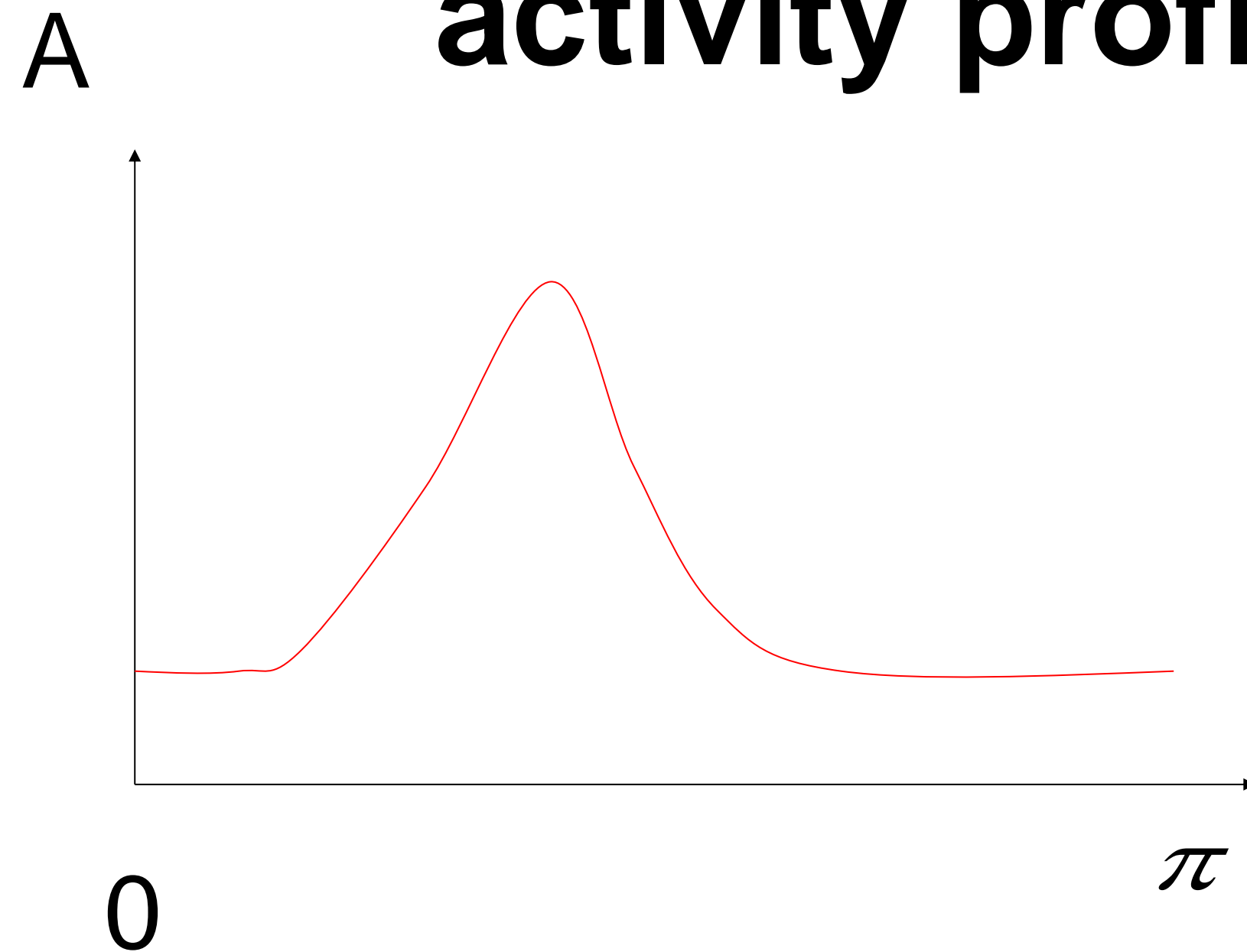
Field Equations:

*Wilson and Cowan, 1973*

**Bump formation:**

**activity profile in the absence of input**

strong lateral connectivity



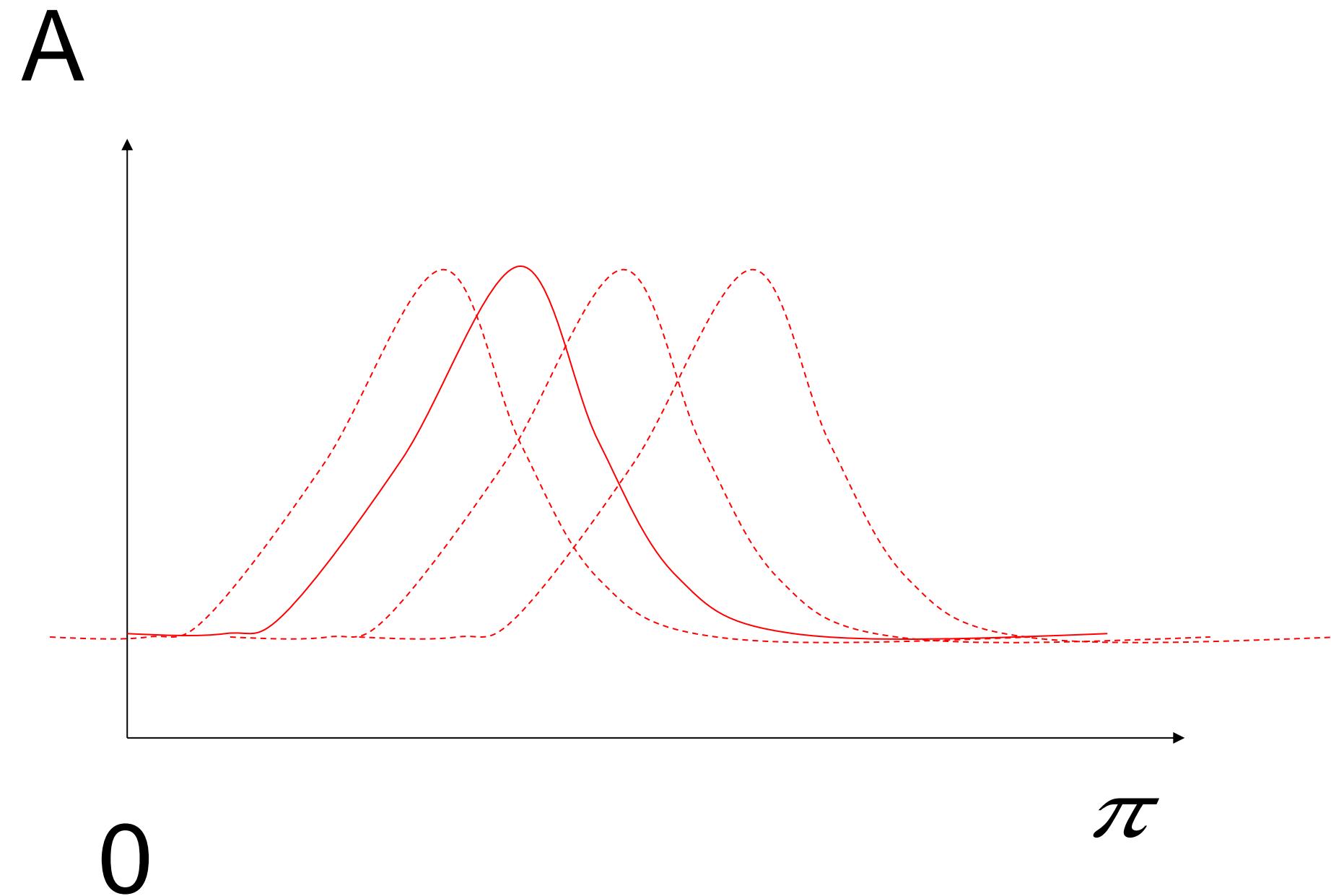


## 5. Solution type B: bump solution

**Bump formation:  
activity profile in the absence of input**

Field Equations:

*Wilson and Cowan, 1973*



- strong lateral connectivity;
- long-range inhibition

**Possible application**

- head direction cells

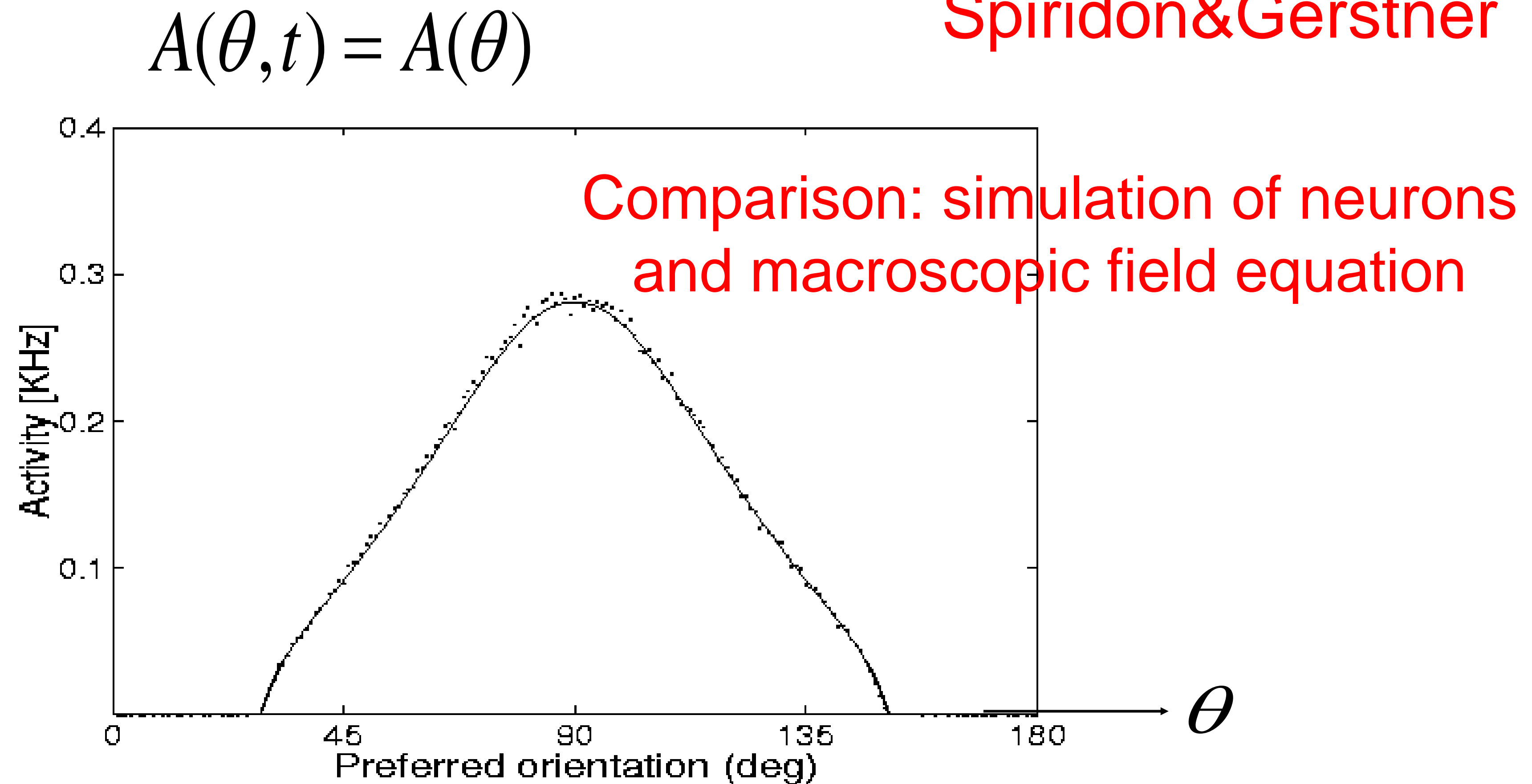
→ (see part 7)

- spatial working memory

A Compte, N Brunel, PS Goldman-Rakic, XJ Wang (2000) Synaptic mechanisms and network dynamics underlying spatial working memory, Cerebral Cortex 10 (9), 910-923

