Computational Neuroscience: Neuronal Dynamics of Cognition

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

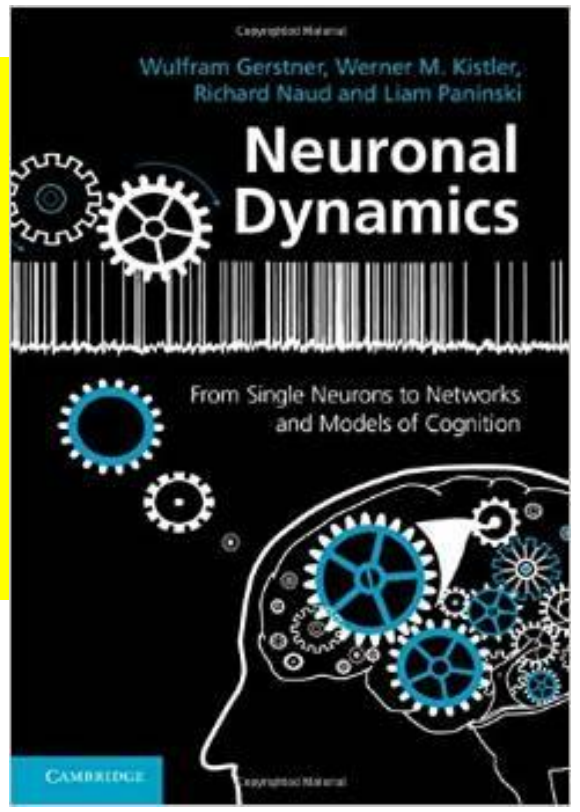
A: ASSOCIATIVE MEMORY
in a Network of Neurons

Wulfram Gerstner

EPFL, Lausanne, Switzerland

Reading for this week:
NEURONAL DYNAMICS
- Ch. 17.1 - 17.2.4

Cambridge Univ. Press



1 Introduction
- networks of neuron
- systems for computing
- associative memory

2 Classification by similarity

3 Detour: Magnetic Materials

4 Hopfield Model

5 Learning of Associations

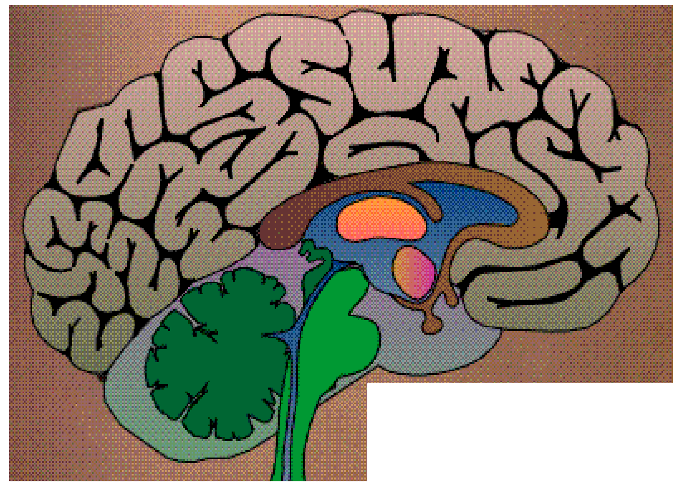
6 Storage Capacity

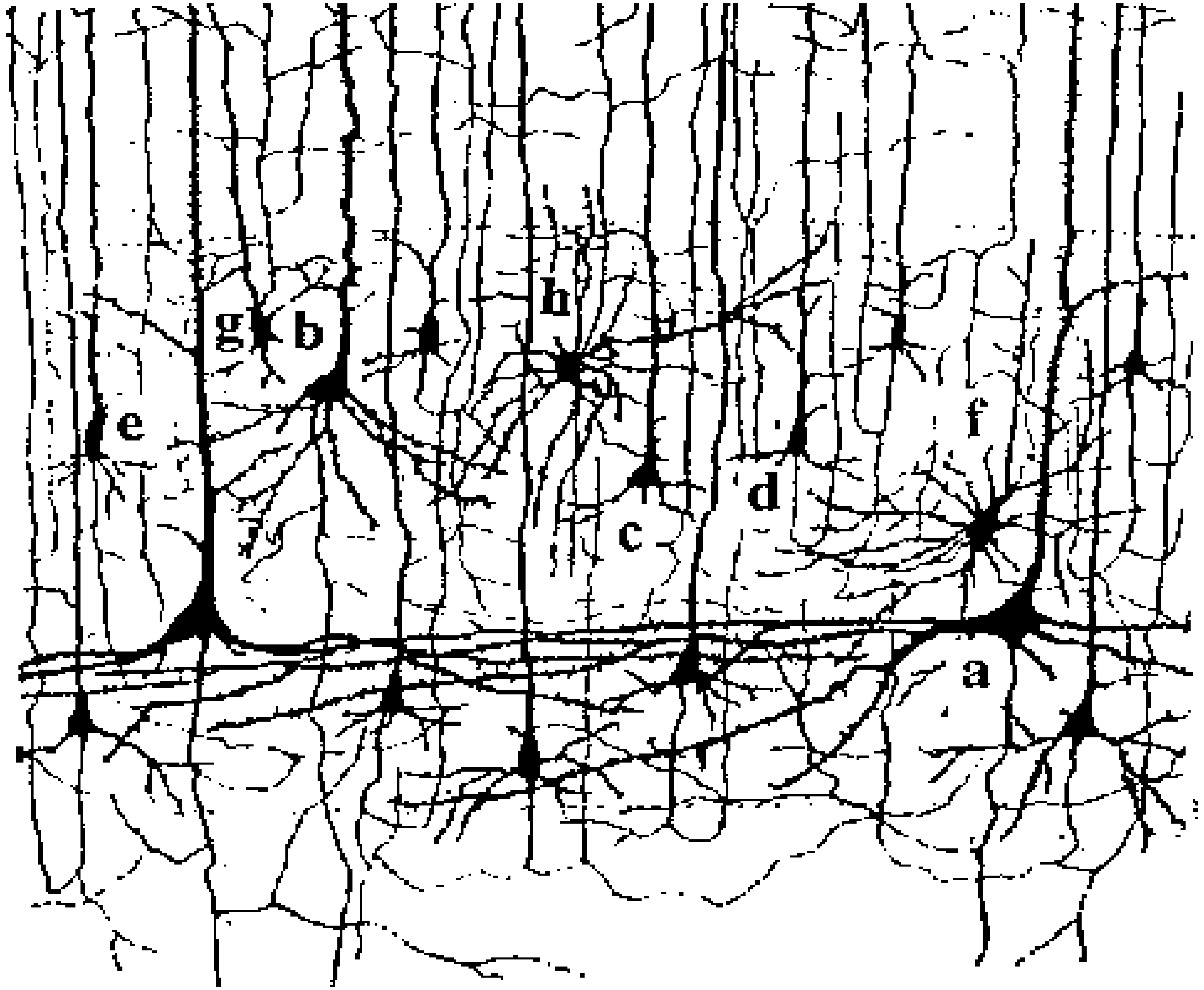
1. memory in the brain

-president
-first day of undergraduate
-apple

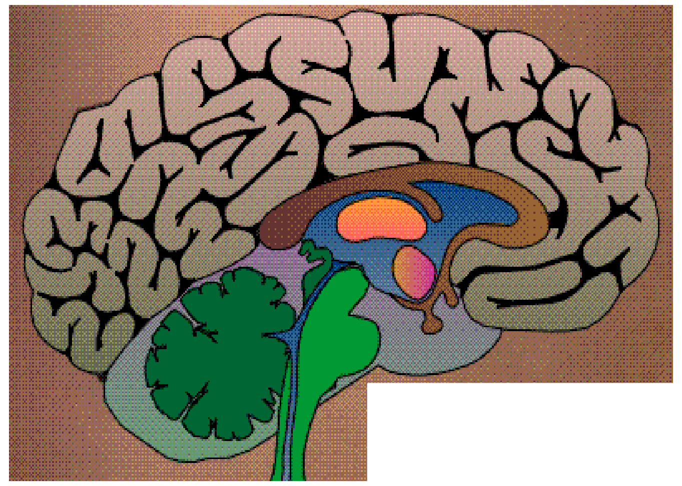
Our memory has multiple aspects
- recent and far-back
- events, places, facts, concepts

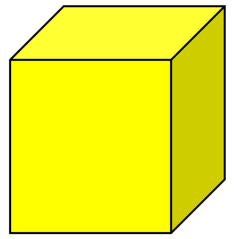
1. memory in the brain





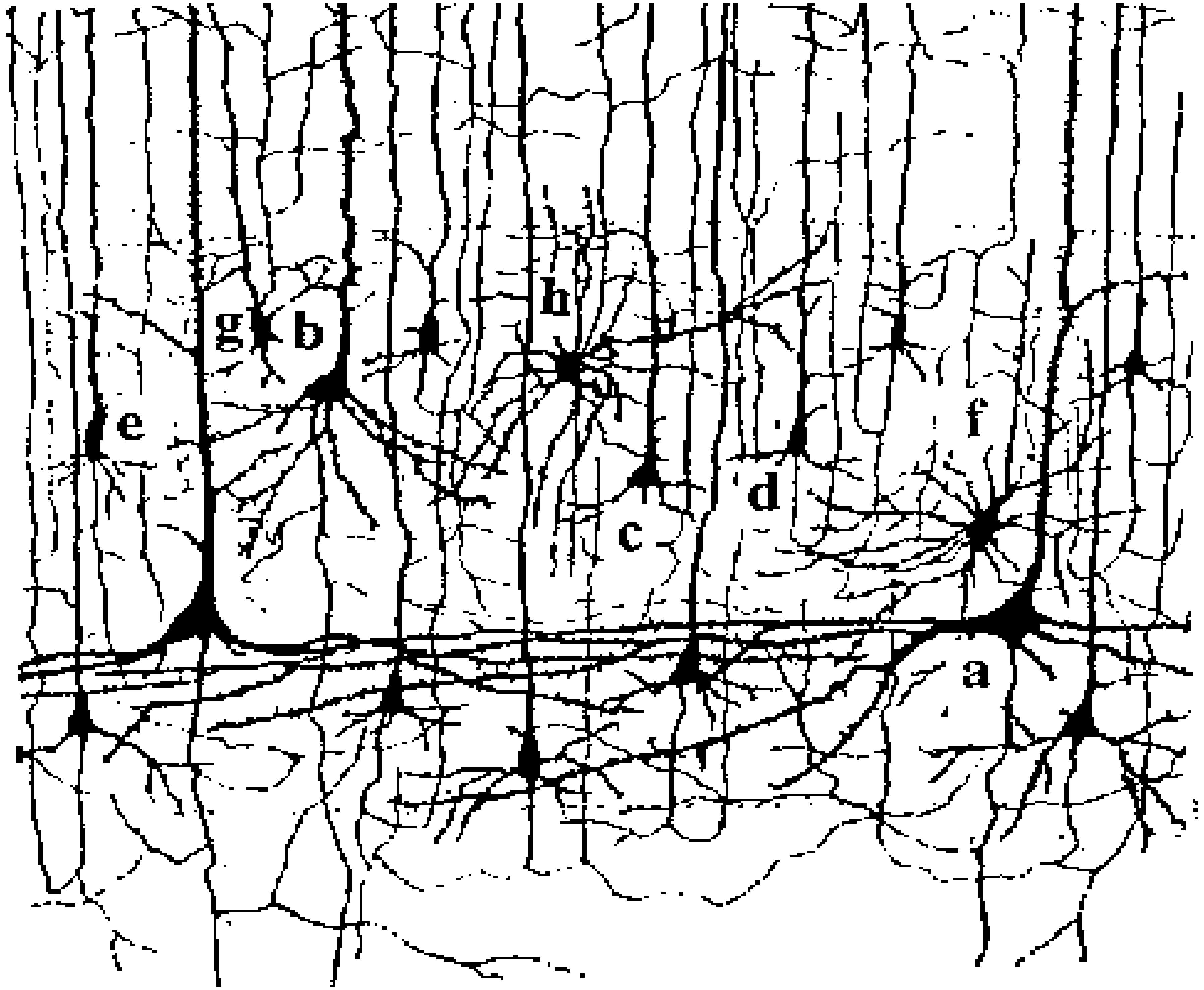
1. Neuronal Networks in the Brain



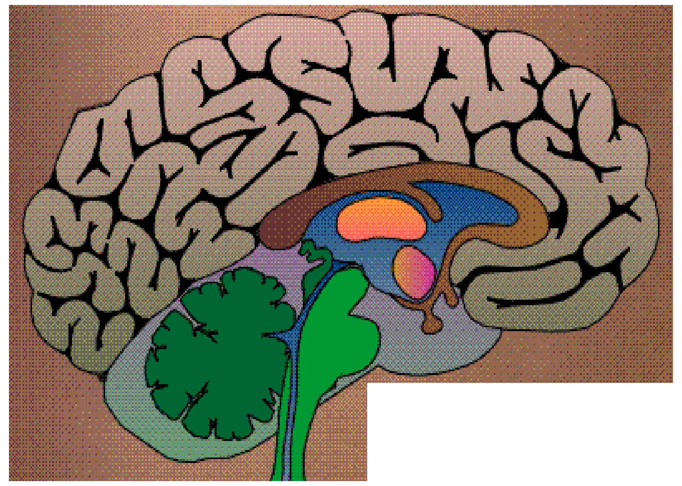


1mm

10 000 neurons
3 km of wire

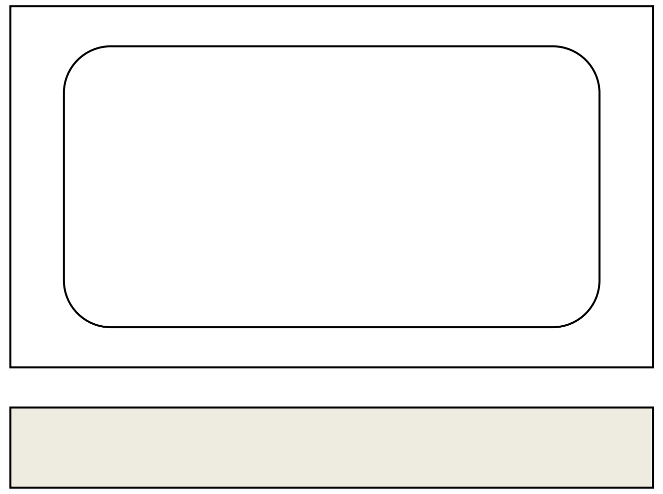


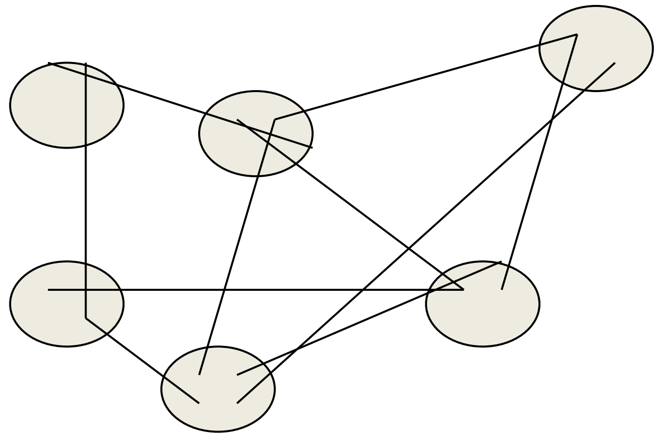
1. Systems for computing and information processing



Brain

Computer

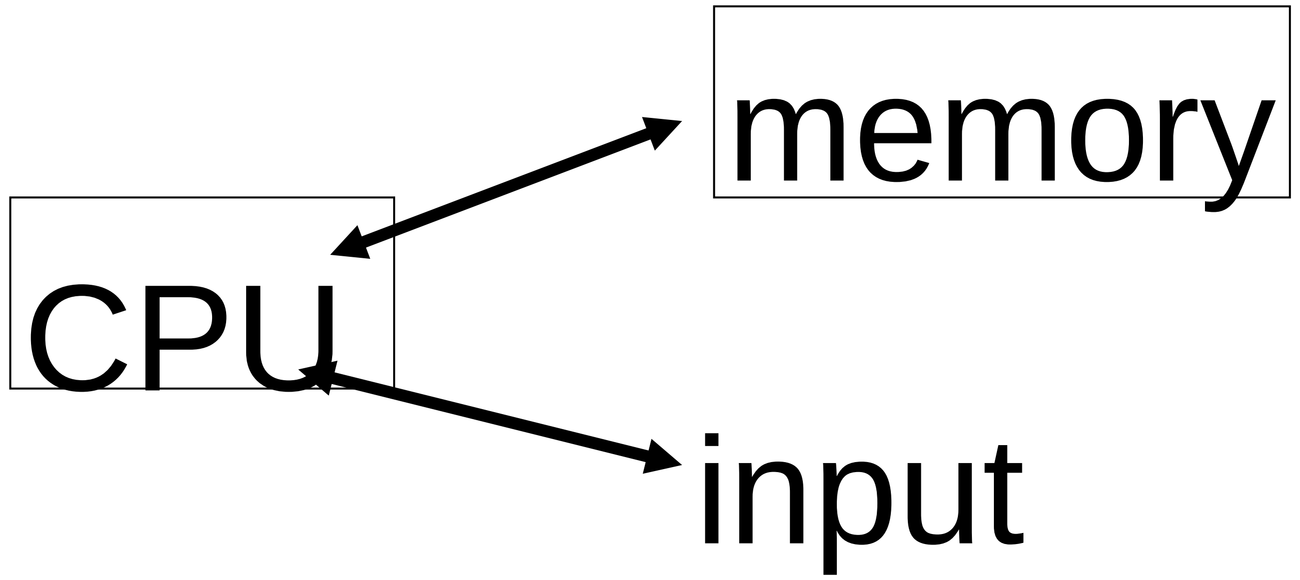




Distributed architecture

(10^{10} proc. Elements/neurons)

No separation of processing and memory

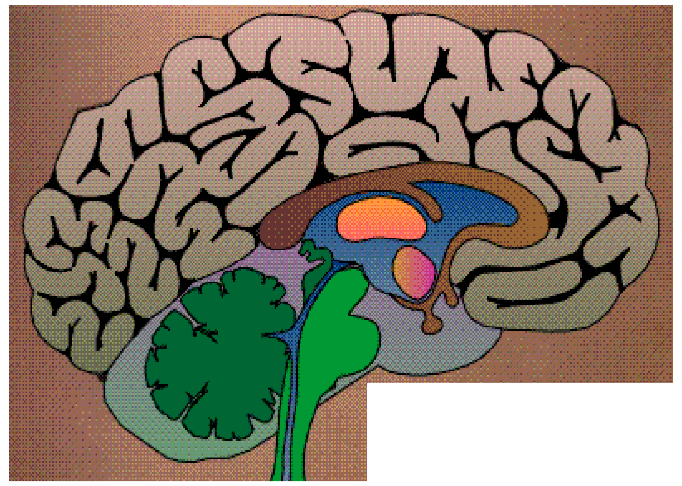


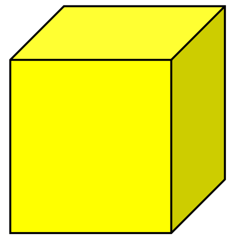
Von Neumann architecture

1 CPU

(10^{10} transistors)

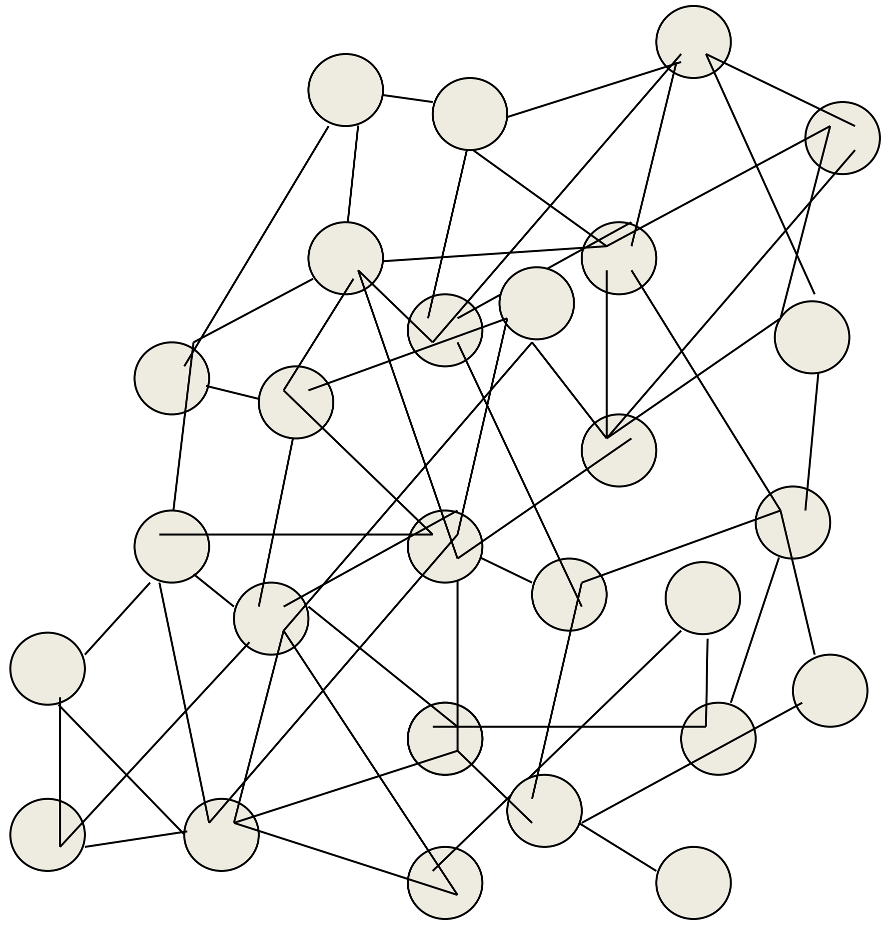
1. Systems for computing and information processing





