Week 5 – part 1 :Variability

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Neuronal Dynamics:
Computational Neuroscience of Single Neurons

Week 5 – Variability and Noise:
The question of the neural code

Wulfram Gerstner

EPFL, Lausanne, Switzerland

5.1 Variability of spike trains
- experiments

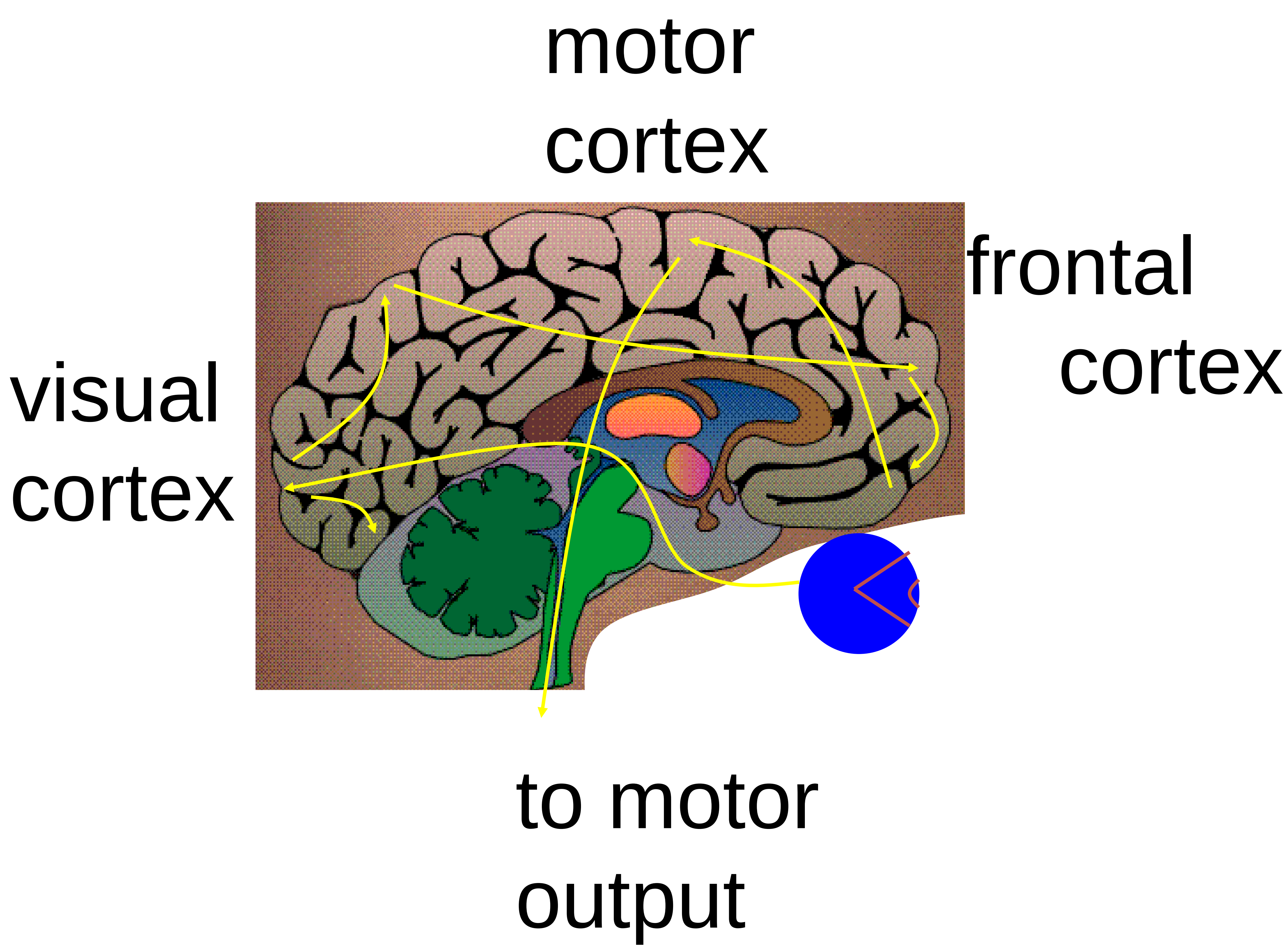
5.2 Sources of Variability?
- Is variability equal to noise?

5.3 Three definitions of Rate code
- Poisson Model
- **Detour**: Poisson model, a modern approach

5.4 Stochastic spike arrival
- Membrane potential fluctuations

5.5. Stochastic spike firing
- subthreshold and superthreshold

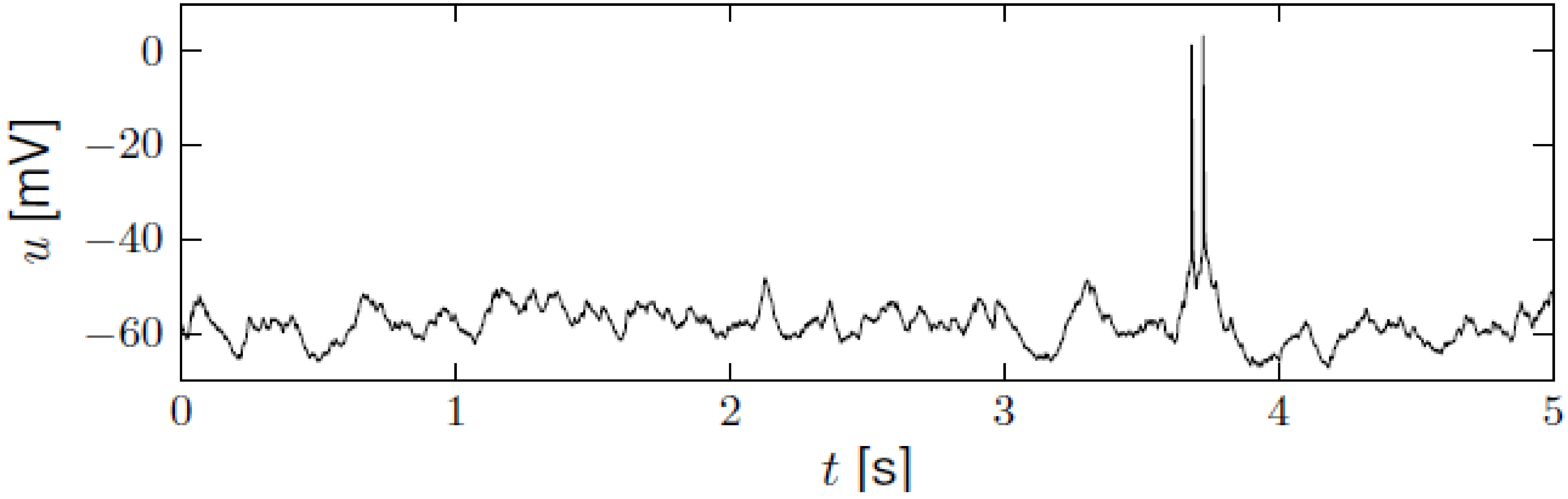
Neuronal Dynamics – 5.1. Variability



Spontaneous activity *in vivo*

Variability
- of membrane potential?
- of spike timing?