## 5. Solution type B: bump solution

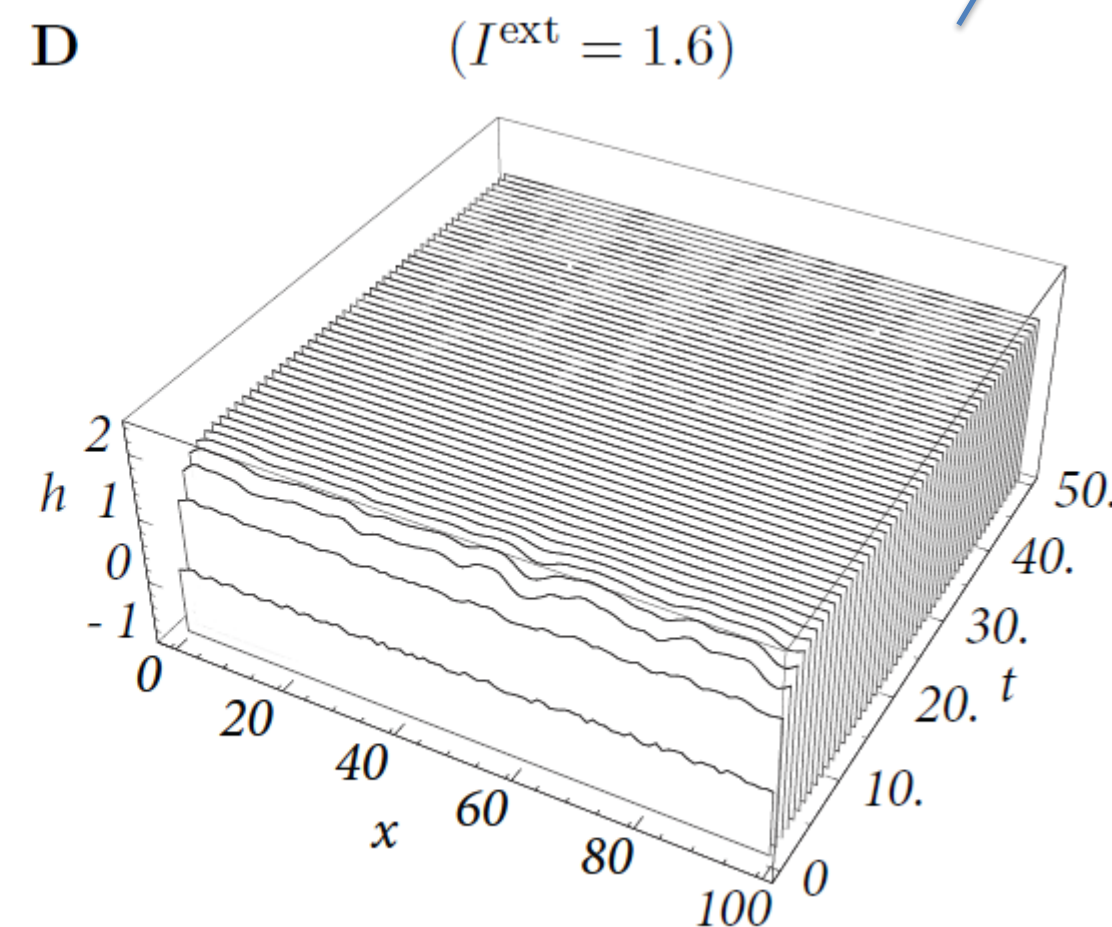
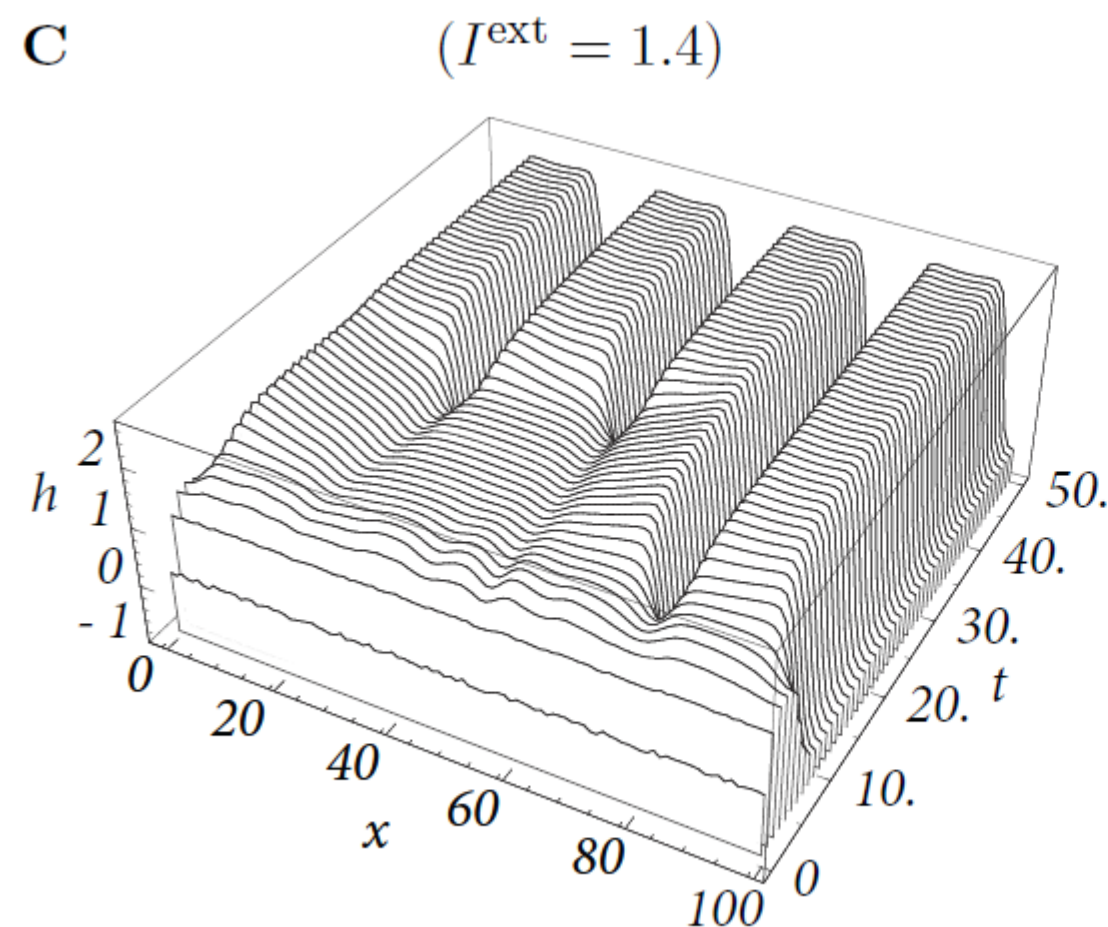
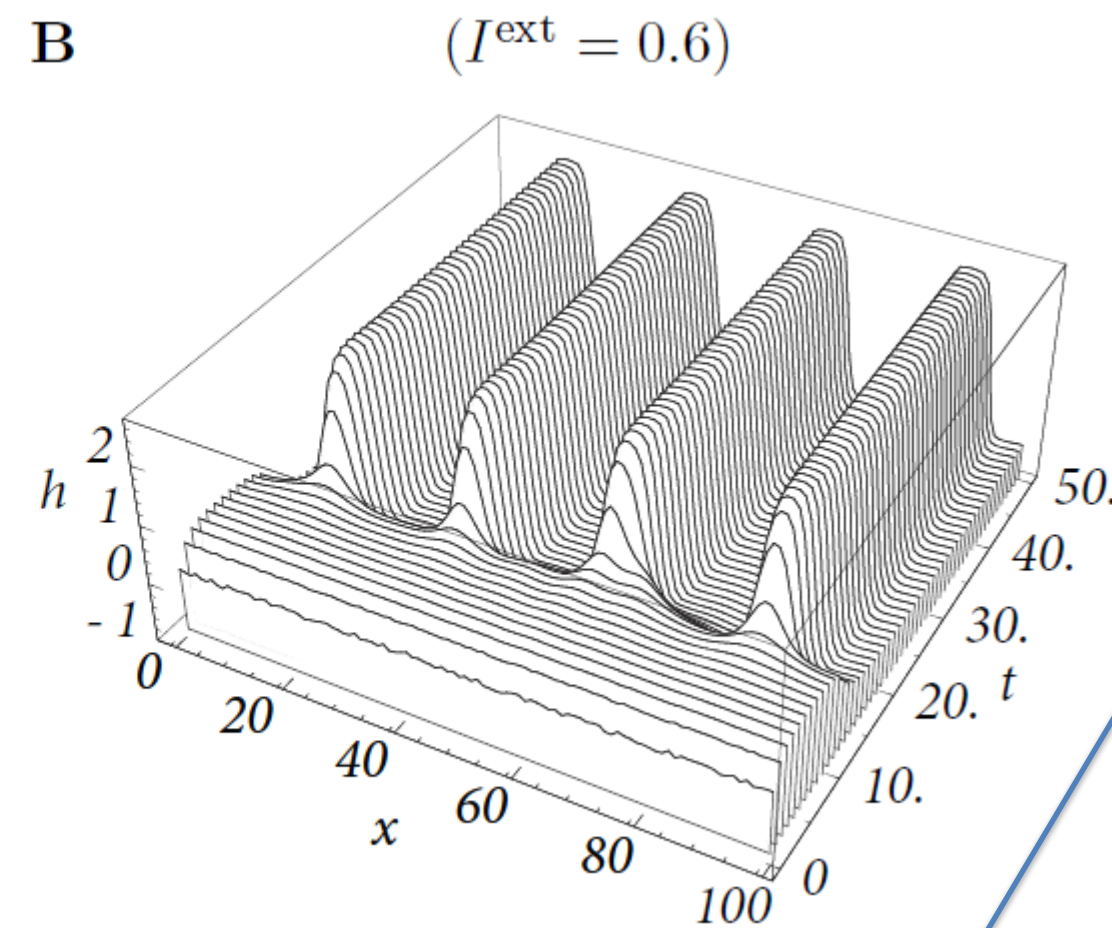
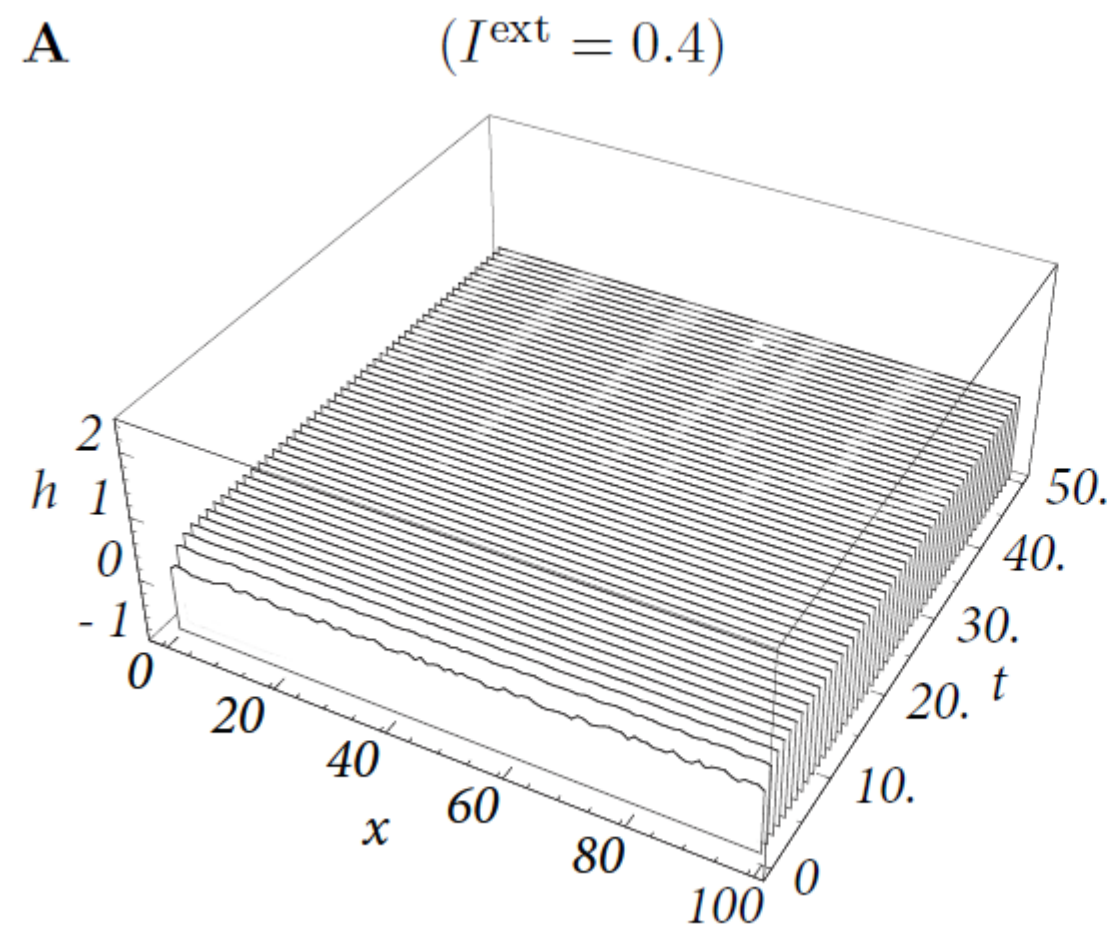
Spiridon&Gerstner



Continuum: stationary profile

*Image: Neuronal Dynamics,  
Gerstner et al.,  
Cambridge Univ. Press (2014),*

# 5. Solution types: multiple bump solutions with local interaction



time

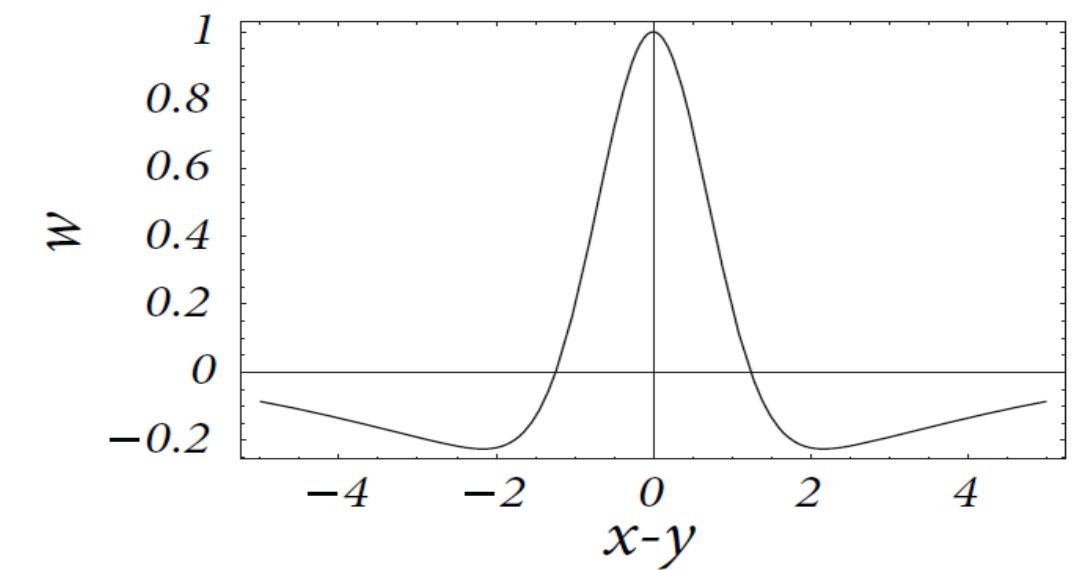


Image: Neuronal Dynamics,  
Gerstner et al.,  
Cambridge Univ. Press (2014),

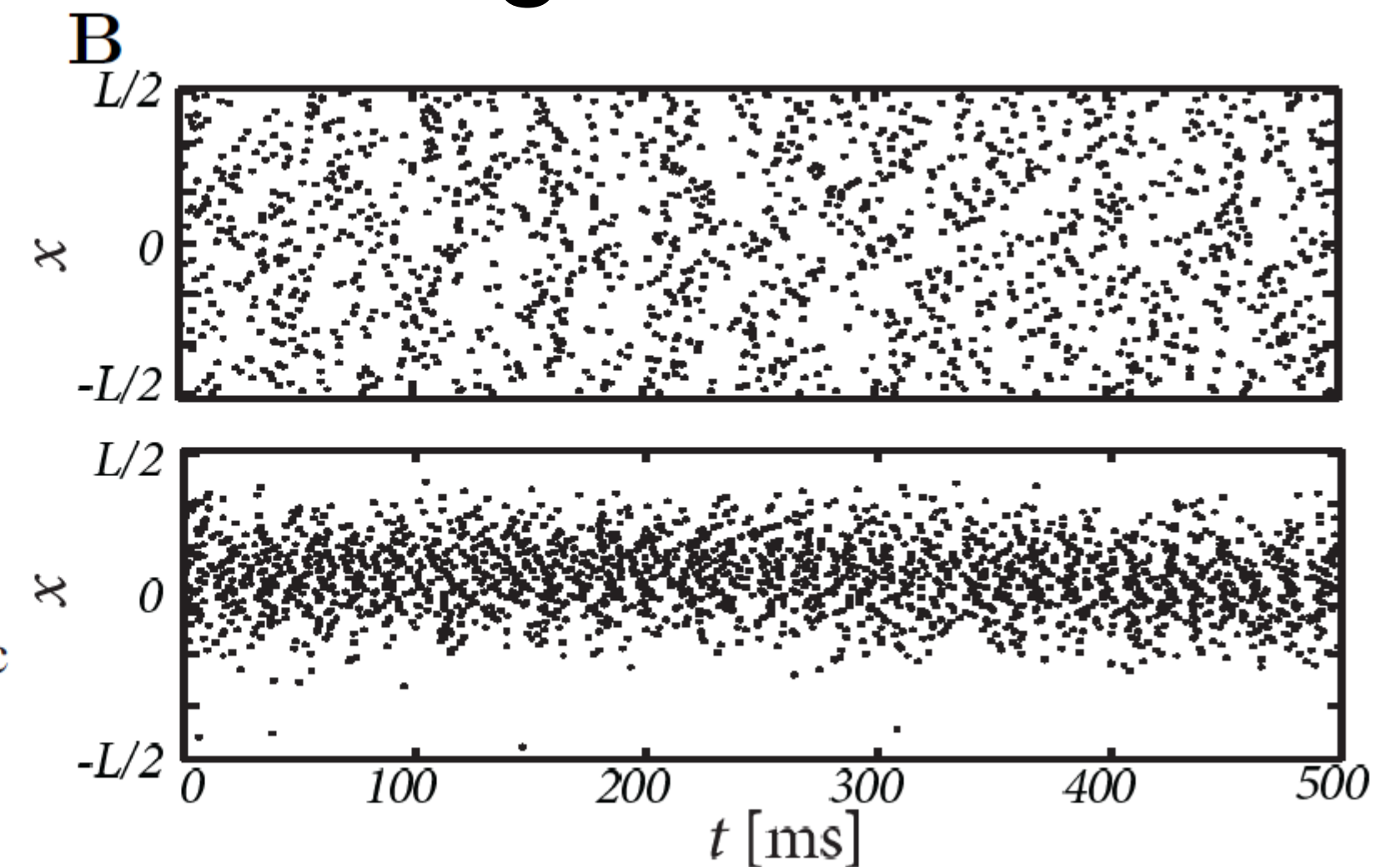


## 5. Two Solution Types (ring model)

### Two stationary solution types:

- homogeneous for flat input  
→ responds to input
- bump attractor for flat input  
→ moves to location of input

Input-driven regime:  
homogeneous solution



Bump attractor regime

*Image: Neuronal Dynamics,  
Gerstner et al.,  
Cambridge Univ. Press (2014),*

## Quiz

### **Solution of Field equations (1-dimensional ring model)**

- [ ] If a solution exists with a single bump localized around  $x_0$ , there are also bump solutions at other locations.
- [ ] If the interaction is Mexican hat, a stationary solution can have at most a single bump
- [ ] A homogeneous solution (constant in time and space) always exists
- [ ] A homogeneous solution (constant in time and space) is always stable
- [ ] If I increase in a model the spatial scale of inhibition, the activity profile of an existing bump solution becomes broader
- [ ] If I increase in a model the amplitude of excitation and the spatial scale of inhibition, a bump solution is more likely to exist