1mm

10 000 neurons
3 km of wire



Distributed architecture

10^{10} neurons

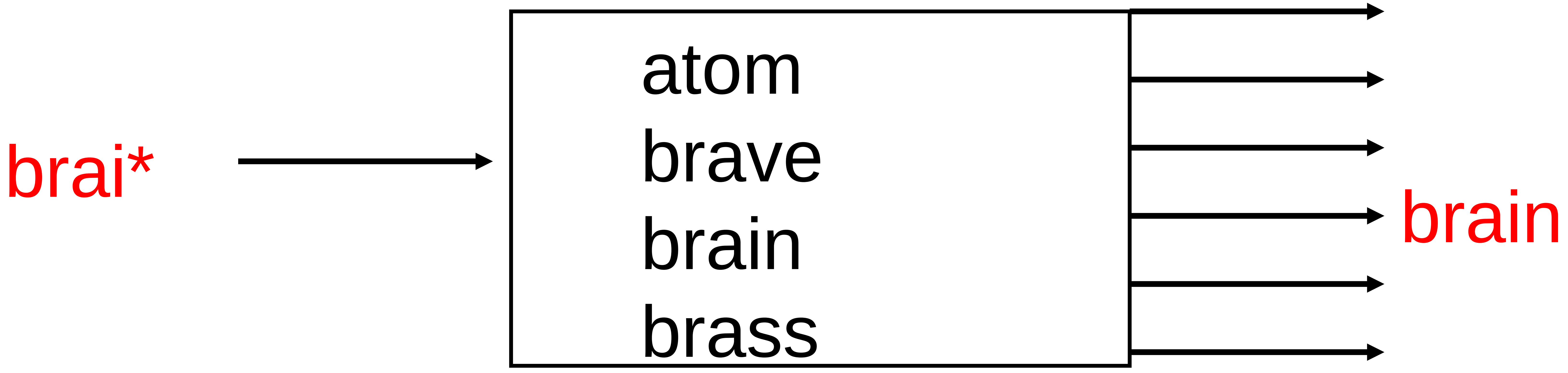
10^{4} connections/neurons

No separation of processing and memory

1. Associations, Associative memory

Read this text ***NOW!***

pattern completion/word recognition



Noisy word

List of words

Output the closest one

Your brain fills in missing information: 'auto-associative memory'

1. Associations, Associative memory

brai* $\longrightarrow$ brain *'auto-associative memory'*

bird $\longrightarrow$ swan

vacation $\longrightarrow$ beach

'associative memory'

Quiz 1: Connectivity and Associations

Tick one or several answers

A typical neuron in the brain makes connections

[ ] To 6-30 neighbors

[ ] To 100-500 neurons nearby

[ ] To more than 1000 neurons nearby

[ ] To more than 1000 neurons nearby or far away.

Associative memory is involved

[ ] If you think of palm trees when you think of a beach

[ ] If partial information helps you to recall a complicated concept

[ ] If a cue helps you to recall a memory

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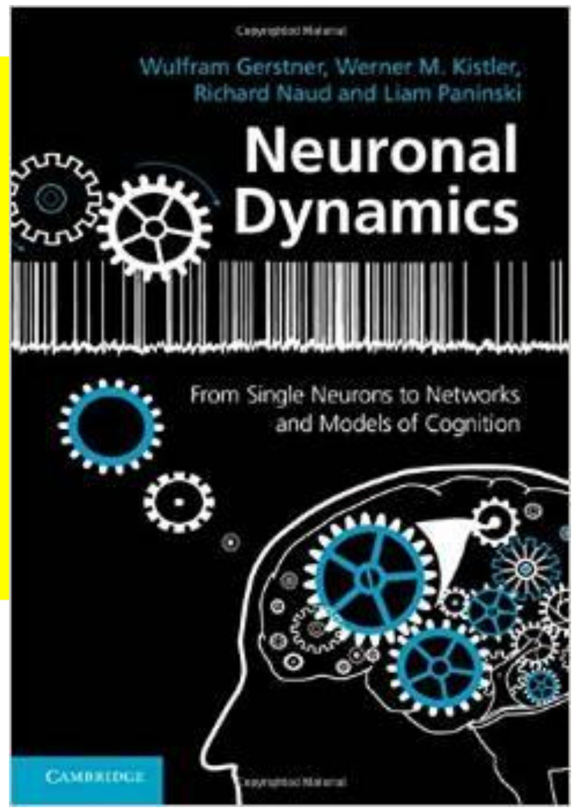

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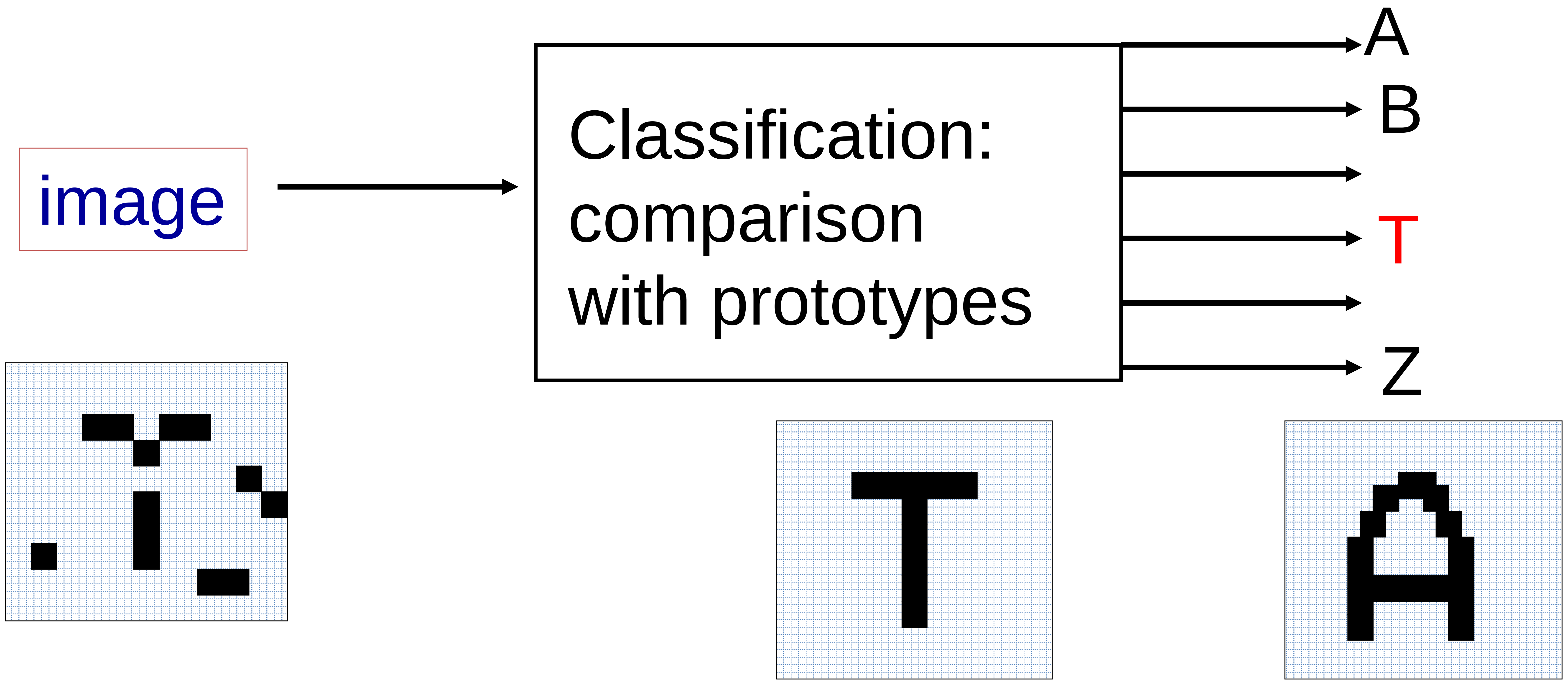
2. Classification by similarity: **pattern recognition**



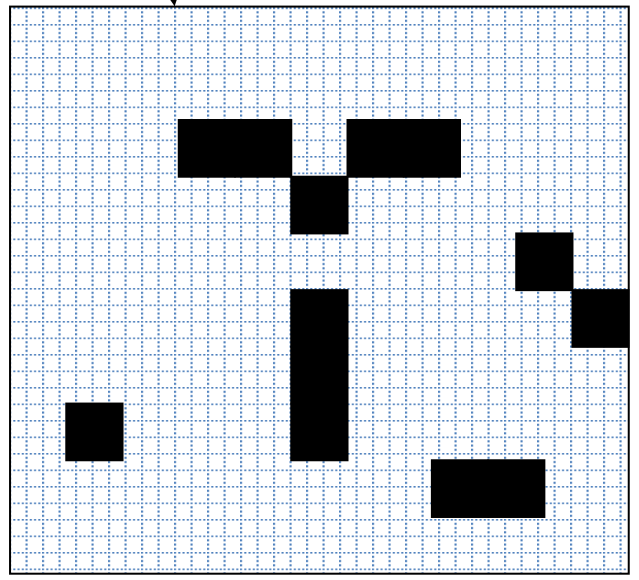
Noisy image

Prototypes

2. Classification by similarity: **pattern recognition**

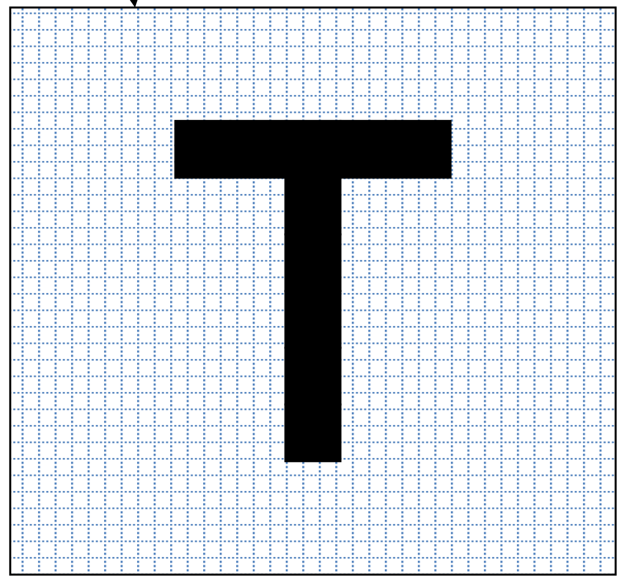
Classification by closest prototype

x

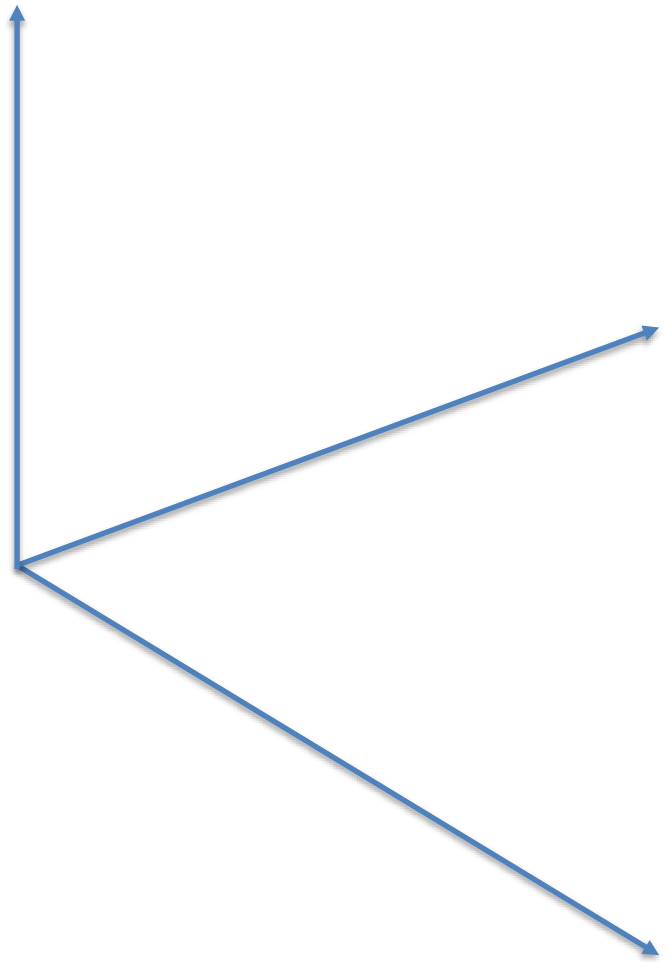


Noisy image

p^T



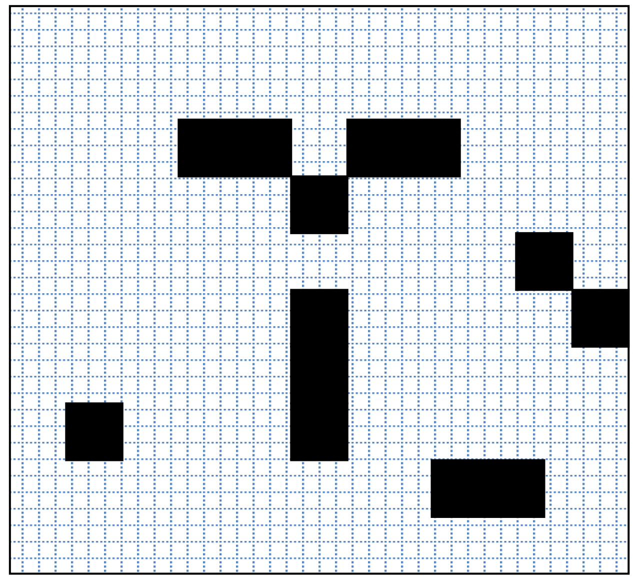
Prototype

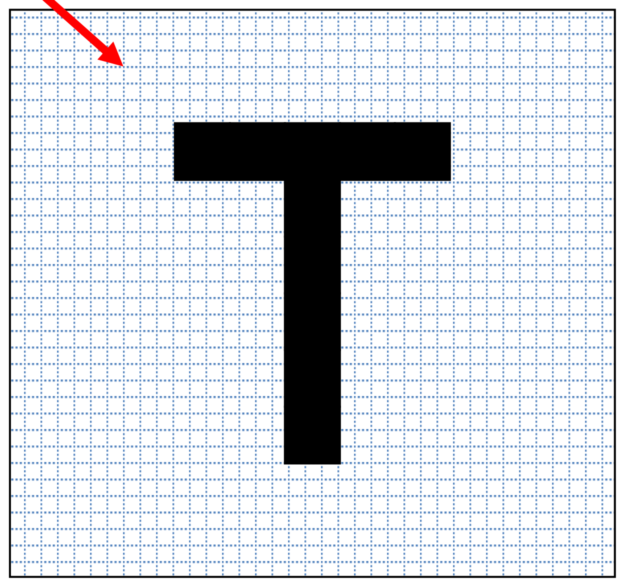


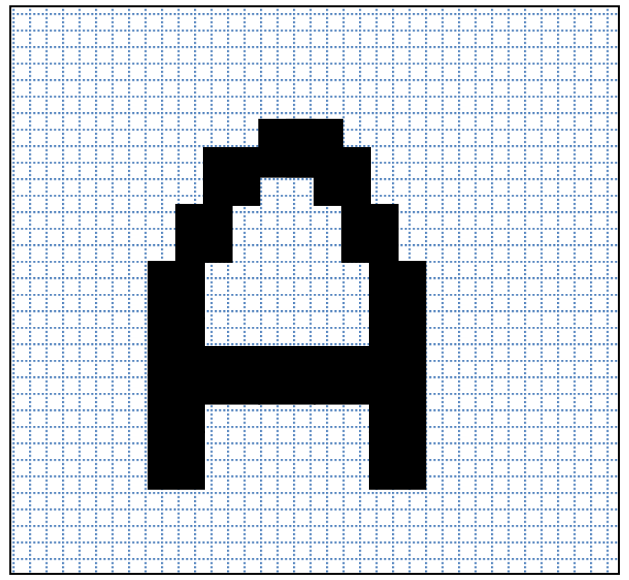
2. Classification by similarity: **pattern recognition**

Classification by closest prototype

$$\left| x - p^{T} \right| \leq \left| x - p^{A} \right|$$







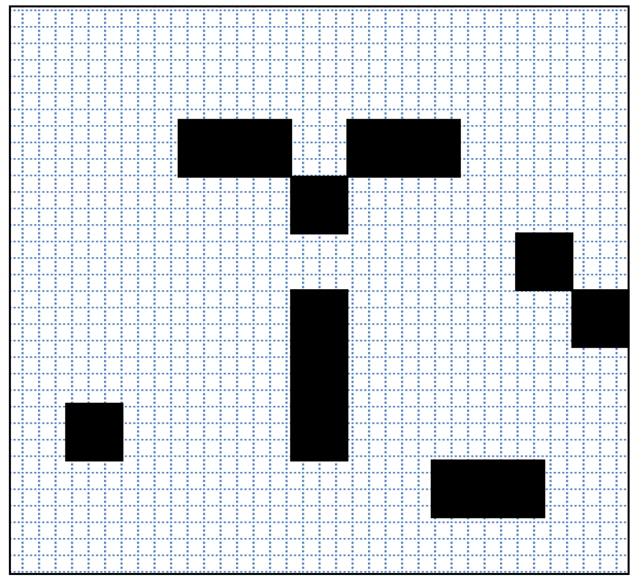
Noisy image

Prototypes

2. pattern recognition and Pattern completion

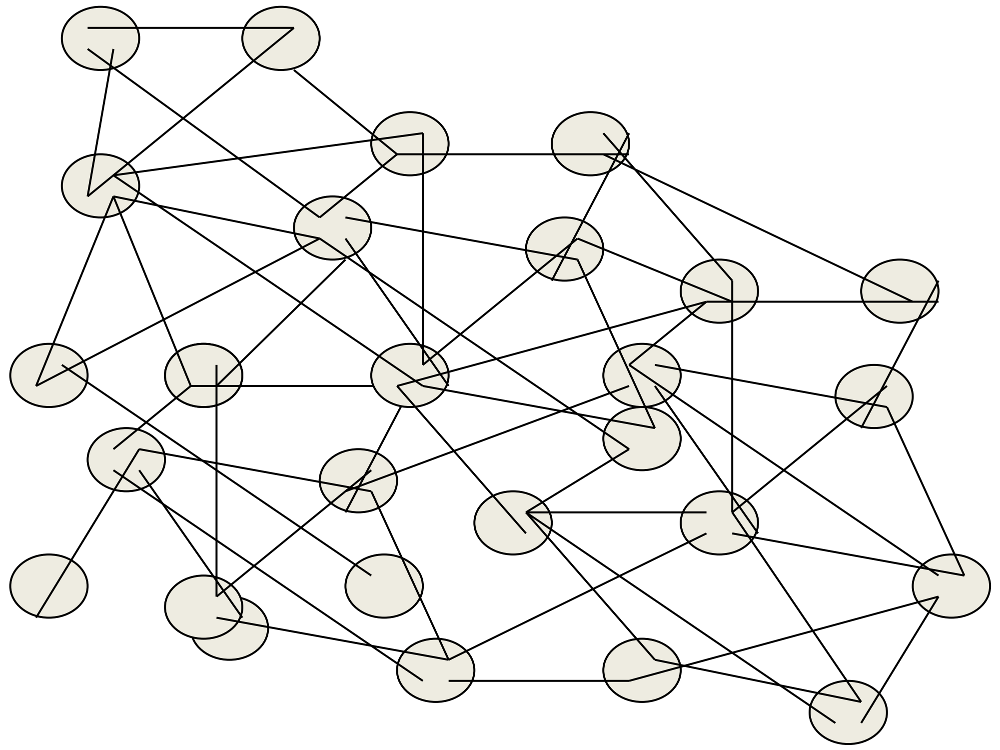
Aim: Understand Associative Memory

Noisy image

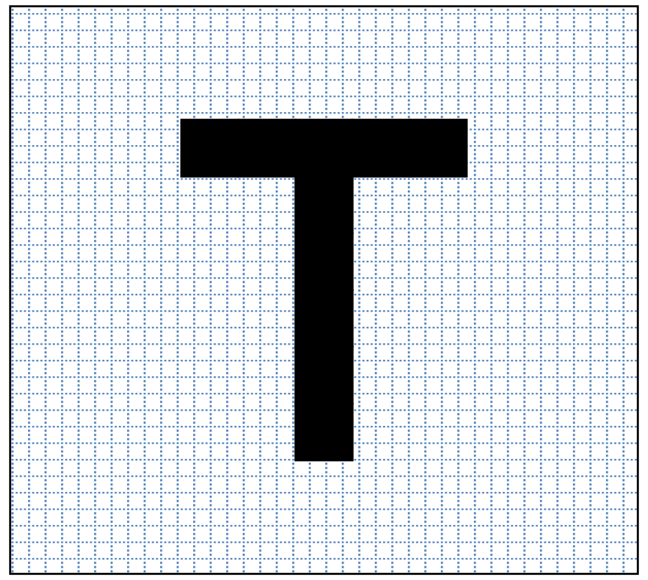


Partial word

Associative memory/ collective computation



Full image



Full word

Brain-style computation

Quiz 2: Closest prototype

Classification by closest prototype (tick one or several answers)