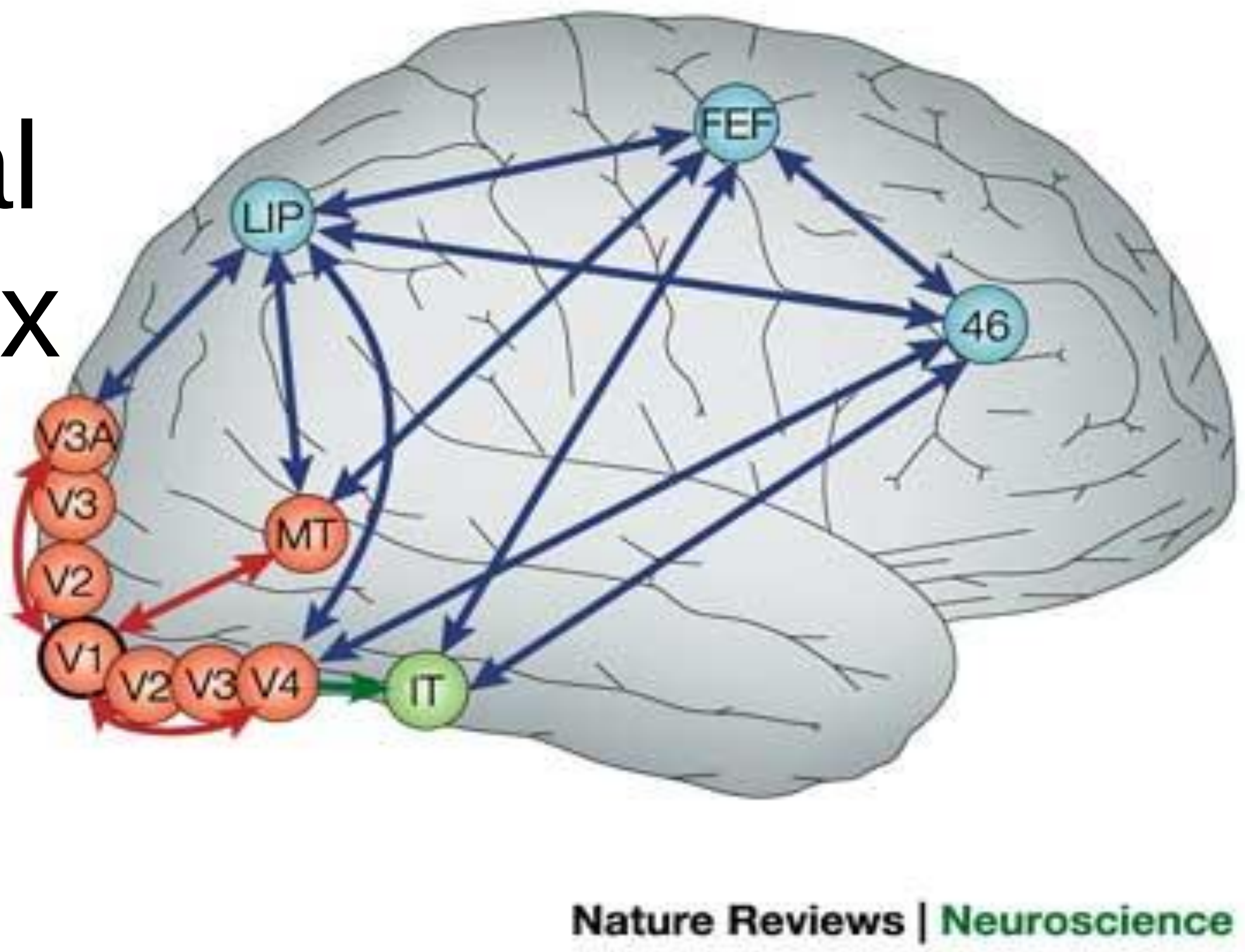
awake mouse, cortex, freely whisking,

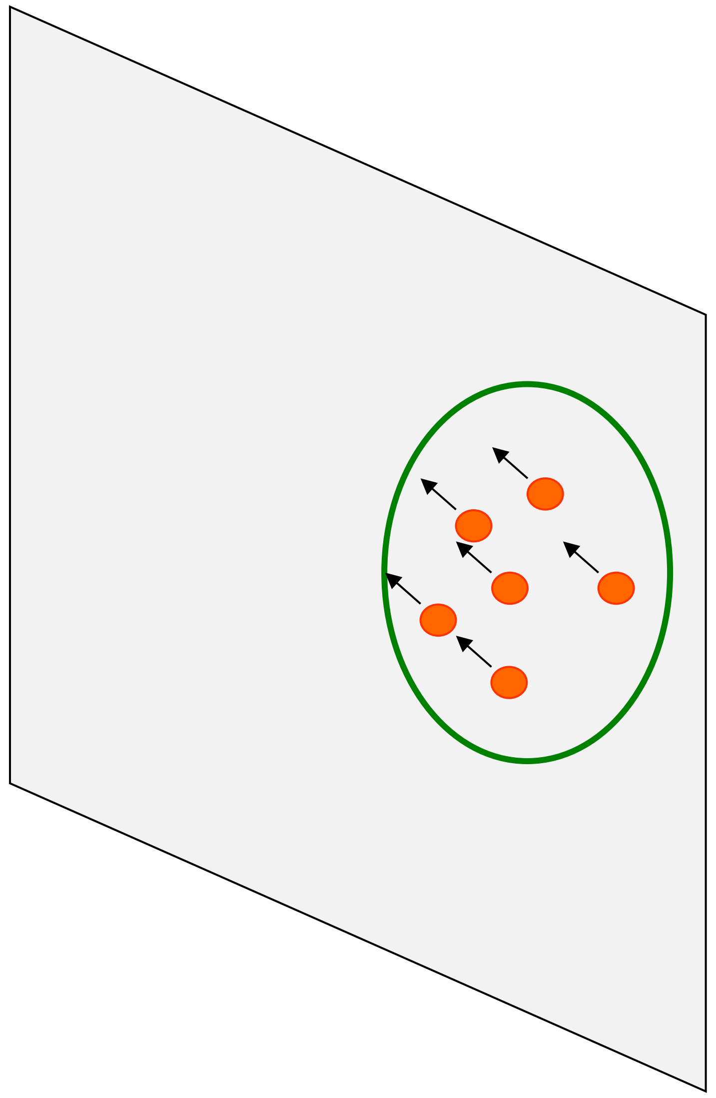


Crochet et al., 2011

Detour: Receptive fields in V5/MT

visual cortex

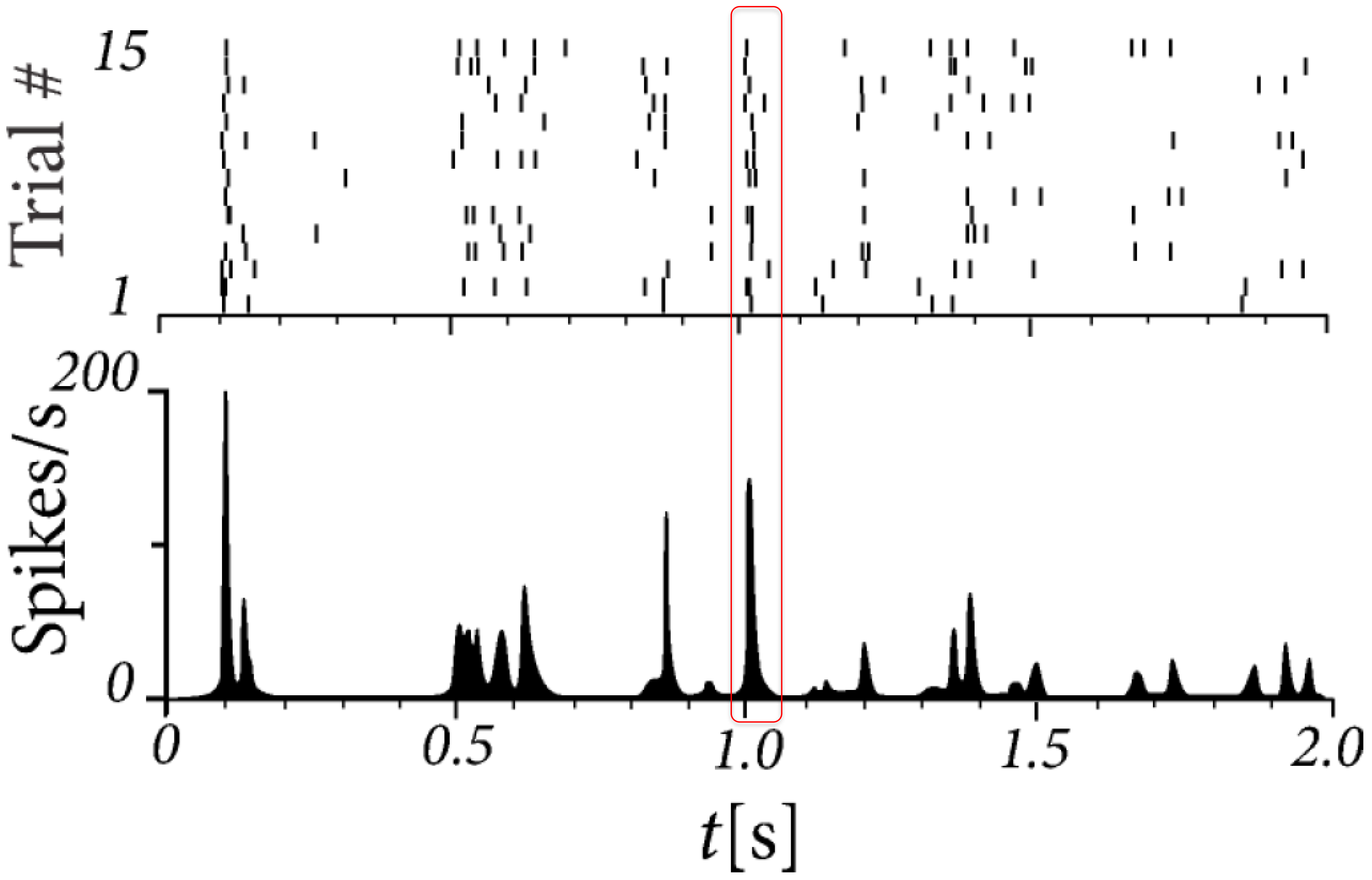


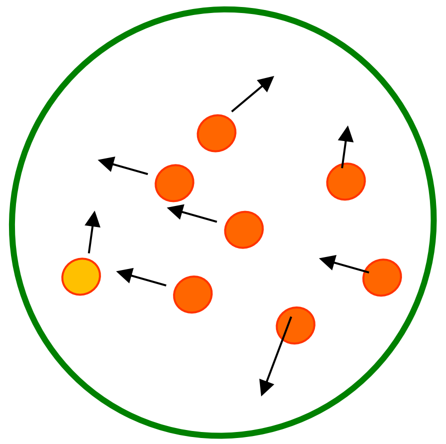


cells in visual cortex MT/V5 respond to motion stimuli

Neuronal Dynamics – 5.1 Variability in vivo

15 repetitions of the **same** random dot motion pattern

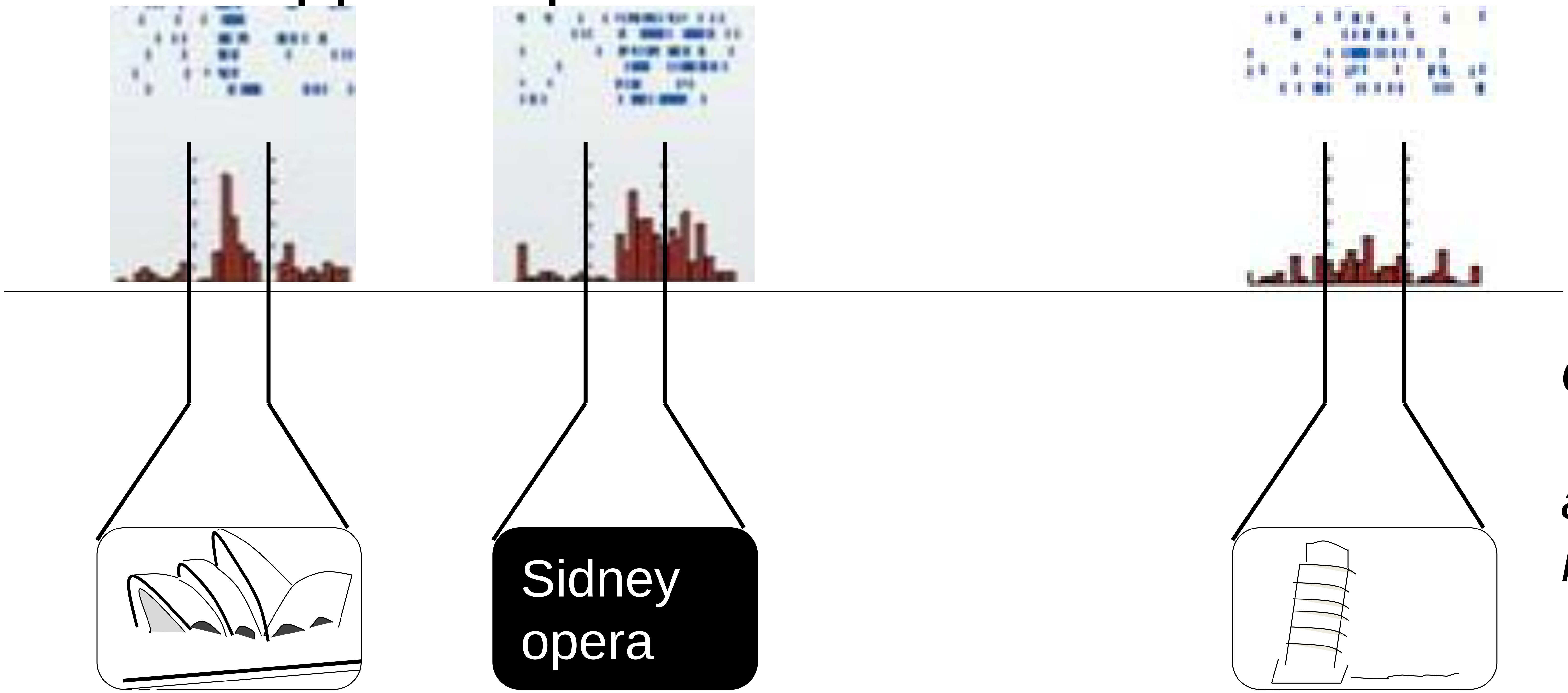




adapted from Bair and Koch 1996;
data from Newsome 1989

Neuronal Dynamics – 5.1 Variability in vivo

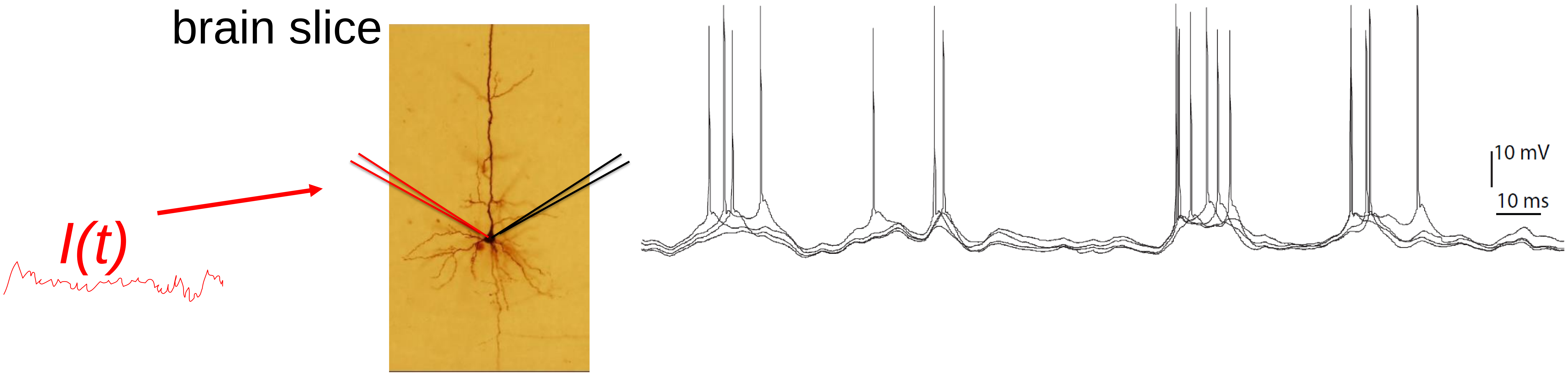
Human Hippocampus



Quiroga, Reddy, Kreiman, Koch, and Fried (2005). Nature, 435:1102-1107.

Neuronal Dynamics – 5.1 Variability in vitro

4 repetitions of the same time-dependent stimulus,



Fluctuations

-of membrane potential

-of spike times

fluctuations=noise?

relevance for coding?

source of fluctuations?

model of fluctuations?

Week 5 – part 2 : Sources of Variability

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Week 5 – Variability and Noise: The question of the neural code

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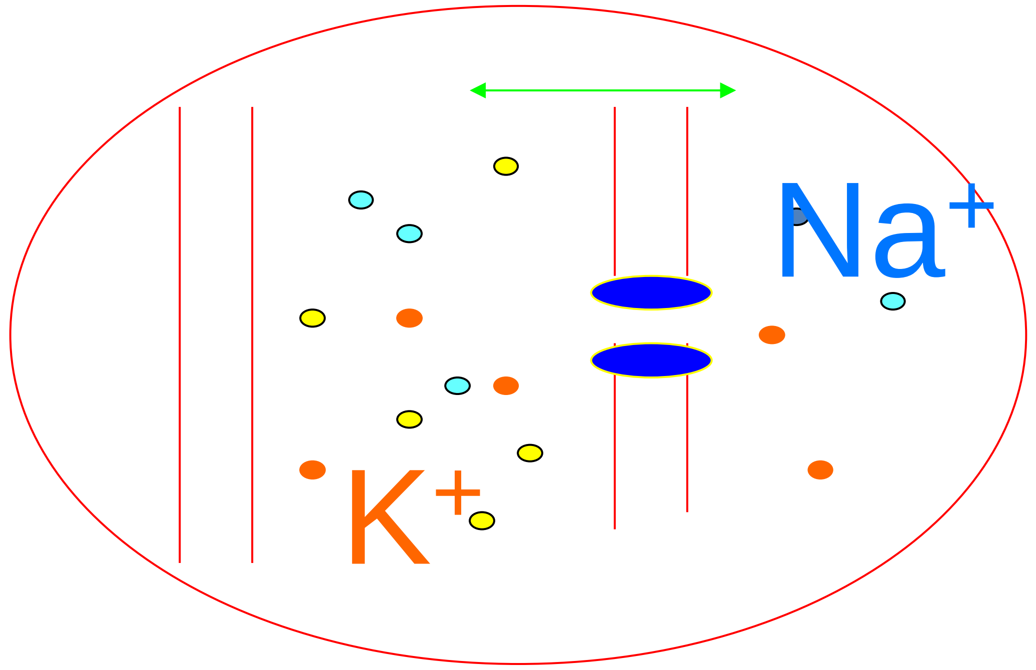
EPFL, Lausanne, Switzerland

Neuronal Dynamics – 5.2. Sources of Variability

- Intrinsic noise (ion channels)



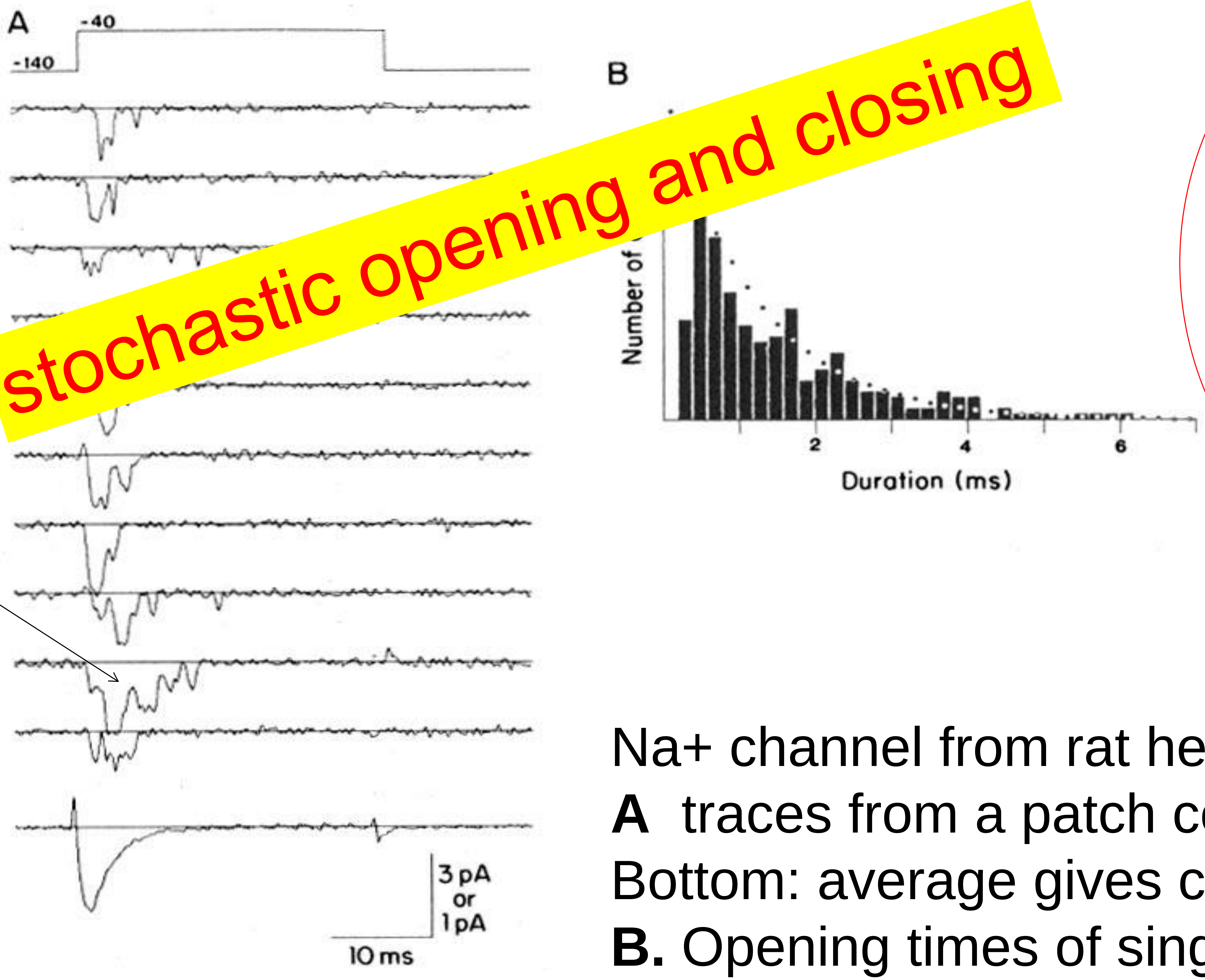
-Finite number of channels

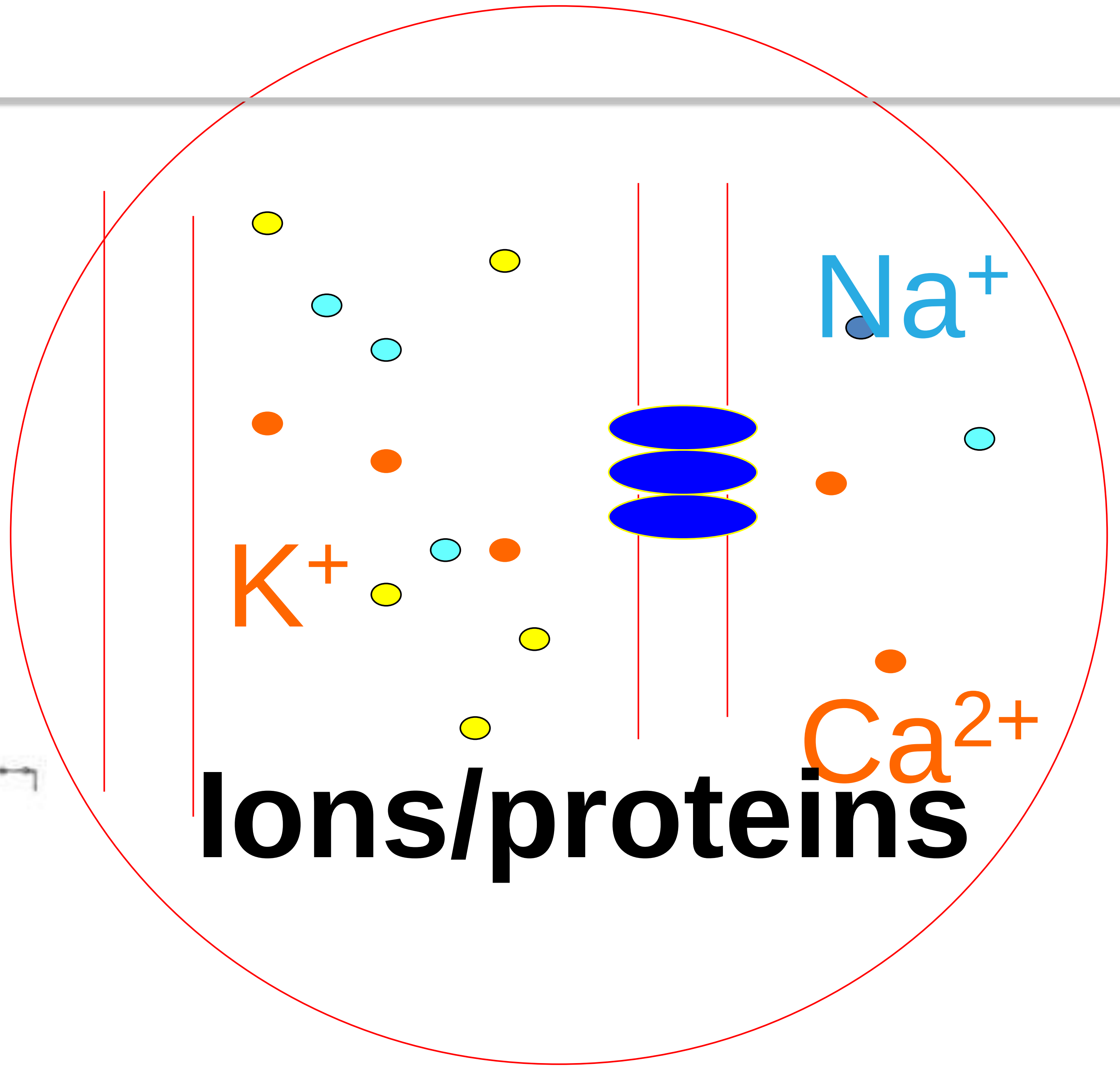
-Finite temperature

Review from 2.5 Ion channels

stochastic opening and closing

Steps:
Different number
of channels





Na+ channel from rat heart *(Patlak and Ortiz 1985)*

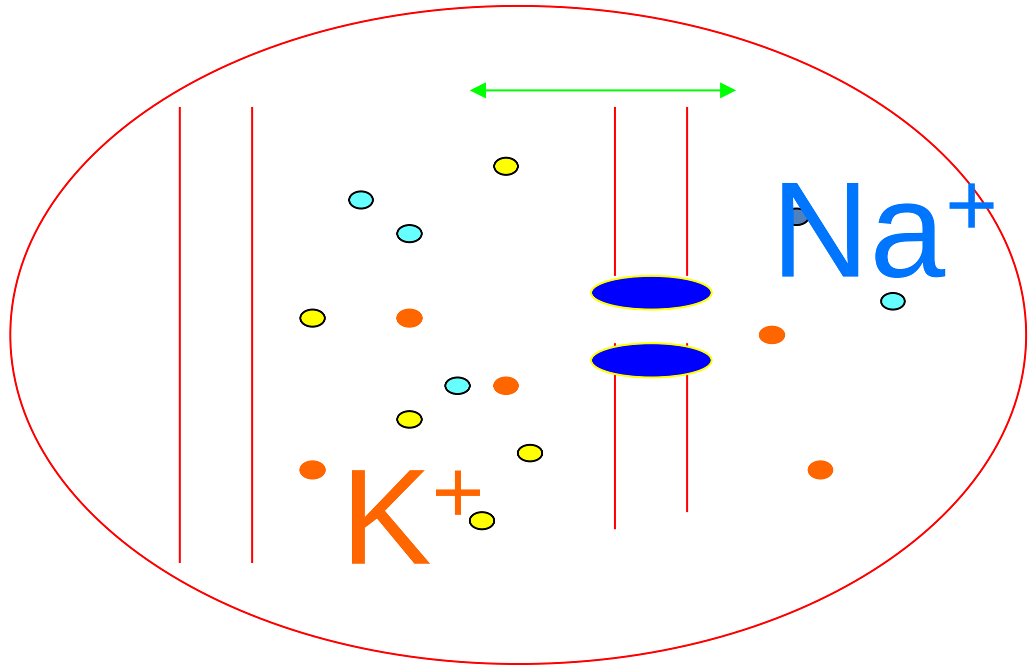
A traces from a patch containing several channels.

Bottom: average gives current time course.

B. Opening times of single channel events

Neuronal Dynamics – 5.2. Sources of Variability

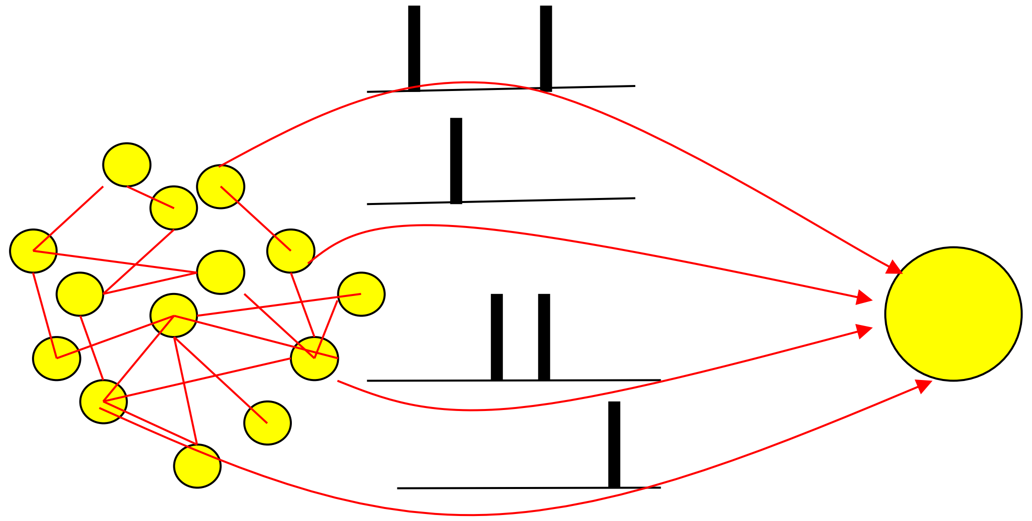
- Intrinsic noise (ion channels)



-Finite number of channels

-Finite temperature

-Network noise (background activity)