# Computational Neuroscience: Neuronal Dynamics of Cognition



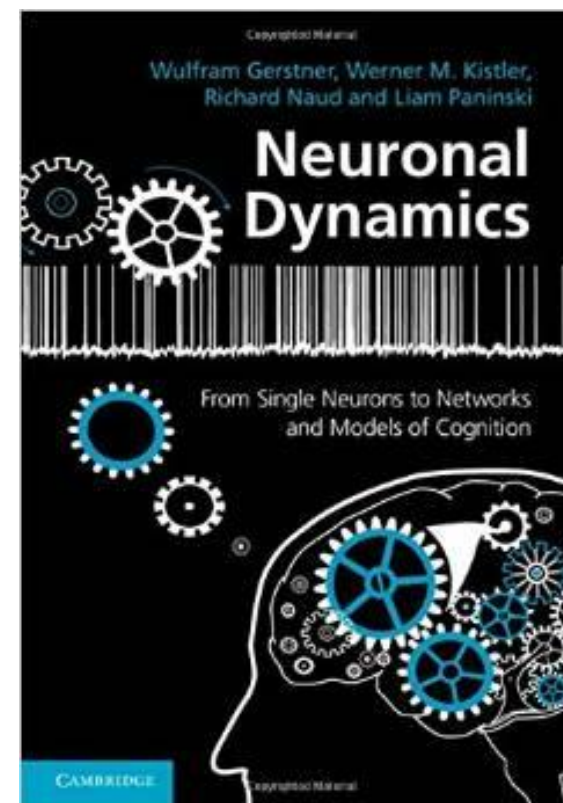
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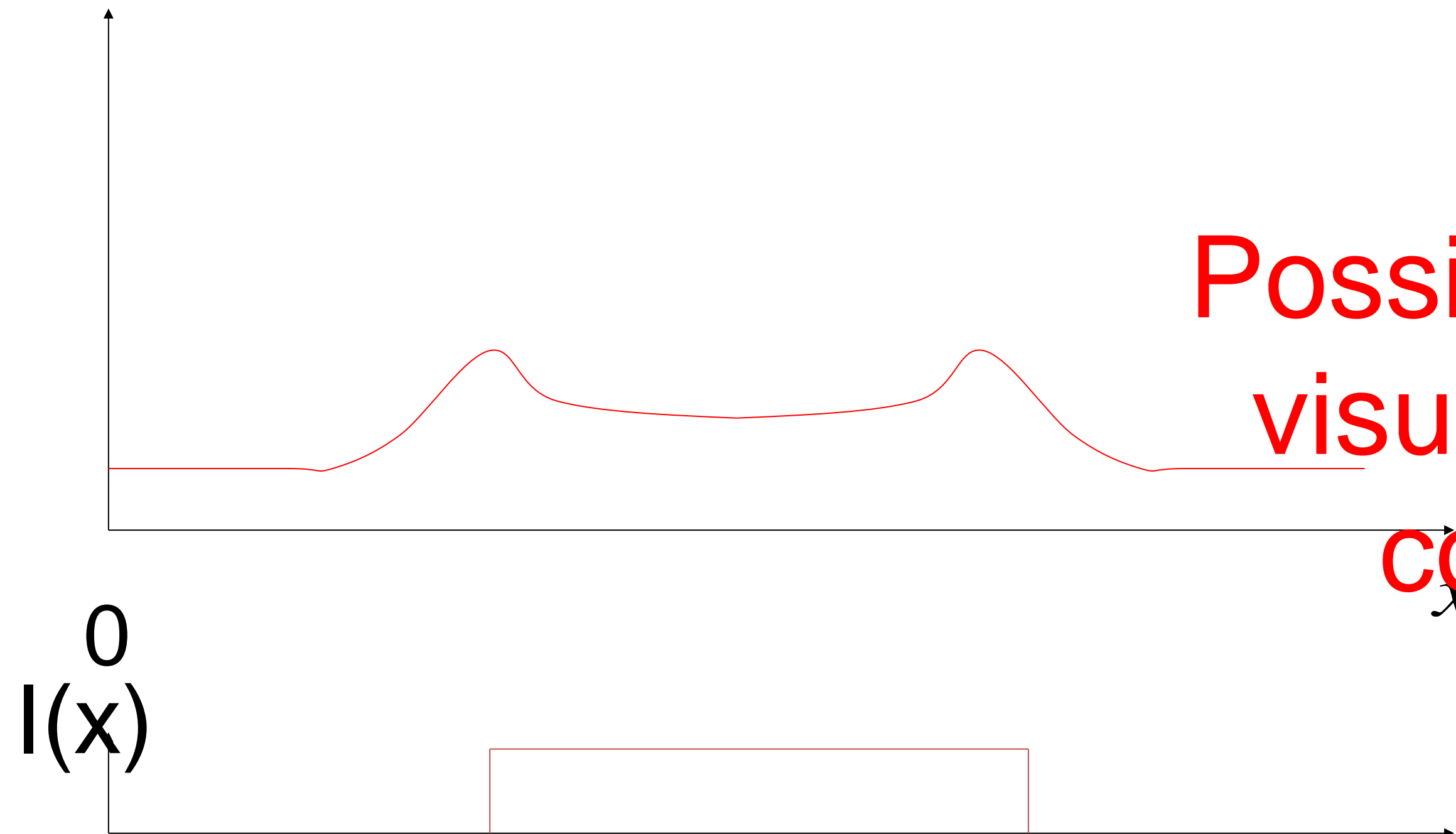
### 7. Head direction cells



## 6. homogeneous/input driven solution

### Edge enhancement

$A(x)$  (Weak lateral connectivity)



Field Equations

for edge enhancement

*Wilson and Cowan, 1973*

*Grossberg, 1973*

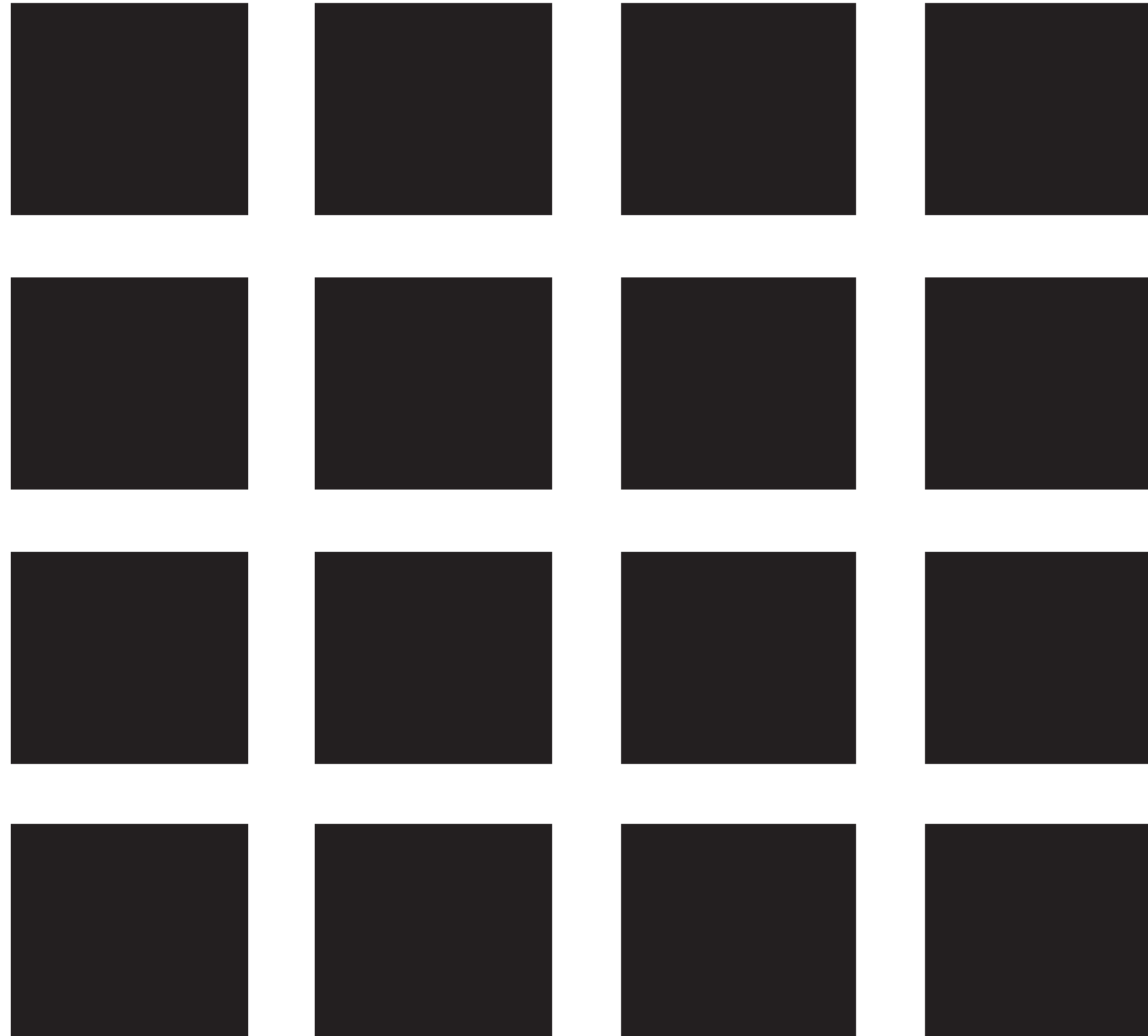
Possible application to  
visual cortex cells:

contrast enhancement in

- orientation

- location

## 6. Perception - grid illusion



*Image: Neuronal Dynamics,  
Gerstner et al.,  
Cambridge Univ. Press (2014),*

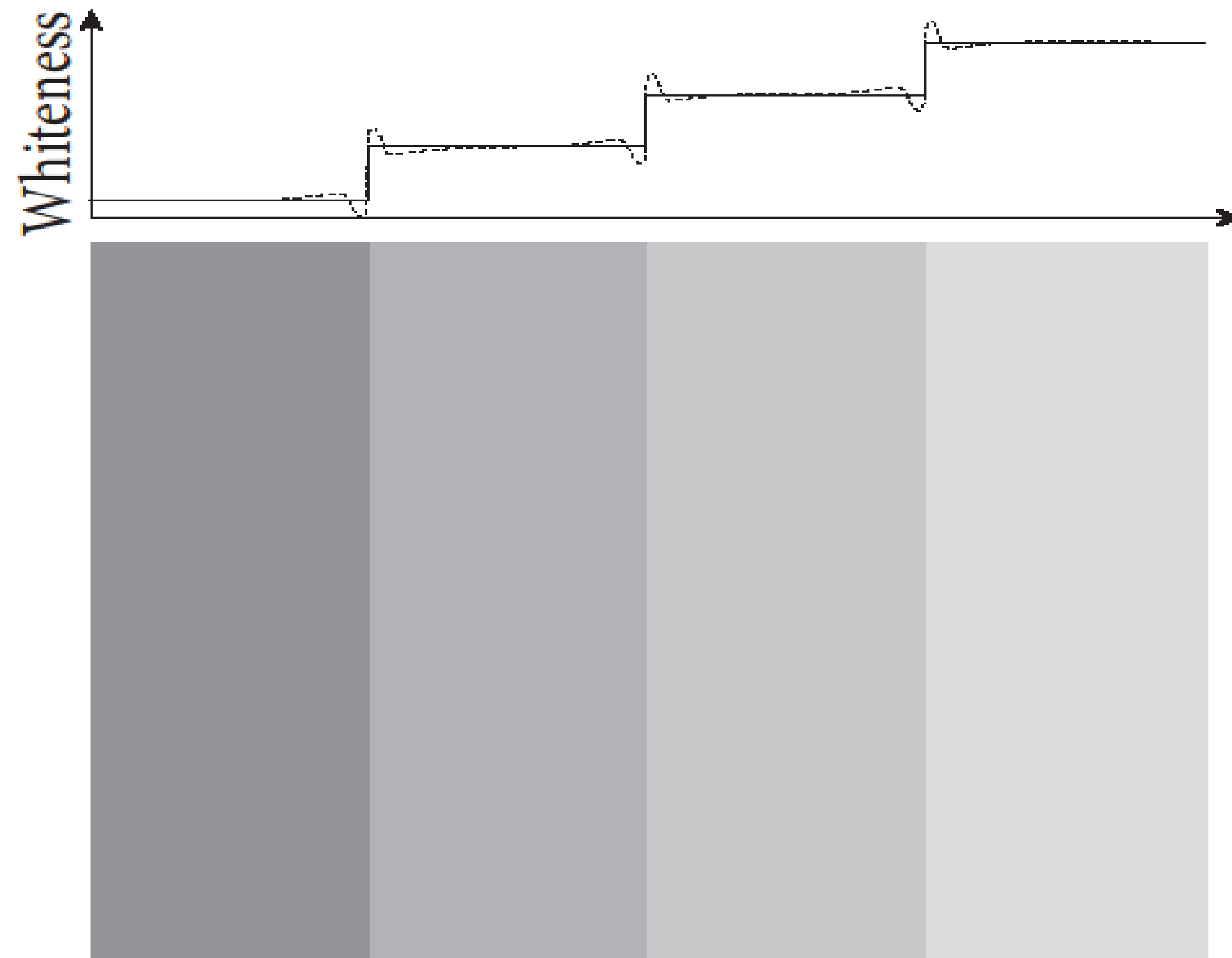
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*Image: Neuronal Dynamics,  
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Cambridge Univ. Press (2014),*