[ ] Needs a similarity measure

[ ] Needs a distance measure

[ ] Needs a method to find the maximum or minimum

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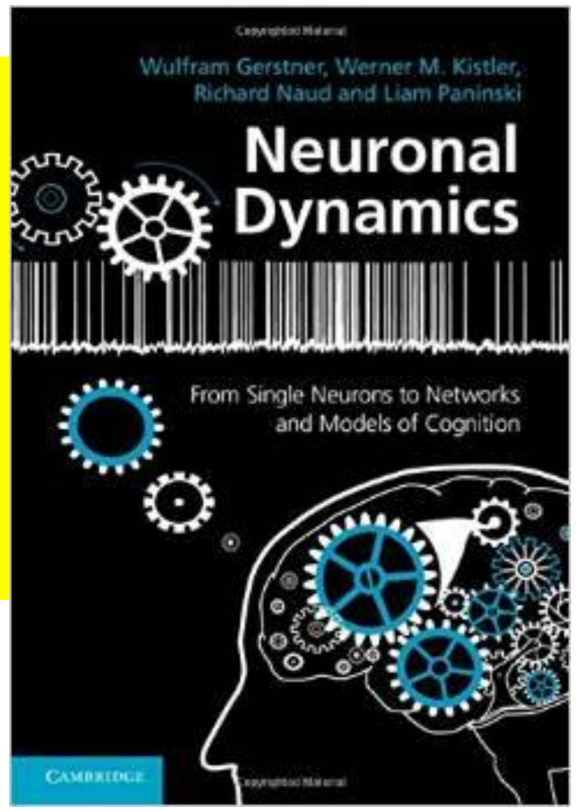

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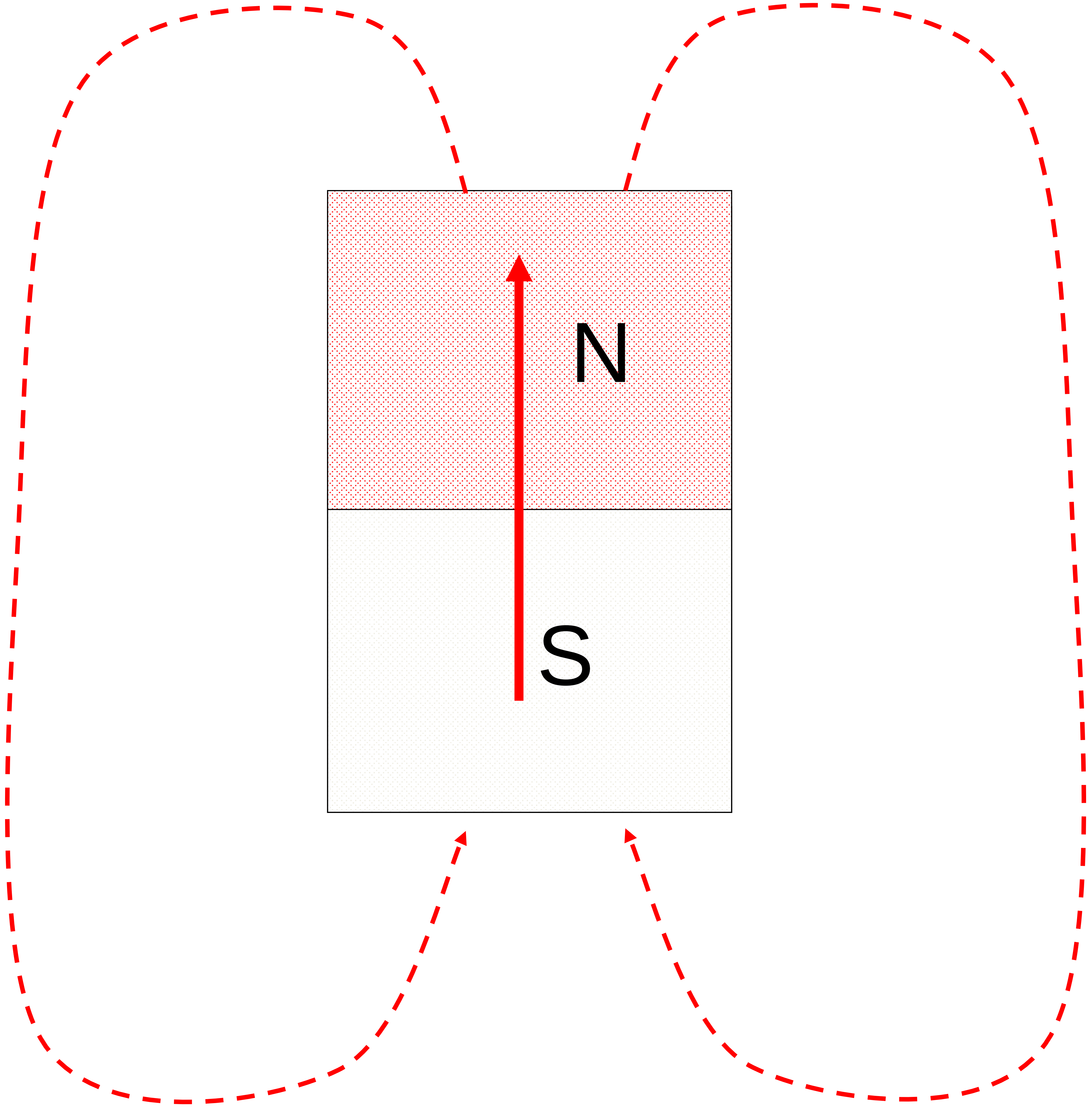
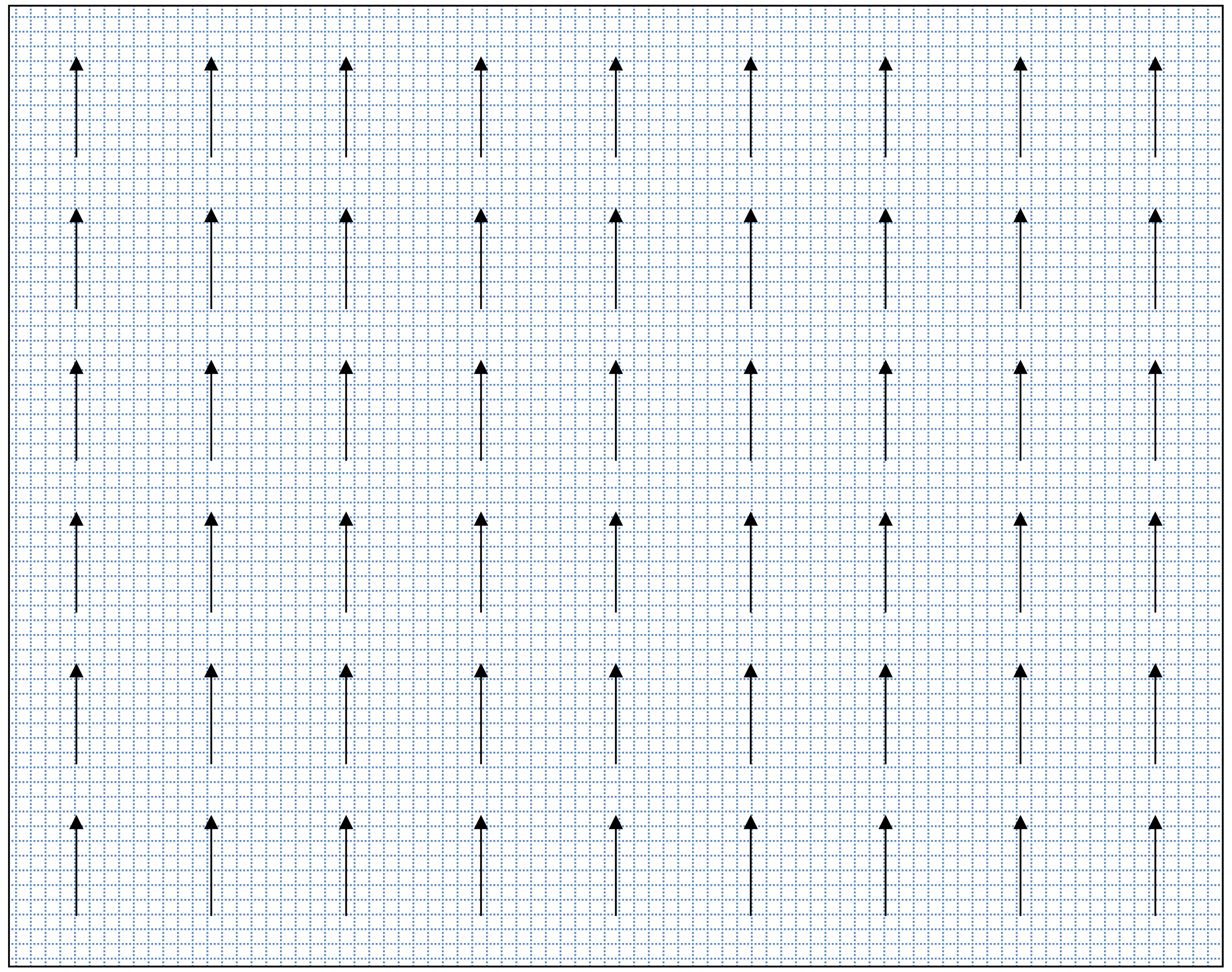
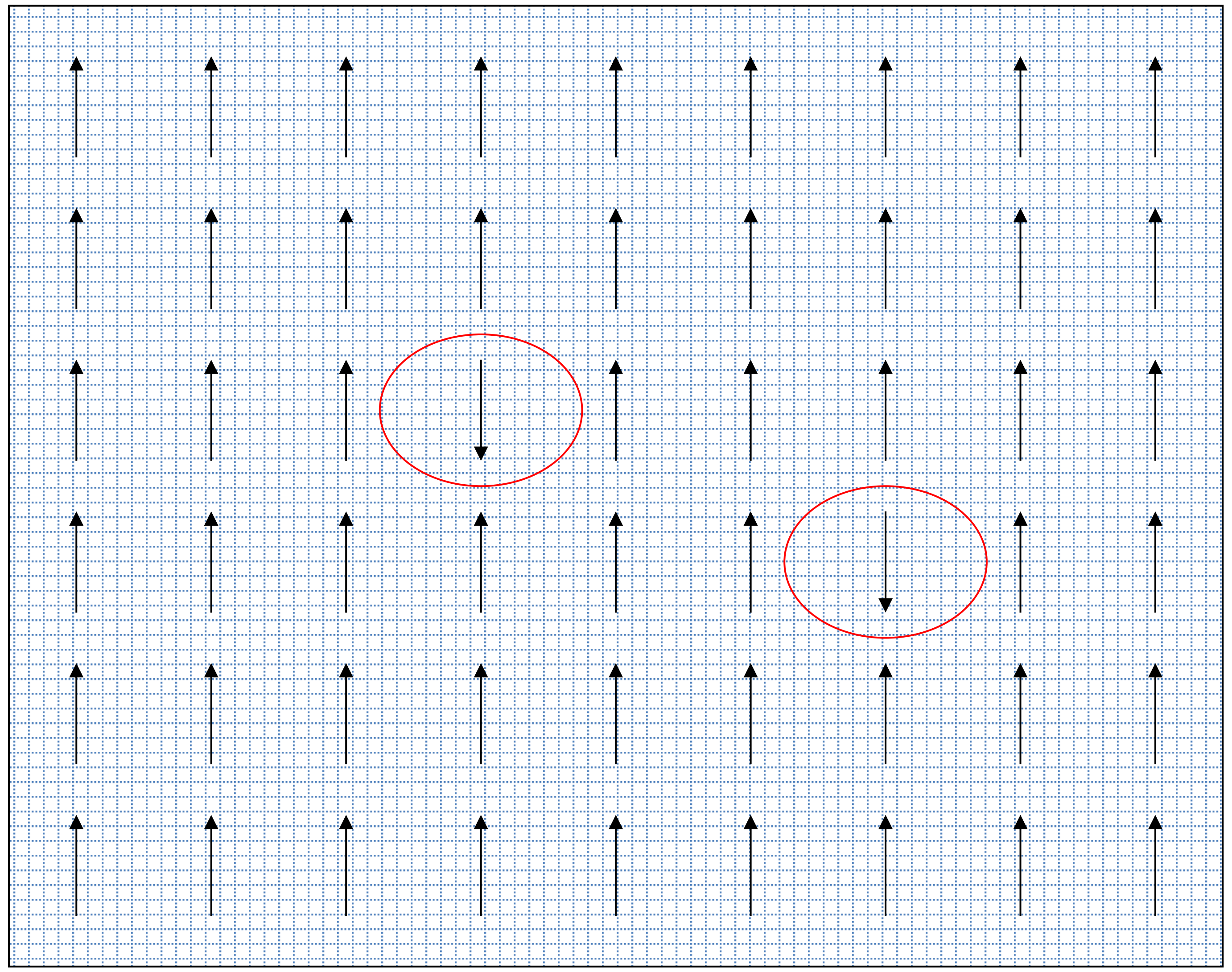
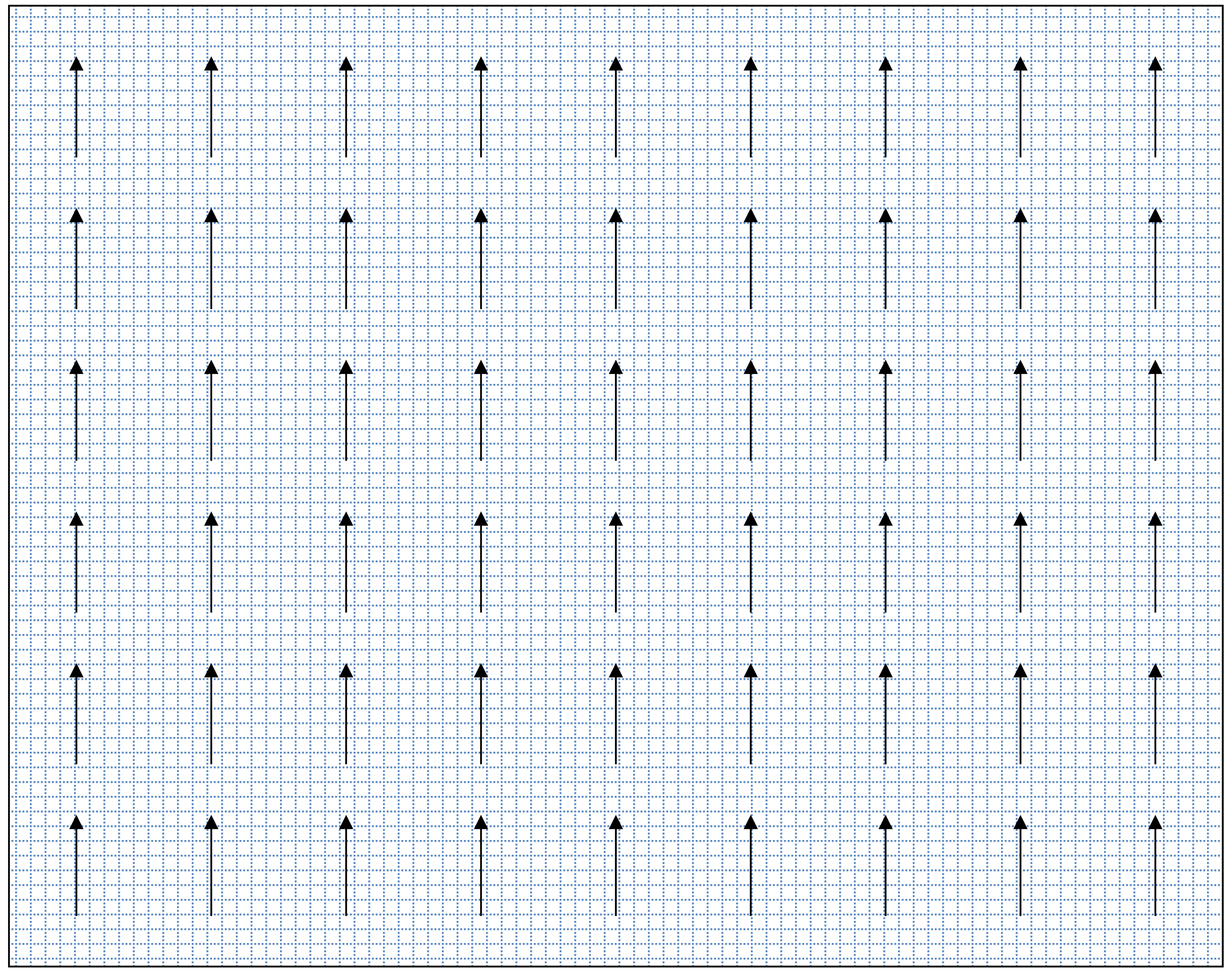
3. Detour: magnetism