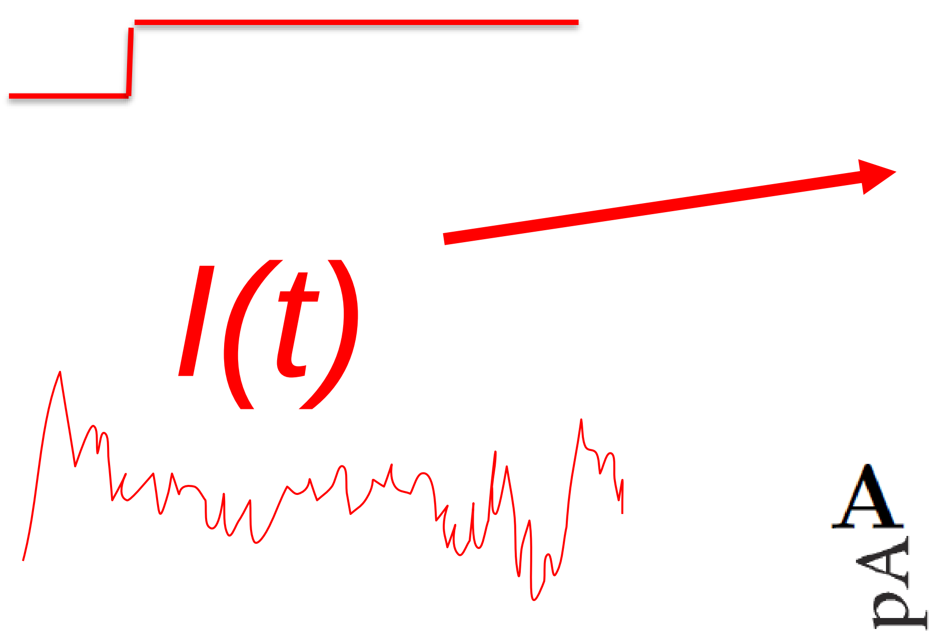


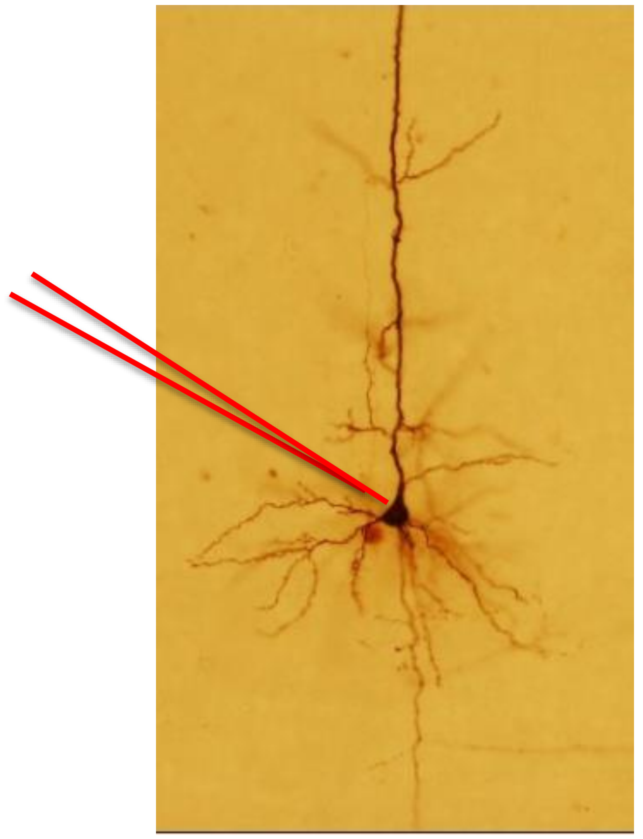
-Spike arrival from other neurons

-Beyond control of experimentalist

→ Check intrinisic noise by removing the network

Neuronal Dynamics – 5.2 Variability in vitro





neurons are fairly reliable

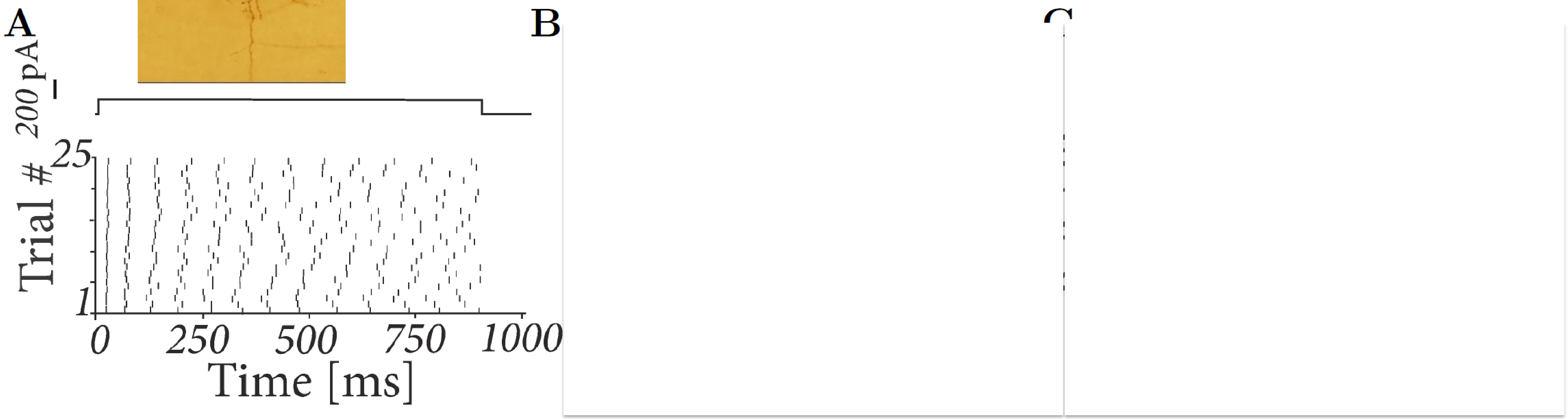
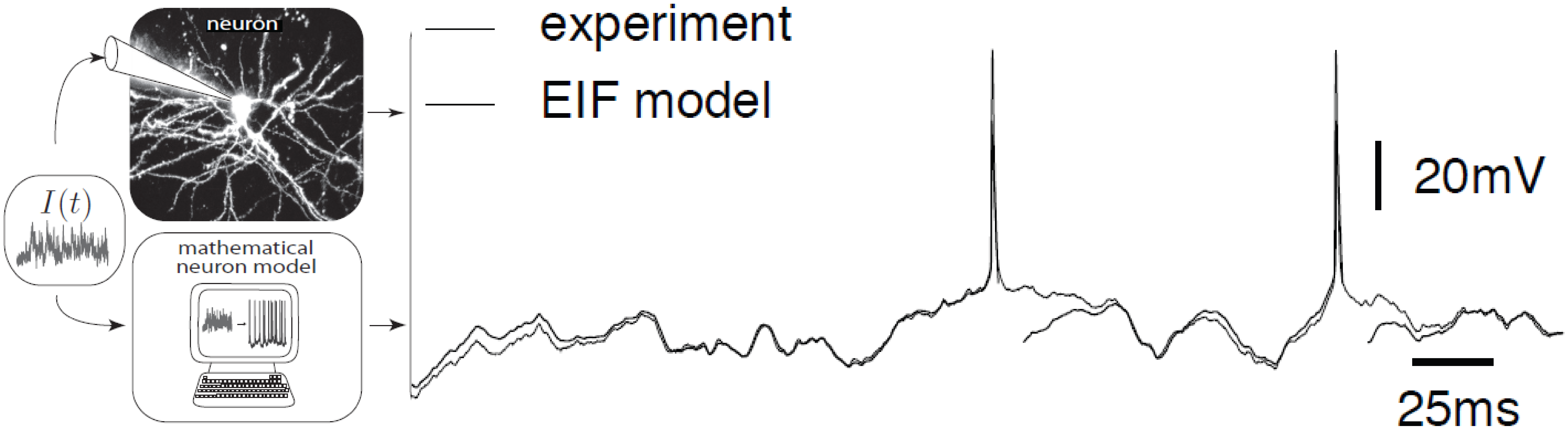


Image adapted from Mainen&Sejnowski 1995

REVIEW from 4.5: How good are integrate-and-fire models?

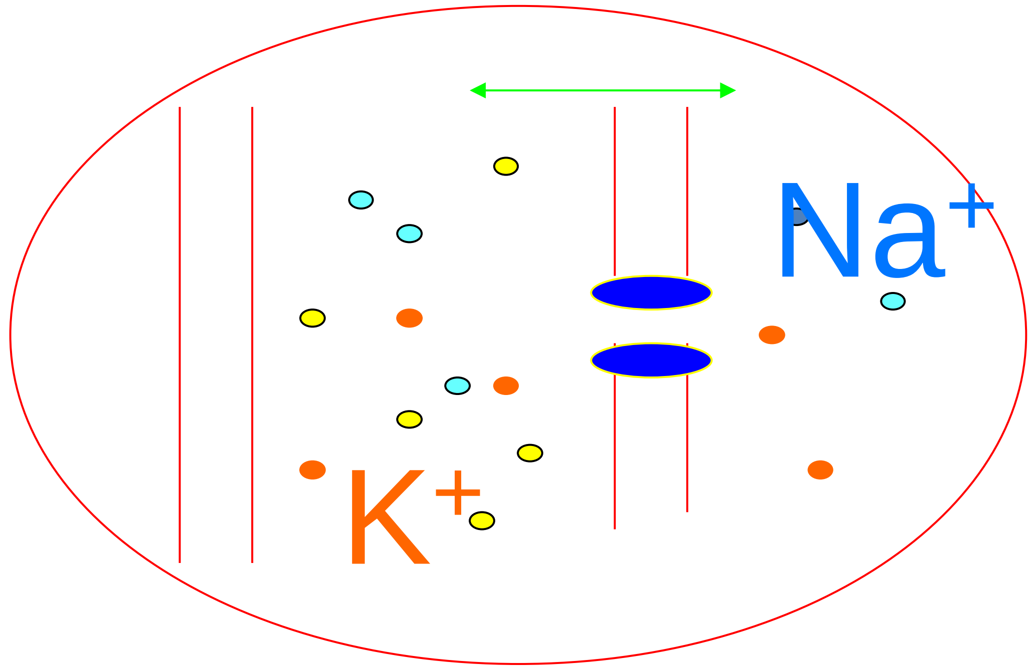


Badel et al., 2008

Aims: - predict spike initiation times
- predict subthreshold voltage

only possible, because neurons are fairly reliable

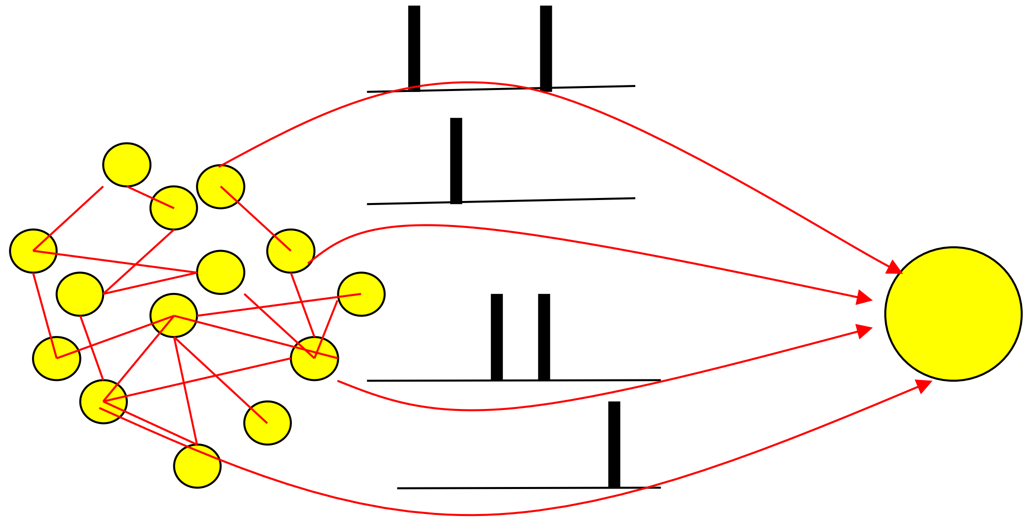
- Intrinsic noise (ion channels)



-Finite number of channels
-Finite temperature

small contribution!

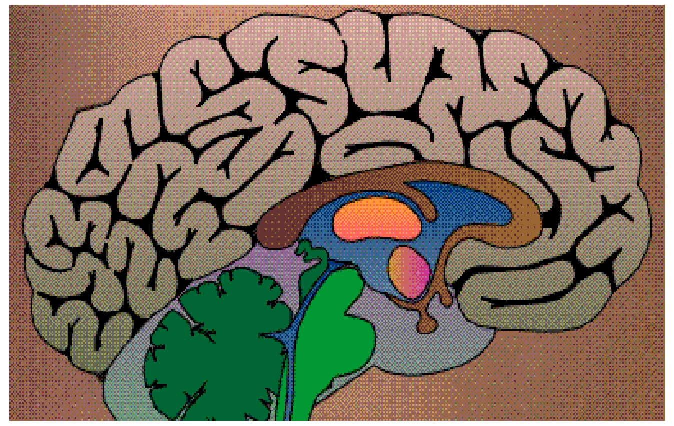
-Network noise (background activity)



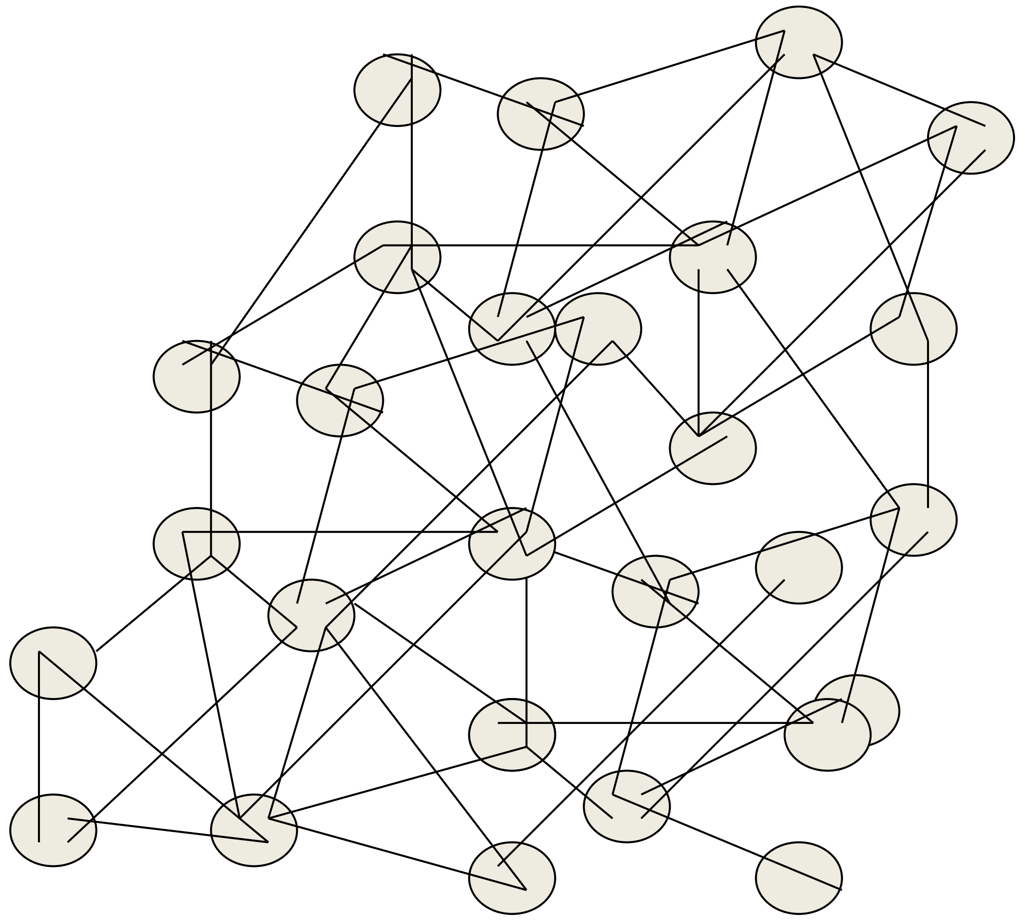
-Spike arrival from other neurons
-Beyond control of experimentalist

→ Check network noise by simulation!

The Brain: a highly connected system



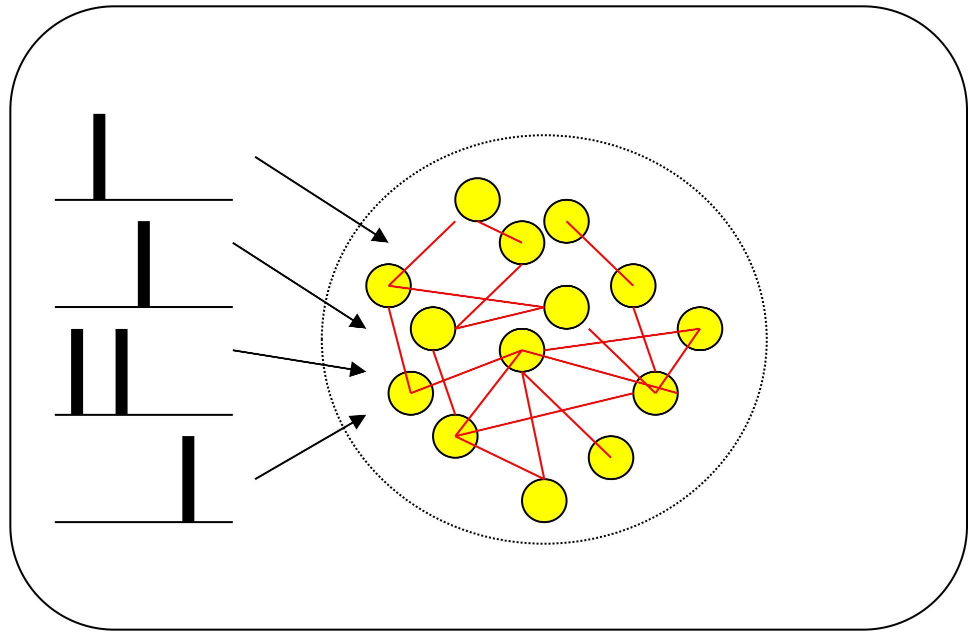
Brain



High connectivity:
systematic, organized in local populations
but **seemingly random**

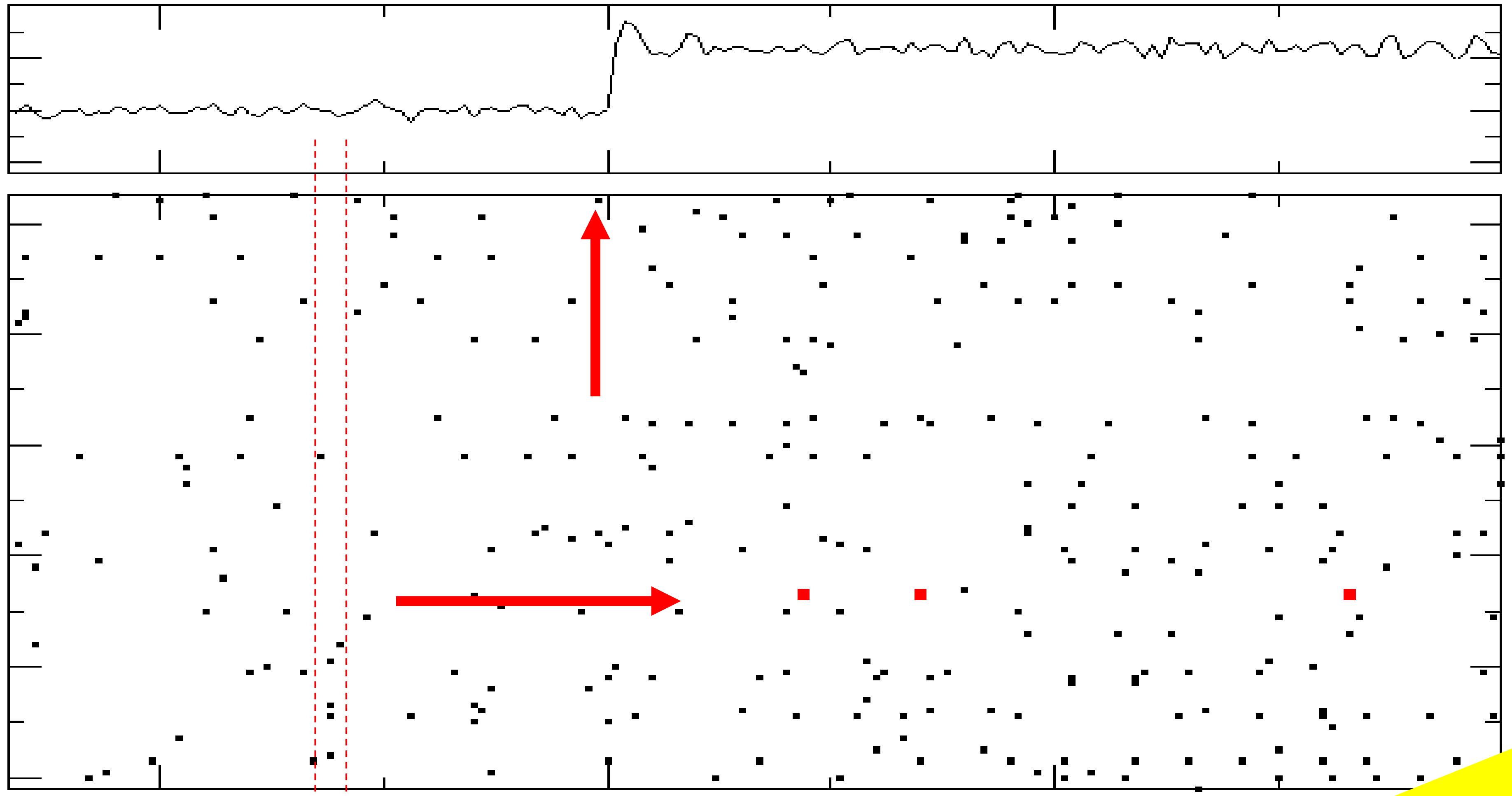
Distributed architecture
10^{10} neurons
10^{4} connections/neurons