# 6. Perception – Mach bands

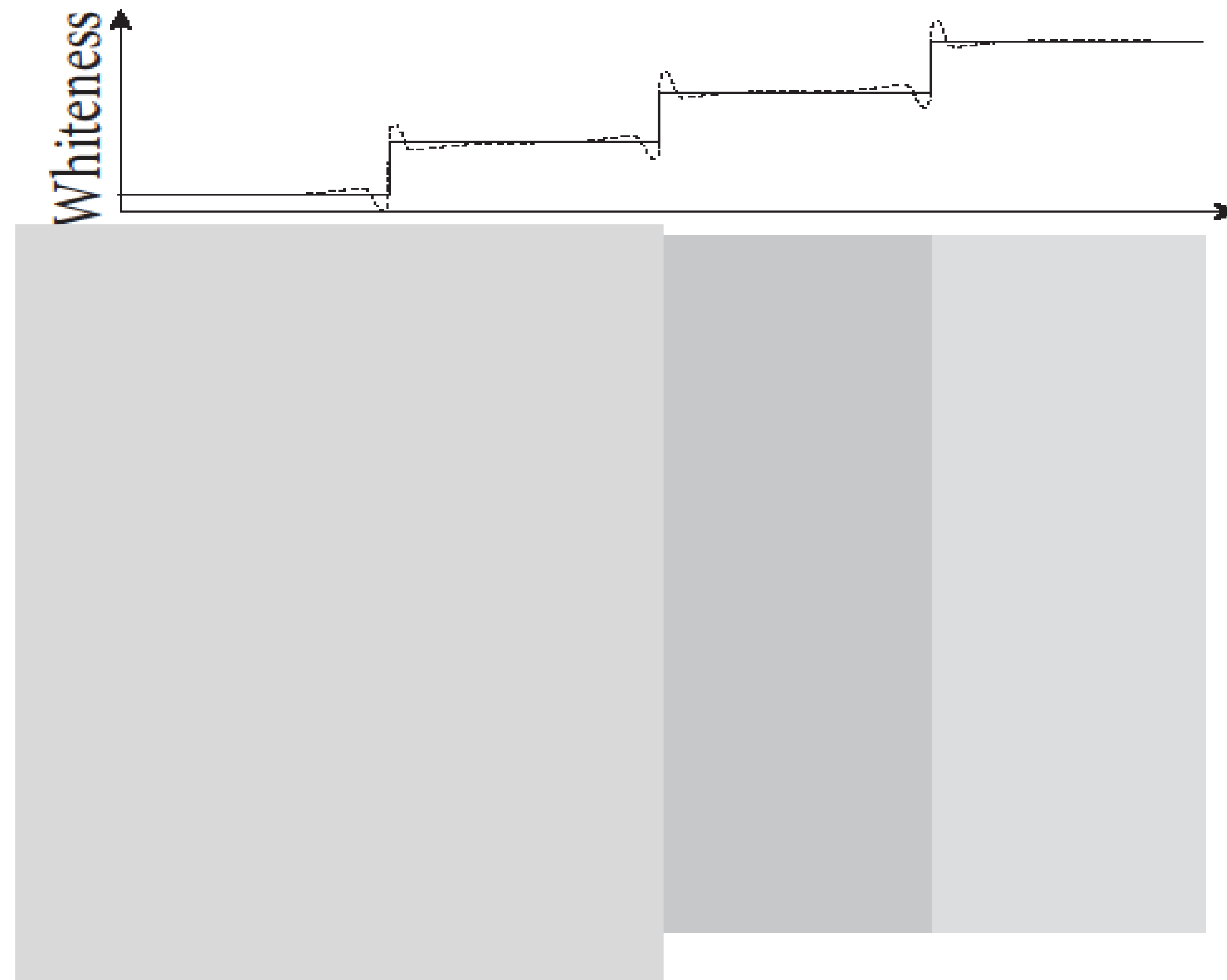
*Mach, 1865, 1906*



*Image: Neuronal Dynamics,  
Gerstner et al.,  
Cambridge Univ. Press (2014),*

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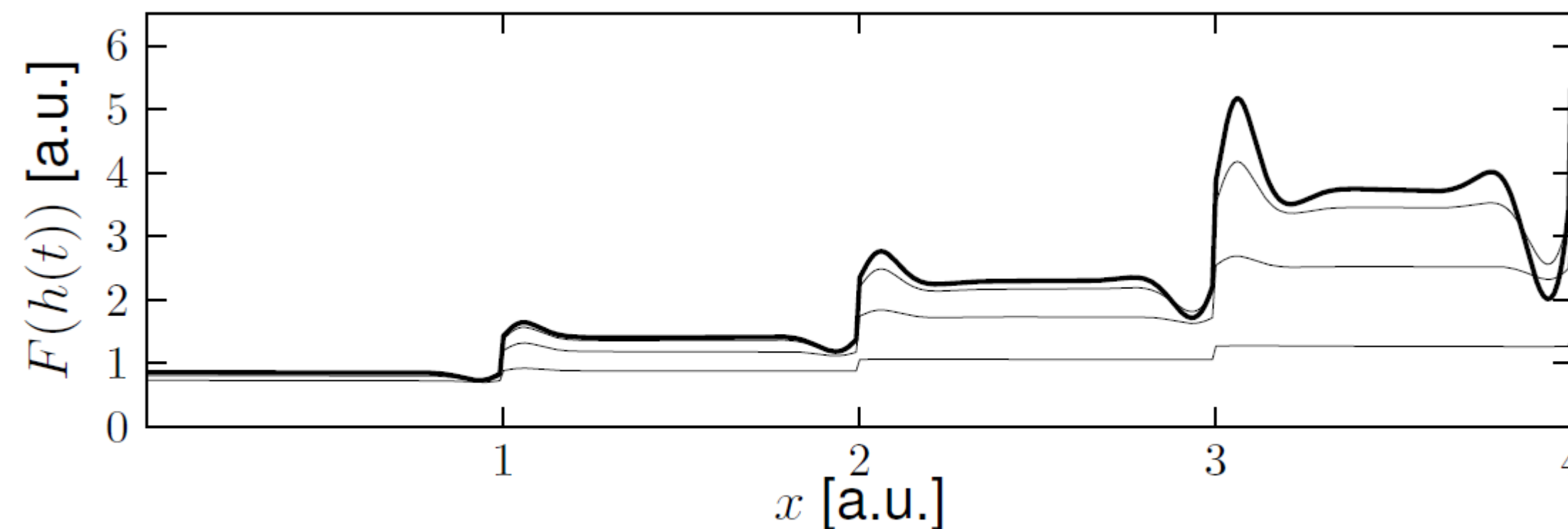
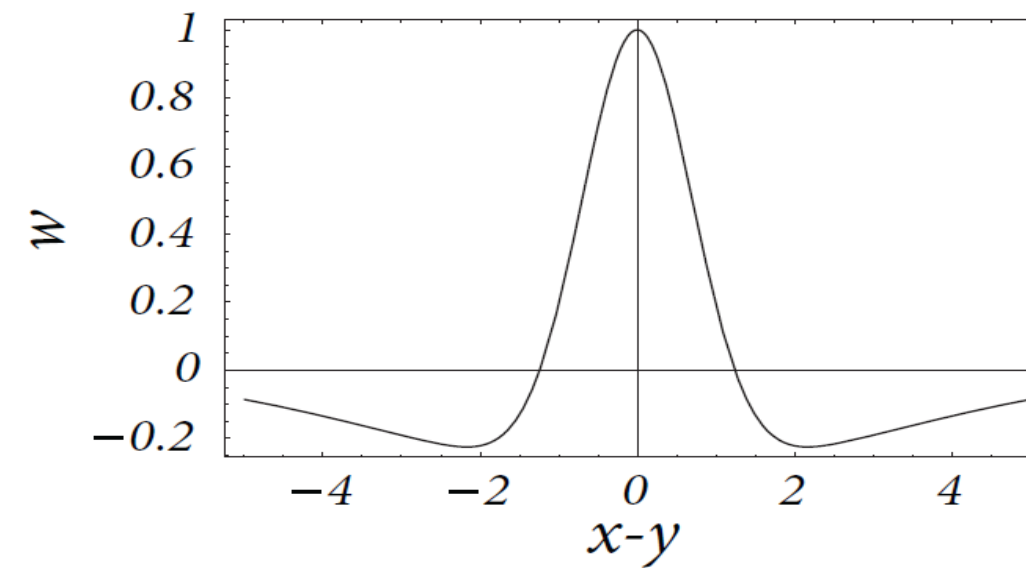
*Mach, 1865, 1906*



*Image: Neuronal Dynamics,  
Gerstner et al.,  
Cambridge Univ. Press (2014),*

## 6. Mach bands in a continuum model

Mexican-hat coupling



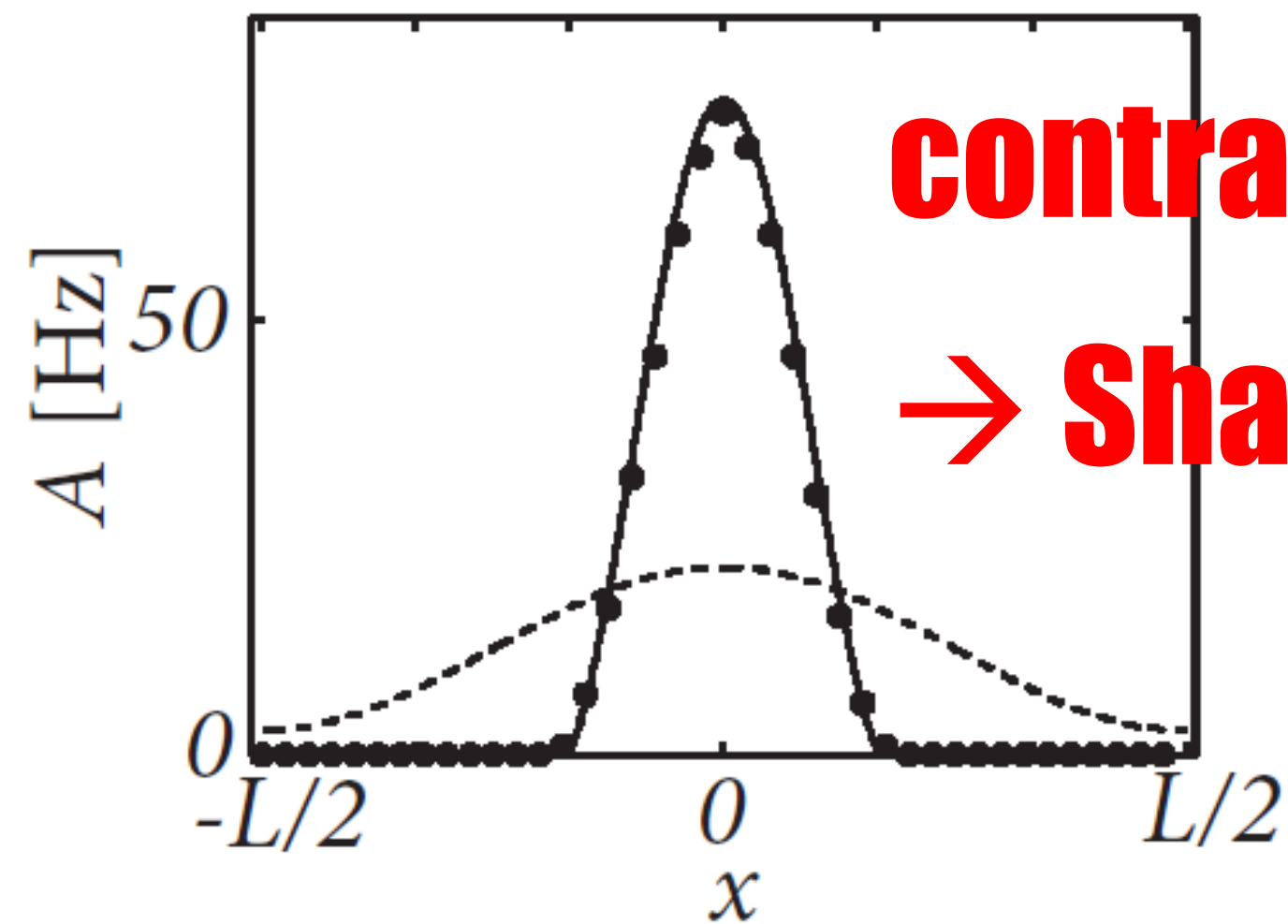
ig. 18.9: **A.** Mach bands in a field model with mexican hat

*Image: Neuronal Dynamics,  
Gerstner et al.,  
Cambridge Univ. Press (2014),*



# 6. Field models and Perception: contrast enhancement

B



**contrast enhancement**

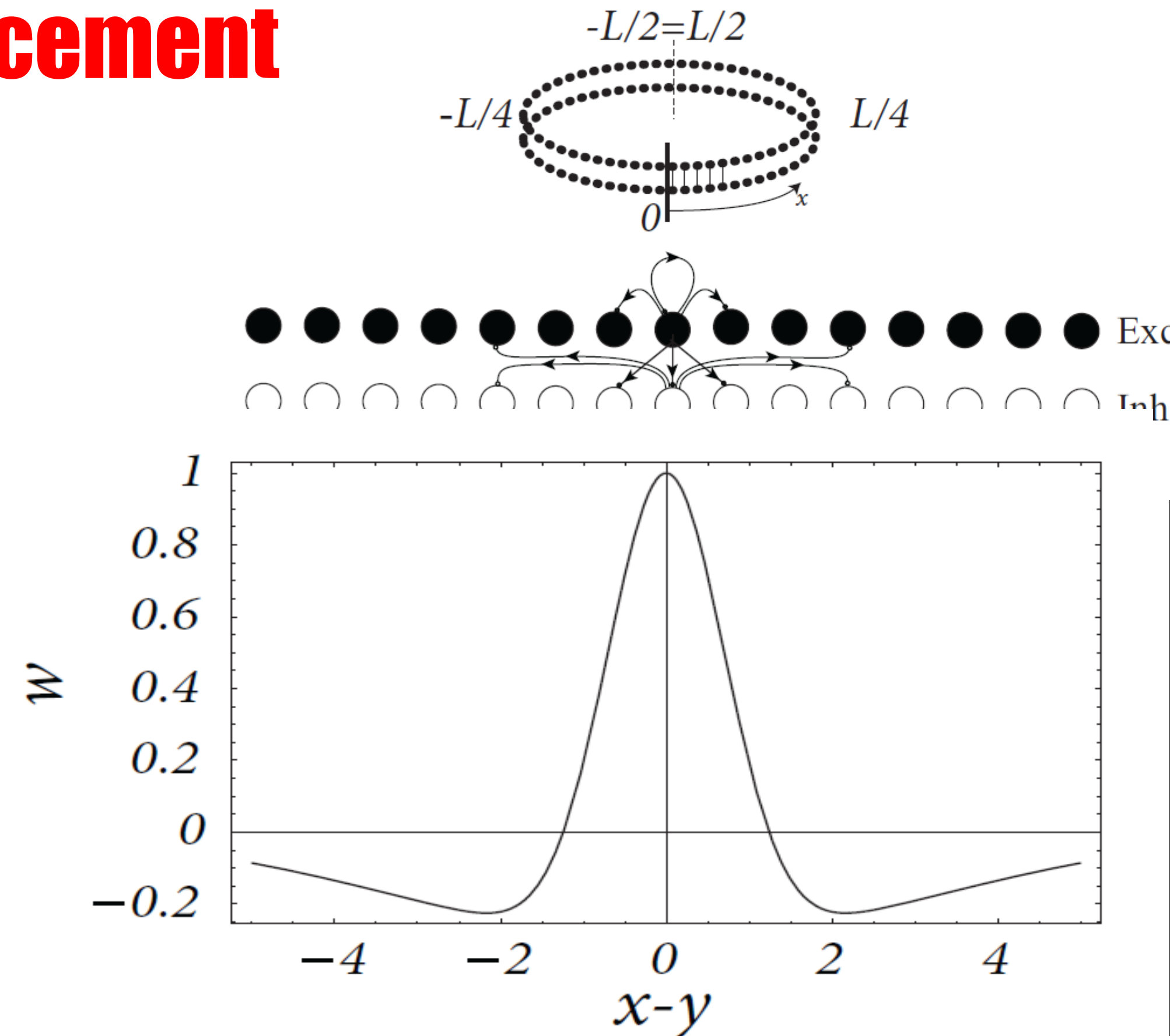
**→ Sharpening**

*Shriki et al. (2003):*

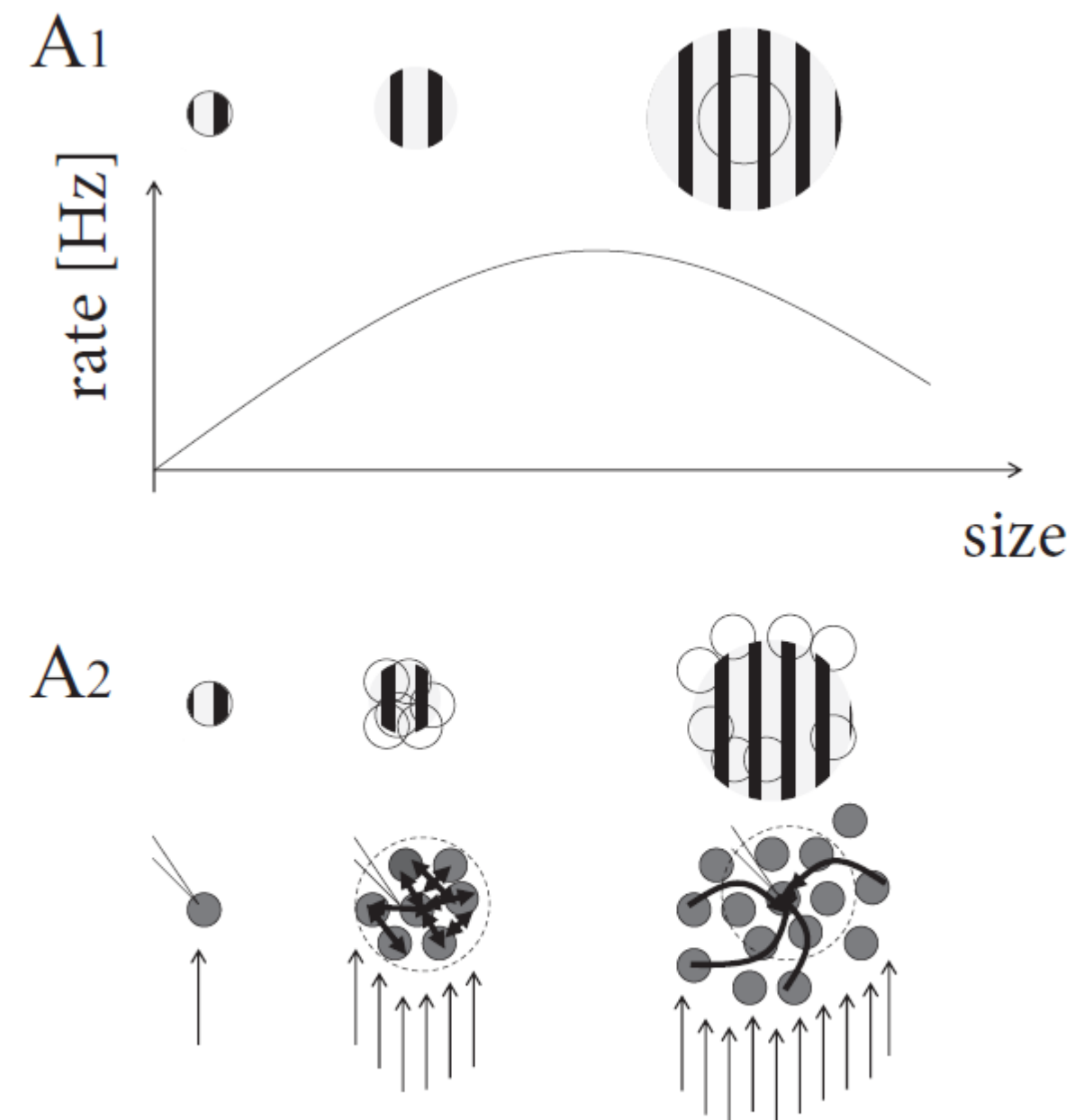
Ring model in input-driven regime,  
driven by broad input (dashed line),  
causes sharp activity bump;