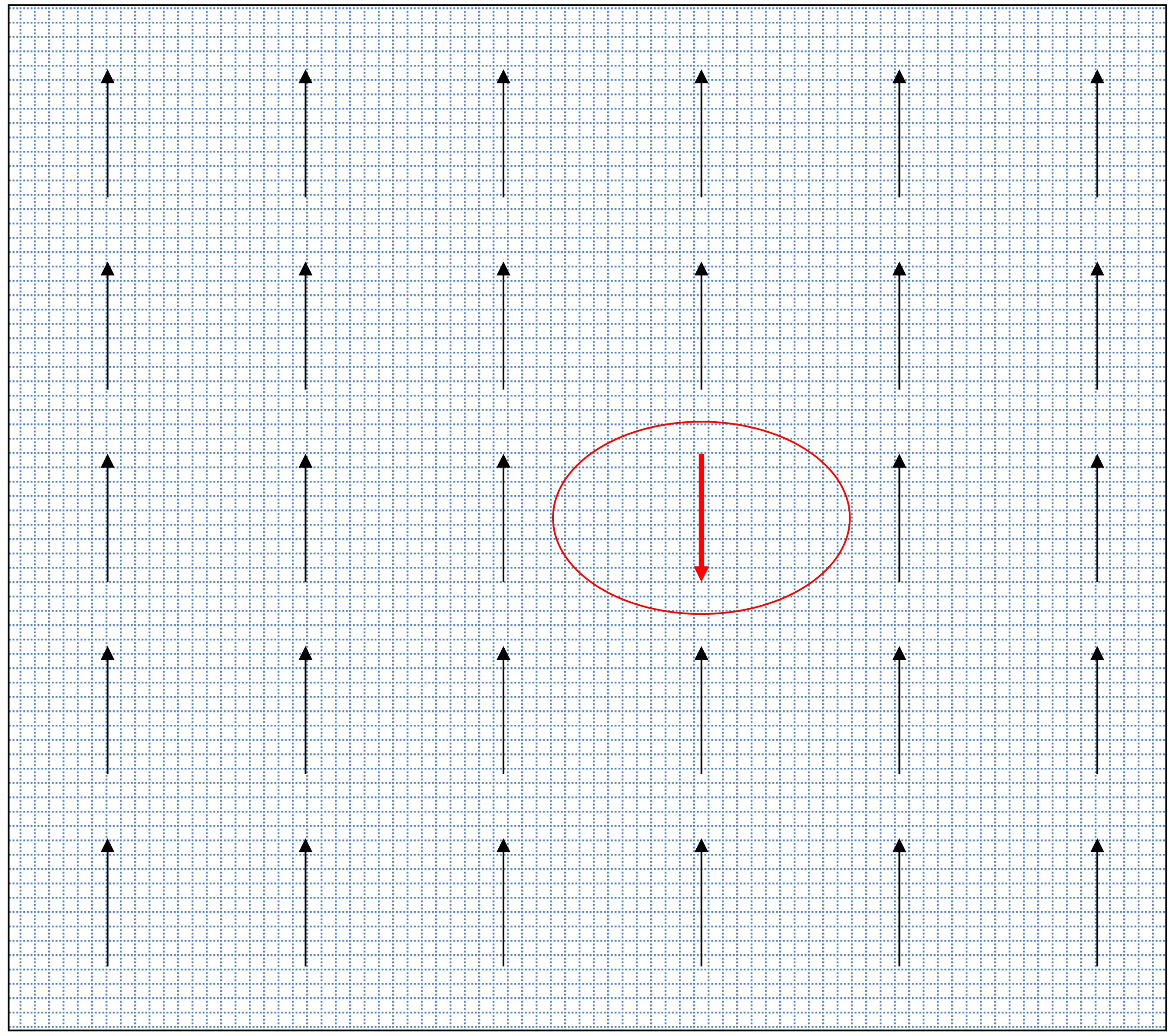
3. Detour: magnetism





Noisy magnet → pure magnet

3. Detour: magnetism



Elementary magnet

↑ $S_i = +1$

↓ $S_i = -1$

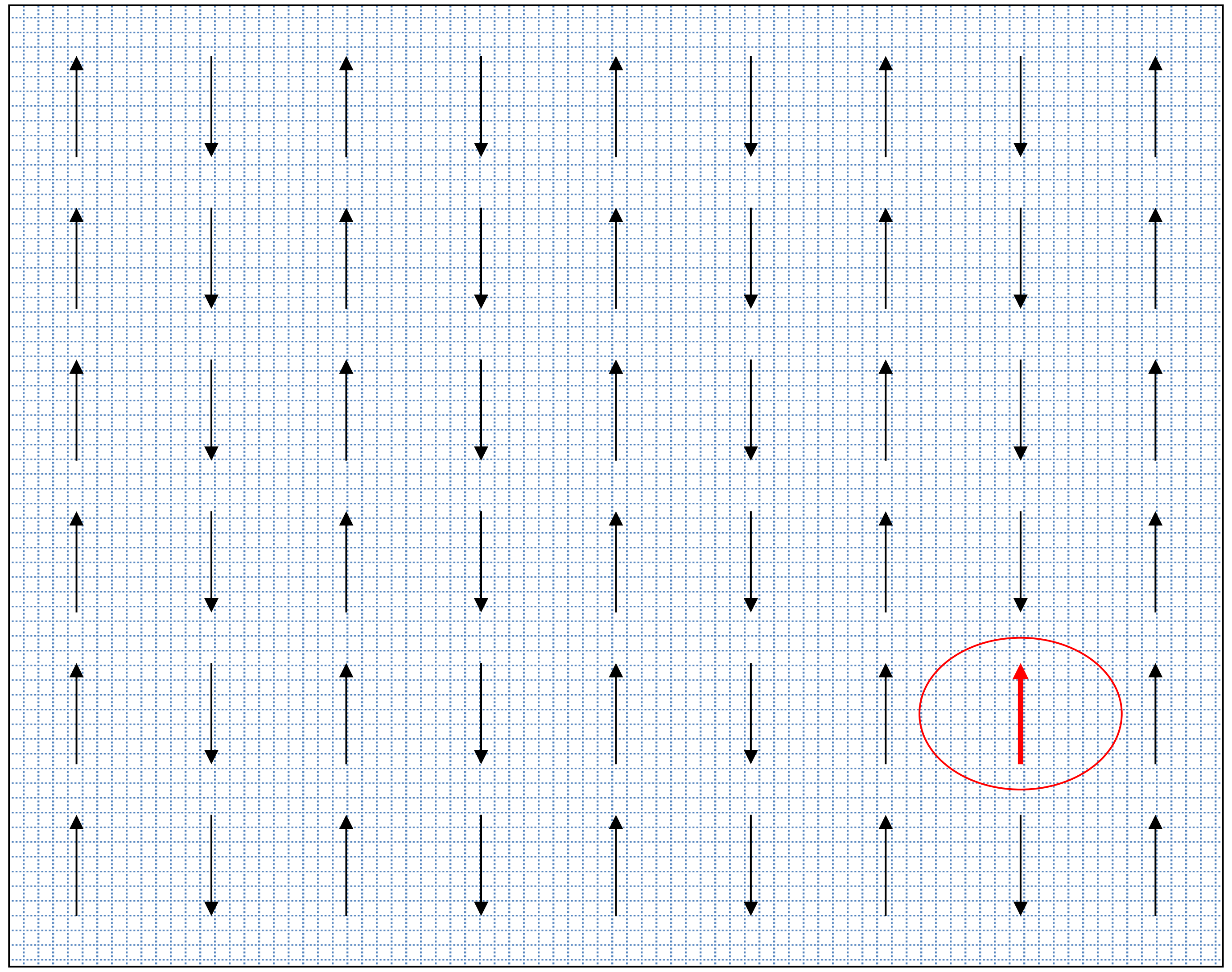
dynamics

$$S_i(t+1) = \mathrm{sgn}\left[\sum_j S_j(t)\right]$$

Sum over all
interactions with i

3. Detour: magnetism

Anti-ferromagnet



Elementary magnet

$S_i = +1$

$S_i = -1$

$w_{ij} = +1$

$w_{ij} = -1$

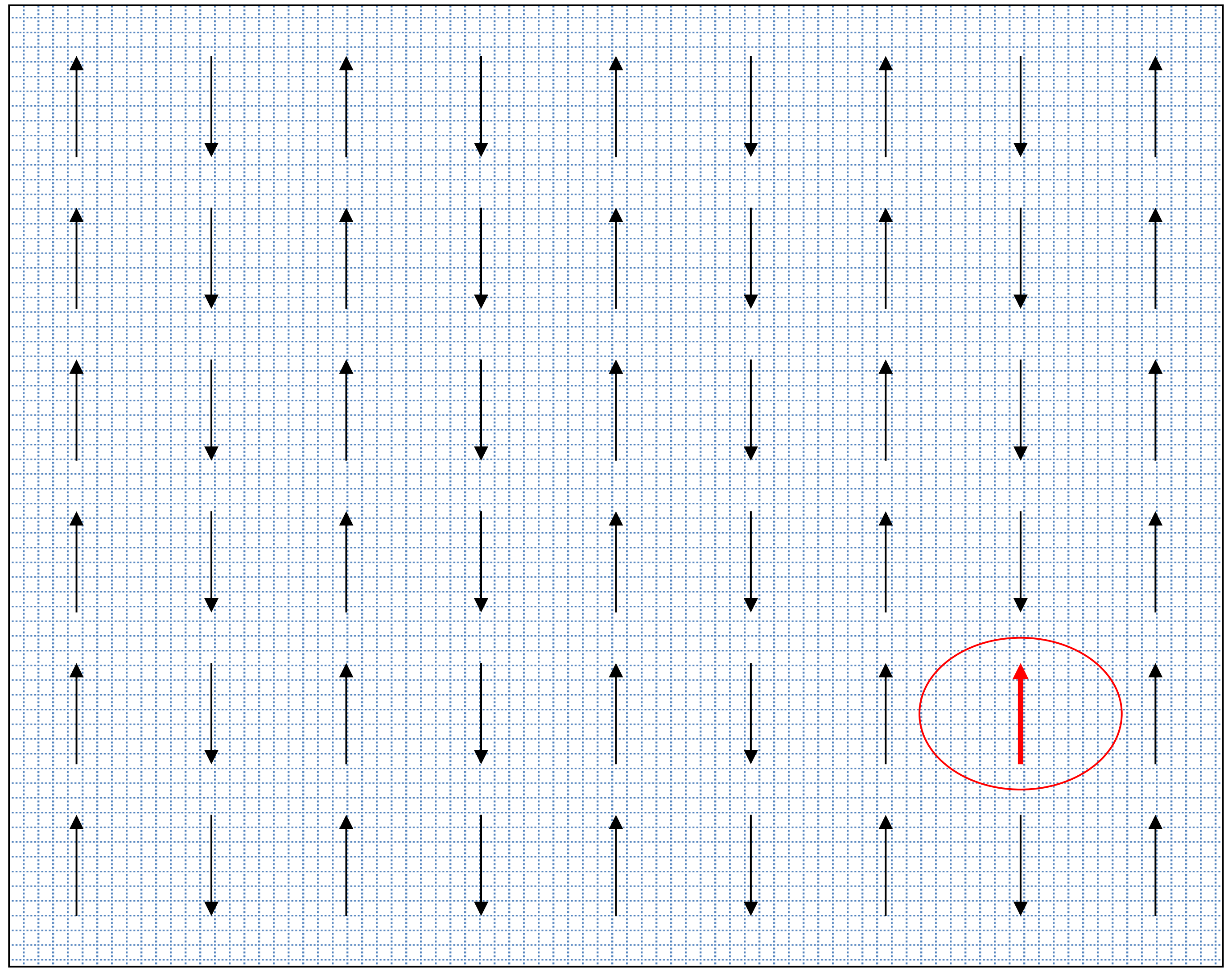
dynamics

$$S_i(t+1) = \mathrm{sgn}[\sum_j w_{ij} S_j(t)]$$

Sum over all interactions with i

3. Detour: magnetism

Anti-ferromagnet



Elementary magnet

$S_i = +1$

$S_i = -1$

$w_{ij} = +1$

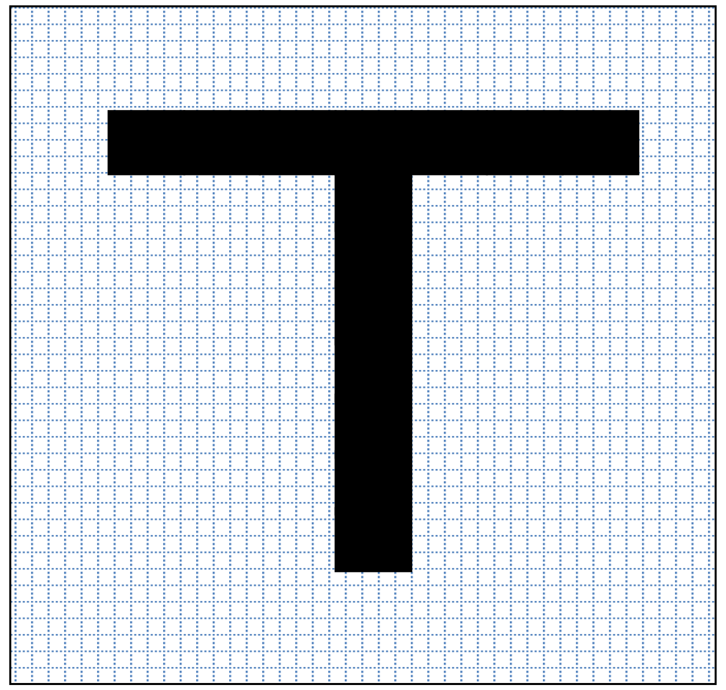
$w_{ij} = -1$

dynamics

$$S_i(t+1) = \mathrm{sgn}\Big[\sum_j w_{ij} S_j(t)\Big]$$

Sum over all
interactions with i

3. Magnetism and memory patterns



Elementary pixel

■ $S_i = +1$

□ $S_i = -1$

■↔■ $w_{ij} = +1$

□↔□ $w_{ij} = +1$

□↔■ $w_{ij} = -1$

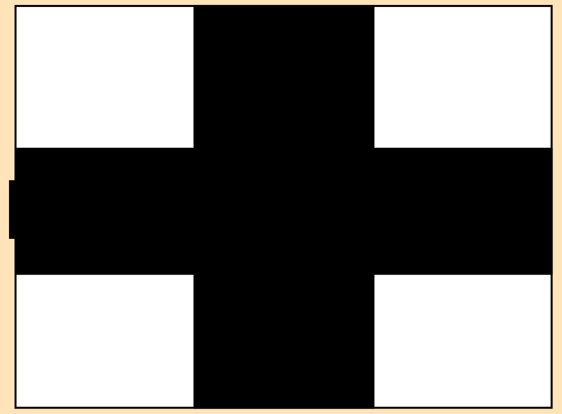
dynamics

$$S_i(t+1) = \mathrm{sgn}[\sum_j w_{ij} S_j(t)]$$

Sum over all interactions with i

Hopfield model:
Several patterns→ next section

Exercise 1: Associative memory (1 pattern)



Elementary pixel

■ $S_i = +1$

□ $S_i = -1$

■ ↔ ■ $w_{ij} = +1$

□ ↔ □ $w_{ij} = +1$

9 neurons, connected all-to-all

- define appropriate weights:
  what is the weight
  $w_{79} = ?$
- what happens if neuron 7 is +1?
- what happens if *3* neurons wrong?

dynamics

$$S_i(t+1) = \mathrm{sgn}[\sum_j w_{ij} S_j(t)]$$

Sum over all interactions with i

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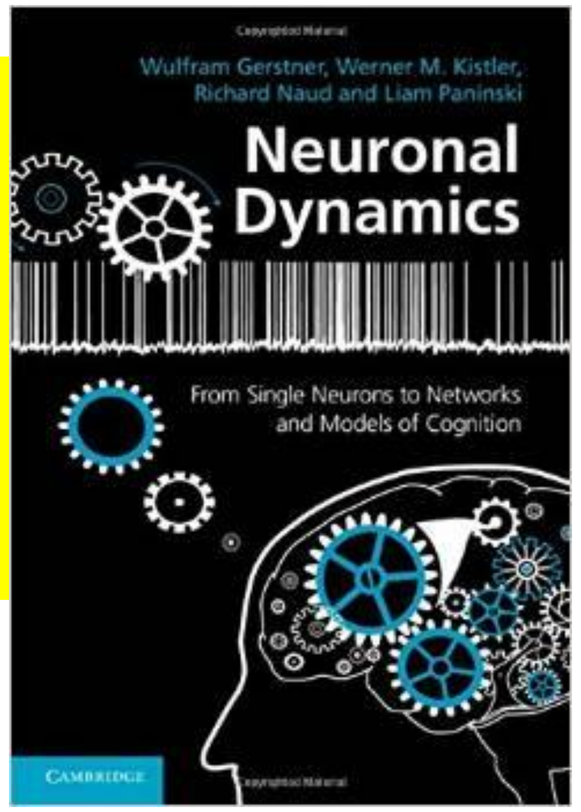

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$\boxed{\text{4 Hopfield Model}}$

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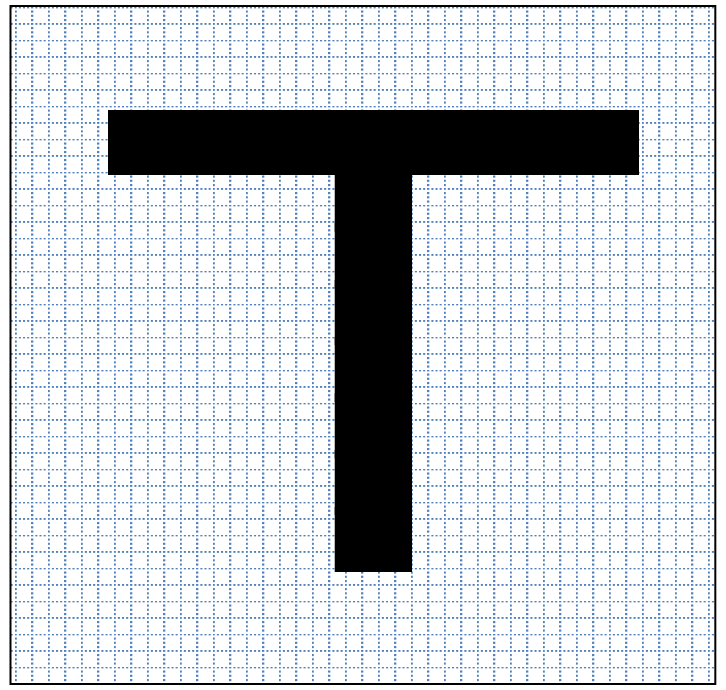
4. Single pattern



Elementary pixel
(target pattern)

■ $p_i = +1$

□ $p_i = -1$

■ ↔ ■ $w_{ij} = +1$

□ ↔ □ $w_{ij} = +1$

□ ↔ ■ $w_{ij} = -1$

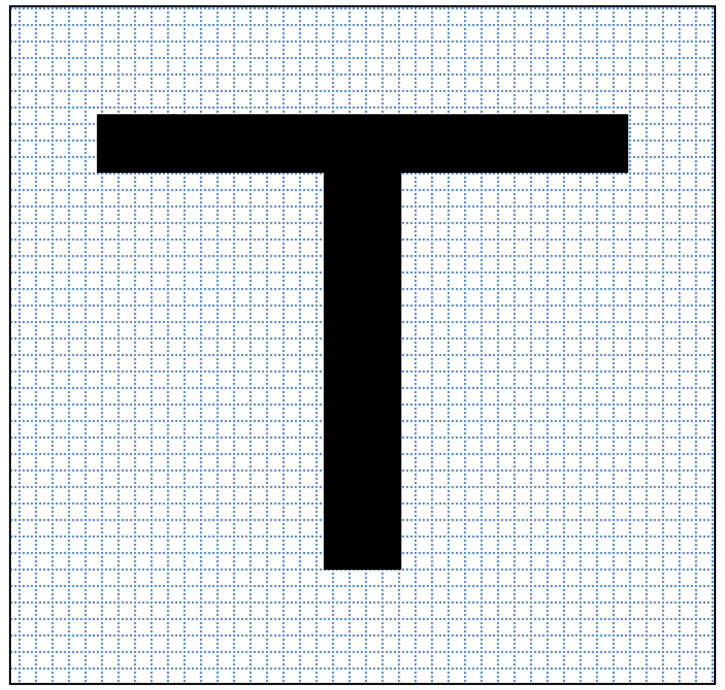
$$w_{ij} =$$

dynamics

$$S_i(t+1) = \mathrm{sgn}[\sum_j w_{ij} S_j(t)]$$

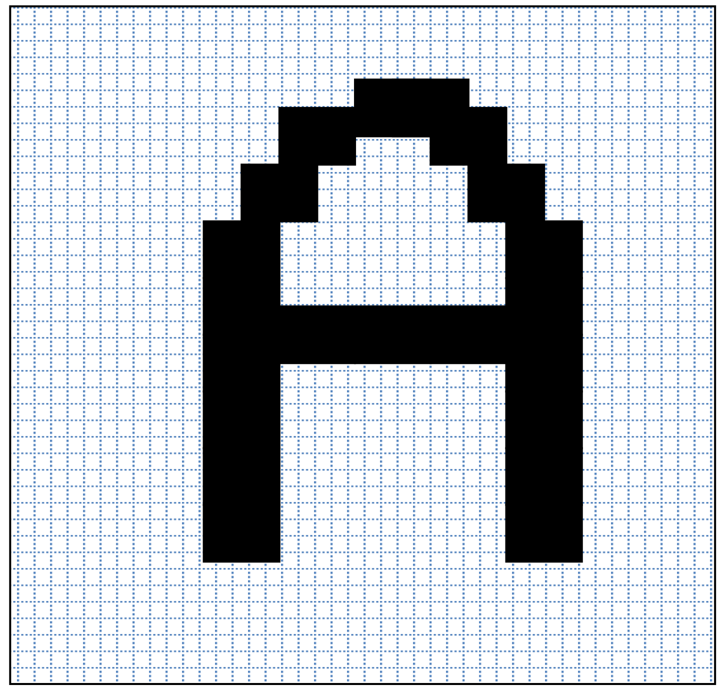
Sum over all
interactions with i

4. Hopfield Model of Associative Memory



Prototype

$\vec{p}^1$



Prototype

$\vec{p}^2$

several patterns

interactions

$$w_{ij} = \sum_{\mu} p_i^{\mu} p_j^{\mu}$$

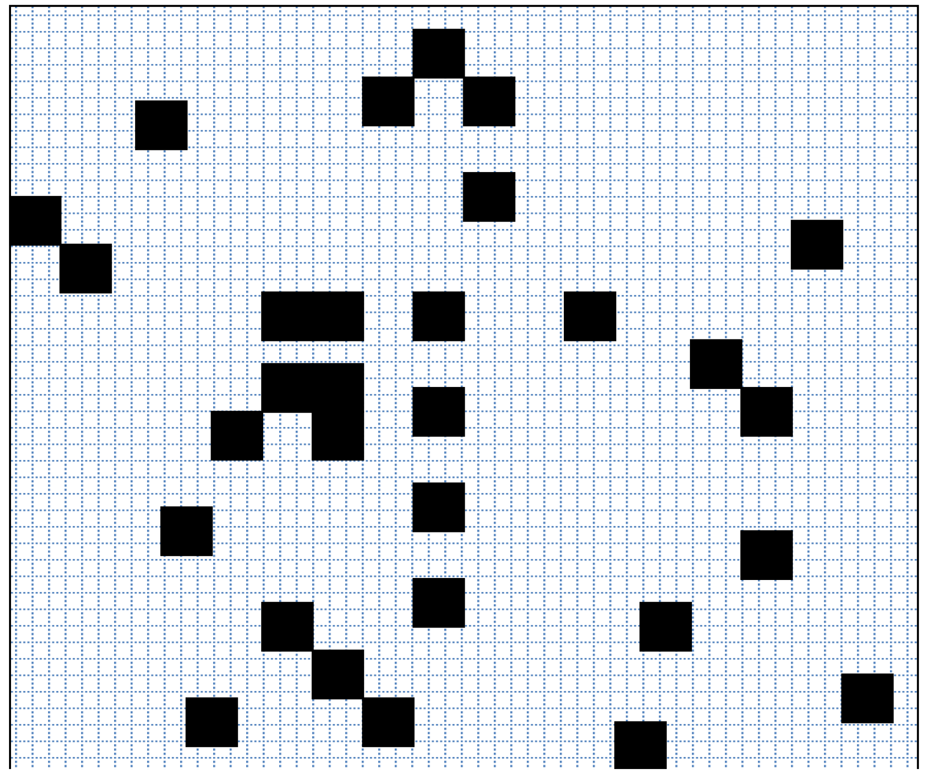
Sum over all prototypes

dynamics

$$S_i(t+1) = \mathrm{sgn}[\sum_j w_{ij} S_j(t)]$$

Sum over all interactions with i

4. Hopfield Model of Associative Memory



Pattern

$\vec{p}^{1}$

interactions

$$w_{ij} = \sum_{\mu} p_i^{\mu} p_j^{\mu} \quad \textbf{(1)}$$

Sum over all prototypes

This rule is very good for **random** patterns

It does not work well for correlated patters

Hopfield model (1982)
- **several random patterns**
- **fully connected network**
- **binary neurons**
- **weights (1); dynamics (2)**