Random firing in a population of LIF neurons



input {low rate / high rate

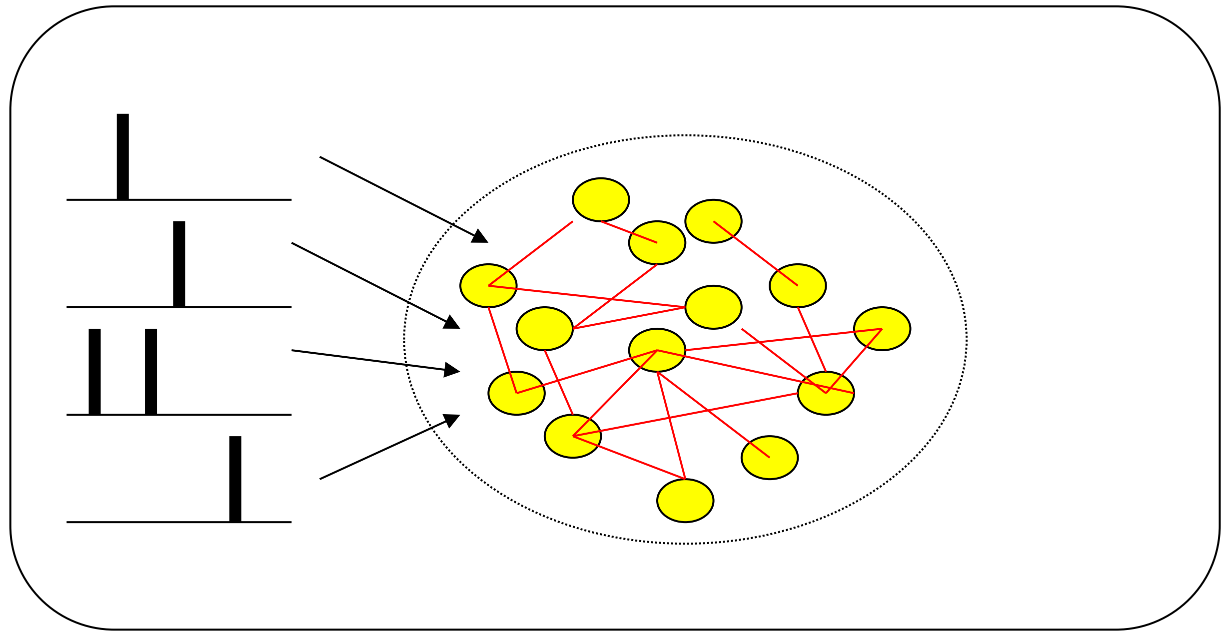
Population
- 50 000 neurons
- 20 percent inhibitory
- **randomly connected**



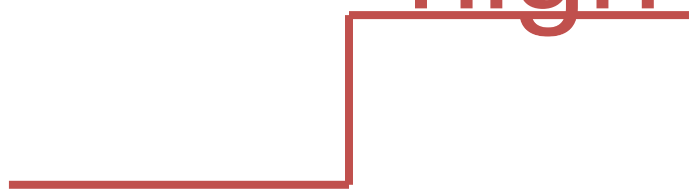
Brunel, J. Comput. Neurosc. 2000
Mayor and Gerstner, Phys. Rev E. 20
Vogels et al., 2005

Network of **deterministic** leaky integrate-and-fire: 'fluctuations'

Random firing in a population of LIF neurons



input {low rate / high rate



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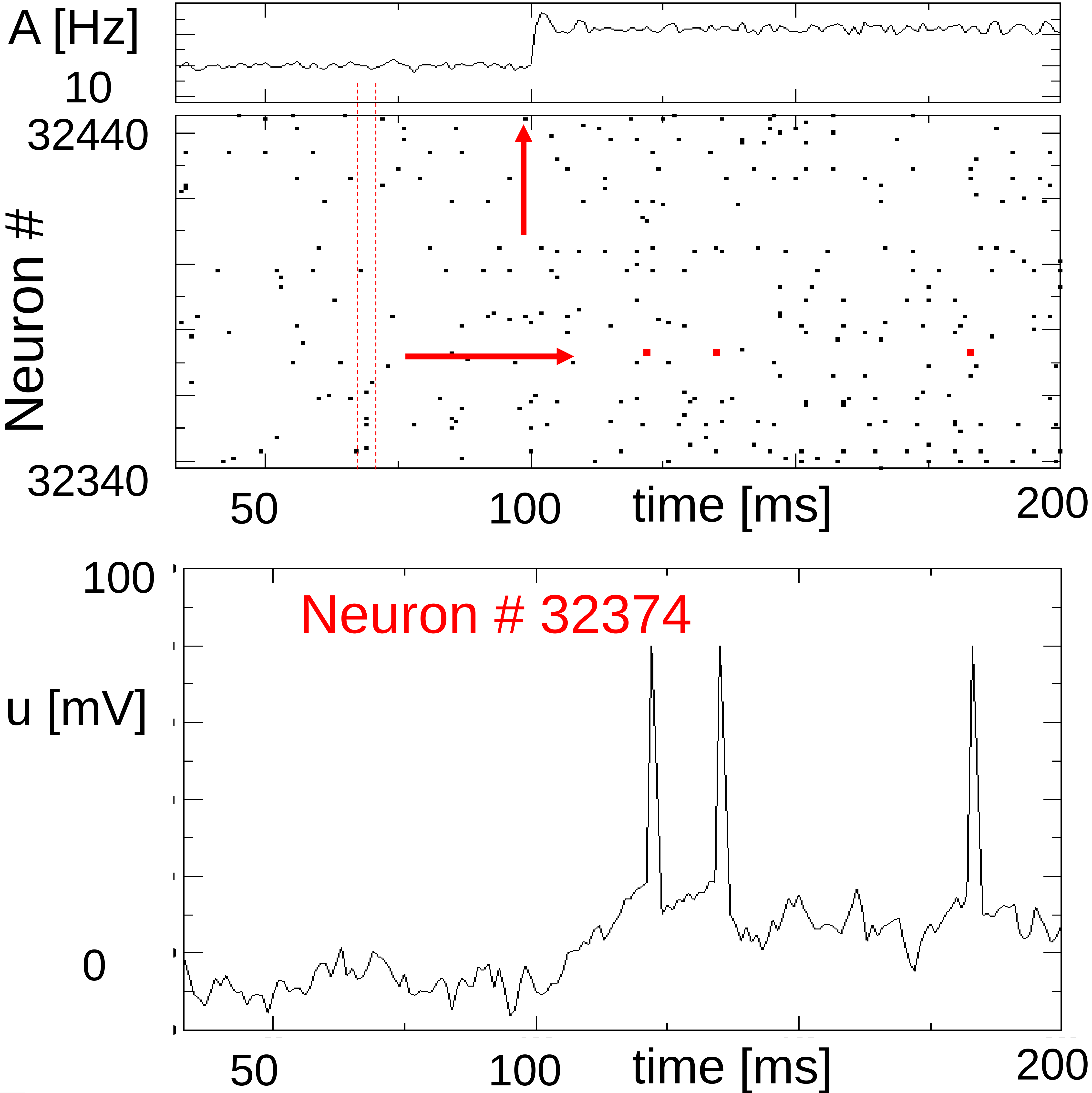
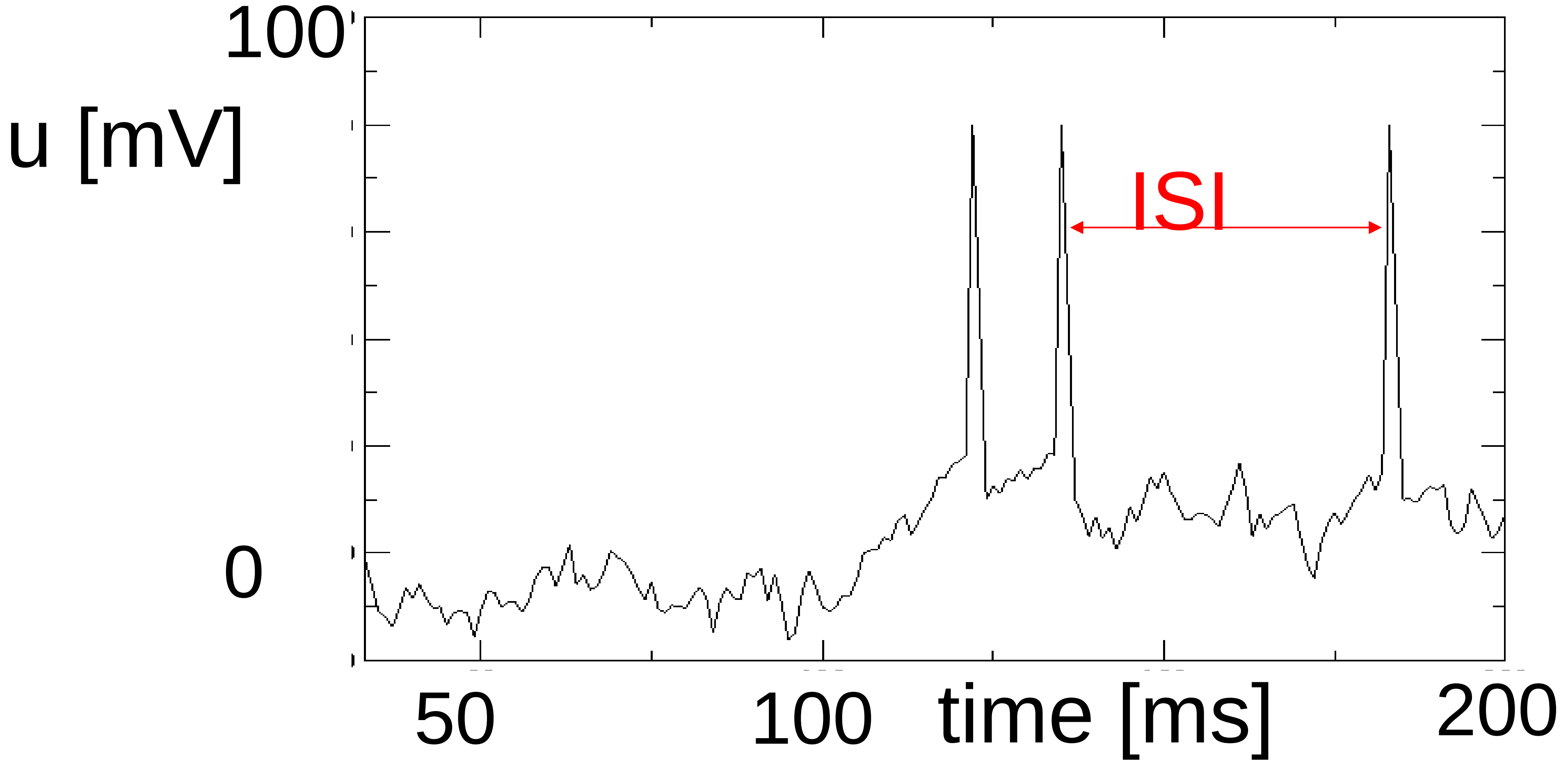
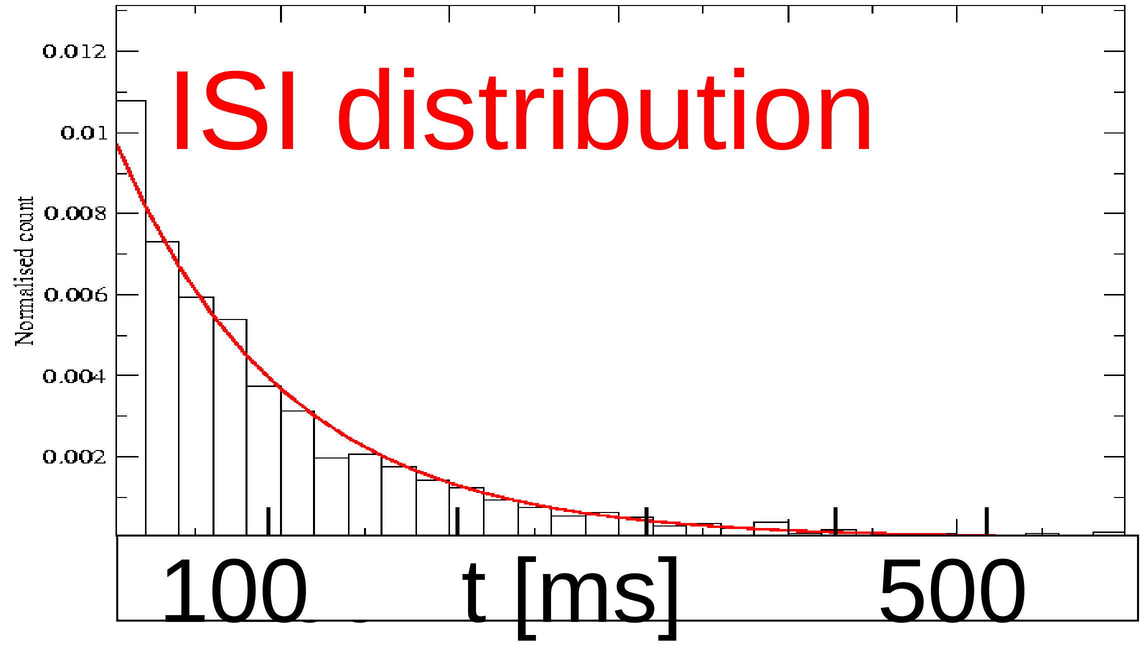


Neuronal Dynamics – 5.2. Interspike interval distribution

- Variability of interspike intervals (ISI)

here in simulations, but also *in vivo*



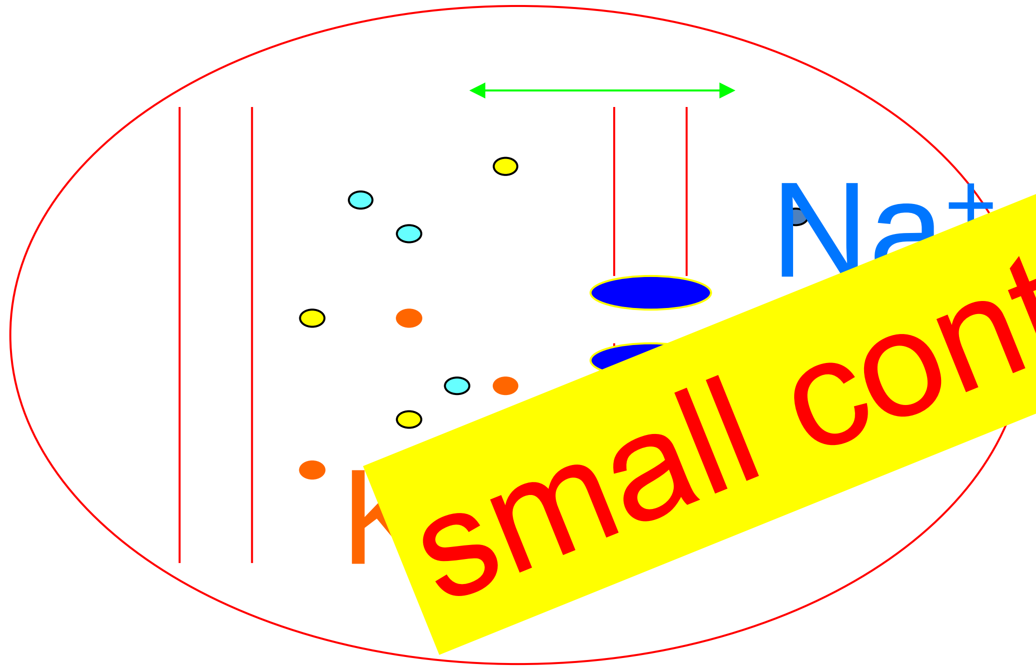


Variability of spike trains: broad ISI distribution

Brunel,
J. Comput. Neurosc. 2000
Mayor and Gerstner,
Phys. Rev E. 2005
Vogels and Abbott,
J. Neuroscience, 2005

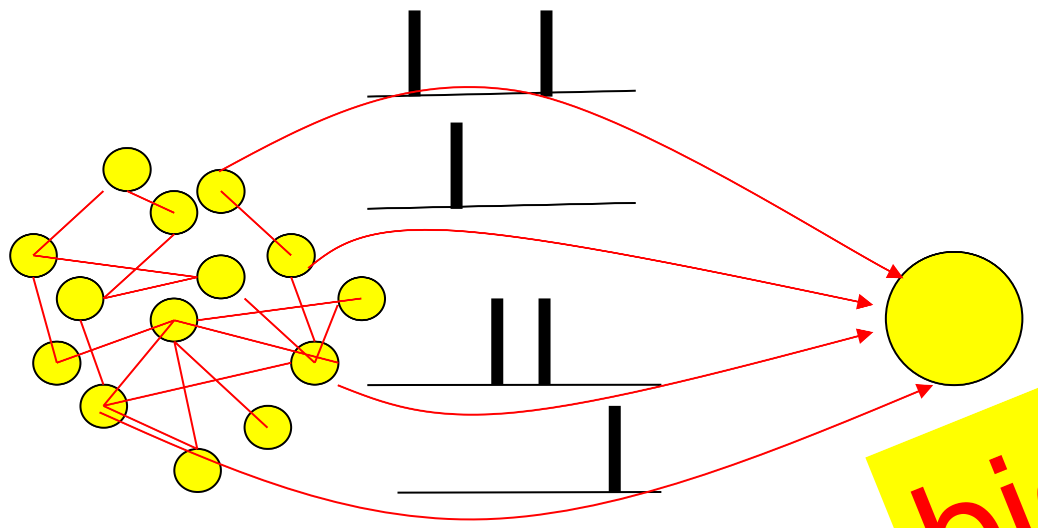
Neuronal Dynamics – 5.2. Sources of Variability

- Intrinsic noise (ion channels)



small contribution!

-Network noise



big contribution!

Neuronal Dynamics – Quiz 5.1.

A- Spike timing in vitro and in vivo

[ ] Reliability of spike timing can be assessed by repeating several times the same stimulus

[ ] Spike timing in vitro is more reliable under injection of constant current than with fluctuating current

[ ] Spike timing in vitro is less reliable under injection of constant current than with fluctuating current

[ ] Spike timing in vitro is more reliable than spike timing in vivo

[ ] Nothing is known about spike timing in humans in vivo

B – Interspike Interval Distribution (ISI)

[ ] An isolated deterministic leaky integrate-and-fire neuron driven by a constant current can have a broad ISI

[ ] A deterministic leaky integrate-and-fire neuron embedded into a randomly connected network of integrate-and-fire neurons can have a broad ISI

[ ] An isolated deterministic Hodgkin-Huxley model as in week 2 driven by a constant current can have a broad ISI

Week 5 – part 3a :Three definitions of rate code

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- experiments

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- Is variability equal to noise?