*See also: Ben-Yishai et al. 1995;  
Hansel and Sompolinsky, 1998*

A



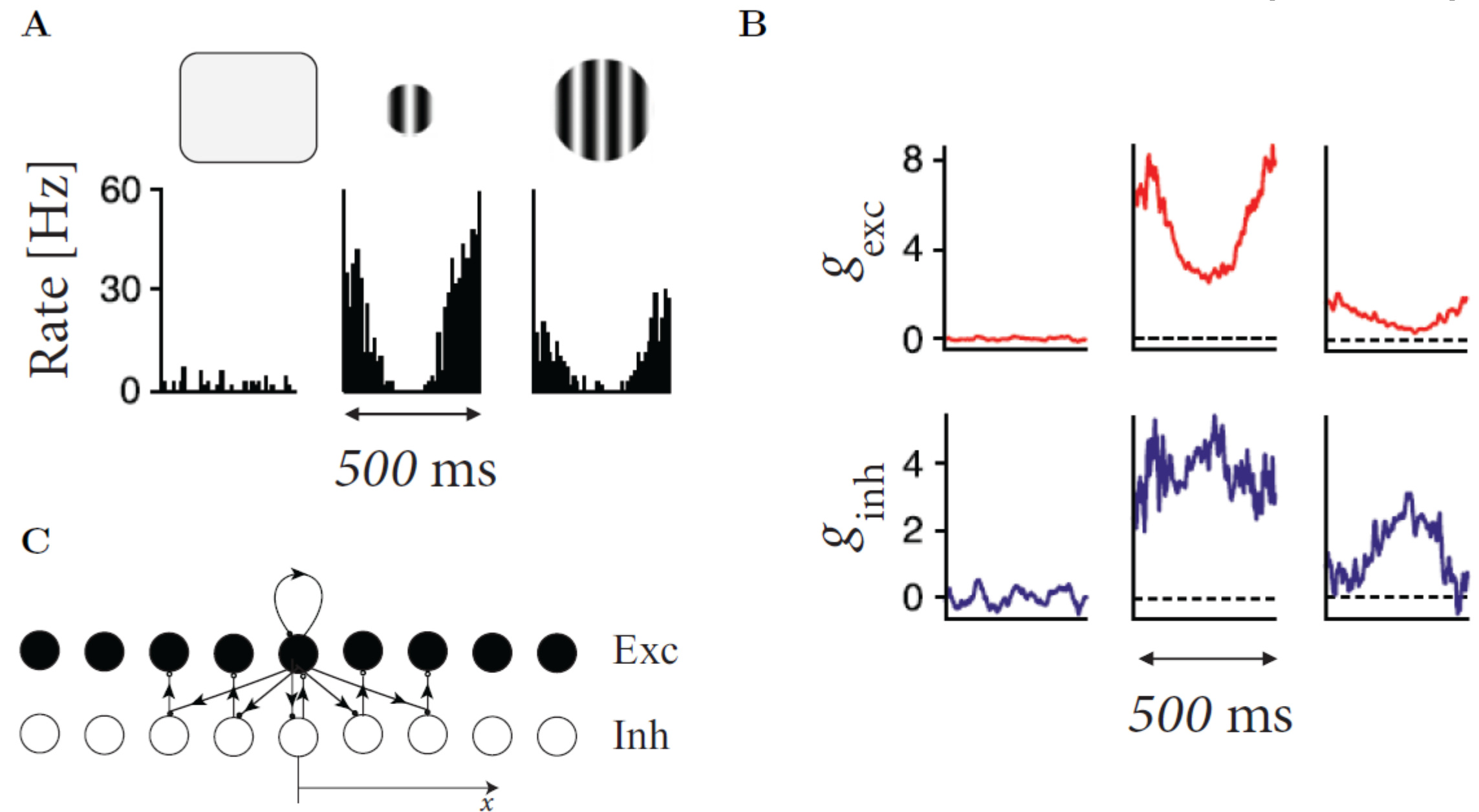
# 6: Field models and Perception: surround suppression



**Fig. 18.12:** Surround suppression.

*Image: Neuronal Dynamics,  
Gerstner et al.,  
Cambridge Univ. Press (2014),*

*Ozeki et al. (2009):*



**Fig. 18.13:** Network stabilized by local inhibition. The schematic model could potentially explain why larger gratings lead not only to less excitatory input  $g_{exc}$ , but also to less inhibitory input  $g_{inh}$ . **A.** The firing rate as a function of the phase of the moving grating for the three stimulus conditions (blank screen, small and large grating). **B.** Top: Excitatory input into the cell. Bottom: Inhibitory input into the same cell. As in A, left, middle and right correspond to a blank screen, a small grating and or a large grating. Note that the larger grating leads to a reduction of both excitation and inhibition; adapted from (Ozeki et al., 2009). **C.** Network model with long range excitation and local inhibition. Excitatory neurons within a local population excite themselves (feedback arrow), and also send excitatory input to inhibitory cells (downward arrows). Inhibitory neurons project to local excitatory neurons.

## 6. Field models and Perception

### **Psychophysics:**

- contrast enhancement is a stable psychophysical phenomenon
- Mach bands are but one example

### **Neuronal:**

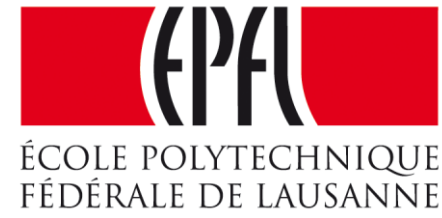
- the activity of V1 cell first increases and then decreases with size of stimulus
- both excitatory and inhibitory input into a cell show similar changes

### **Modeling**

- continuum model with Mexican-hat interaction in the input-driven regime for Mach bands
- Receptive Field tuning:  
contrast enhancement = 'sharpening'



# Computational Neuroscience: Neuronal Dynamics of Cognition



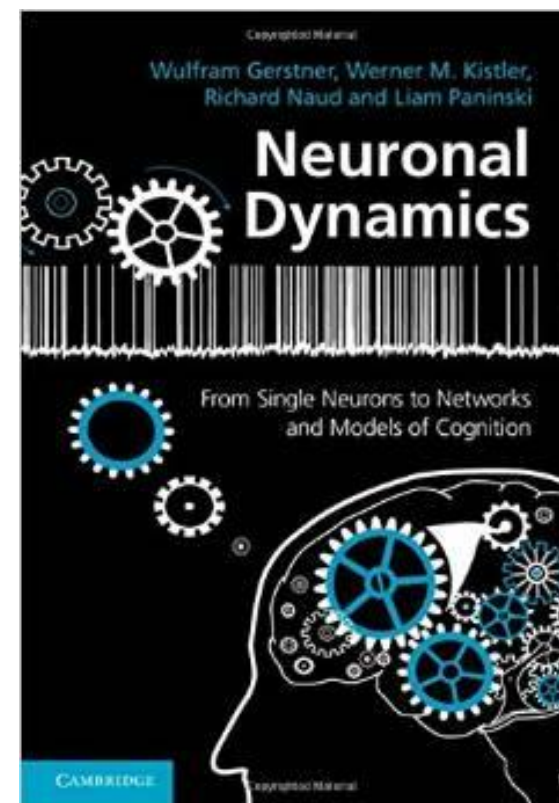
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### 7. Head direction cells

## 7. Head direction cells: aims

sense of direction



### Model

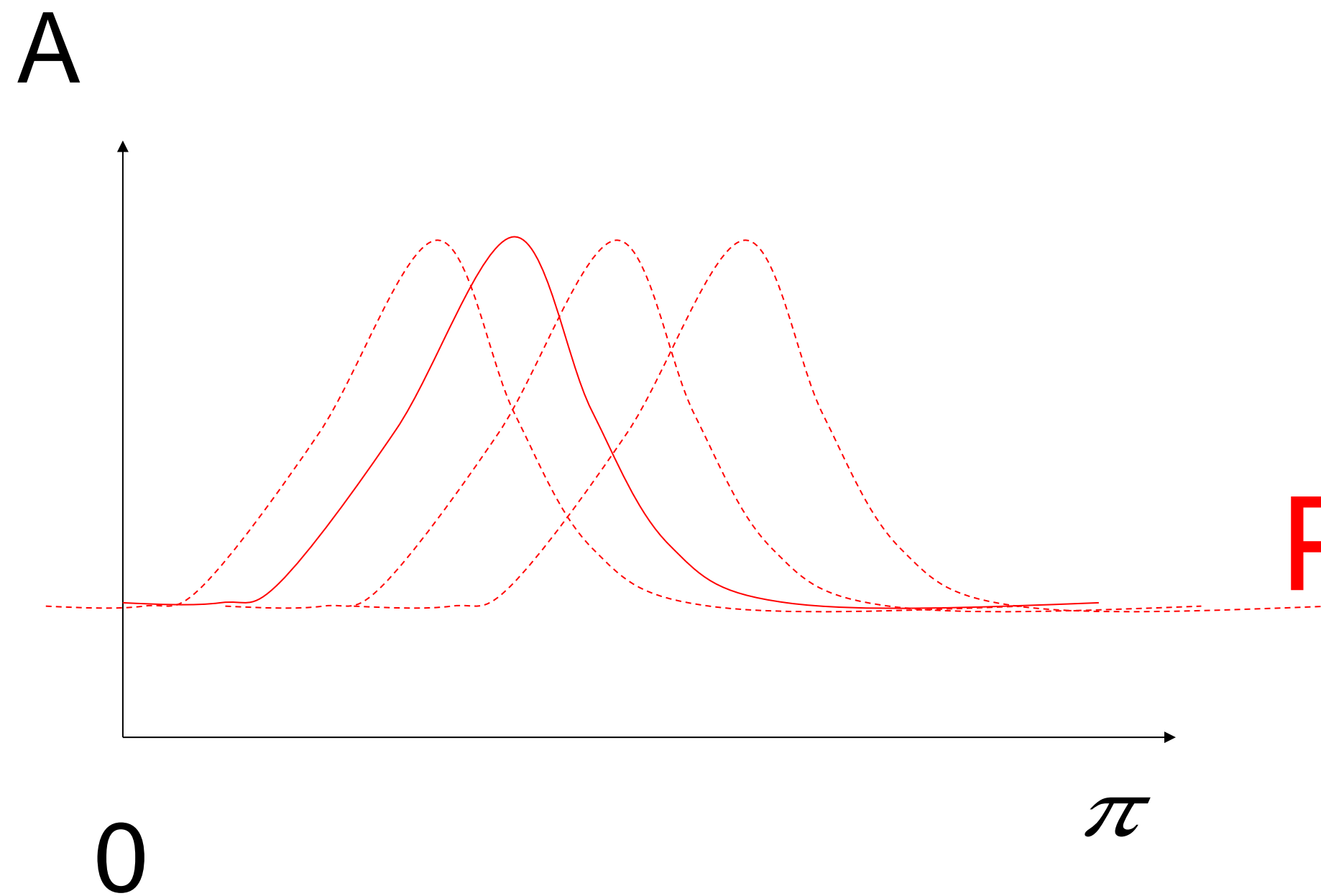
memory of direction related  
to bump solution of ring model

# 7. Bump solution

Basic phenomenology

## Bump formation

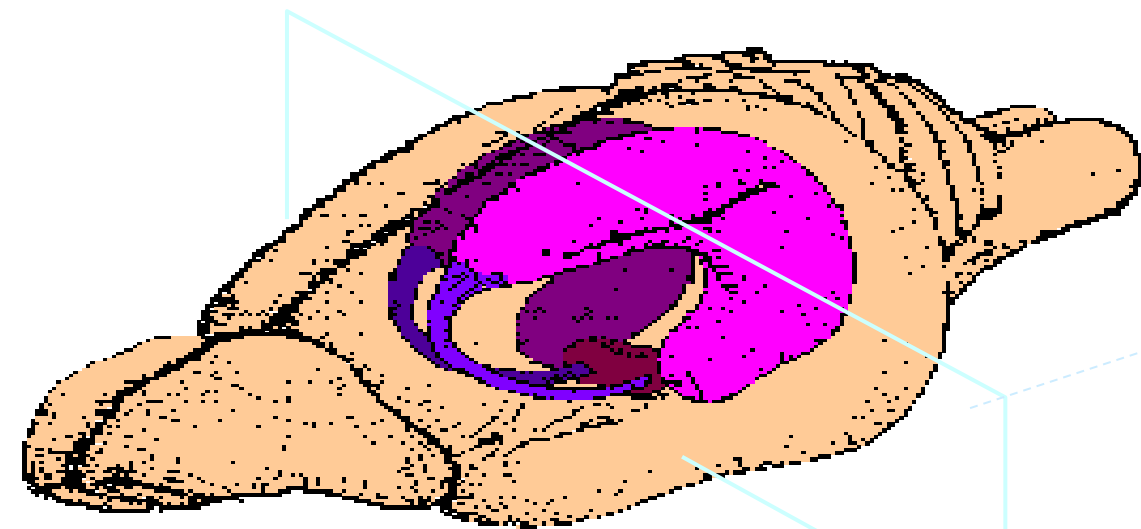
strong lateral connectivity



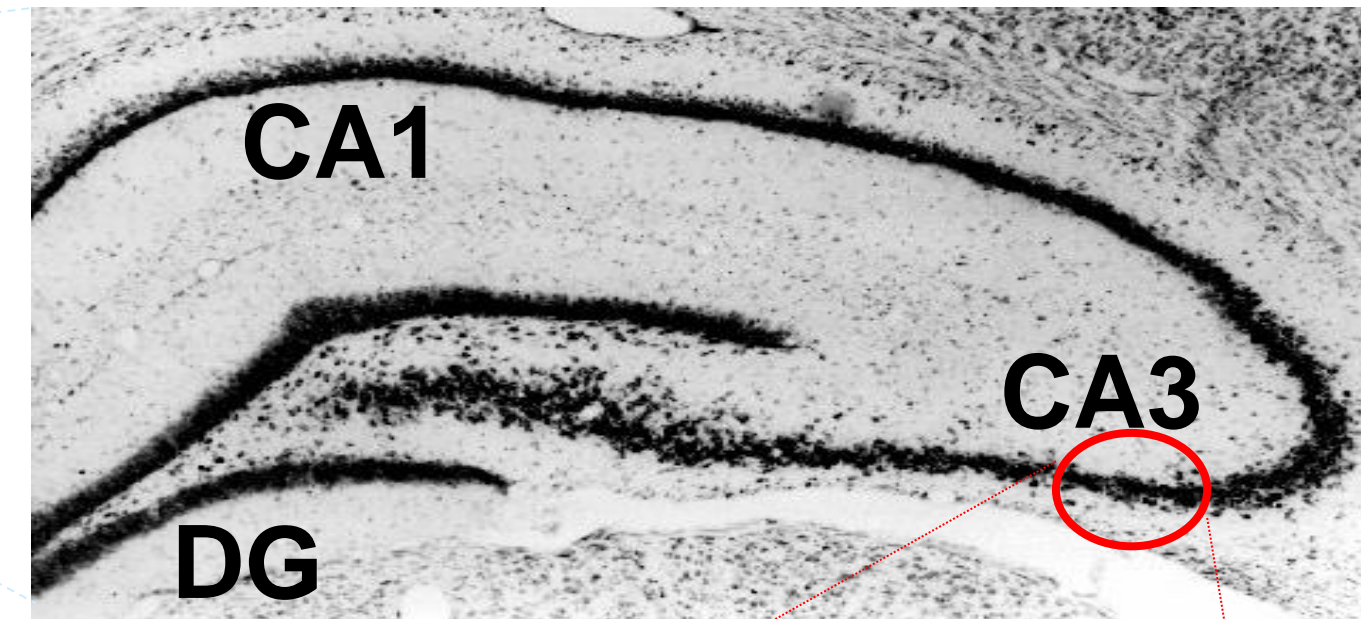
Possible application:  
head direction cells -  
bump of active cells  
→ indicate current orientation