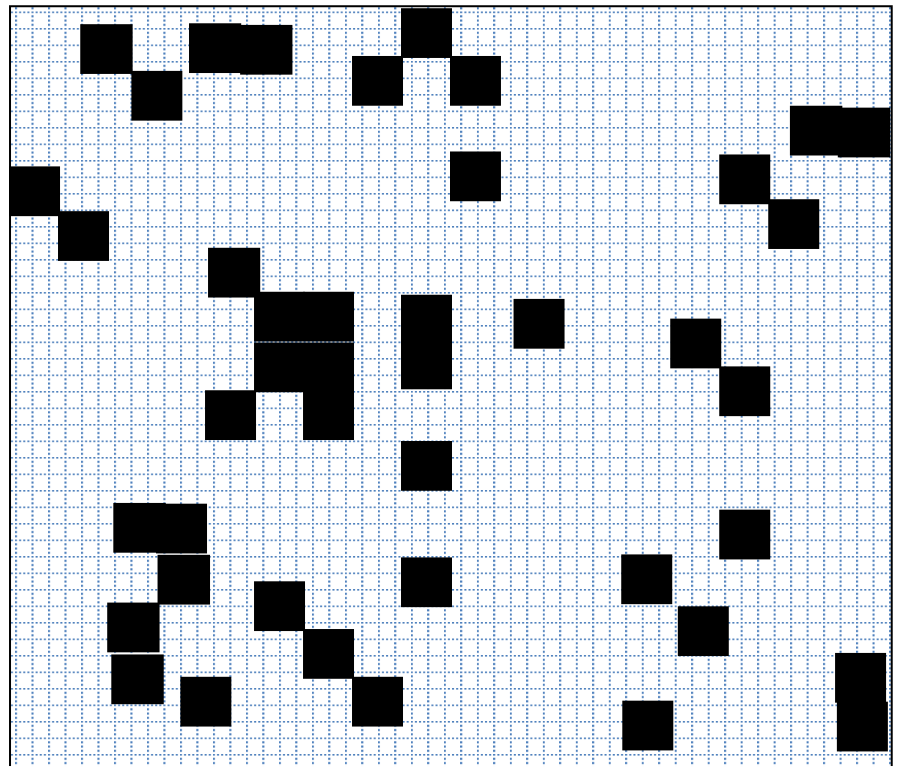
dynamics

$$S_i(t+1) = \mathrm{sgn}[\sum_j w_{ij} S_j(t)] \quad \textbf{(2)}$$

all interactions with i

J. Hopfield, 1982

4. Overlap: a measure of similarity



current state: (+1,-1,-1,+1,-1,+1,+1,-1)

target pattern, (+1,+1,-1,+1,-1,-1,-1,-1)
prototype

overlap $$m^{\mu}(t) = \frac{1}{N} \sum_j p_j^{\mu} S_j(t)$$

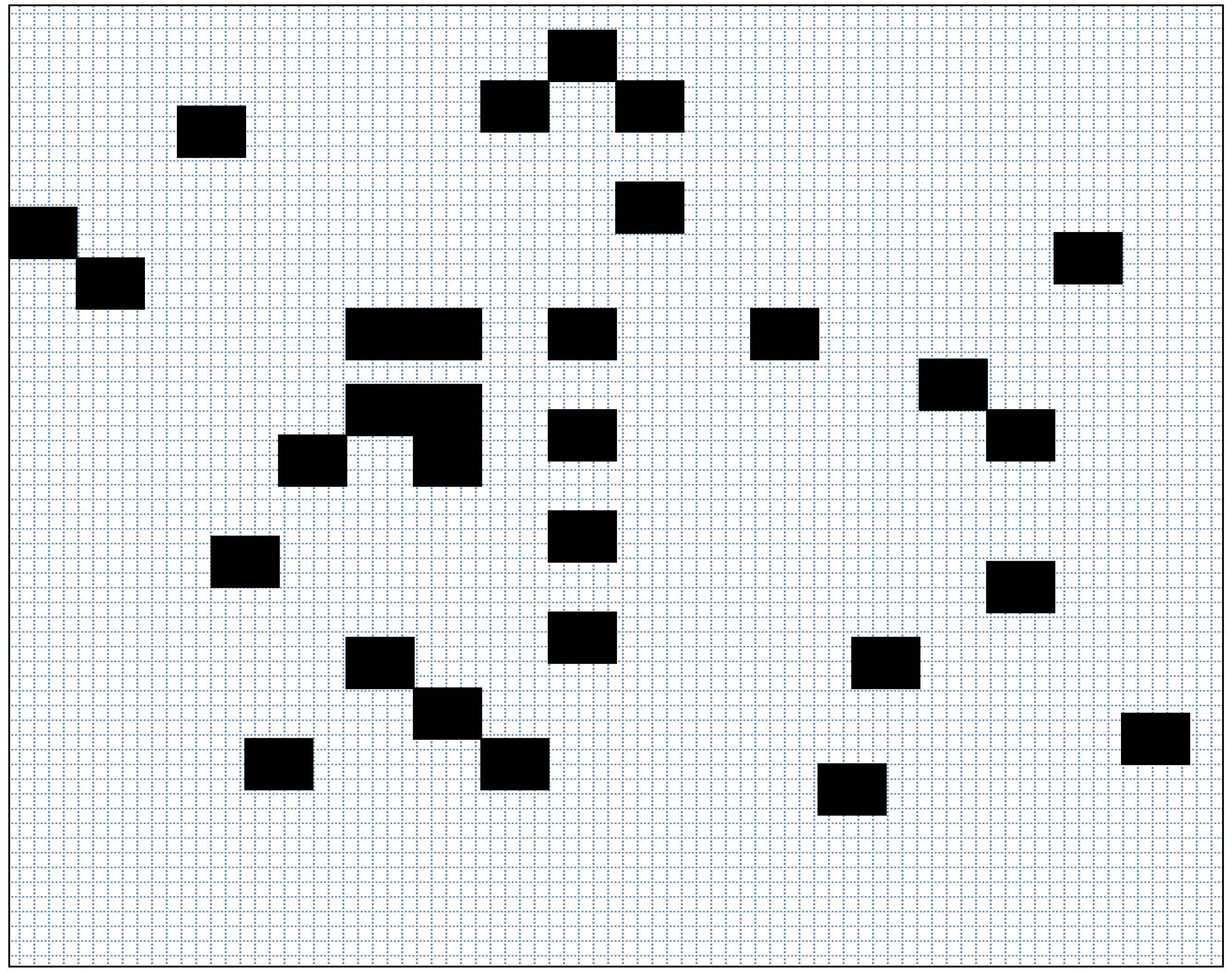
4. Hopfield Model of Associative Memory

$$S_i(t+1) = \mathrm{sgn}[\sum_j w_{ij} S_j(t)]$$

$$w_{ij} = \sum_\mu p_i^\mu p_j^\mu$$

$$m^\mu(t+1) = \frac{1}{N} \sum_j p_j^\mu S_j(t+1)$$

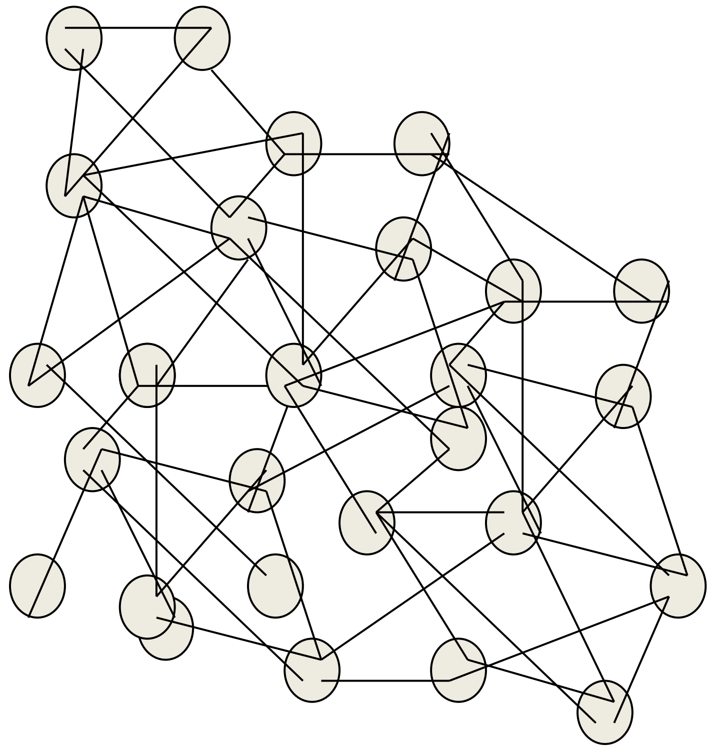
4. Hopfield Model of Associative Memory



Prototype

$\vec{p}^1$

Interacting neurons



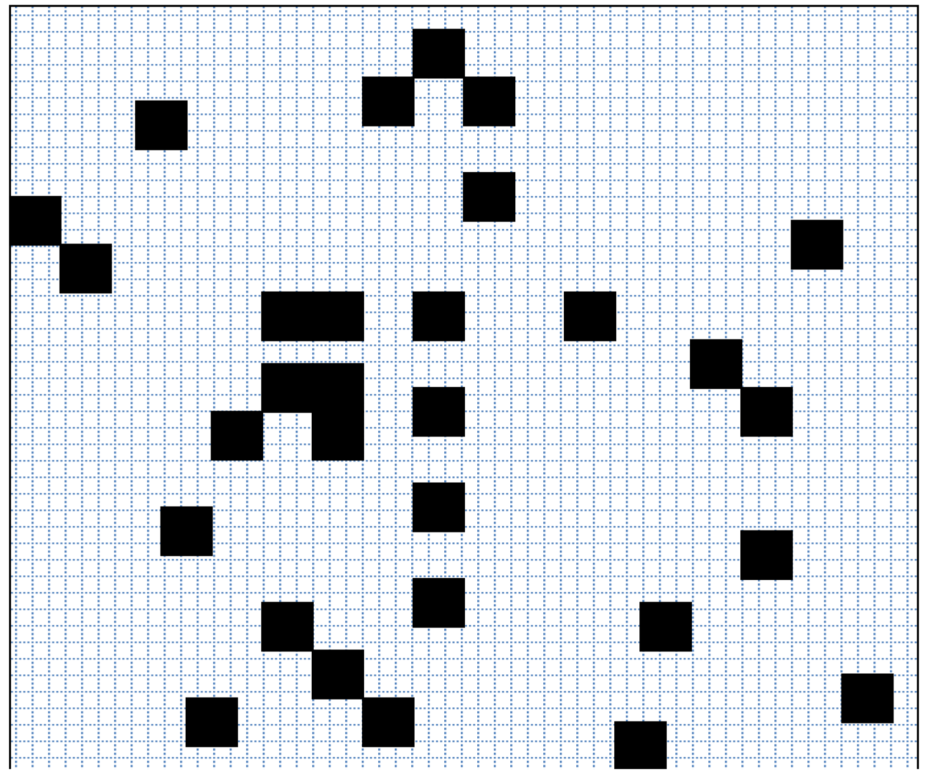
Finds the closest prototype
i.e. maximal overlap
(similarity) m^{μ}

Hopfield model

Computation

- without CPU,
- without explicit memory unit

4. Correlated patterns, orthogonal patterns



target pattern, prototype 3 (+1,-1,+1,+1,-1,+1,+1,-1)

target pattern, prototype 7 (+1,+1,-1,+1,-1,-1,-1,-1)

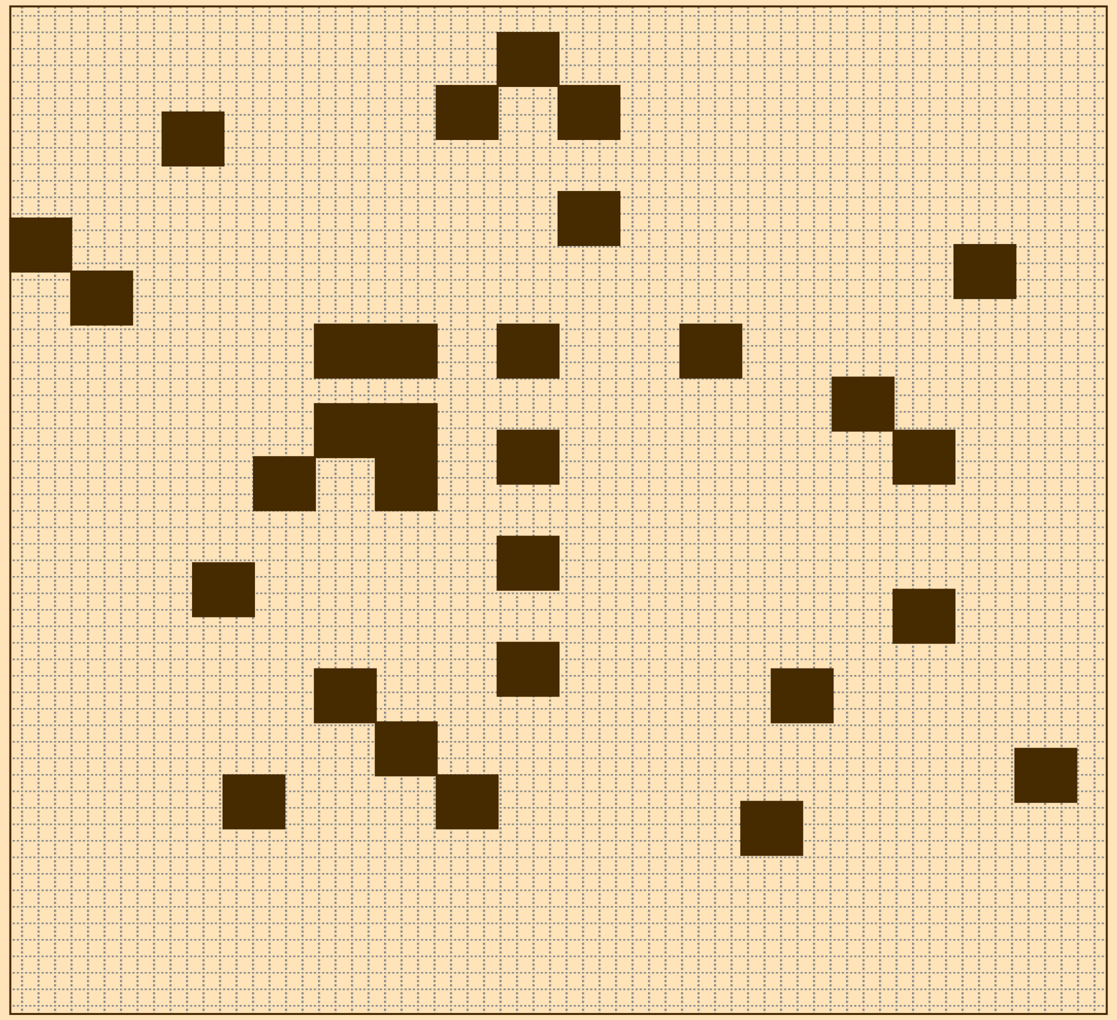
Similarity of two patterns:

Orthogonal patterns:

Random patterns

overlap $m^{\mu}(t) = \frac{1}{N} \sum_j p_j^{\mu} S_j(t)$

Exercise 2 (now)



$$w_{ij} = \tfrac{1}{N} \sum_{\mu} p_i^{\mu} p_j^{\mu}$$

$$S_i(t+1) = \mathrm{sgn}[\sum_j w_{ij} S_j(t)]$$

Sum over all interactions with i

Assume 4 orthogonal patterns.
At time t=0, overlap with pattern 3, no overlap with other patterns.

Calculate the overlap at t=1!

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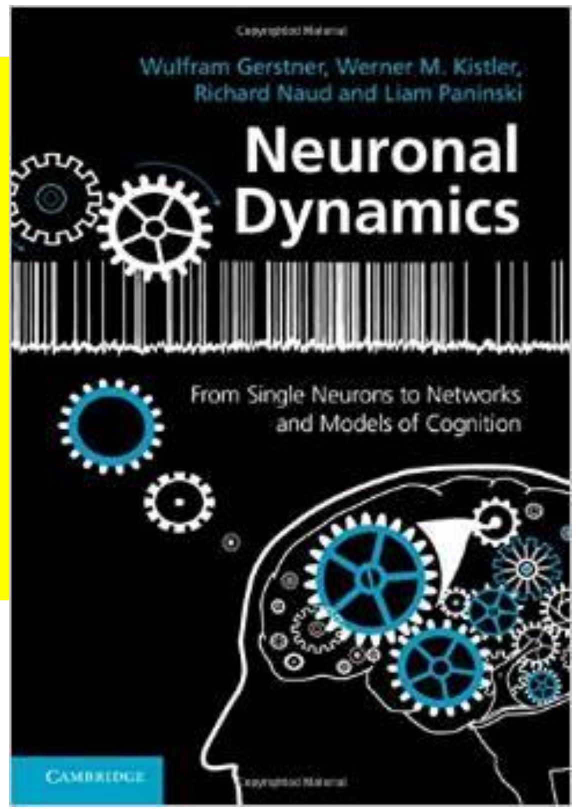

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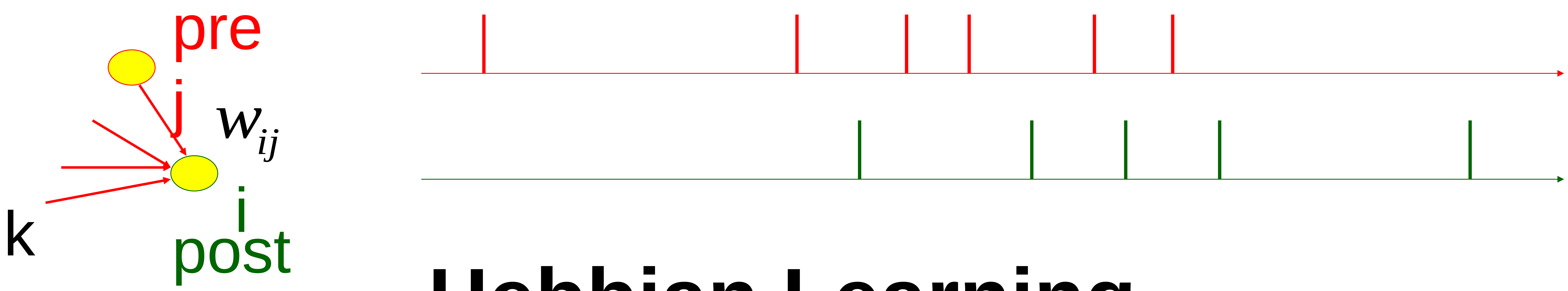
5. Learning of Associations

Where do the connections come from?