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5.5. Stochastic spike firing
- subthreshold and superthreshold

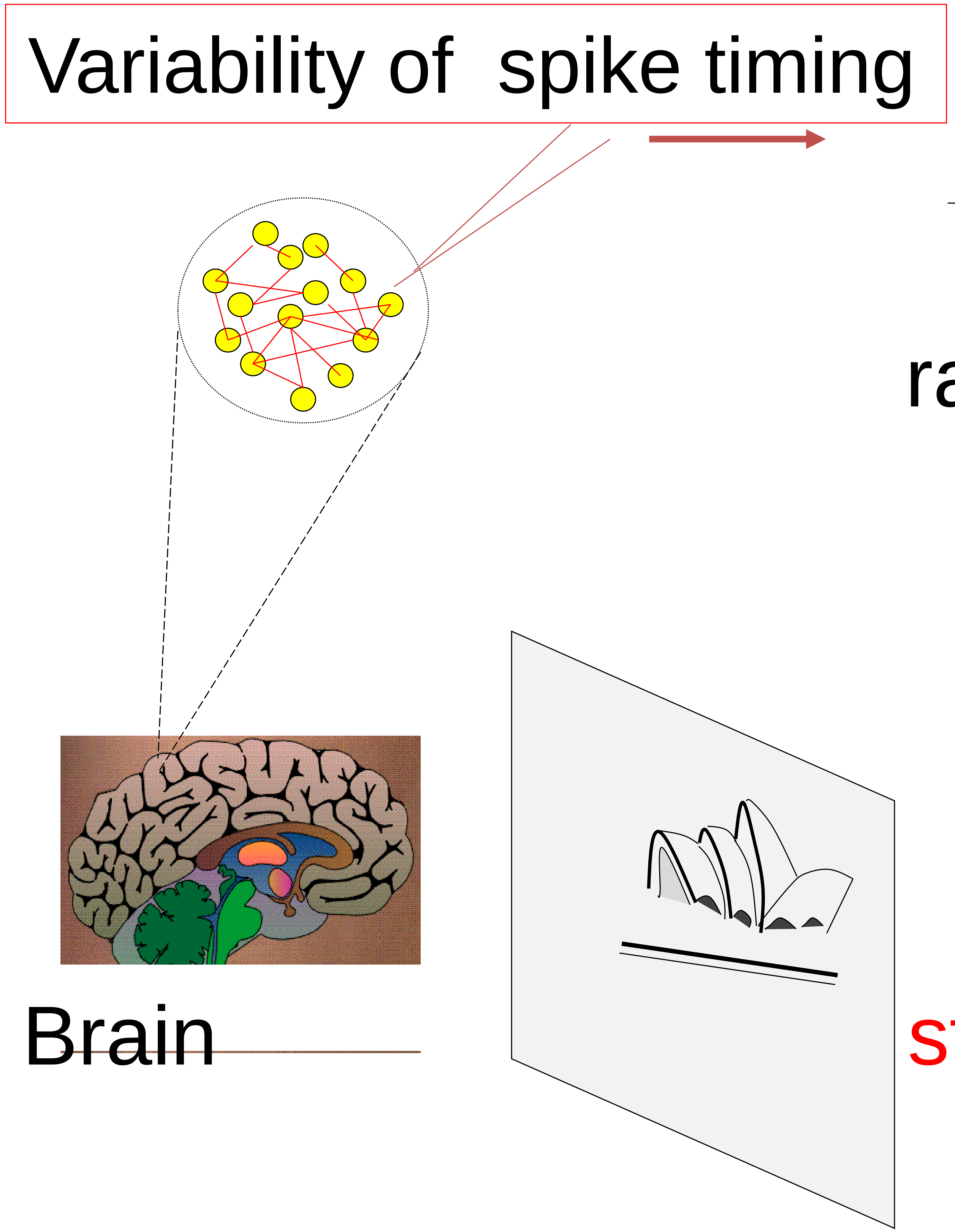
3 definitions

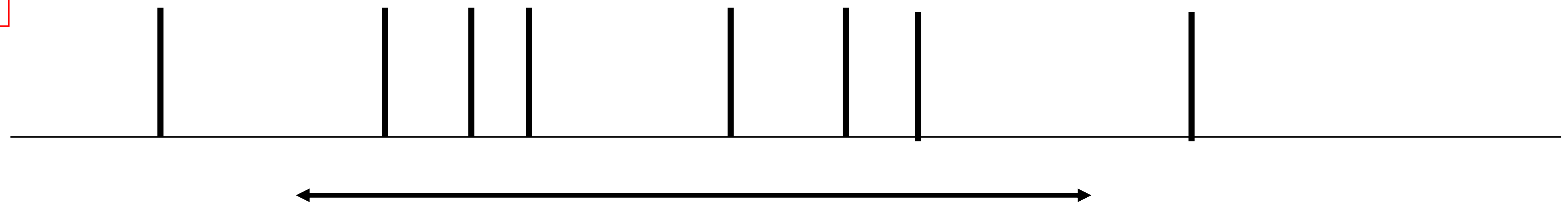
-Temporal averaging

- Averaging across repetitions

- Population averaging (‘spatial’ averaging)

Neuronal Dynamics – 5.3. Rate codes: spike count





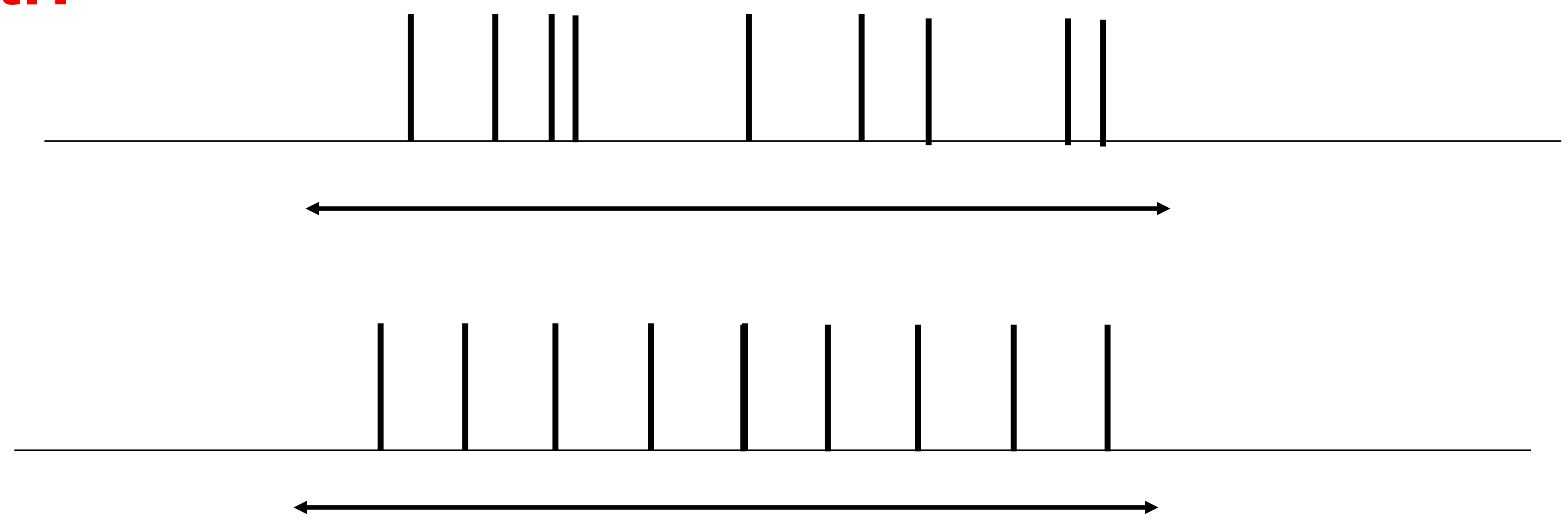
rate as a (normalized) spike count:

$$\nu(t) = \frac{n^{sp}}{T}$$

single neuron/single trial:
temporal average

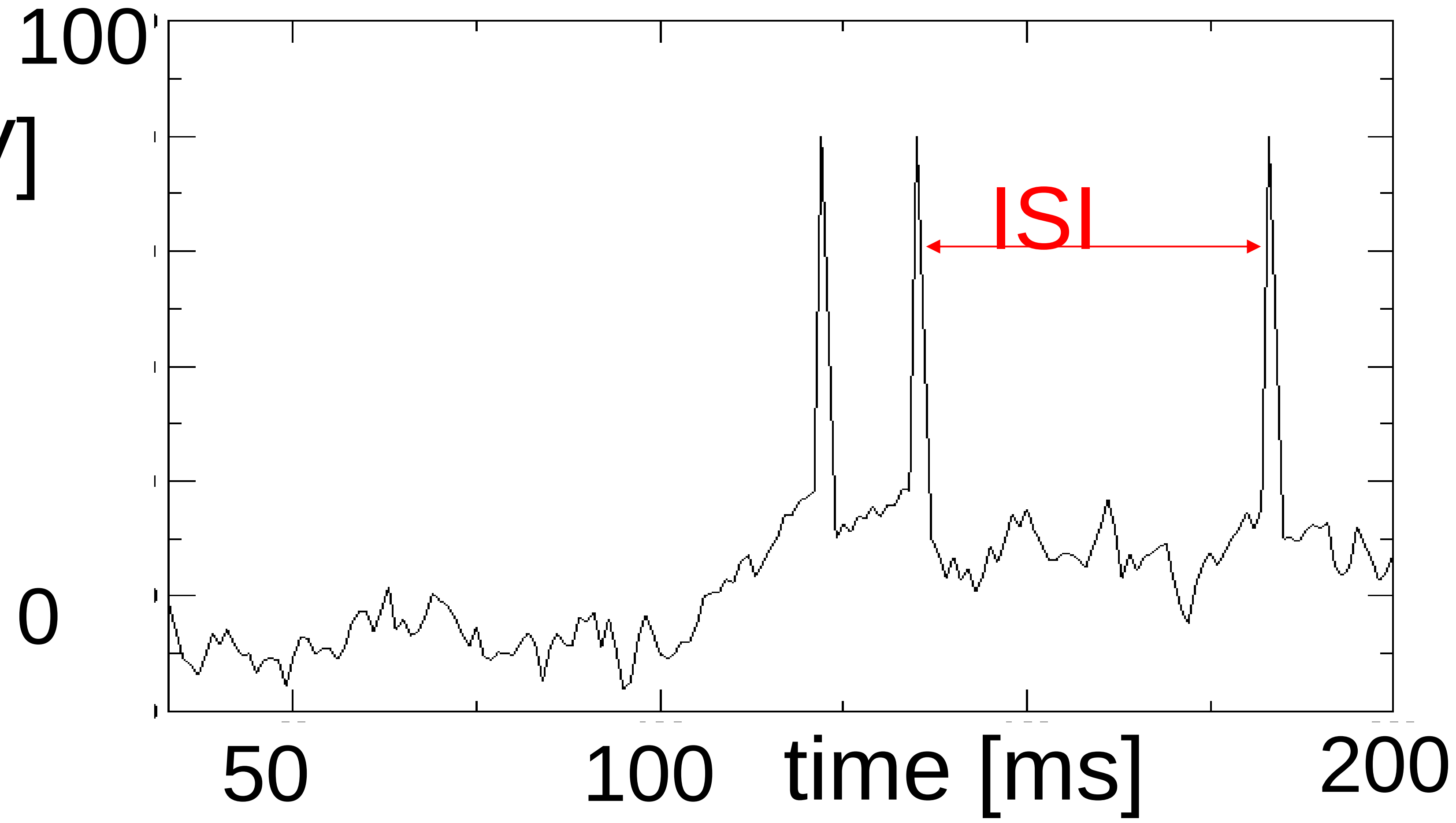
Neuronal Dynamics – 5.3. Rate codes: spike count

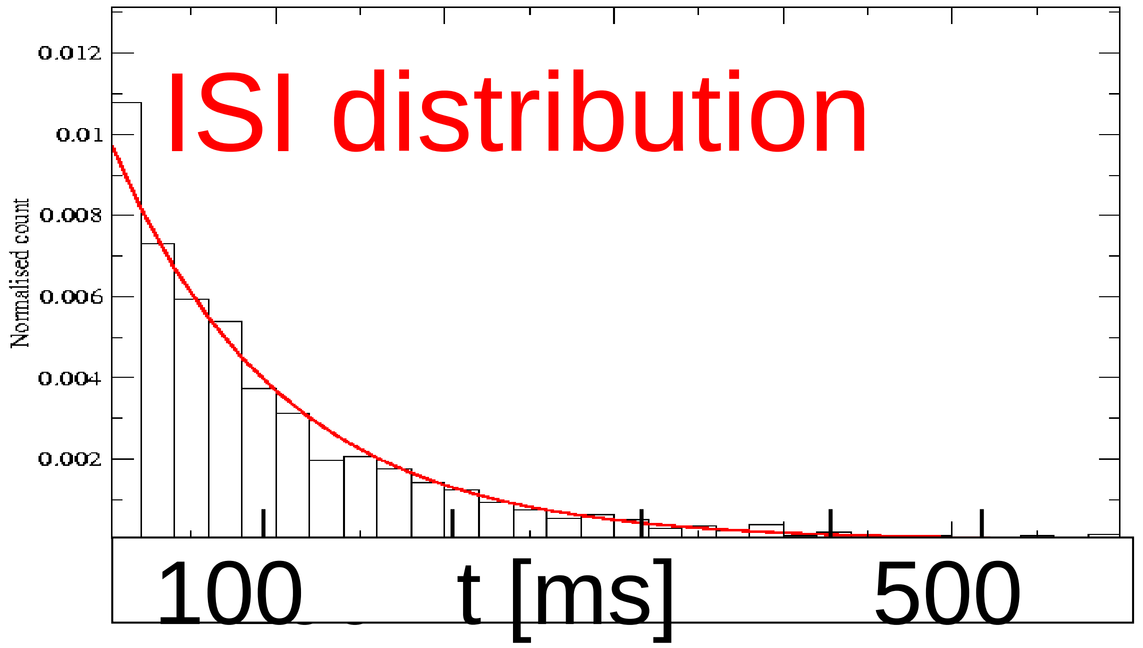
single neuron/single trial:
temporal average



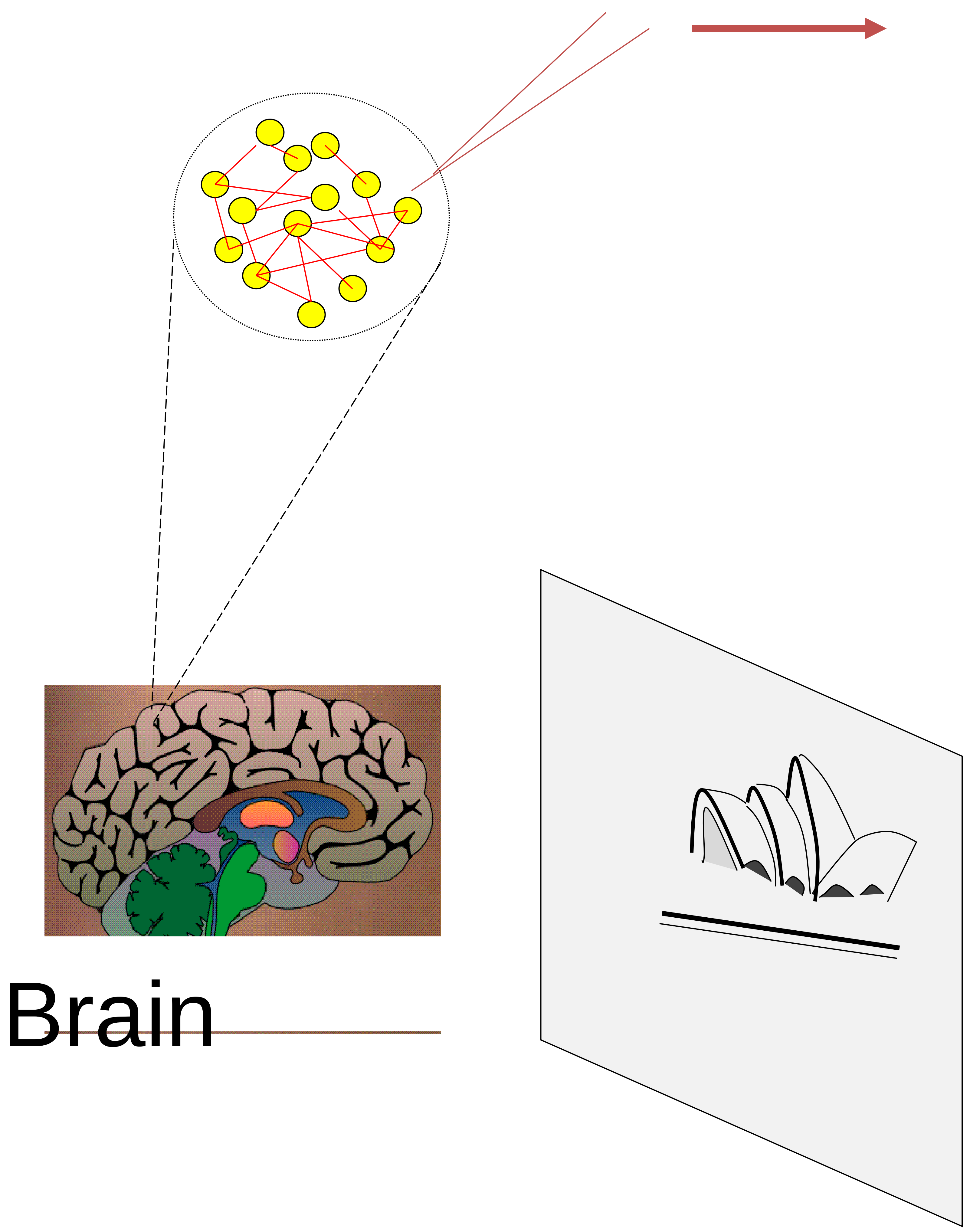
$$\nu(t) = \frac{n^{sp}}{T}$$

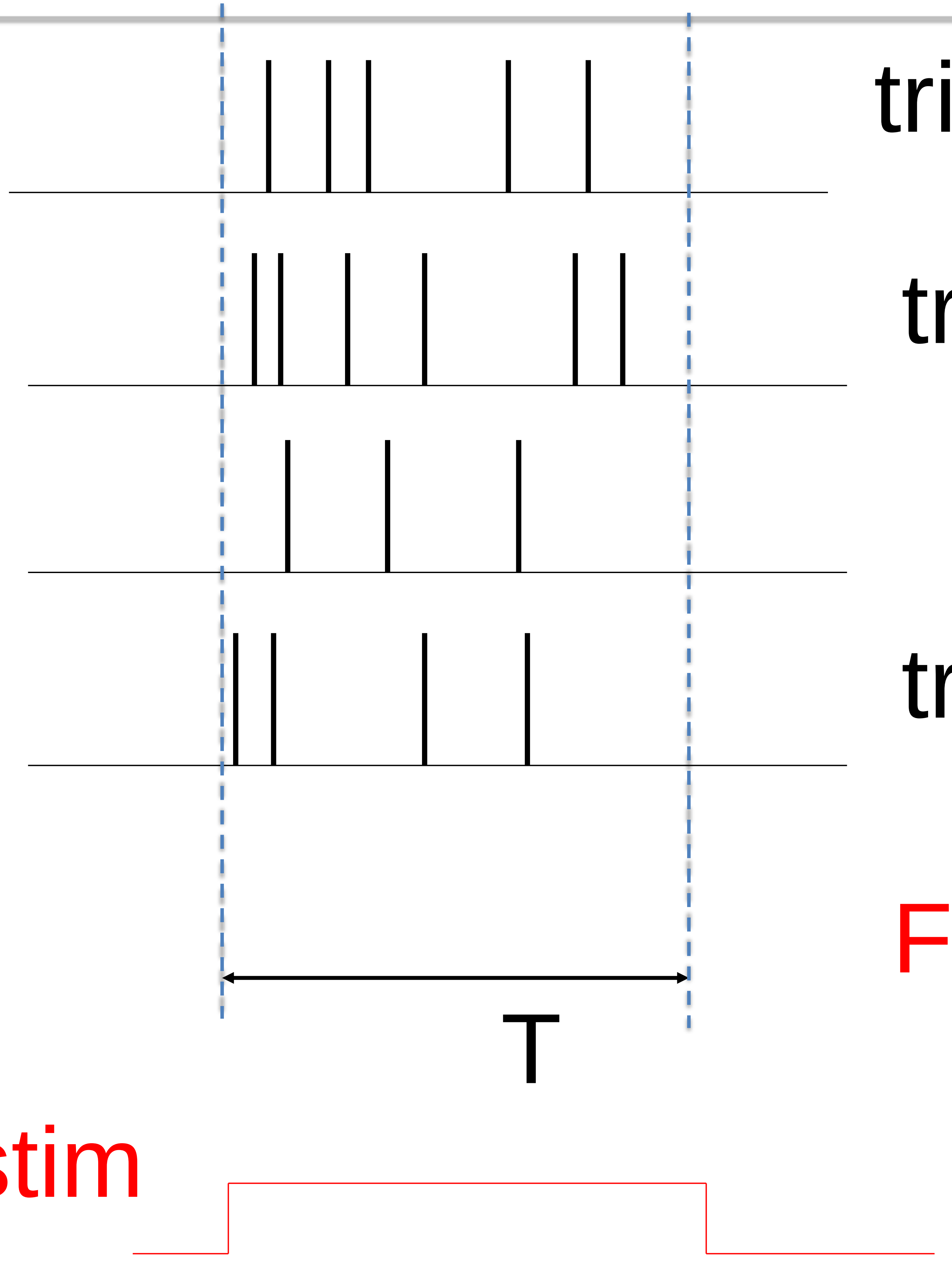
Variability of interspike intervals (ISI) measure regularity





Neuronal Dynamics – 5.3. Spike count: FANO factor





trial 1 $\quad n_1^{sp} = 5$

trial 2 $\quad n_2^{sp} = 6$

trial K $\quad n_K^{sp} = 4$

Fano factor

$$F = \frac{\left\langle \left(n_k^{sp} - \left\langle n_k^{sp} \right\rangle \right)^2 \right\rangle}{\left\langle n_k^{sp} \right\rangle}$$

3 definitions

√ -Temporal averaging (spike count)

Problem: slow!!!