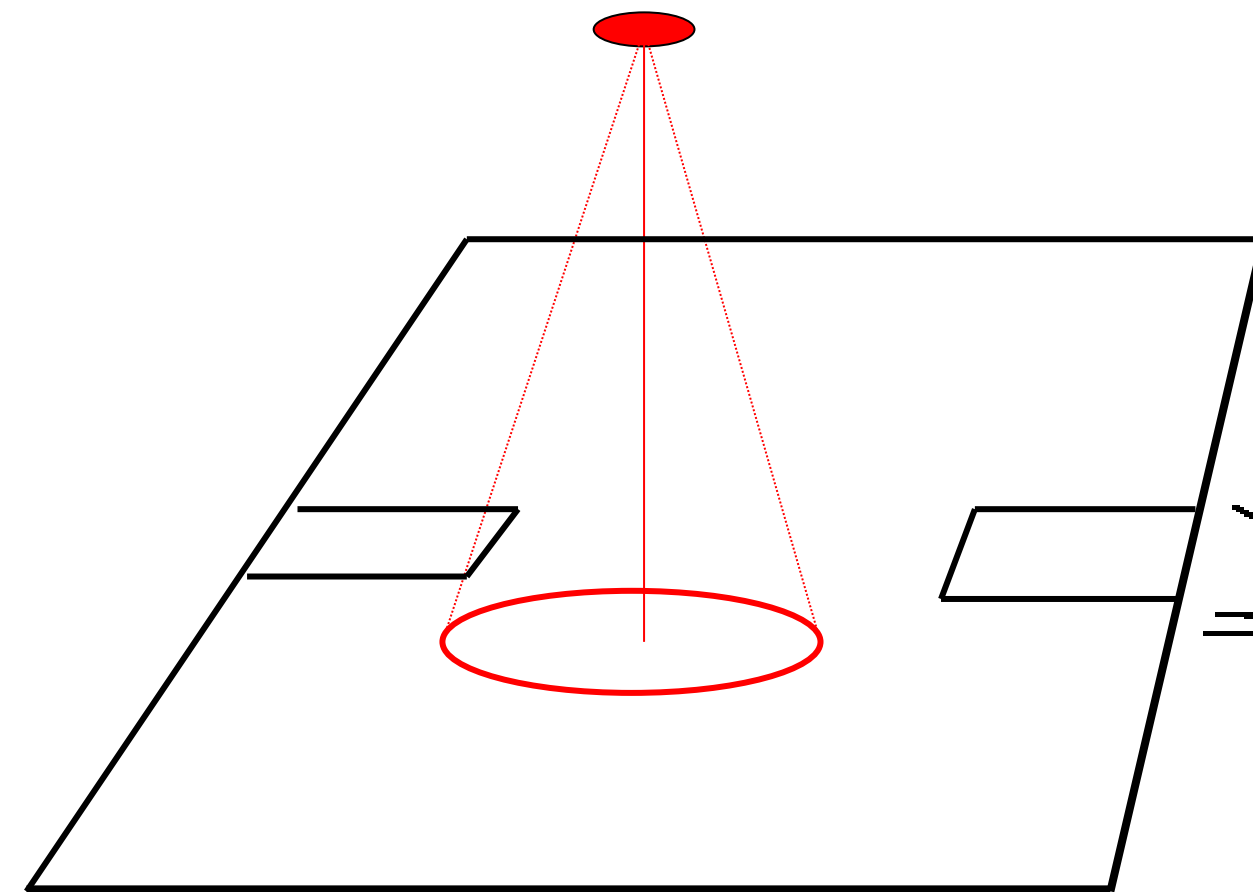
# 7. Hippocampal place cells



rat brain

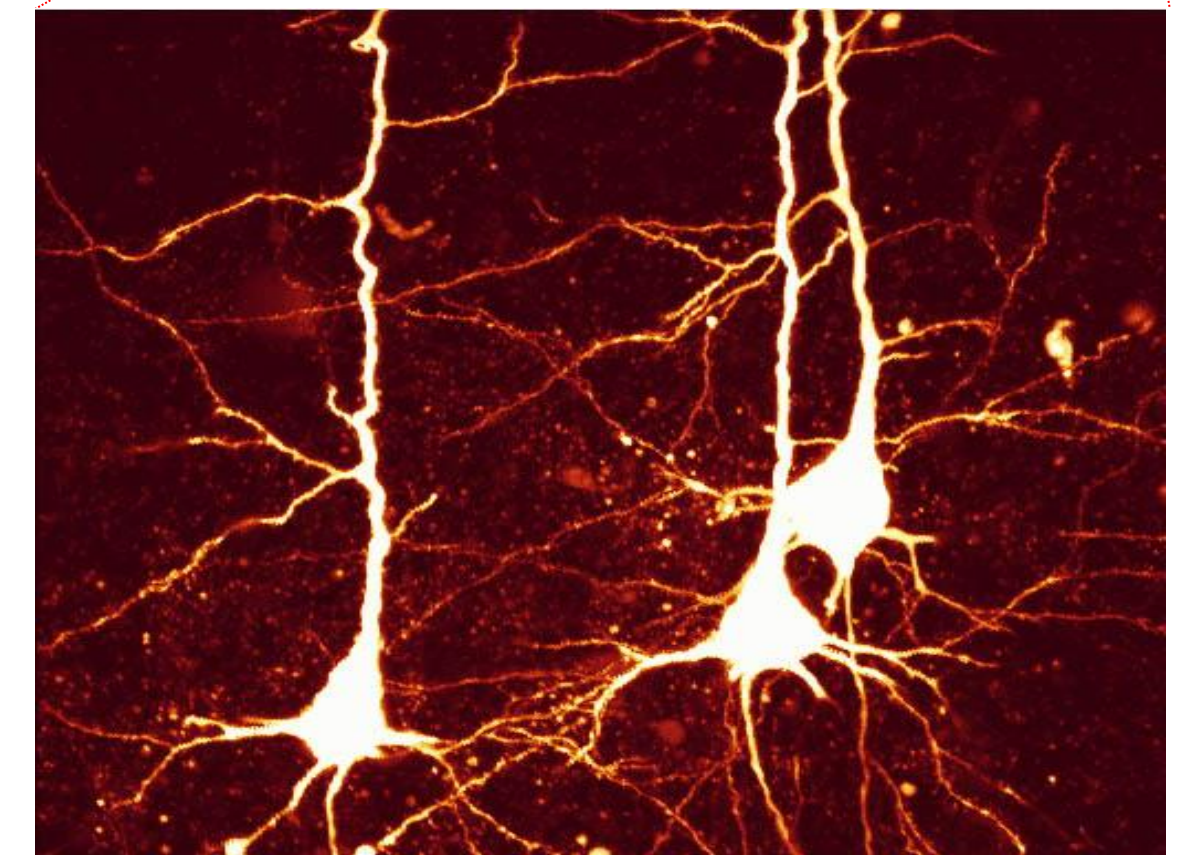
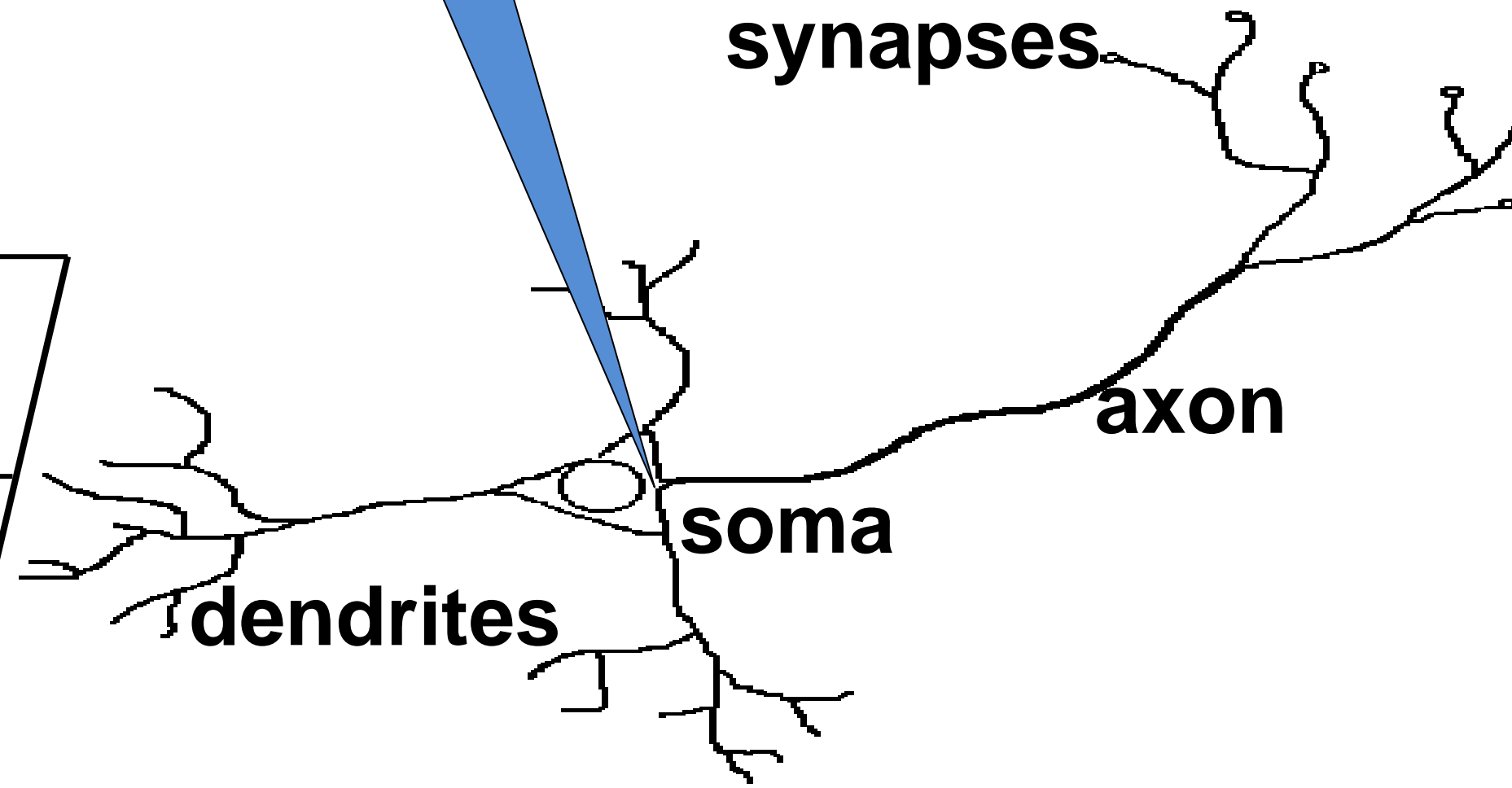