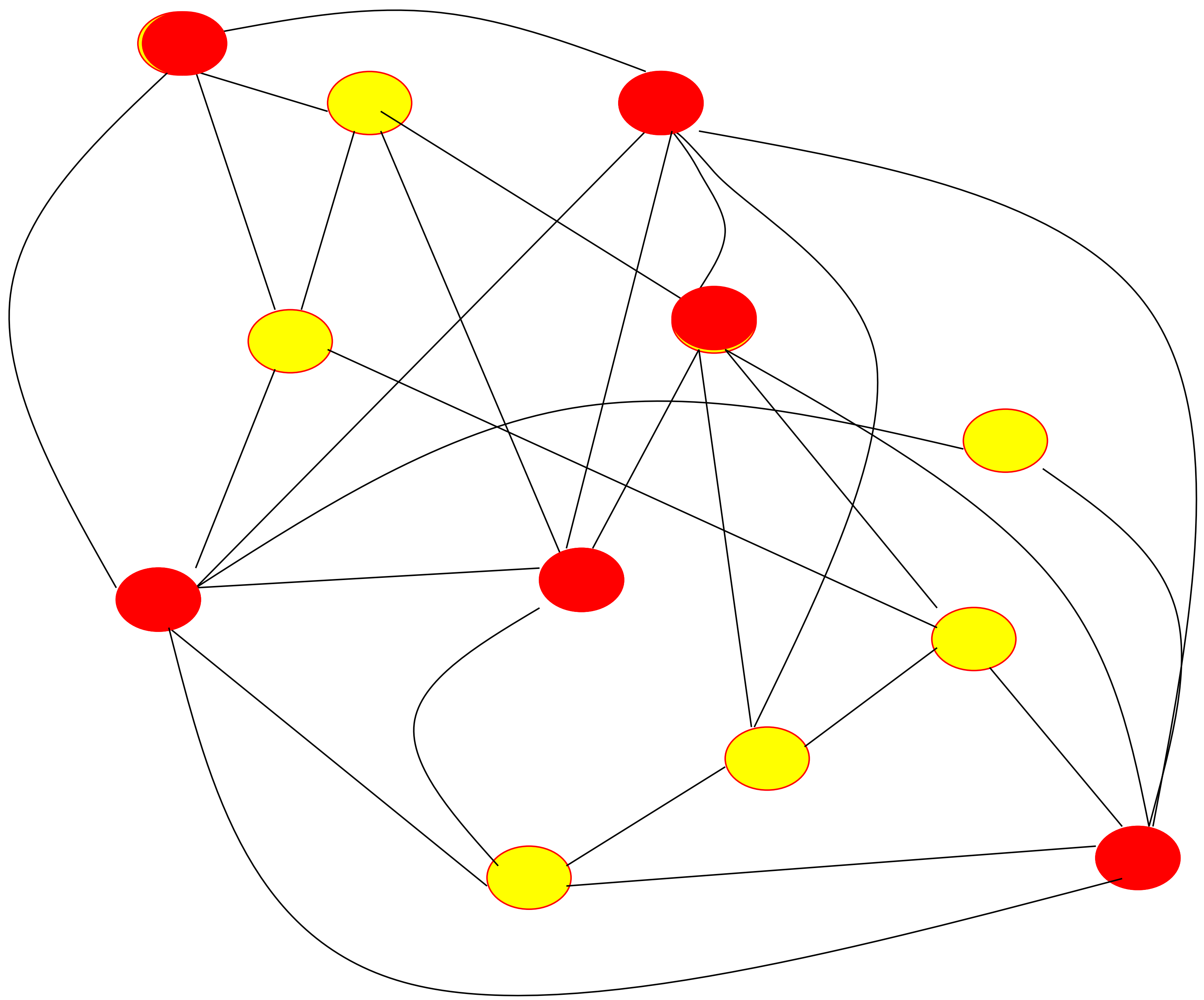
Hebbian Learning

When an axon of cell **j** repeatedly or persistently takes part in firing cell **i**, then j's efficiency as one of the cells firing i is increased

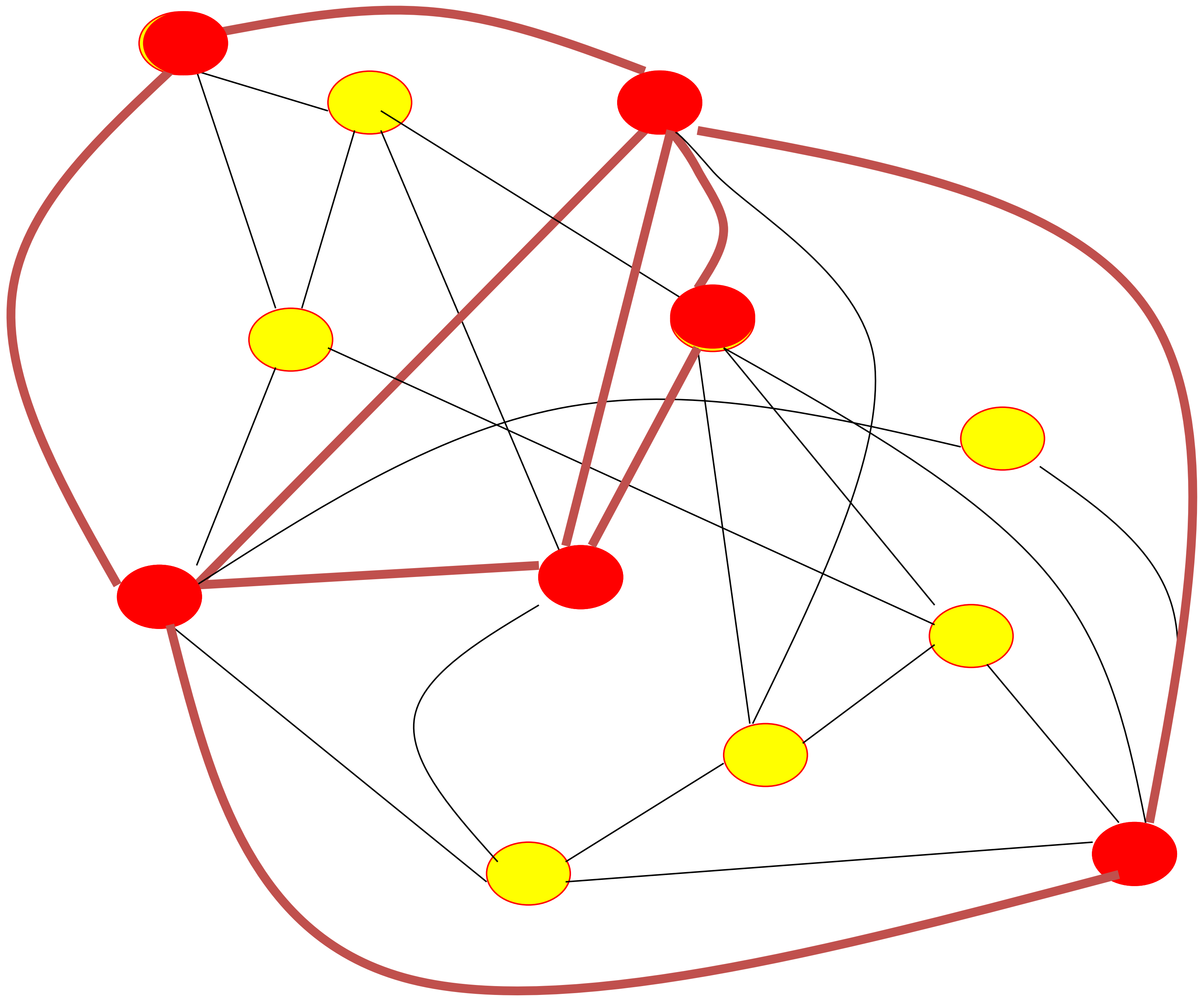
Hebb, 1949

- local rule
- simultaneously active (correlations)

5. Hebbian Learning of Associations



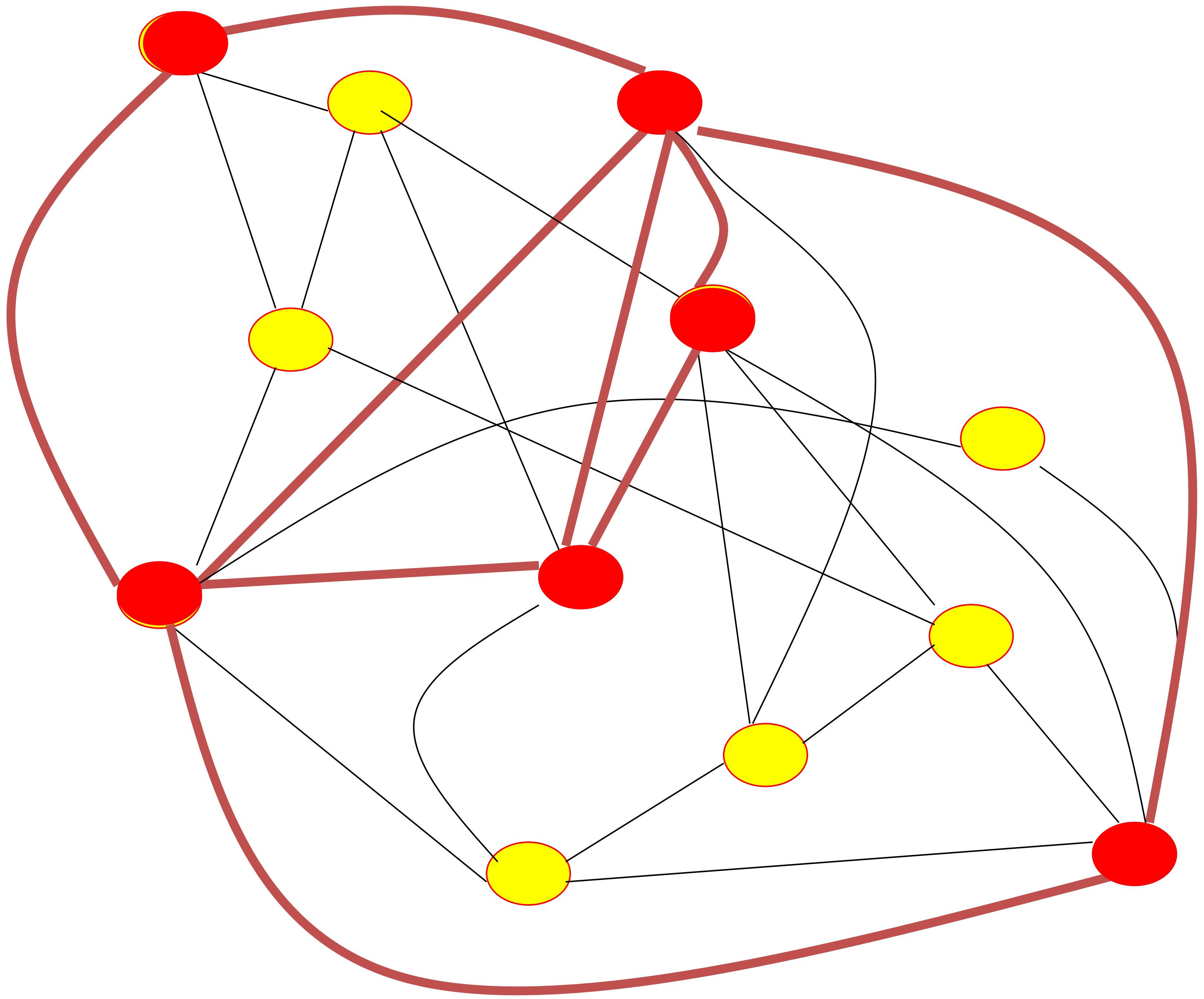
5. Hebbian Learning of Associations

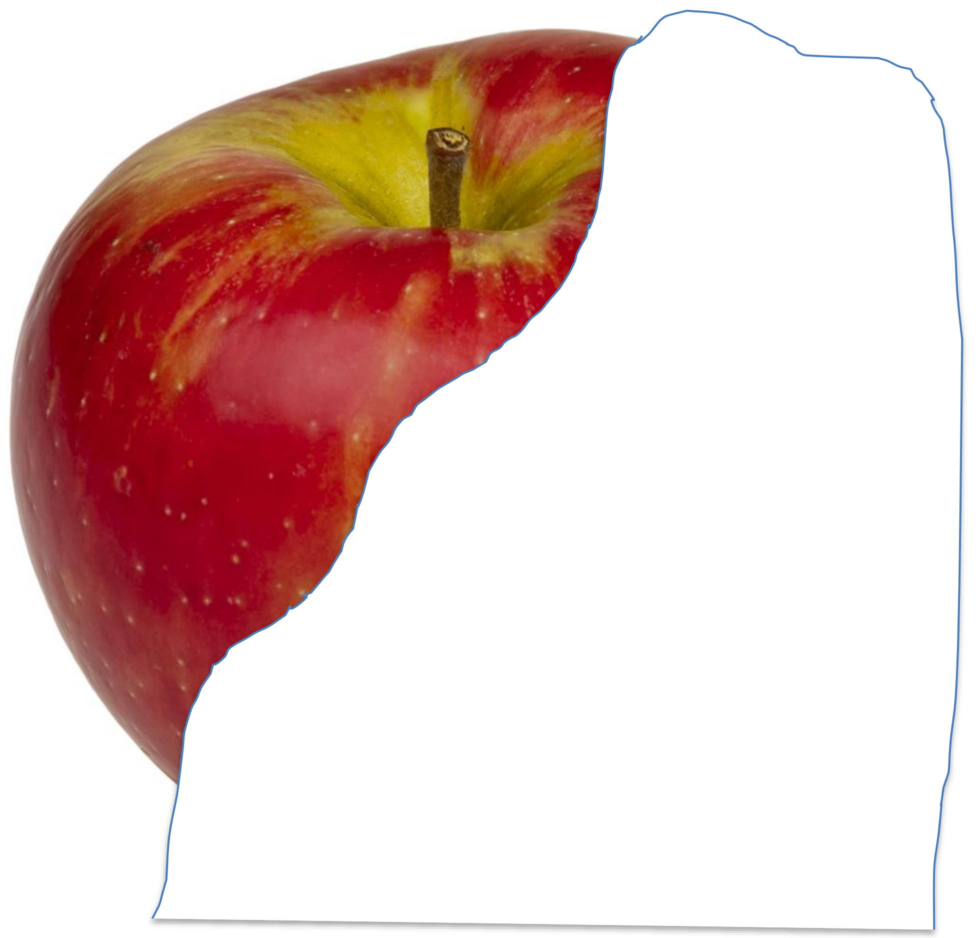


item memorized

5. Hebbian Learning: Associative Recall

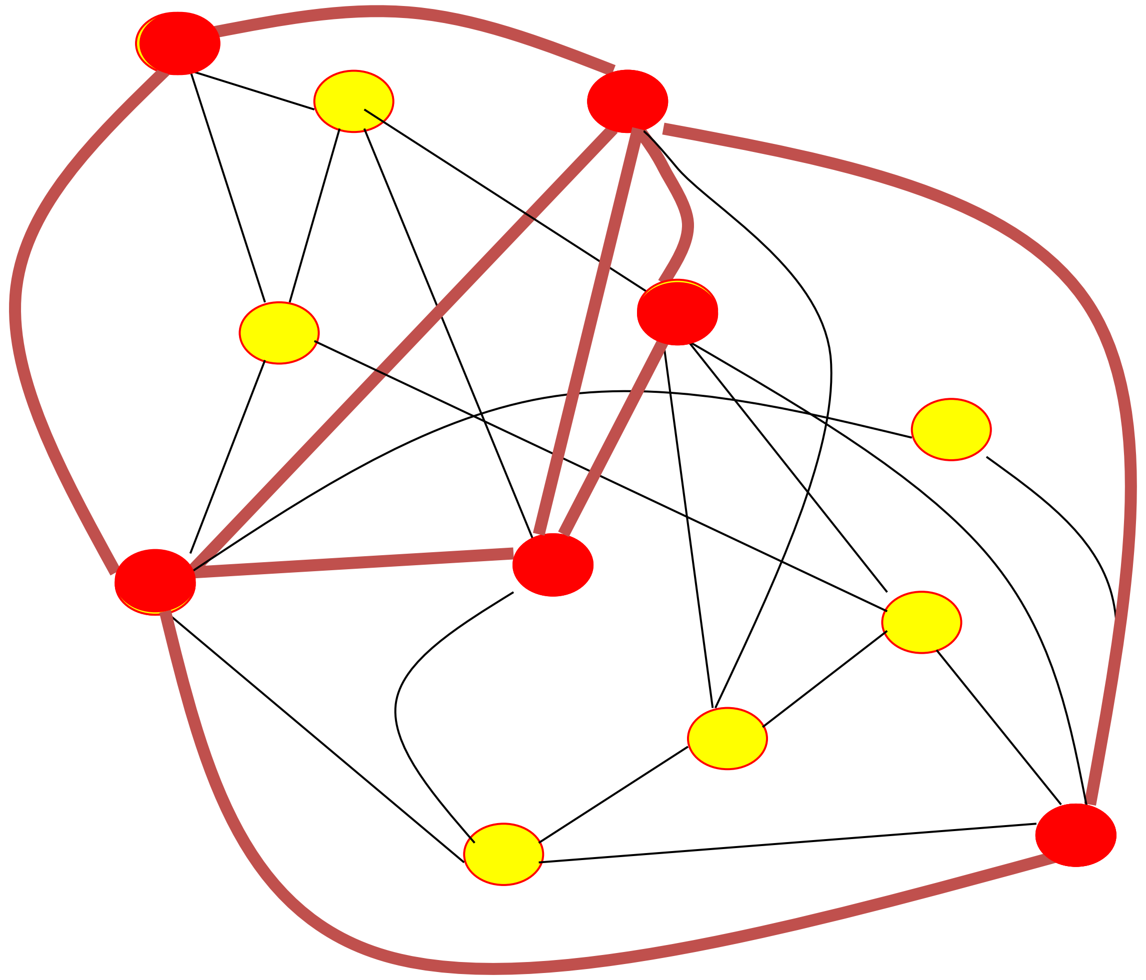
Recall:
Partial info



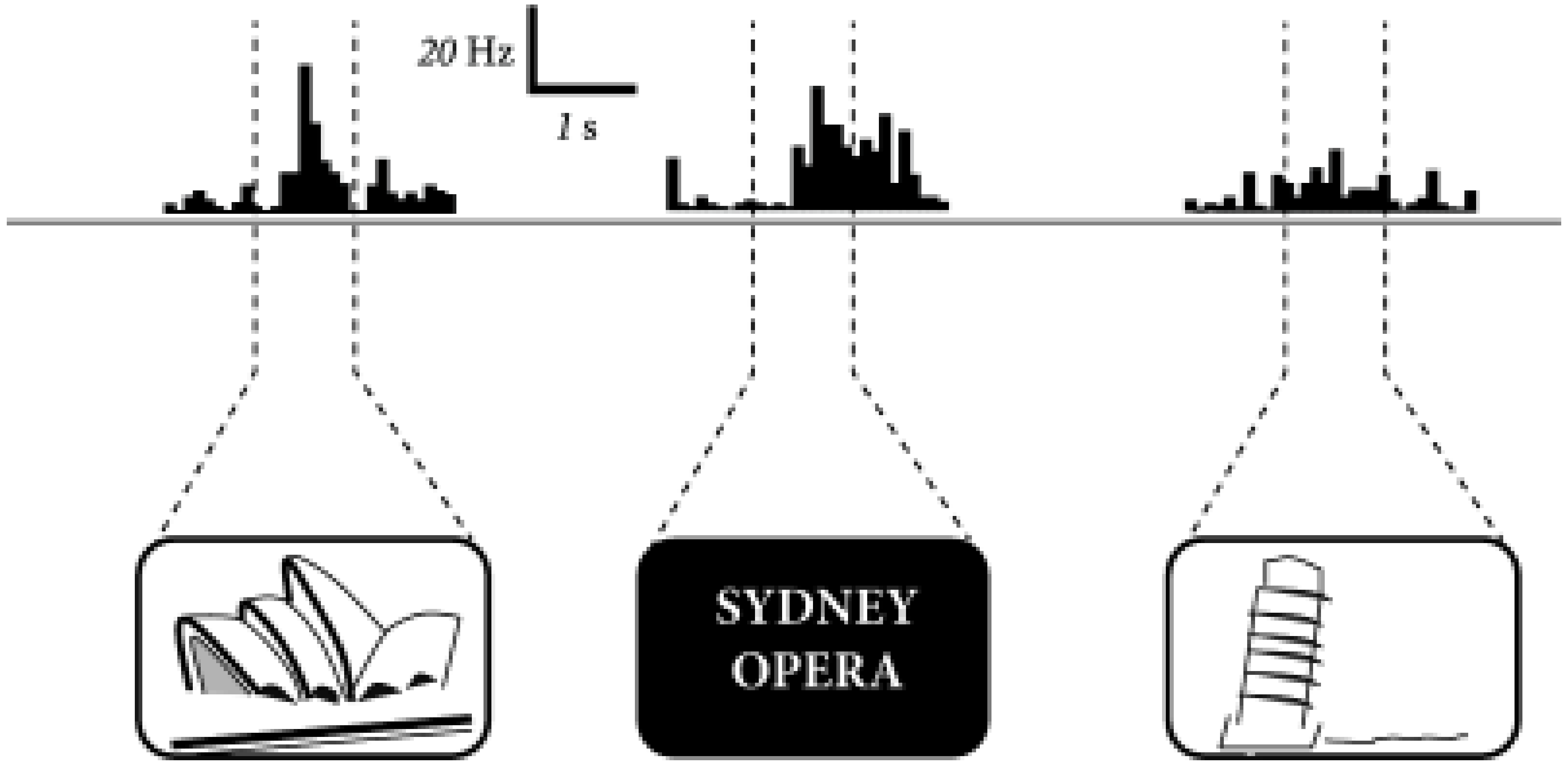


item recalled

5. Learned concepts



Activity of neurons in human brain



assembly of neurons

*Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014), Adapted from **Quiroga et al. (2005),** Nature 435:1102-1107*