ISI distribution (regularity of spike train)

Fano factor (repeatability across repetitions)

- Averaging across repetitions

- Population averaging (‘spatial’ averaging)

3 definitions

√ -Temporal averaging

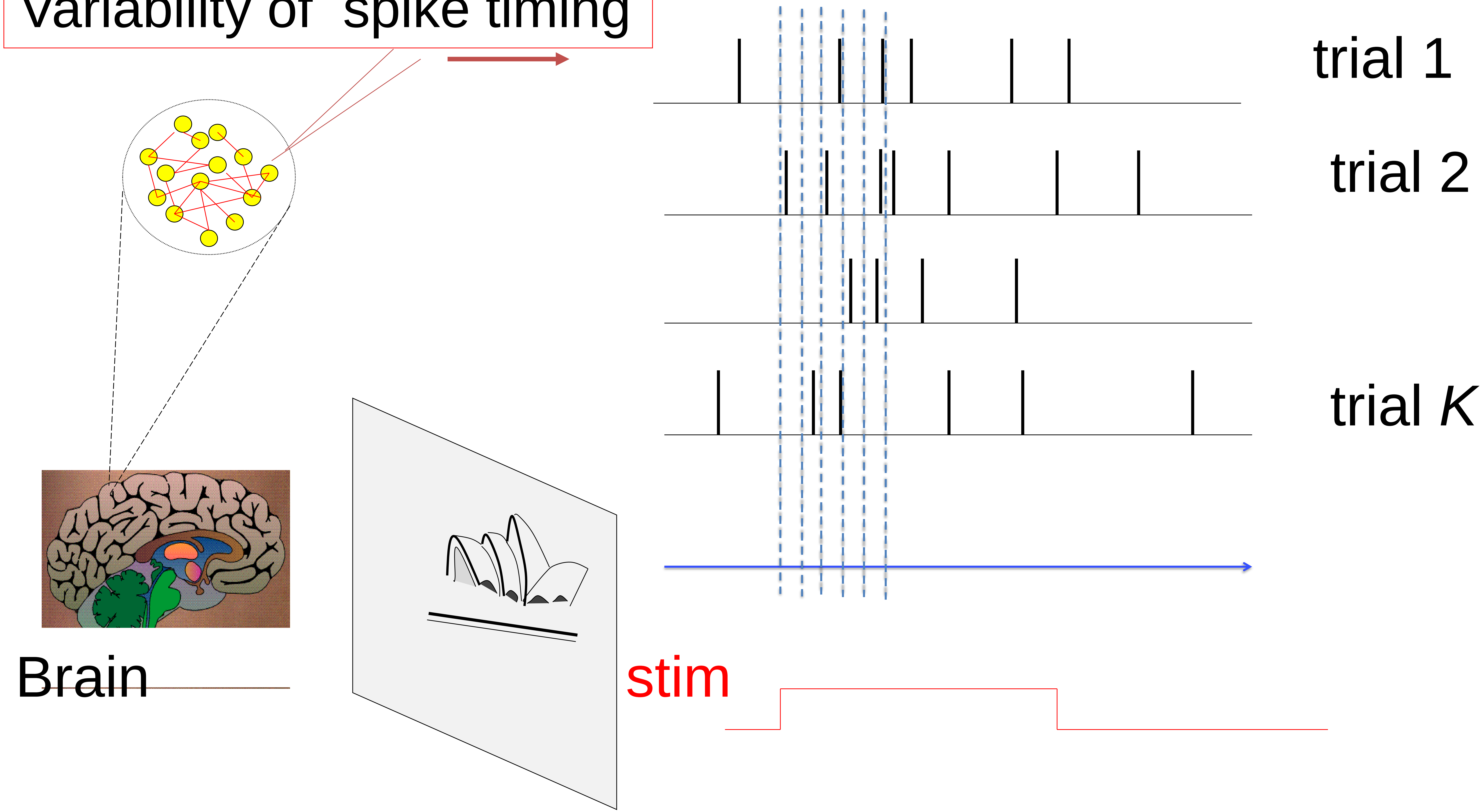
Problem: slow!!!

- Averaging across repetitions

- Population averaging

Neuronal Dynamics – 5.3. Rate codes: PSTH

Variability of spike timing



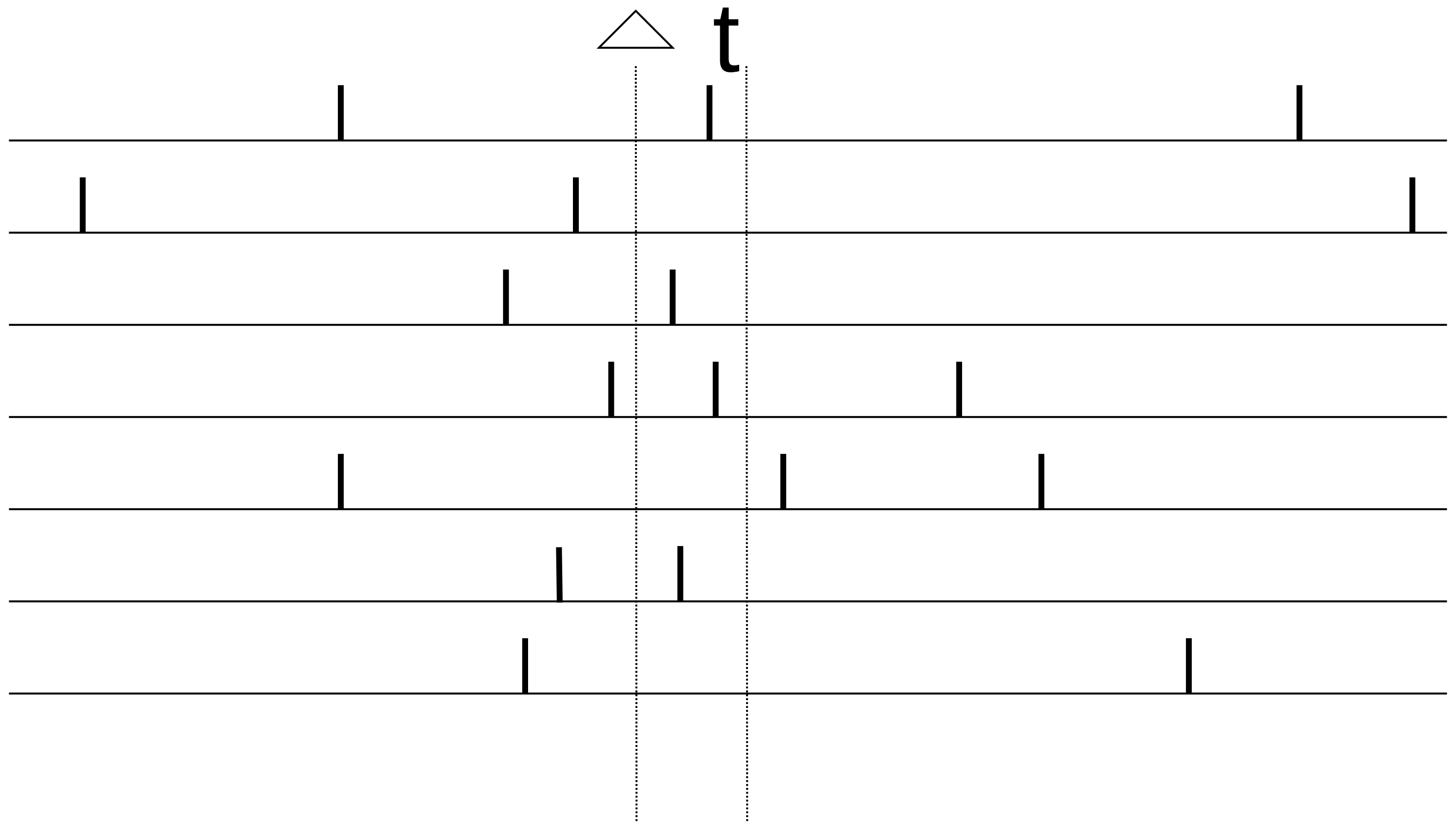
Neuronal Dynamics – 5.3. Rate codes: PSTH

Averaging across repetitions

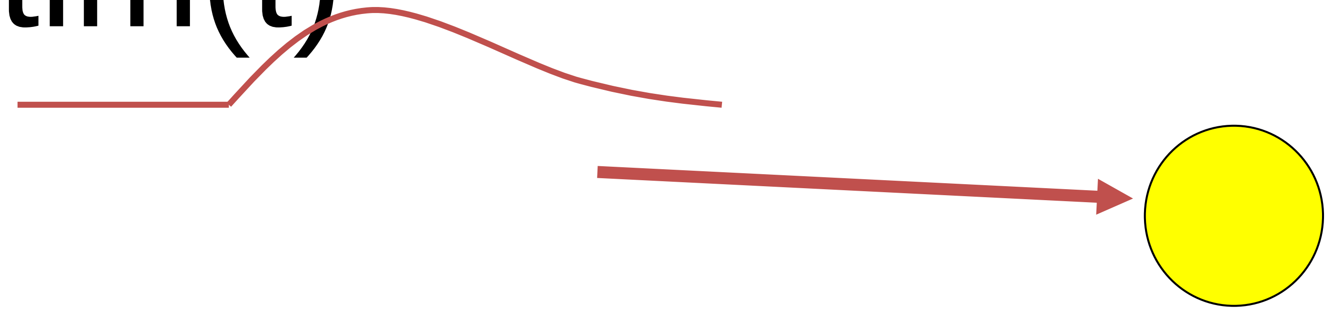
single neuron/many trials: average across trials

$$PSTH(t) = \frac{n(t;t+\Delta t)}{K\,\Delta t}$$

K repetitions

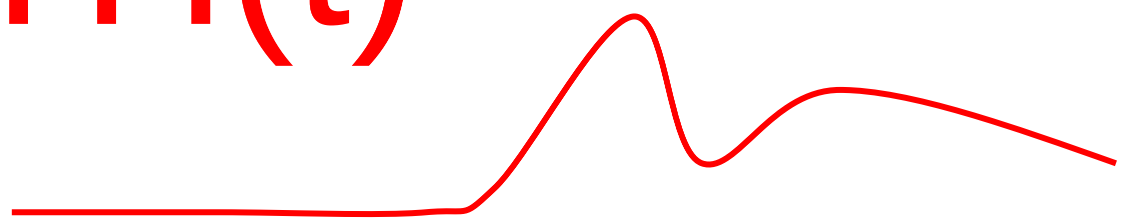


Stim(t)



K=50 trials

PSTH(t)



3 definitions

√ -Temporal averaging

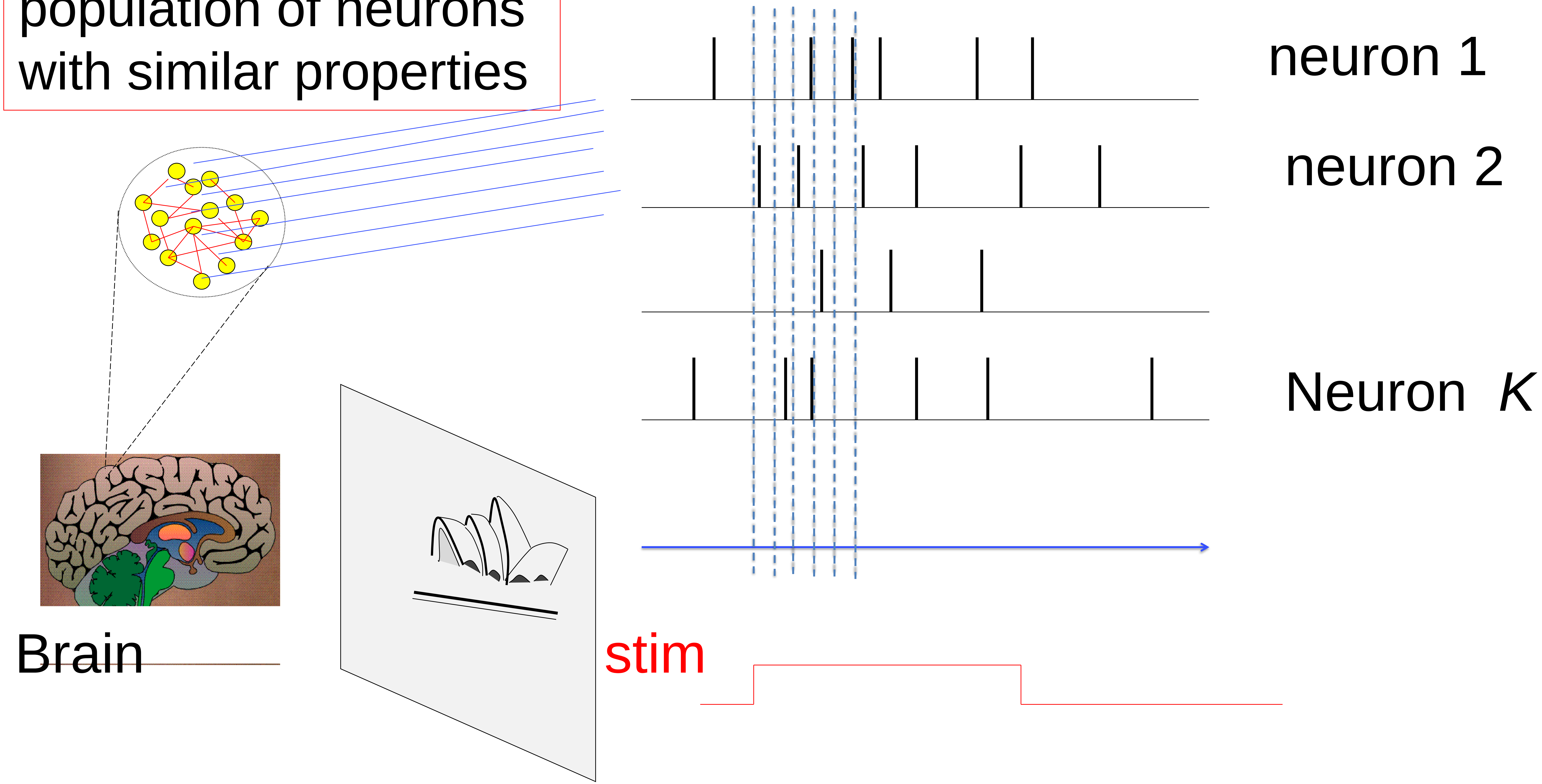
√ - Averaging across repetitions

Problem: not useful for animal!!!