Place fields



electrode

synapses

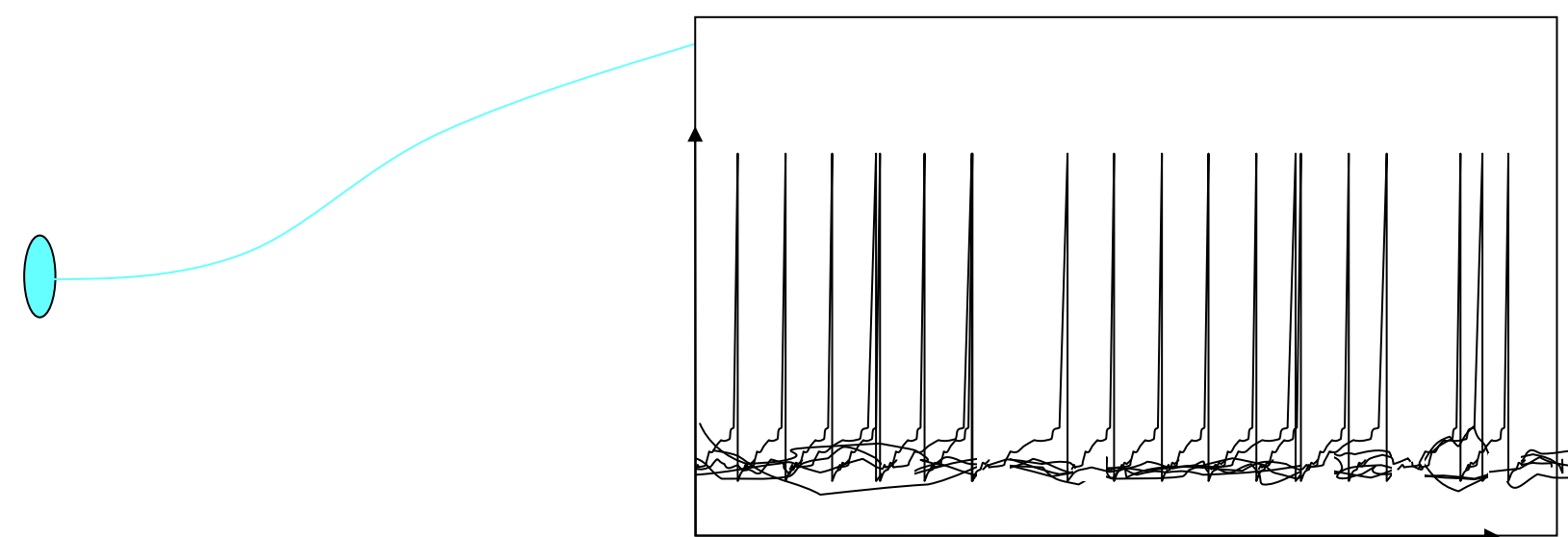
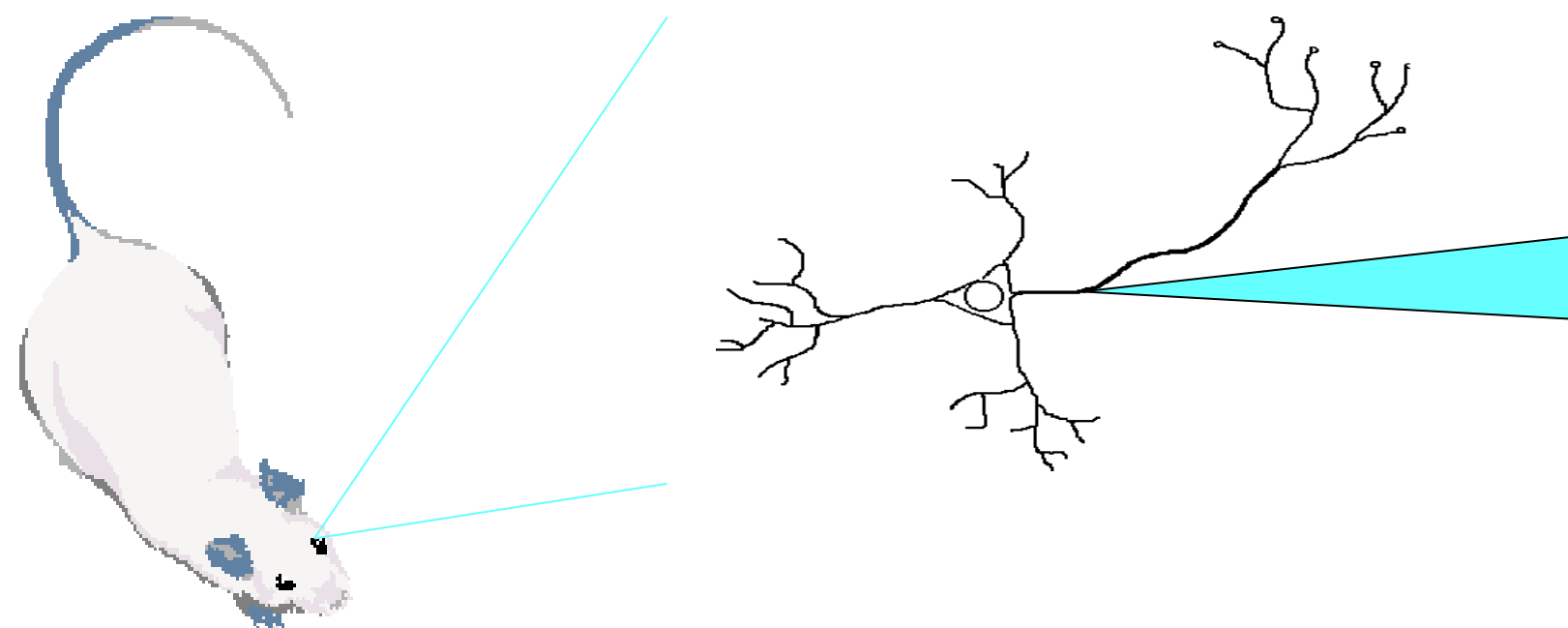
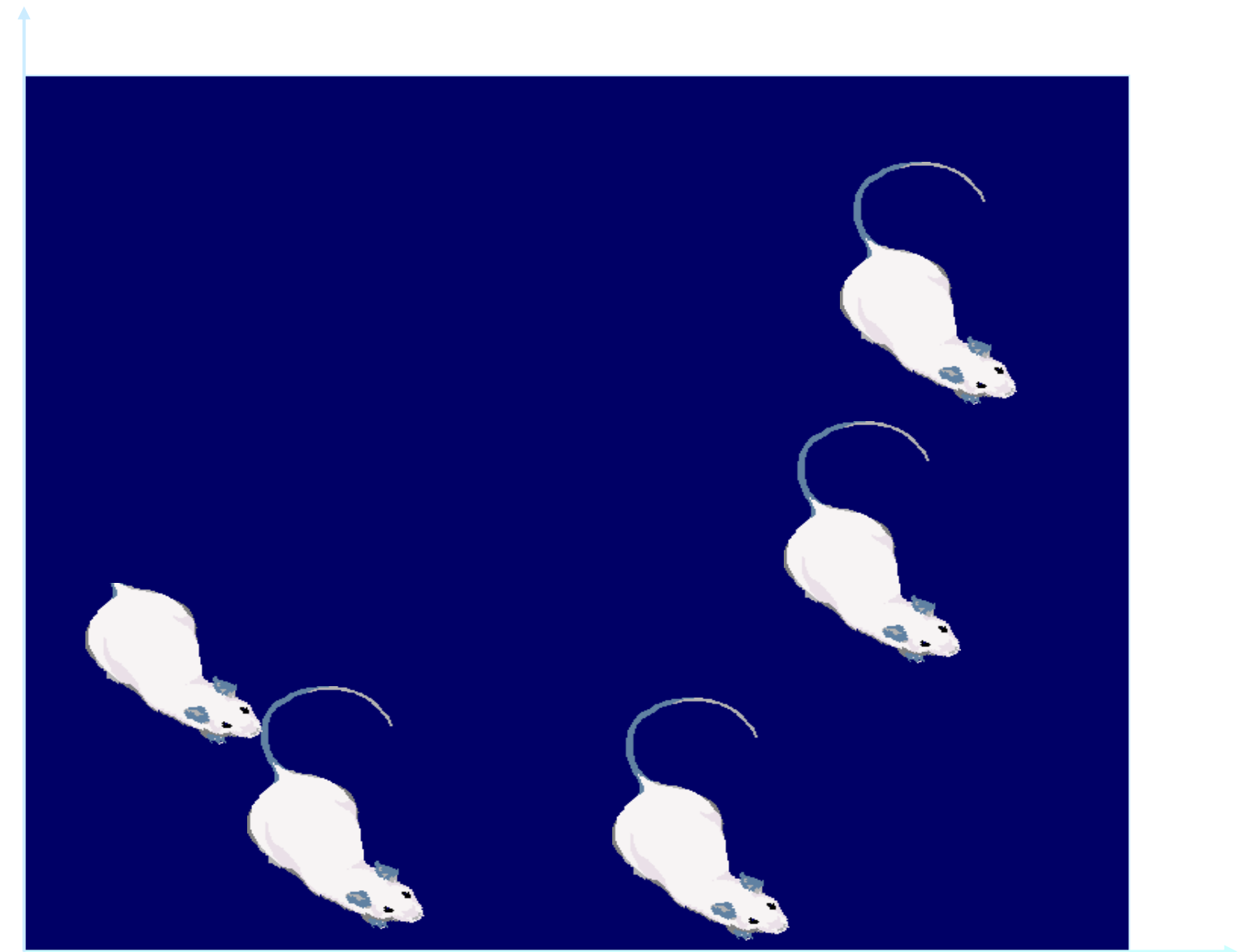
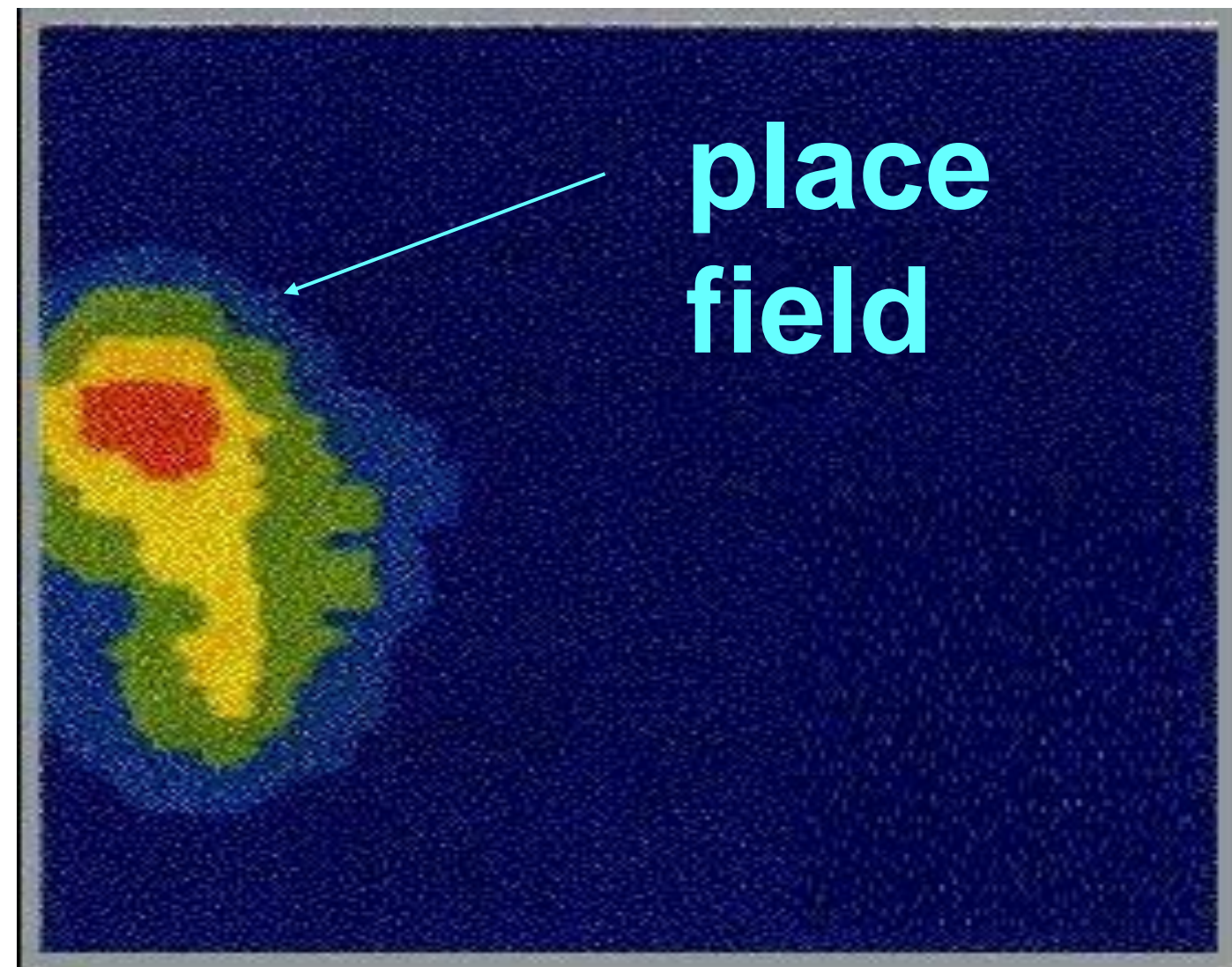


pyramidal cells



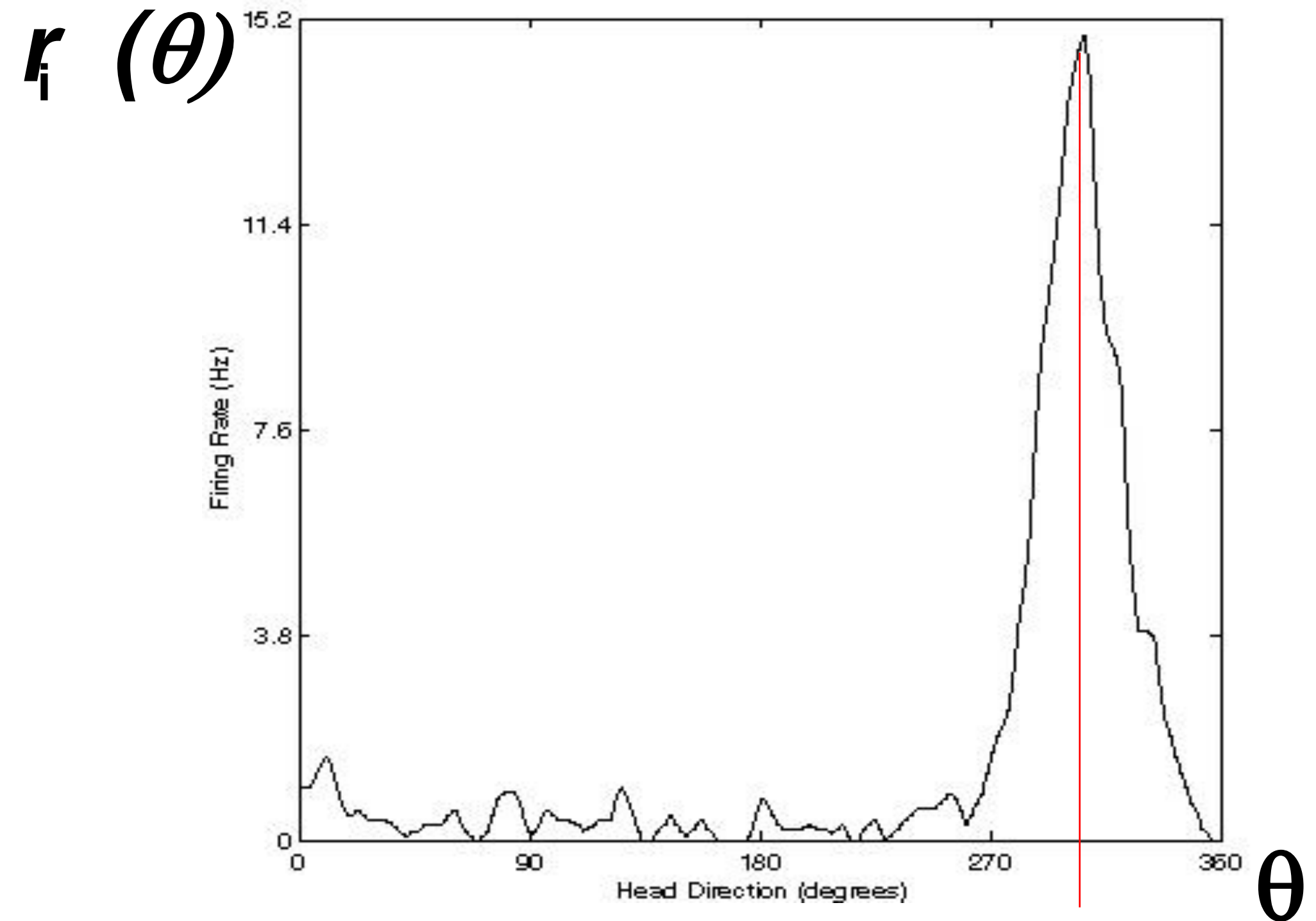
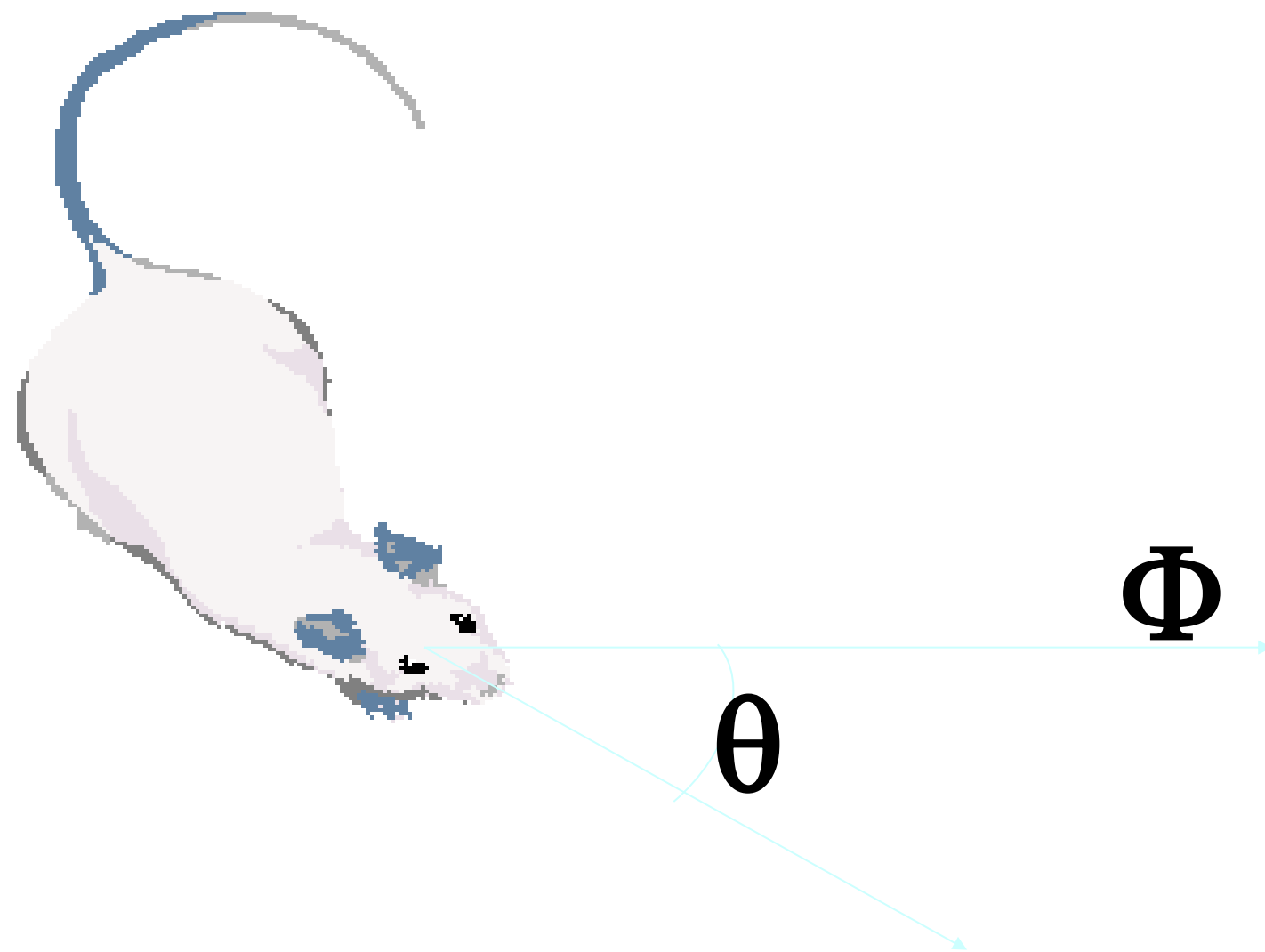
# .7. Hippocampal place cells