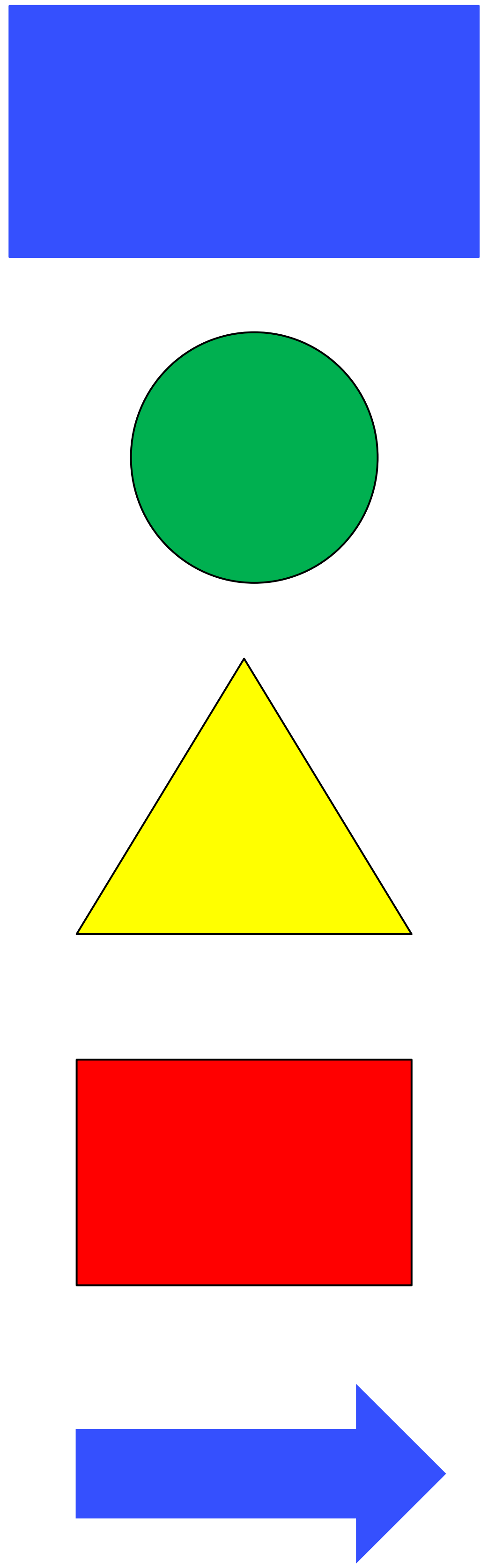
5. Associative Recall

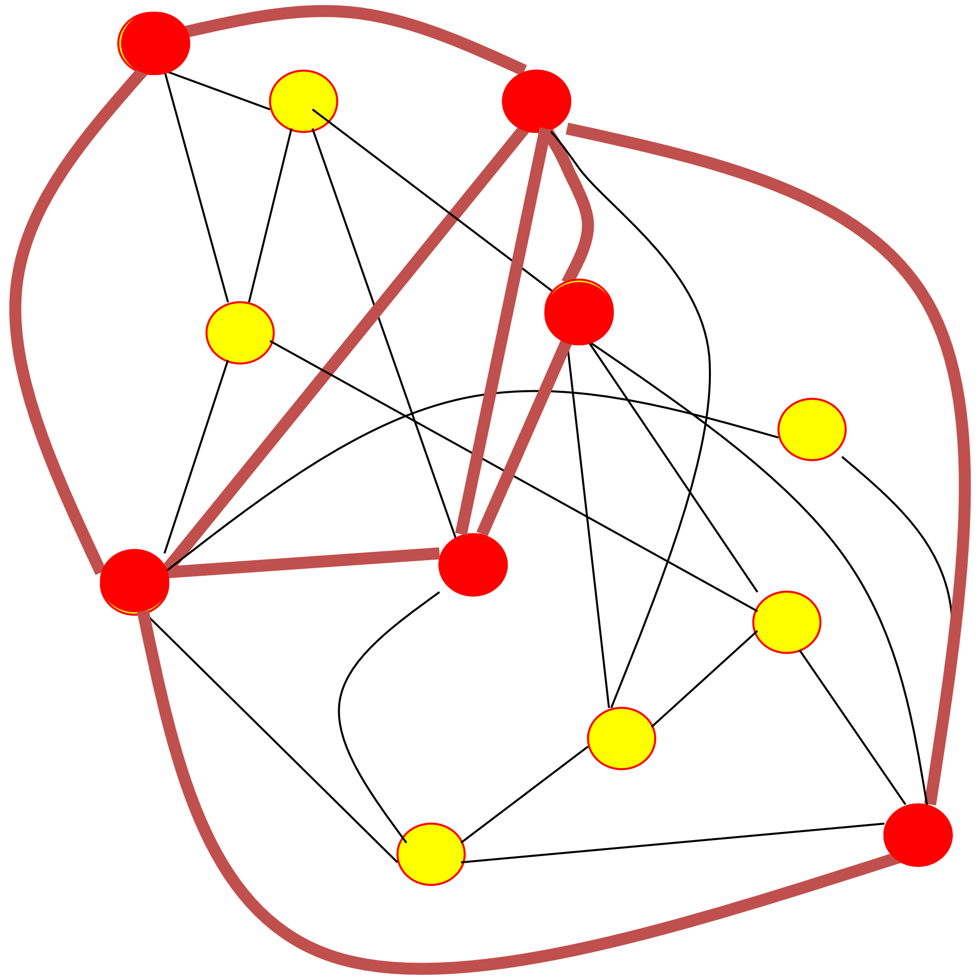
Tell me the object **shape**

for the following list of 5 items:

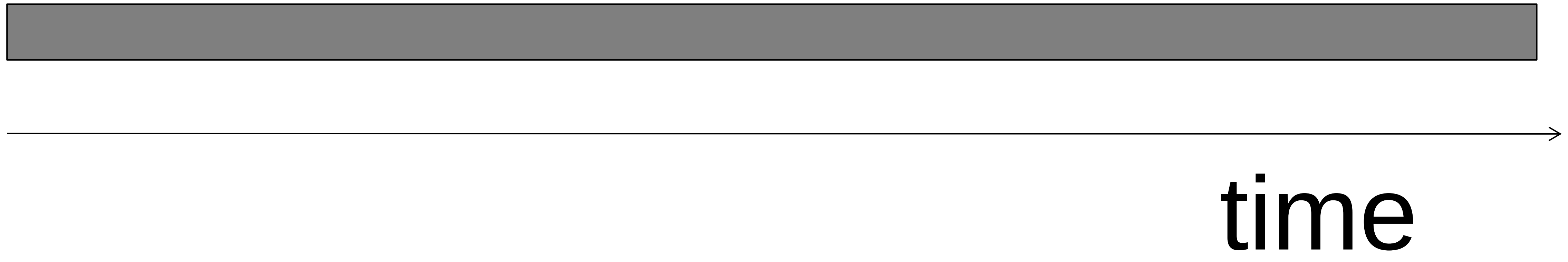
Tell me the **color**

for the following list of 5 items:





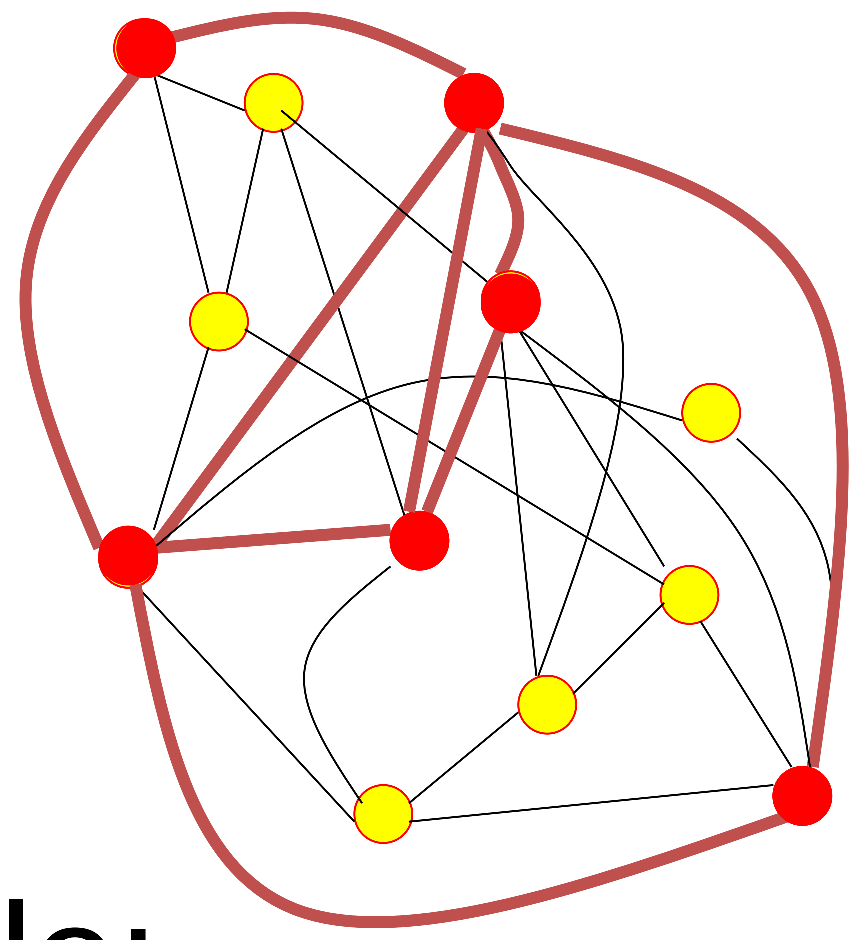
be as fast as possible:



5. Associative Recall

Tell me the **color**

for the following list of 5 items:



be as fast as possible:

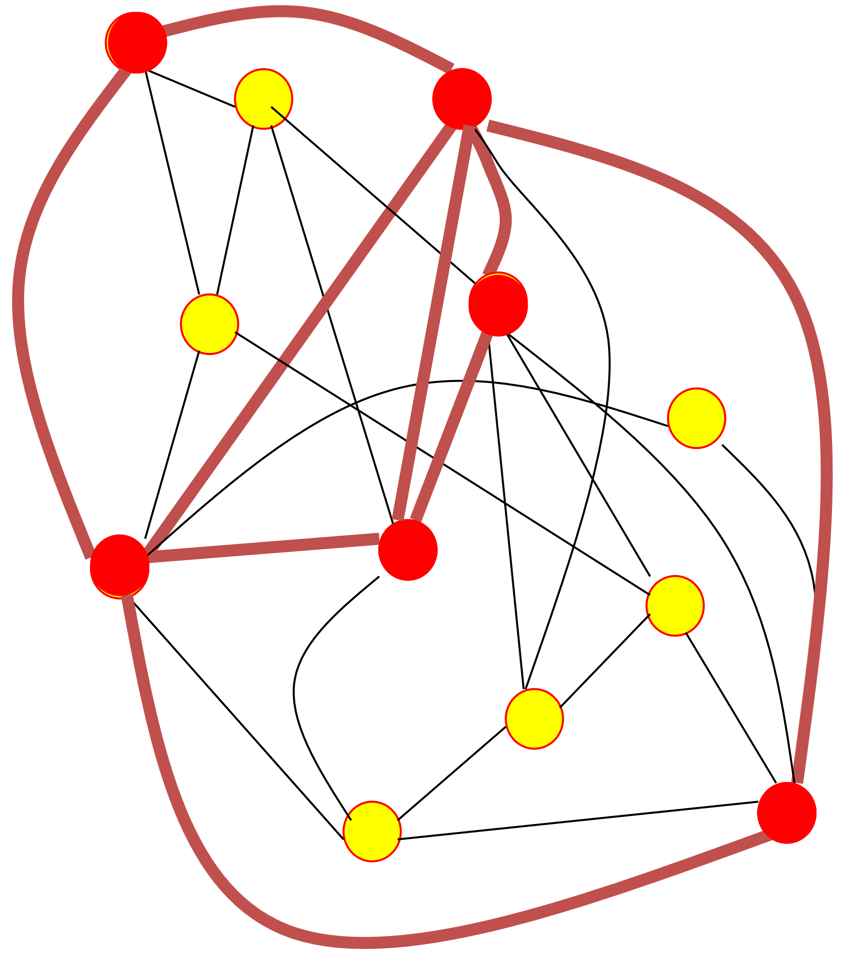
Stroop effect:

Slow response: hard to work

Against natural associations

5. Associative Recall

Hierarchical organization of
Associative memory



animals

birds fish

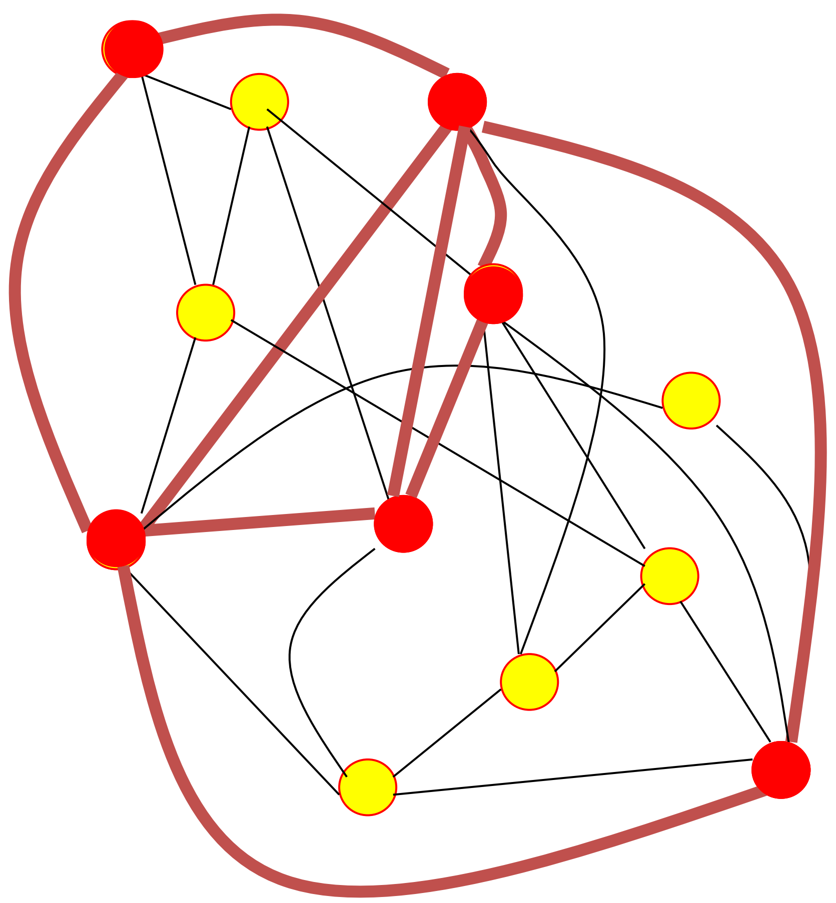
Name as fast as possible an example of a bird

swan (or goose or raven or …)

Write down first letter: *s* for *swan* or *r* for *raven* …

5. Associative Recall

name as fast as possible
an example of a

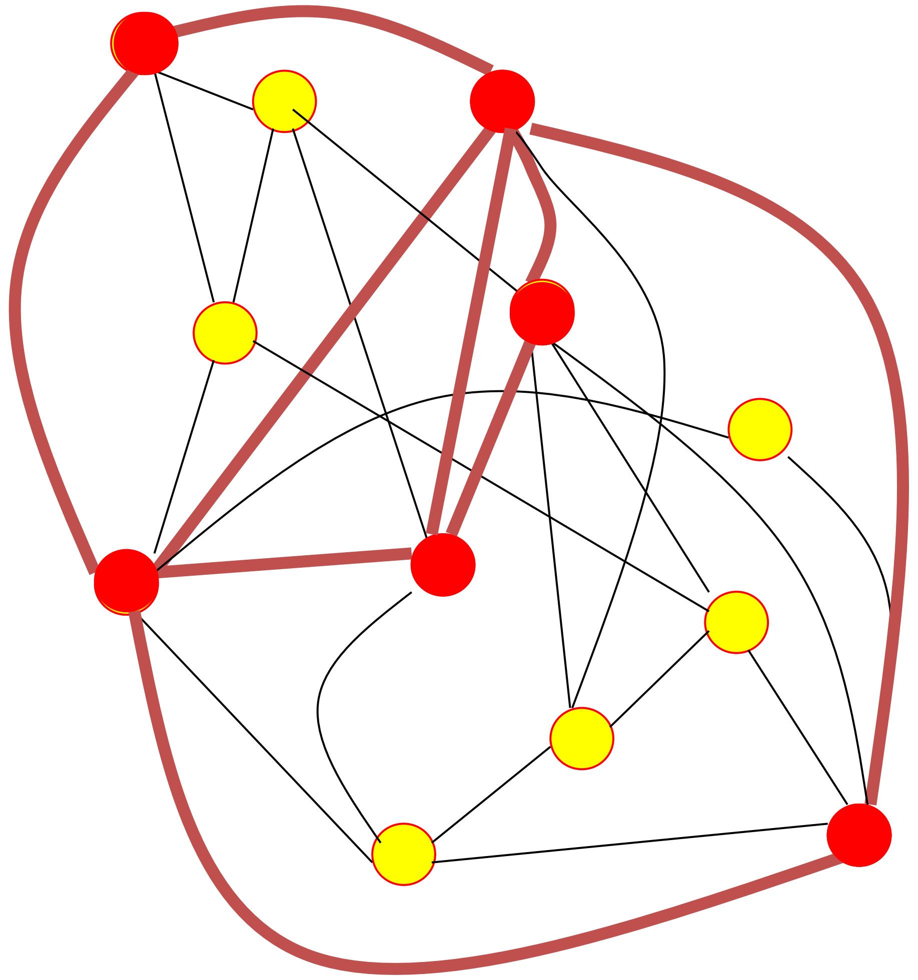


5. Associative Recall

Associative memory

animals

birds fish



- Associations can be very strong!
- It is hard to go against natural associations!
- Different aspects of a ‘concept’ are bound together!
- Assocations have been learned!

Quiz 3: Assocations

The Stroop effect implies that you are faster,
if the color does not match the meaning of the color-word

[ ] Yes

[ ] No

Hebbian learning strengthens links between neurons that

[ ] are simultaneously active

[ ] belong to the same ‘concept’ (assembly)

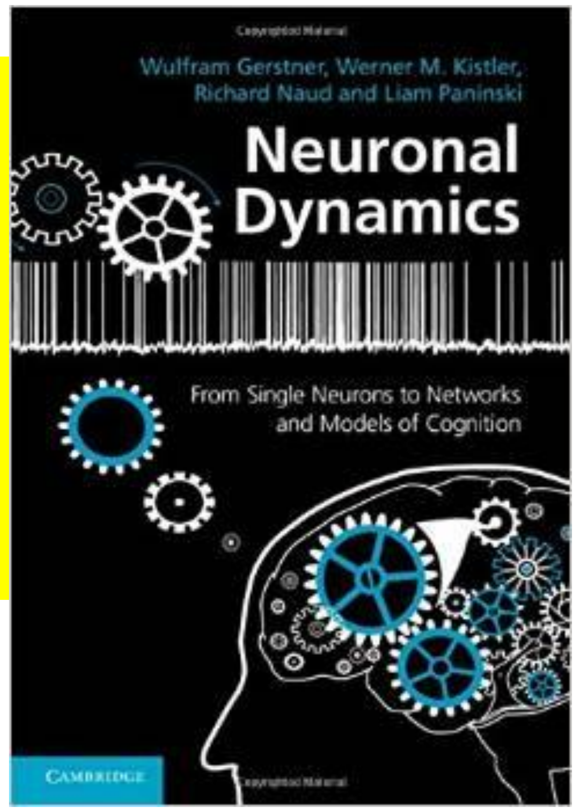
Computational Neuroscience: Neuronal Dynamics of Cognition

A: ASSOCIATIVE MEMORY
in a Network of Neurons

Wulfram Gerstner

EPFL, Lausanne, Switzerland

Reading for this week:
NEURONAL DYNAMICS
- Ch. 17.1 - 17.2.4



Cambridge Univ. Press

1 Introduction
- networks of neuron
- systems for computing
- associative memory

2 Classification by similarity

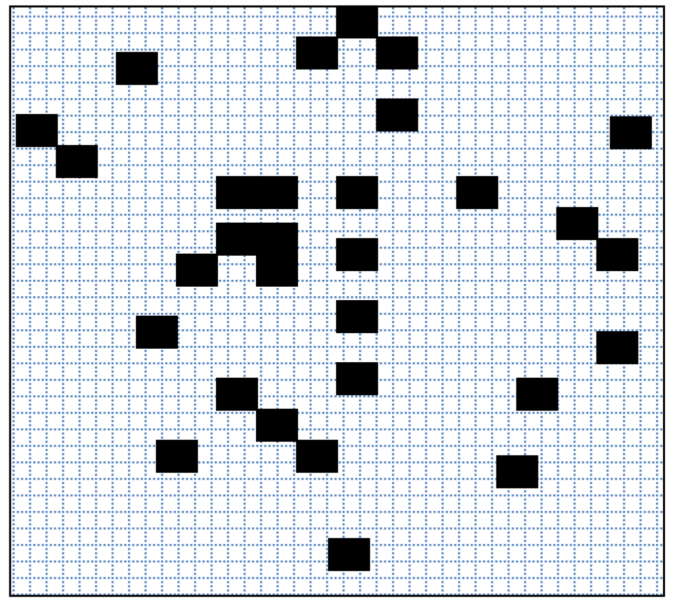
3 Detour: Magnetic Materials

4 Hopfield Model

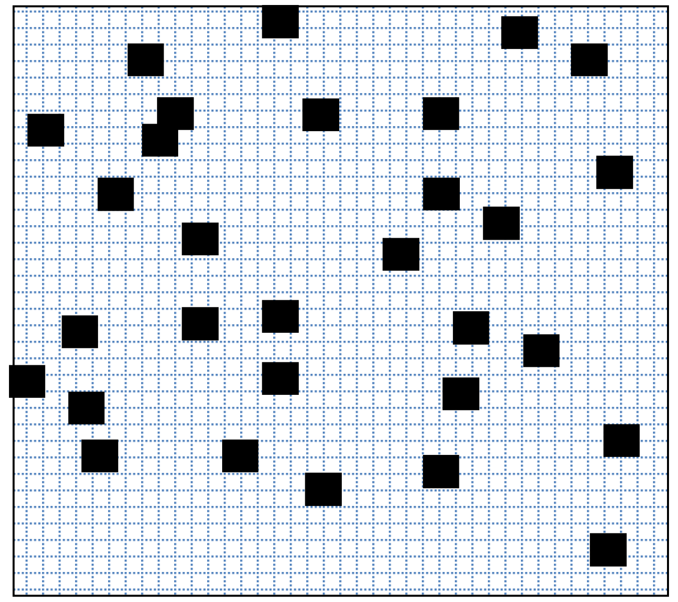
5 Learning of Associations

6 Storage Capacity

6. learning of several prototypes



Prototype $\vec{p}^1$



Prototype $\vec{p}^2$

interactions

$$(1) \quad w_{ij} = \frac{1}{N} \sum_{\mu} p_i^{\mu} p_j^{\mu}$$

Sum over all prototypes

Question: How many prototypes can be stored?