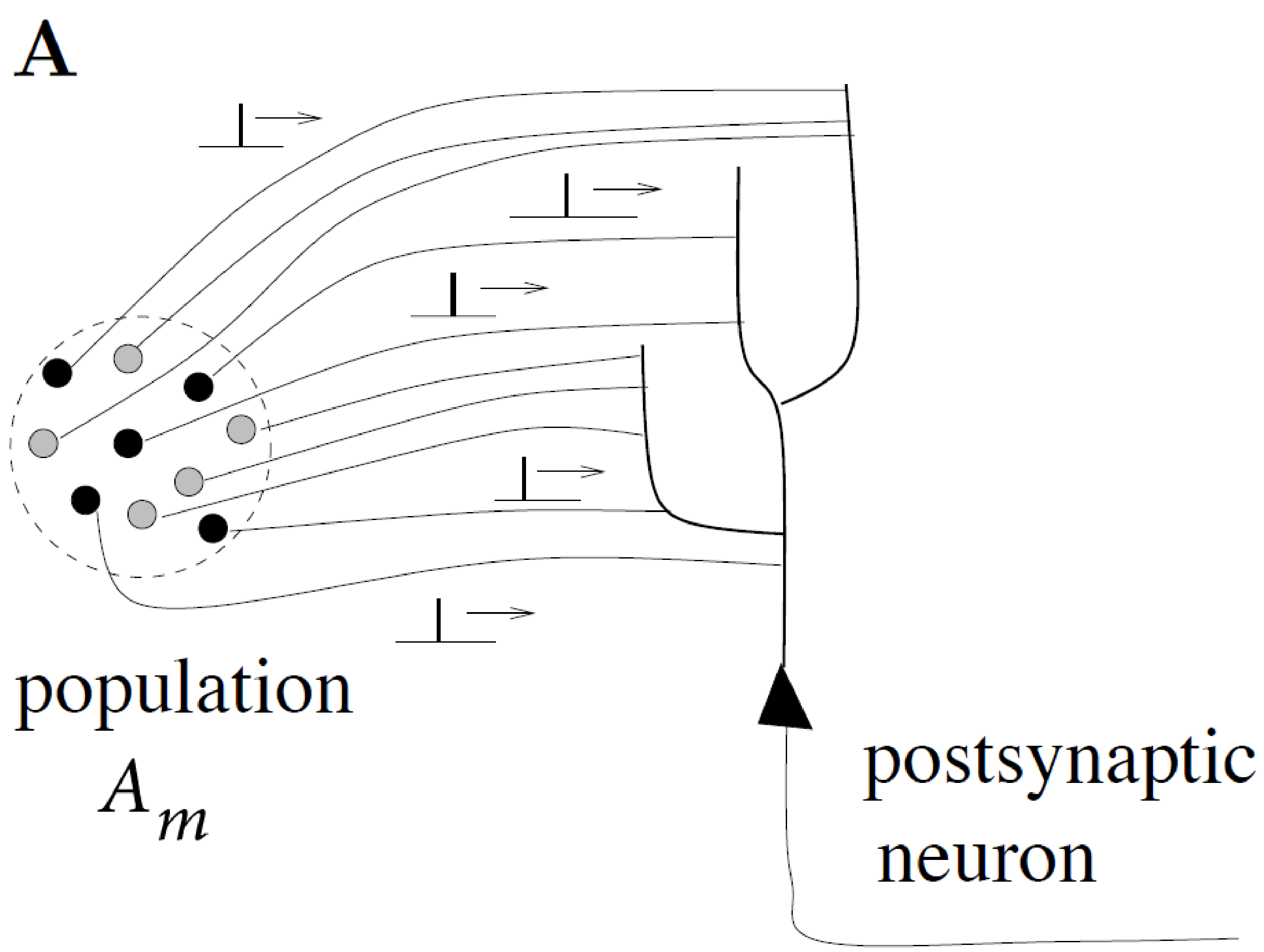
- Population averaging

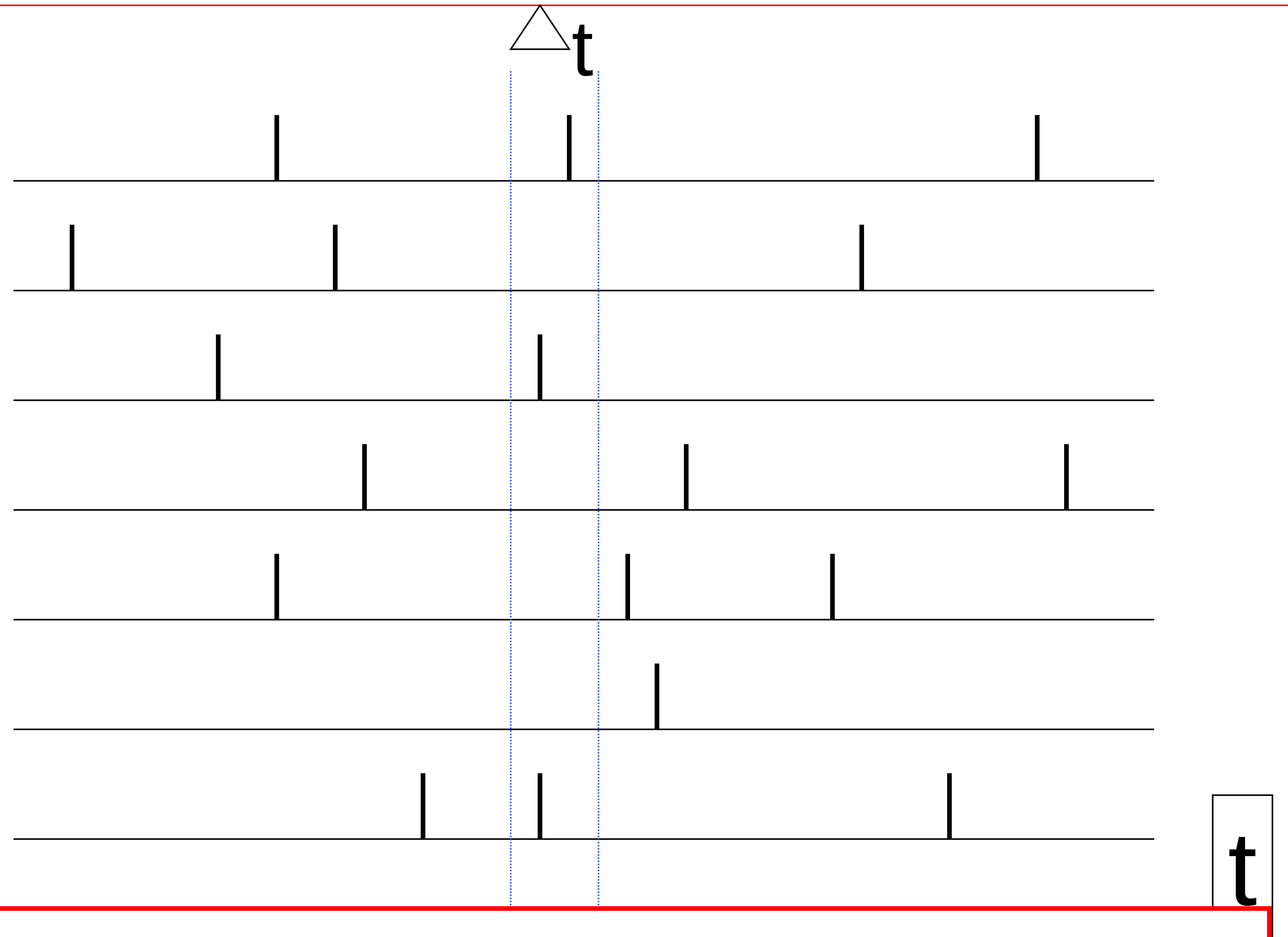
Neuronal Dynamics – 5.3. Rate codes: population activity

population of neurons
with similar properties



population activity - rate defined by population average





‘natural’

population activity

$$A(t) = \frac{n(t;t+\Delta t)}{N\Delta t}$$

Three averaging methods

-over time

Too slow
for animal!!!

- over repetitions

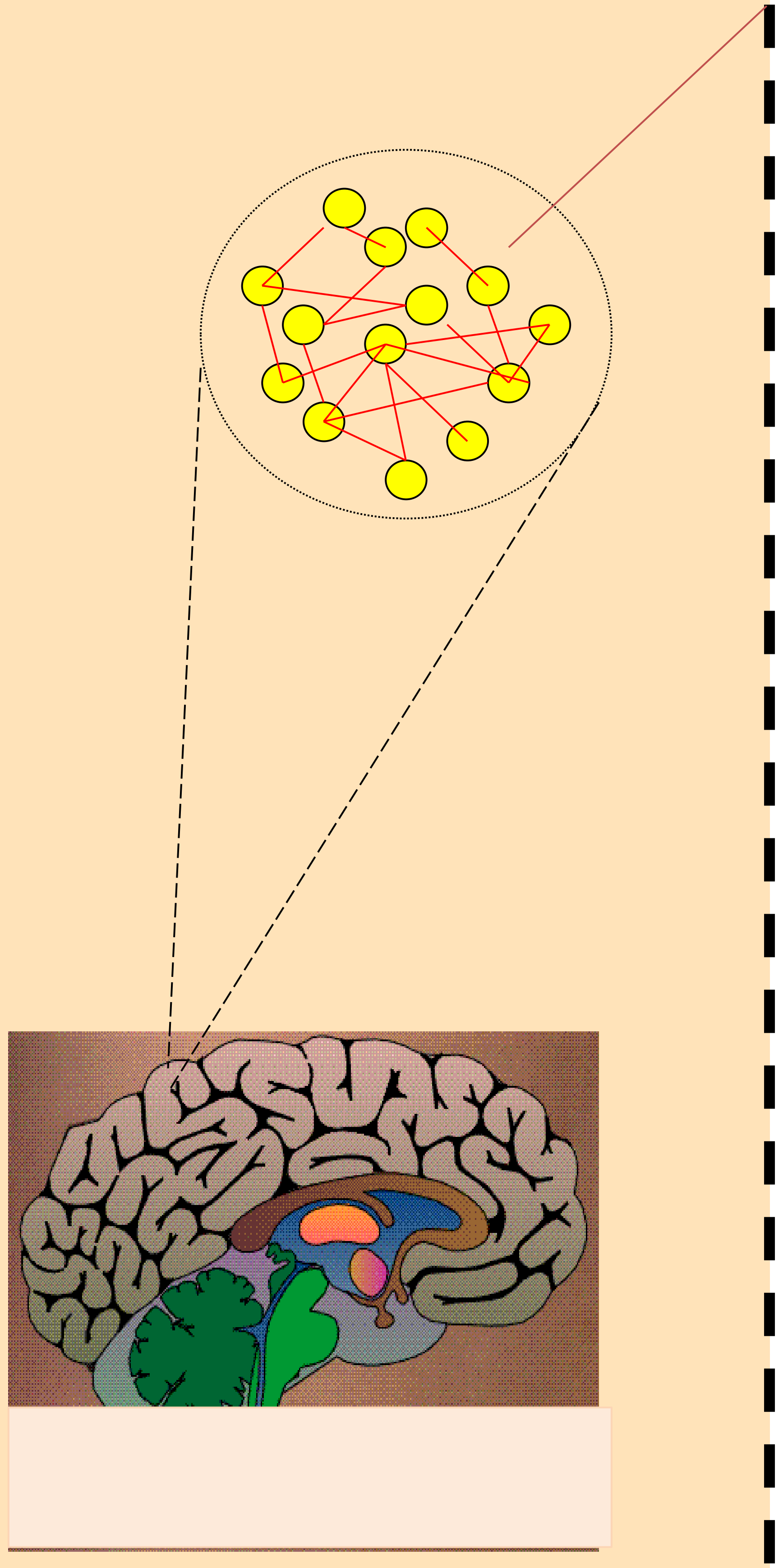
Not possible
for animal!!!

- over population (space)

'natural'

Neuronal Dynamics – Quiz 5.2.

Rate codes. Suppose that in some brain area we have a group of 500 neurons. All neurons have identical parameters and they all receive the same input. Input is given by sensory stimulation and passes through 2 preliminary neuronal processing steps before it arrives at our group of 500 neurons. Within the group, neurons are not connected to each other. Imagine the brain as a model network containing 100 000 nonlinear integrate-and-fire neurons, so that we know exactly how each neuron functions.



Experimentalist A makes a measurement in a single trial on all 500 neurons using a multi-electrode array, during a period of sensory stimulation.

Experimentalist B picks an arbitrary single neuron and repeats the same sensory stimulation 500 times (with long pauses in between, say one per day).

Experimentalist C repeats the same sensory stimulation 500 times (1 per day), but every day he picks a random neuron (amongst the 500 neurons).

All three determine the time-dependent firing rate.

[ ] A and B and C are expected to find the same result.

[ ] A and B are expected to find the same result, but that of C is expected to be different.

[ ] B and C are expected to find the same result, but that of A is expected to be different.

[ ] None of the above three options is correct.

Week 5 – part 3b :Poisson Model

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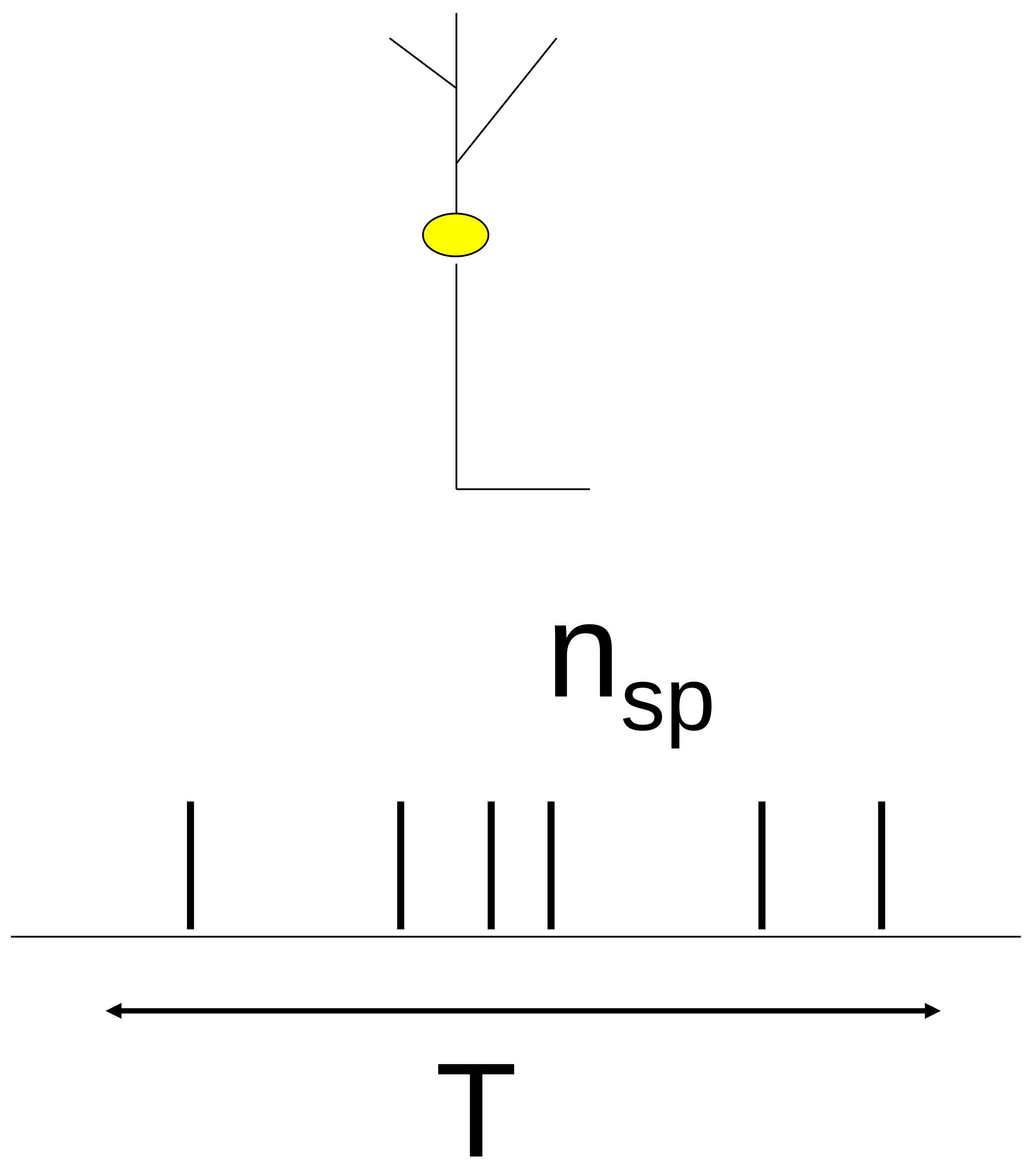
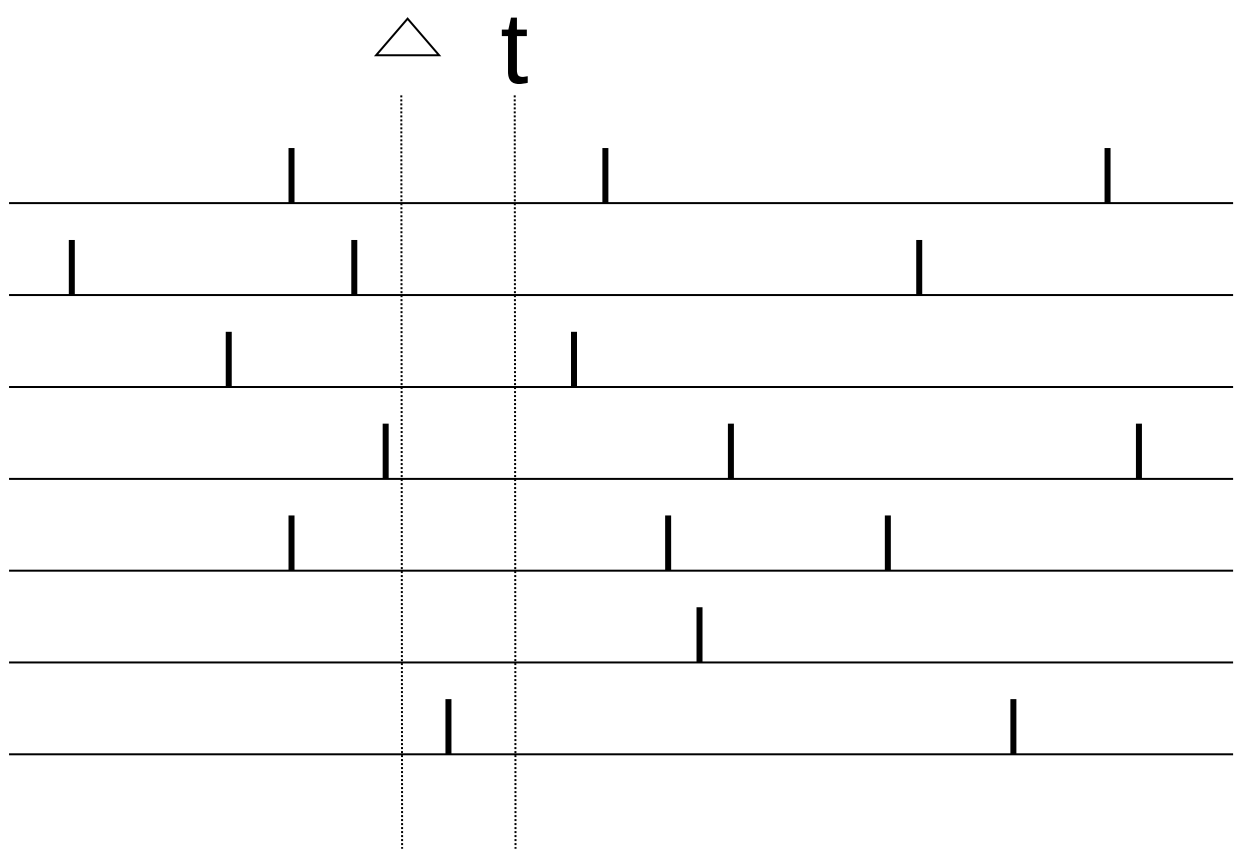
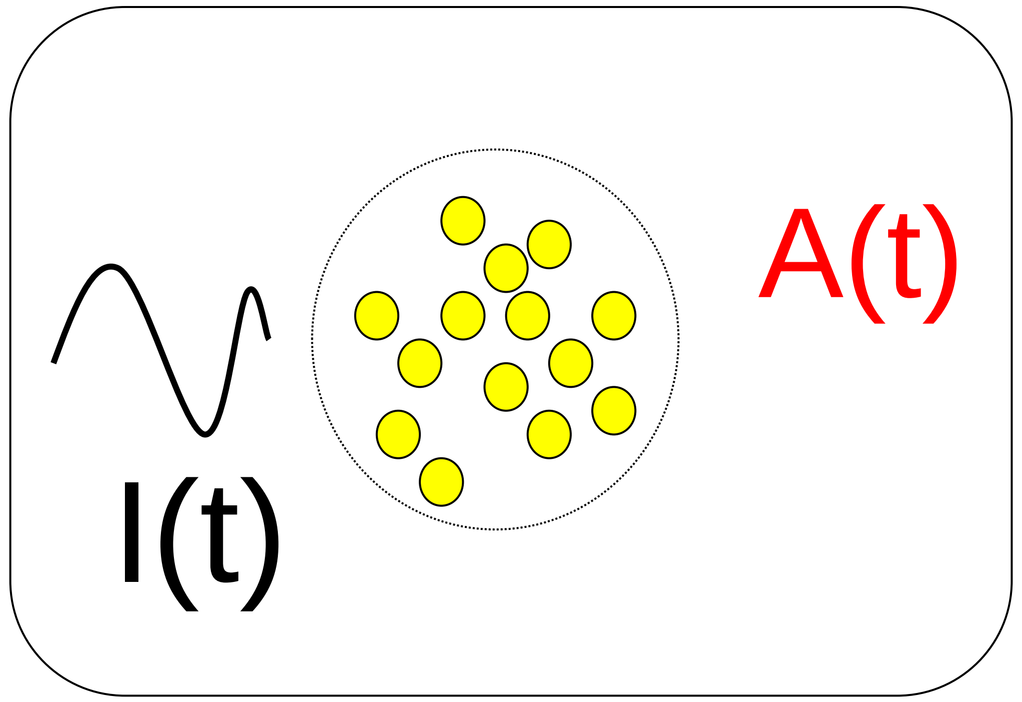
- subthreshold and superthreshold