Main property: encoding the animal's location



# 7. Head direction cells

Main property: encoding the animal's heading

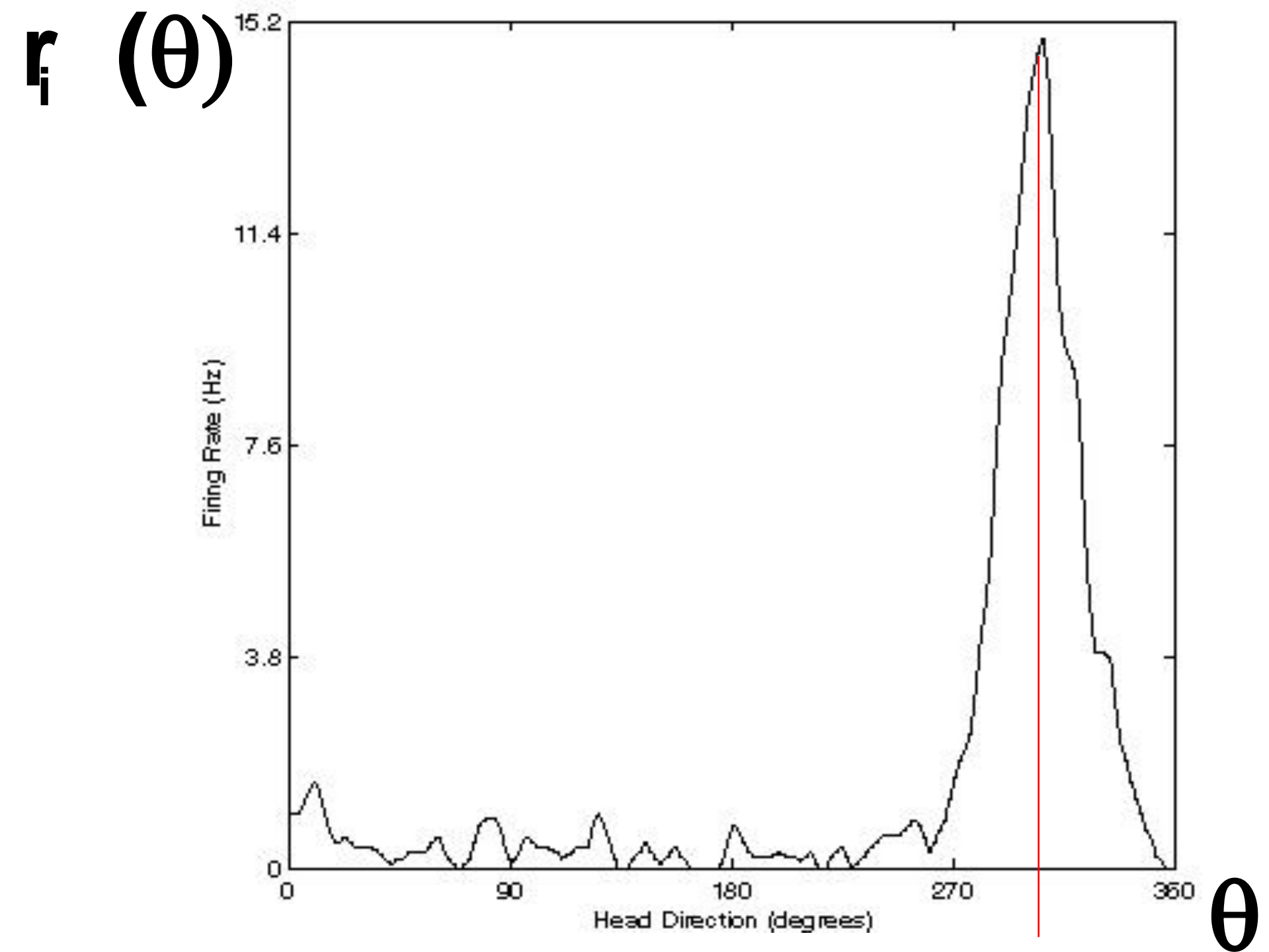
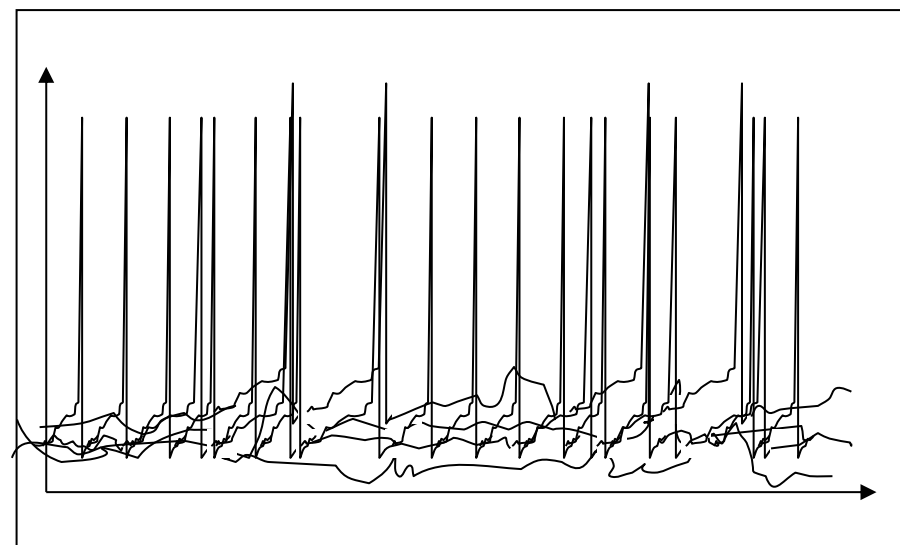
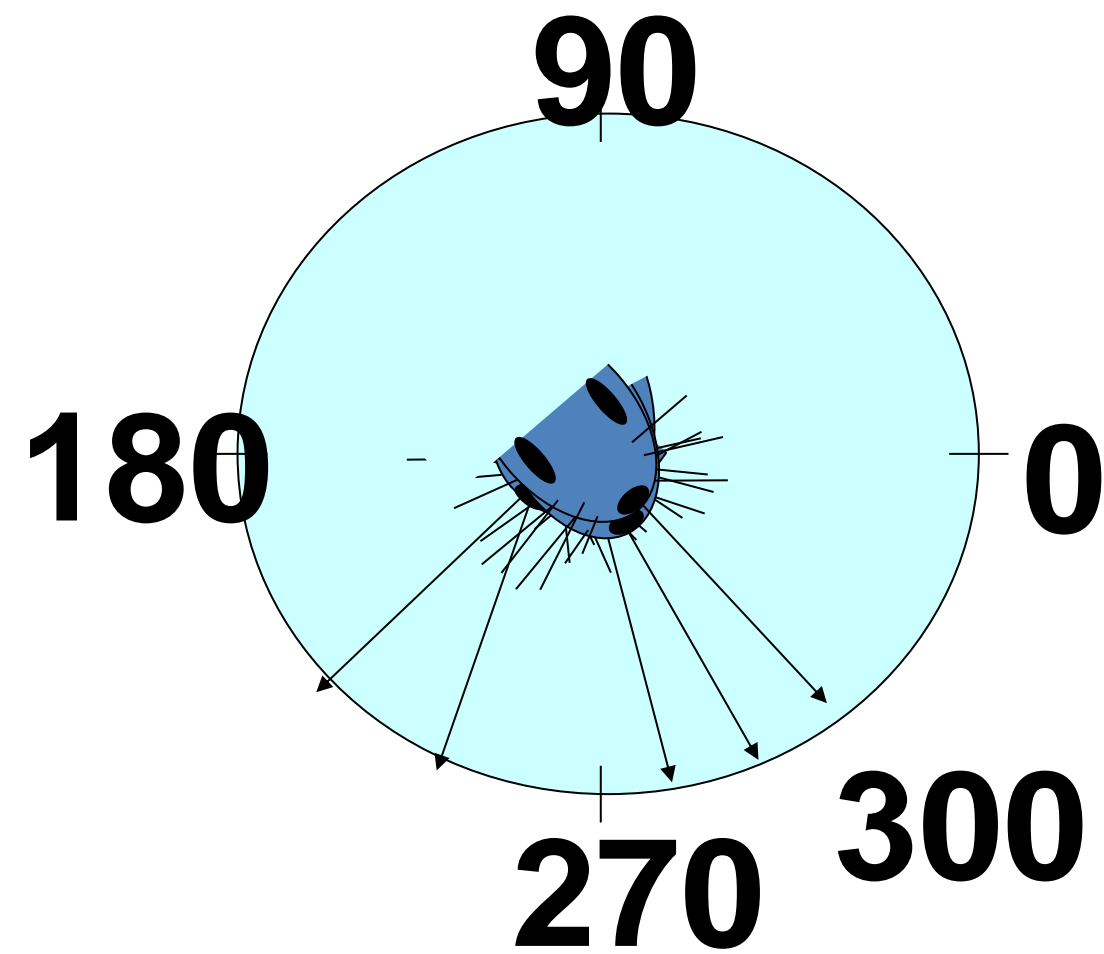


*Taube and Muller,  
Hippocampus 1998,*

**$\theta_i$   
Preferred firing direction**

# 7. Head direction cells

Main property: encoding the animal's allocentric heading



$\theta_i$   
Preferred firing direction

# 7. Head direction cells

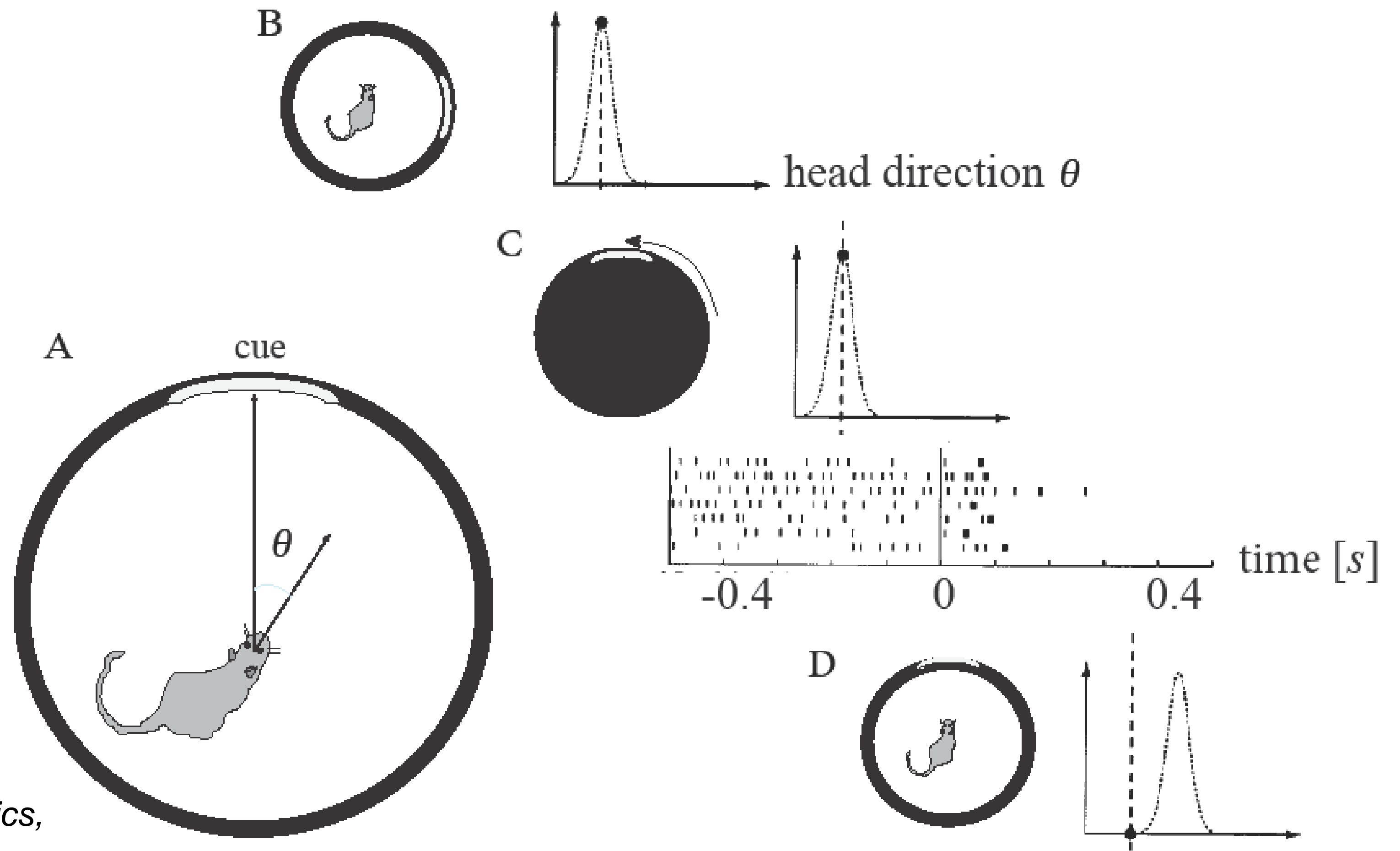


Image: *Neuronal Dynamics*,  
Gerstner et al.,  
Cambridge Univ. Press (2014),  
Adapted from Zugaro et al. (2003), *J. Neurosci.* 23:3478-3482

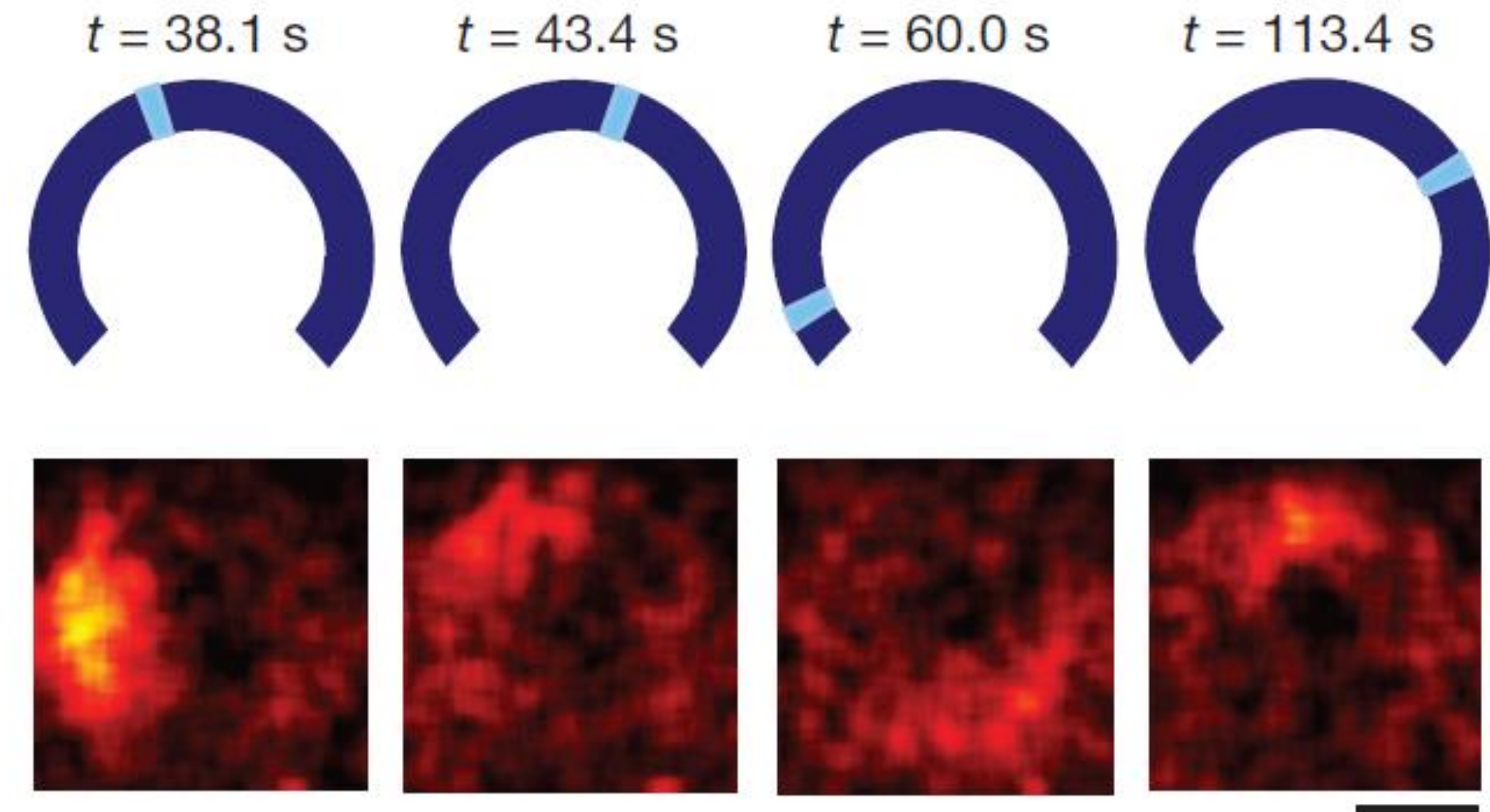


## 7. Head direction cells in the fly brain

Similar to the rat: head direction cells in fly brain (ellipsoid body)

stimulus on screen:

activity in  
ellipsoid body



- bump activity persists in the dark
- cue is landmark configuration

Seelig and Jarayaman, Nature, 2015,

*Neural dynamics for landmark orientation and angular path integration*



## 7. Head direction cells: summary

head direction cells

- are sensitive to direction of head with respect to visual cues
- keep their activity if light is switched off
- exist in rodents and in flies
- can be explained by bump solution in ring model



*Taube and Muller, Hippocampus 1998,*

*Zugaro et al., J. Neurosci. 2003*

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*Redish et al., Network, 1996, Zhang, J. Neurosci. 1996*

## **7. Summary: field models**

**Continuum model provides understanding for:**

- head direction cell
  - bumps of activity
- spatial working memory
  - bumps of activity
- place cells
  - bumps of activity
- contrast enhancement and some visual illusions
  - input driven regime
- receptive field properties
  - input driven regime

# 7. Selected References: Field Models

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