dynamics

$$S_i(t+1) = \mathrm{sgn}[\sum_j w_{ij} S_j(t)]$$

all interactions with i

6. Storage capacity: How many prototypes can be stored?

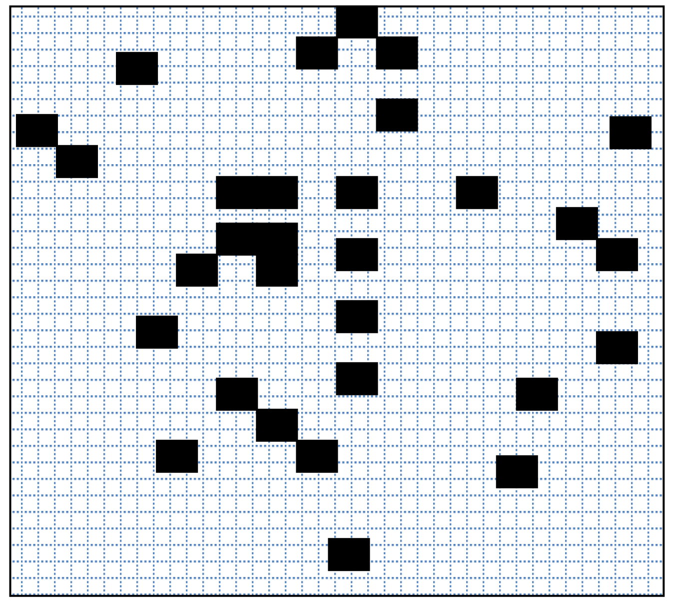
-Assume we start directly in one pattern (say pattern 7)
-Pattern must stay

$$S_i(t+1) = \mathrm{sgn}[\sum_j w_{ij} S_j(t)]$$

Interactions (1)

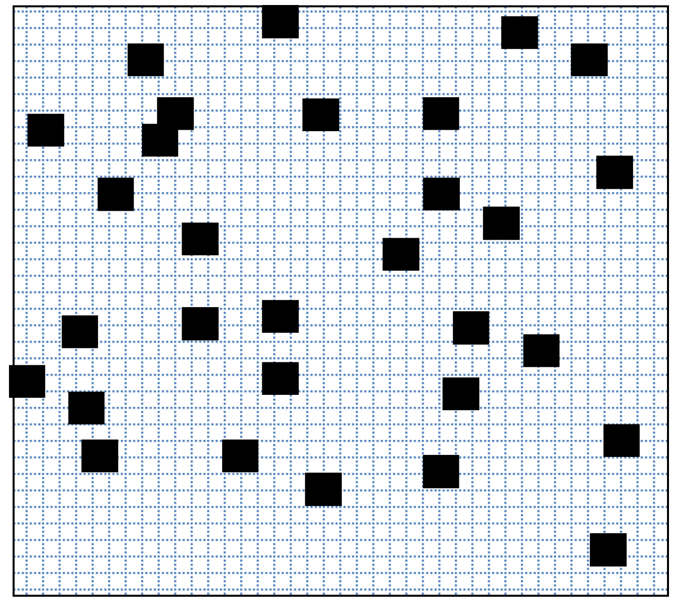
$$w_{ij} = \frac{1}{N} \sum_\mu p_i^\mu p_j^\mu$$

6. Storage capacity: How many prototypes can be stored?



Prototype

$\vec{p}^1$



Prototype

$\vec{p}^2$

Random patterns

Interactions (1) $w_{ij} = \frac{1}{N} \sum_{\mu} p_i^{\mu} p_j^{\mu}$

Dynamics (2) $S_i(t+1) = \mathrm{sgn}[\sum_j w_{ij} S_j(t)]$

Minimal condition: pattern is fixed point of dynamics

-Assume we start directly in one pattern (say pattern ν)

-Pattern must stay

Attention: Retrieval requires more (pattern completion)

Q: How many prototypes can be stored?

A: If too many prototypes, errors (wrong pixels) show up.
The number of prototypes M that can be stored
is proportional to number of neurons N;
memory load = M/N

$$S_i(t+1) = p_i^\nu \, \mathrm{sgn}\Big[1 + \frac{1}{N} \sum_{\mu=1,\mu\neq\nu}^{M} \sum_{j=1}^{N} p_i^\mu p_i^\nu p_j^\mu p_j^\nu \Big]$$

$$= p_i^\nu \, \mathrm{sgn}[1 - a_i^\nu]$$

Error-free if

$$S_i(t+1) = p_i^\nu$$

Gaussian

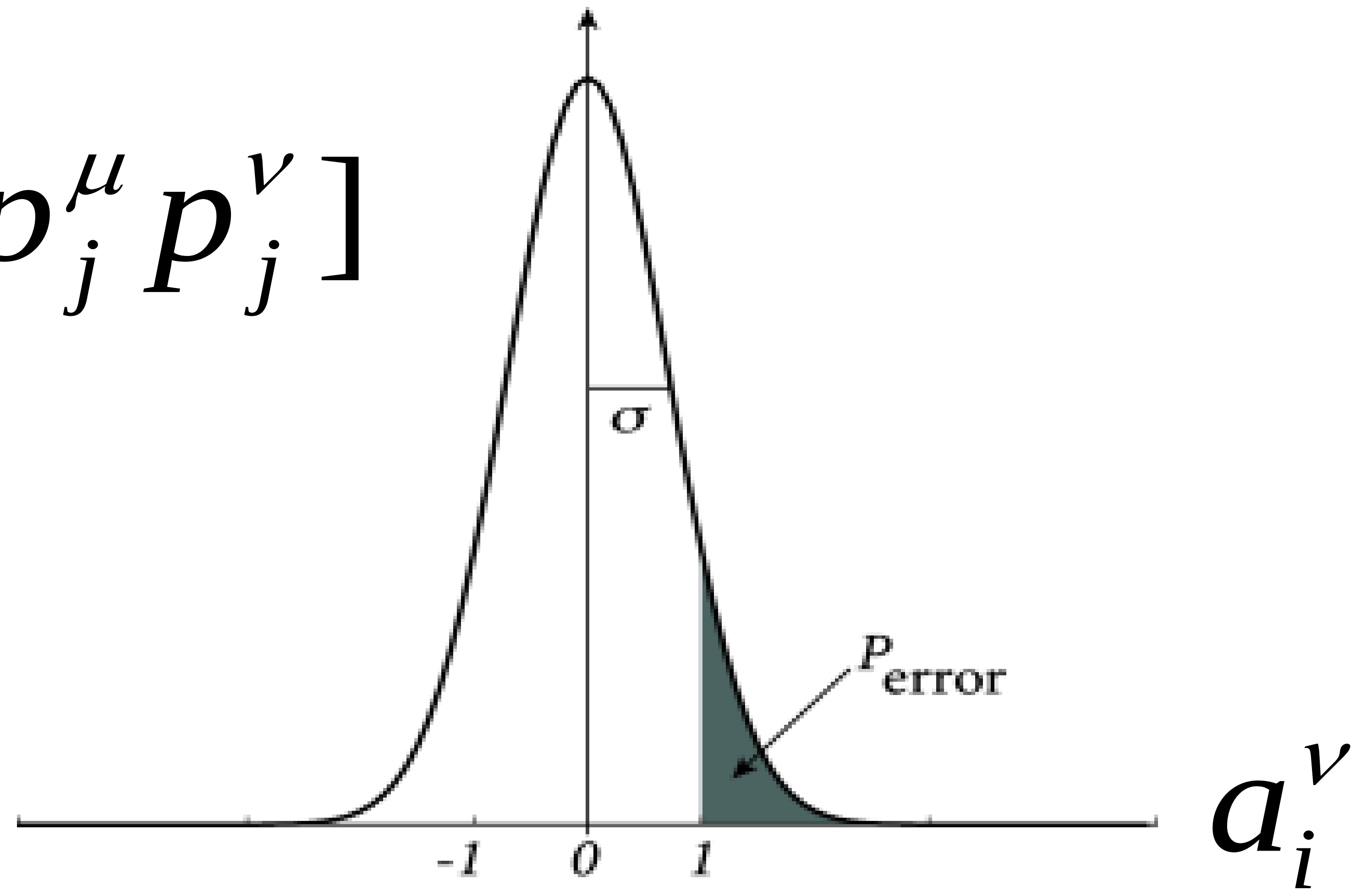


Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014),

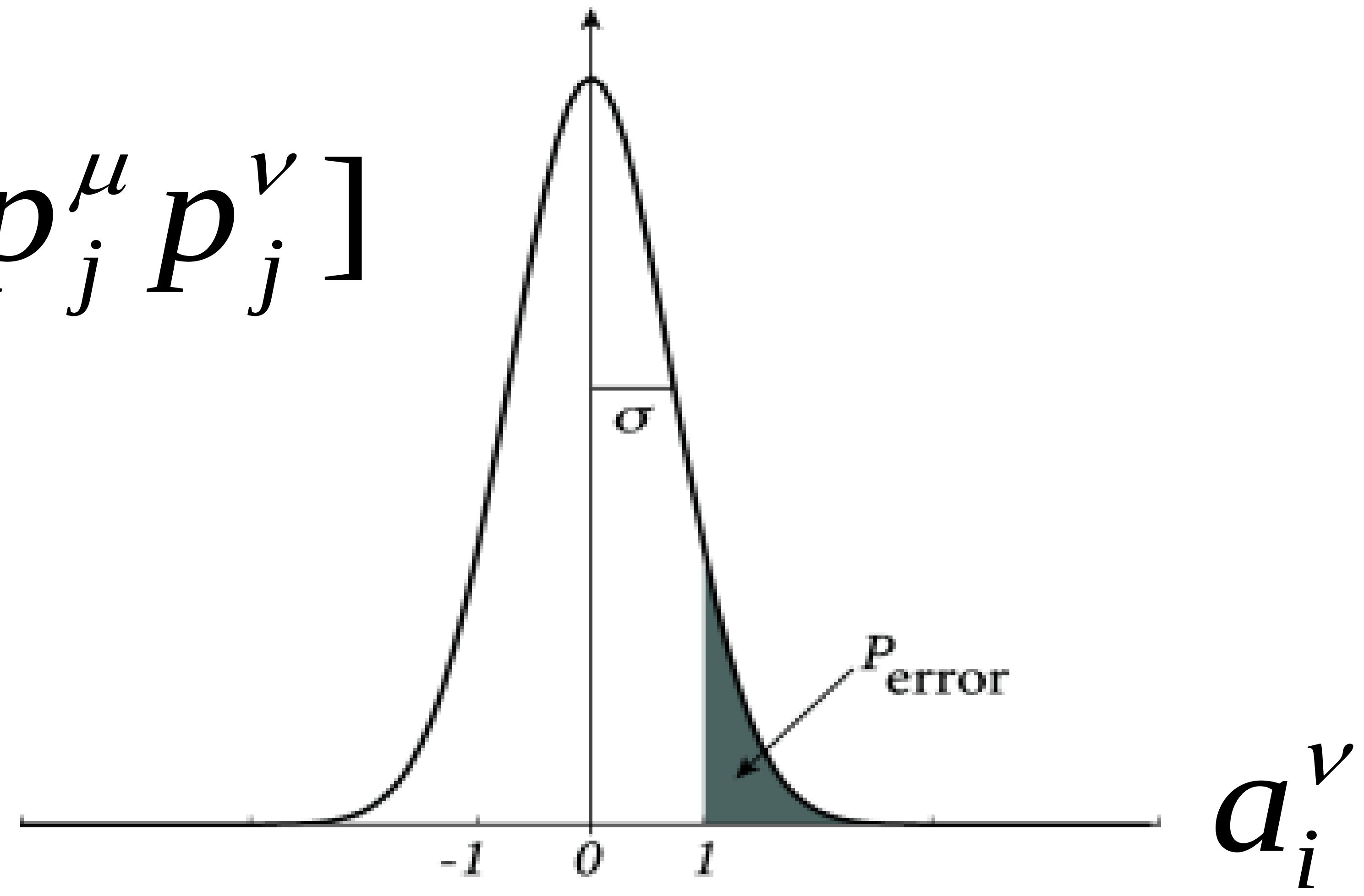
6. Storage capacity: How many prototypes can be stored?

Random walk with steps

Standard deviation

$$S_i(t+1) = p_i^\nu \,\mathrm{sgn}[1 + \tfrac{1}{N} \sum_{\mu=1,\mu\neq\nu}^{M} \sum_{j=1}^{N} p_i^\mu p_i^\nu p_j^\mu p_j^\nu]$$

$$= p_i^\nu \,\mathrm{sgn}[1 - a_i^\nu]$$



Error-free if

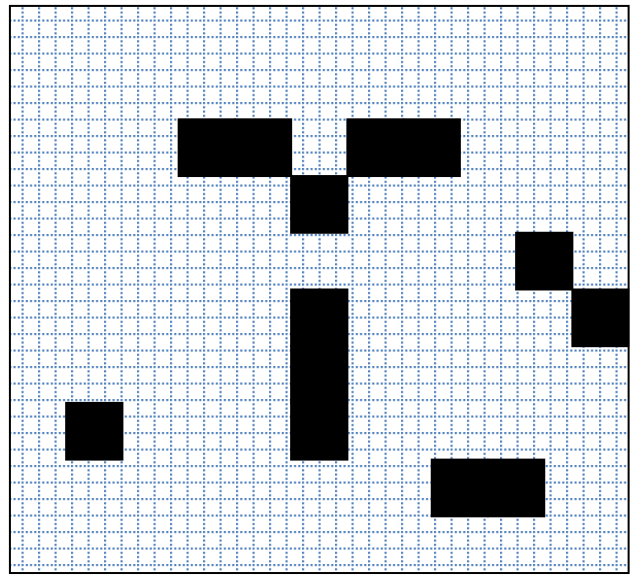
$$S_i(t+1) = p_i^\nu$$

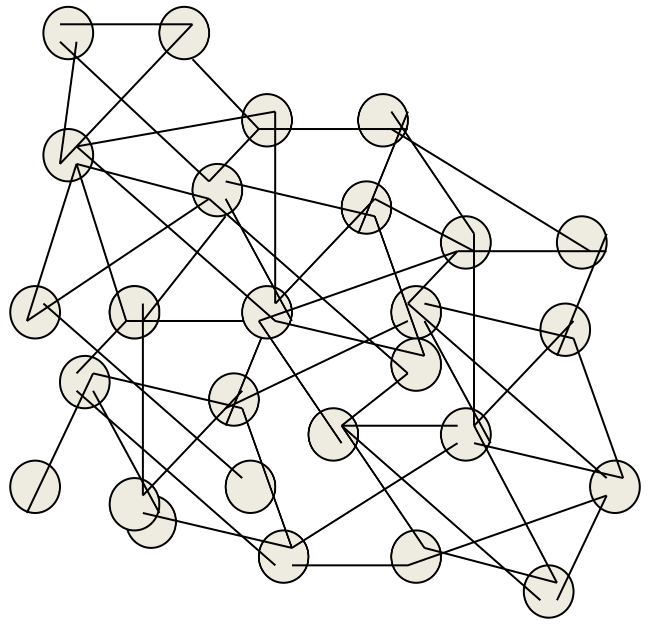
Gaussian

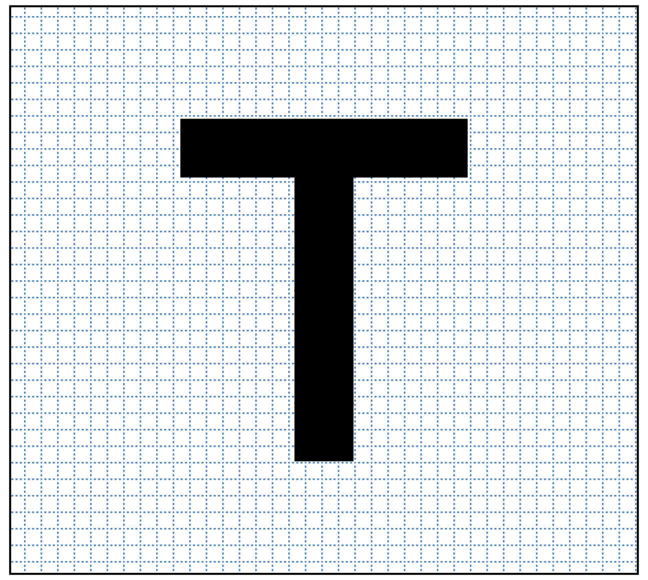
Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014),

This week: Understand Associative Memory

Partial information → Associative memory/ collective computation → Full concept







Brain-style computation

- Memory stored in connections
- Many memories can be stored in same network
- Retrieval of memories without centralized controller
- Interactions of neurons makes network converge to most similar pattern

References: Associative Memory Models

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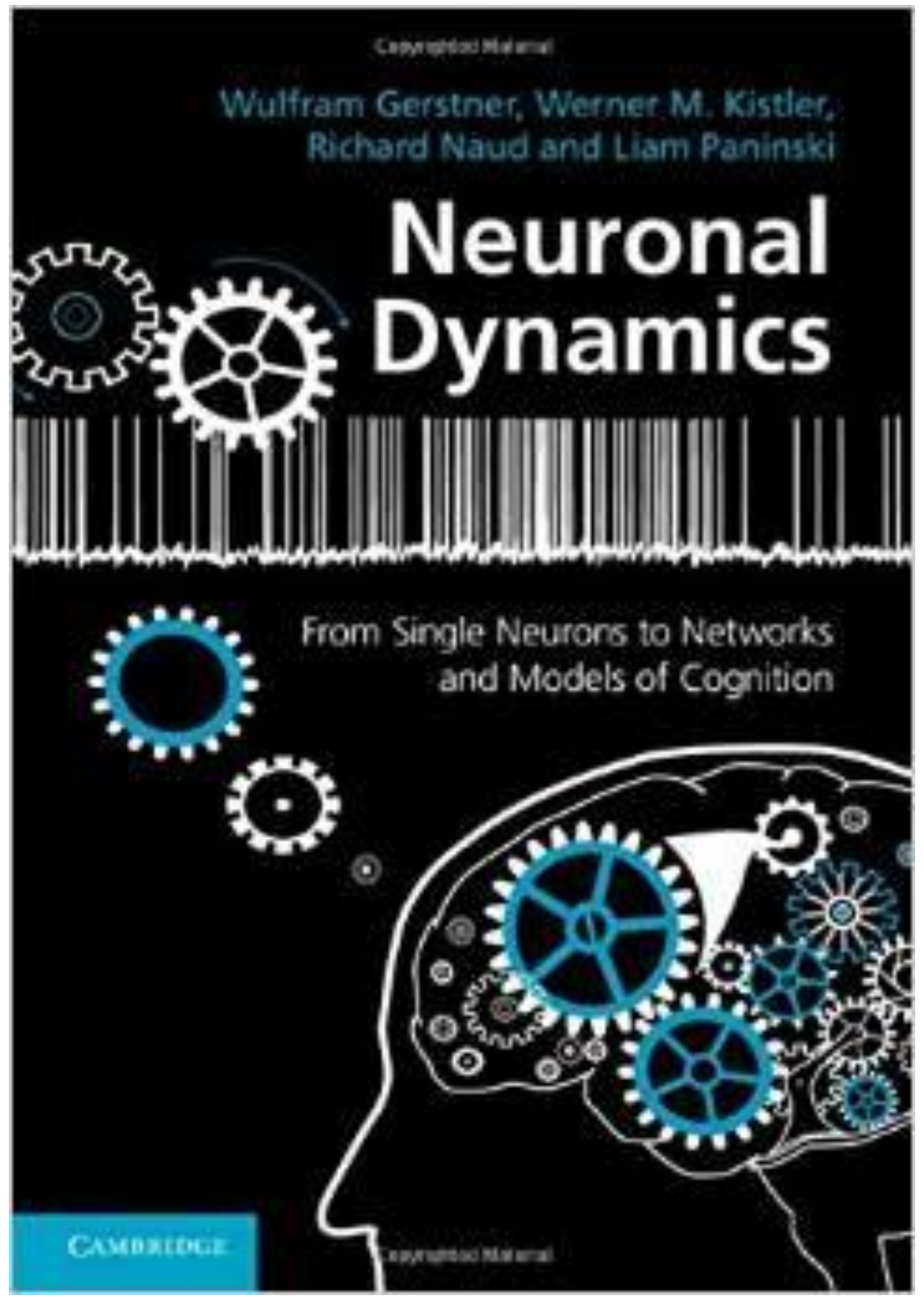
The end

Documentation:
http://neuronaldynamics.epfl.ch/

Online html version available



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NEURONAL DYNAMICS
- Ch. 17.1 - 17.2.4

Cambridge Univ. Press