Neuronal Dynamics – 5.3b. Inhomogeneous Poisson Process





$$PSTH(t) = \frac{n(t;t+\Delta t)}{K\,\Delta t}$$



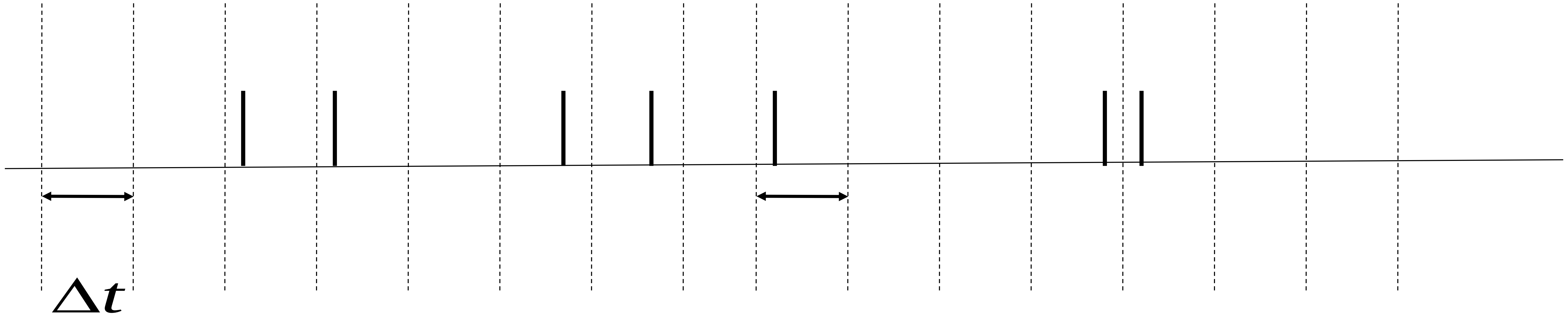
$$A(t) = \frac{n(t;t+\Delta t)}{N\Delta t}$$

population activity

Pure rate code = stochastic spiking → Poisson model

Homogeneous Poisson model: constant rate

Math detour: Poisson model

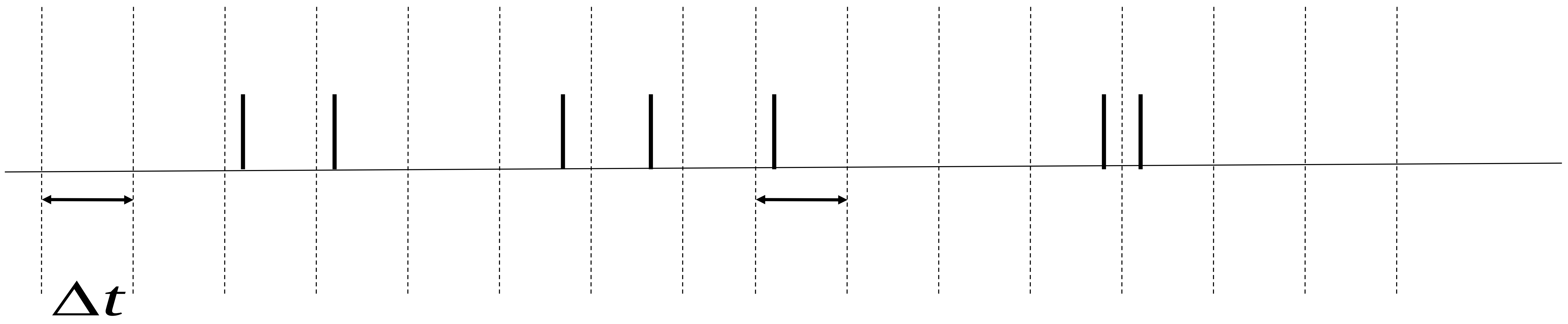


Probability of finding a spike $P_F = \rho_0 \Delta t$

Pure rate code = stochastic spiking → Poisson model

Probability of firing:

$$P_F = \rho_0 \, \Delta t$$

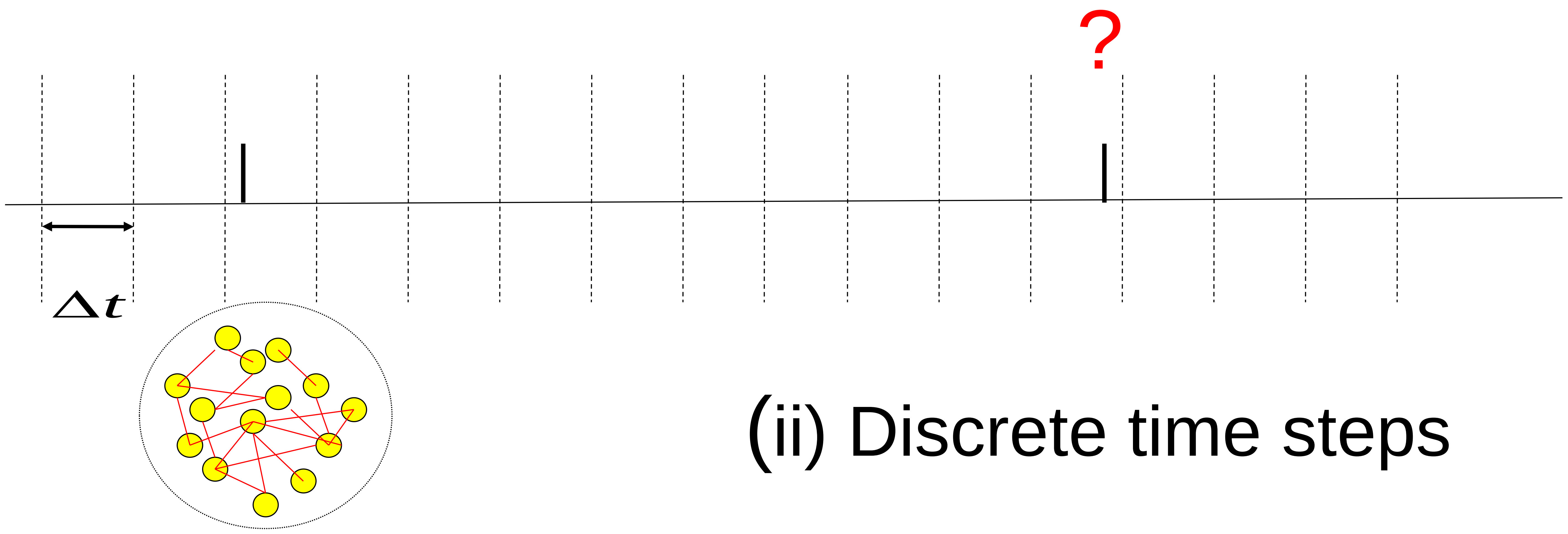


Take $\Delta t \rightarrow 0$

Neuronal Dynamics – 5.3b. Interval distribution

Probability of firing:

$$P_F = \rho_0 \, \Delta t$$



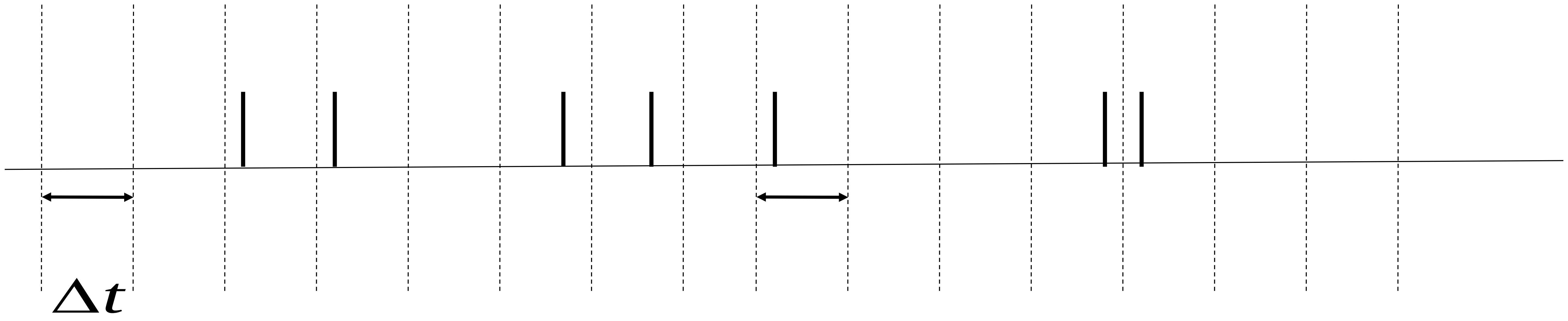
(i) Continuous time

(ii) Discrete time steps

prob to 'survive'

$$\Delta t \to 0$$

rate changes

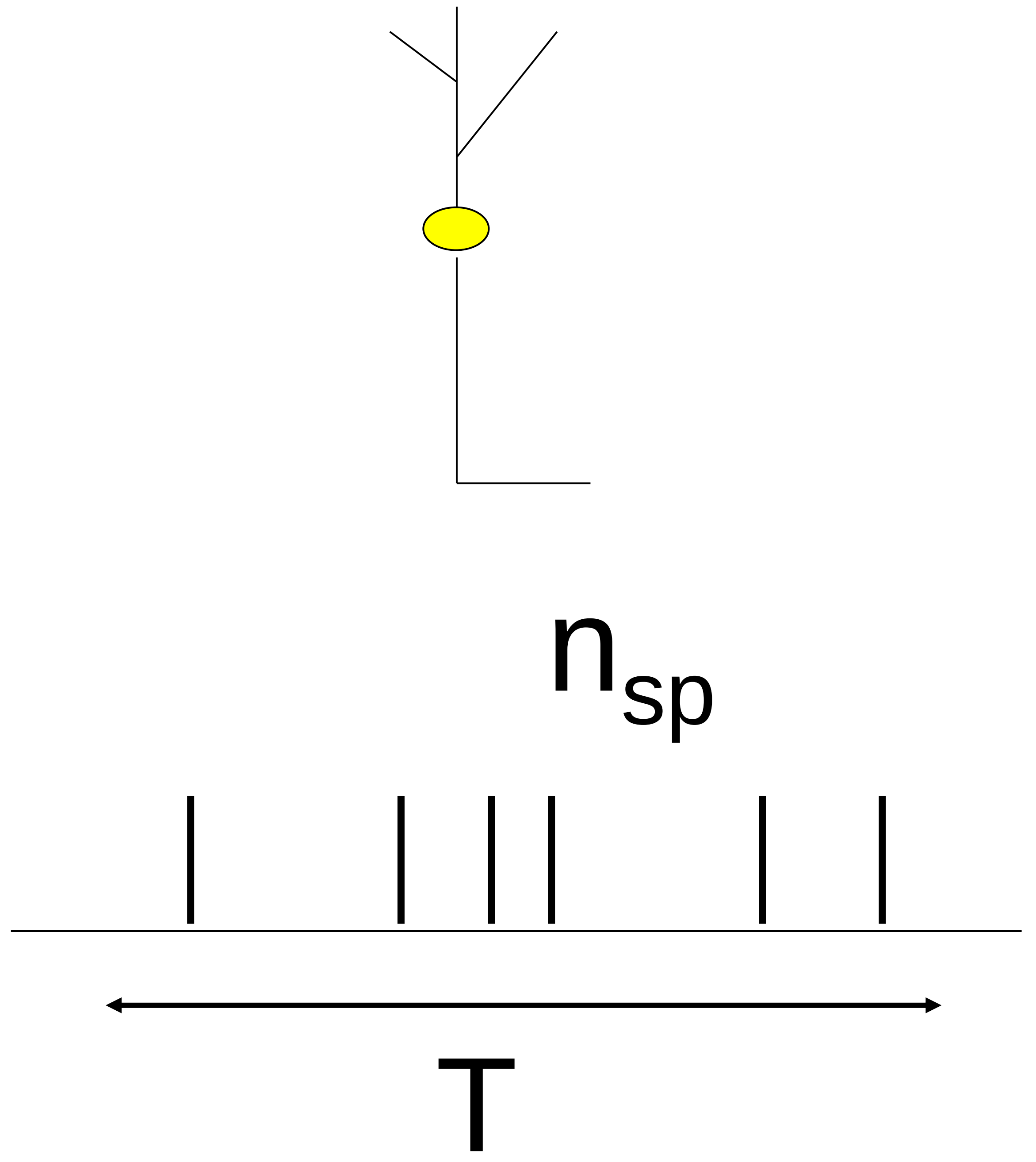


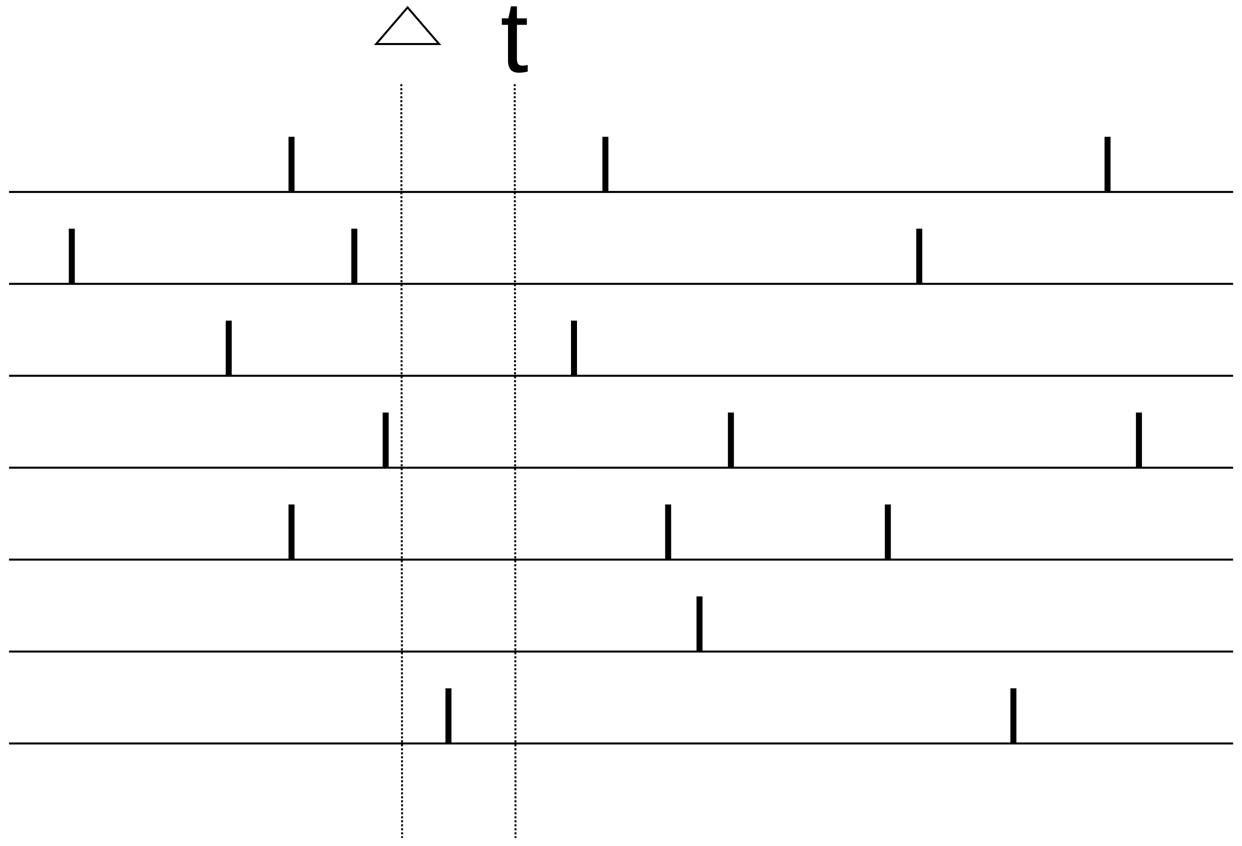
Probability of firing $P_F = \rho(t)\,\Delta t$

Survivor function $S(t\,|\,\hat{t}) = \exp(-\int_{\hat{t}}^{t} \rho(t')\,dt')$

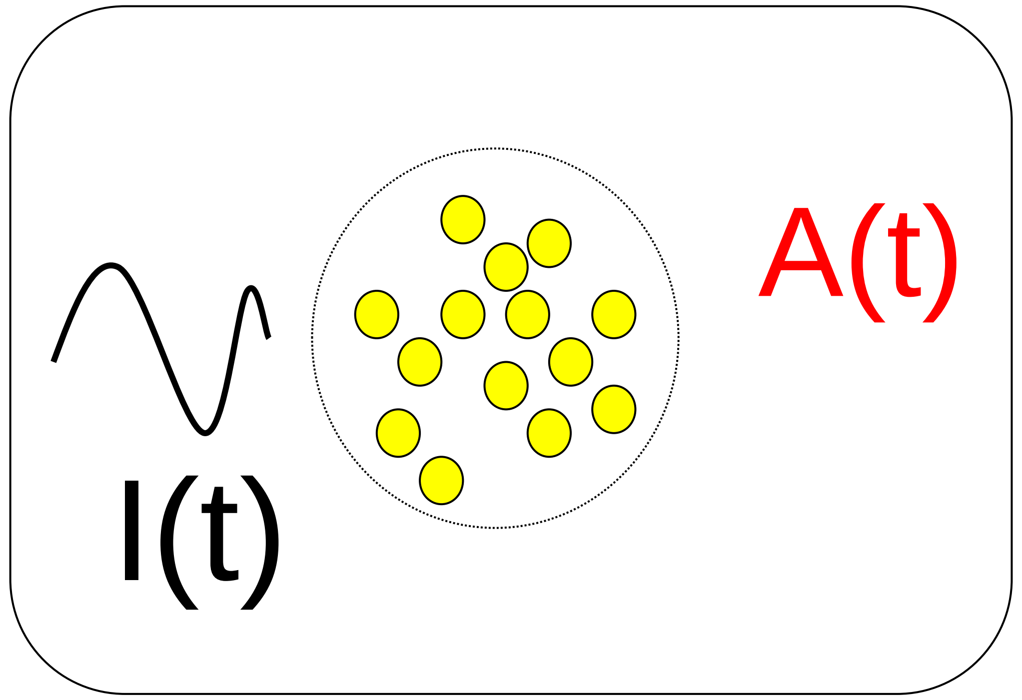
Interval distribution

Neuronal Dynamics – 5.3b. Inhomogeneous Poisson Process





$$PSTH(t) = \frac{n(t;t+\Delta t)}{K\,\Delta t}$$



$$A(t) = \frac{n(t;t+\Delta t)}{N\Delta t}$$

population activity

inhomogeneous Poisson model consistent with rate coding

Probability of firing

$$P_F = \rho(t)\,\Delta t$$

Survivor function

$$S(t\,|\,\hat{t}) = \exp\left(-\int_{\hat{t}}^{t} \rho(t')\,dt'\right)$$

Interval distribution

$$P(t\,|\,\hat{t}) = \rho(t)\exp\left(-\int_{\hat{t}}^{t} \rho(t')\,dt'\right)$$

Neuronal Dynamics – Quiz 5.3.

A Homogeneous Poisson Process:
A spike train is generated by a homogeneous Poisson process with rate 25Hz with time steps of 0.1ms.
[ ] The most likely interspike interval is 25ms.
[ ] The most likely interspike interval is 40 ms.
[ ] The most likely interspike interval is 0.1ms
[ ] We can't say.

B Inhomogeneous Poisson Process:
A spike train is generated by an inhomogeneous Poisson process with a rate that oscillates periodically (sine wave) between 0 and 50Hz (mean 25Hz). A first spike has been fired at a time when the rate was at its maximum. Time steps are 0.1ms.
[ ] The most likely interspike interval is 25ms.
[ ] The most likely interspike interval is 40 ms.
[ ] The most likely interspike interval is 0.1ms.
[ ] We can't say.

Poisson Processes:
A modern approach

Wulfram Gerstner

EPFL, Lausanne, Switzerland

Reading:

[1] A.W. Lewis, G.S. Shedler (1979), Simulation of nonhomogeneous Poisson processes by thinning, Naval Res. Logist. Q. 26: 403–413.

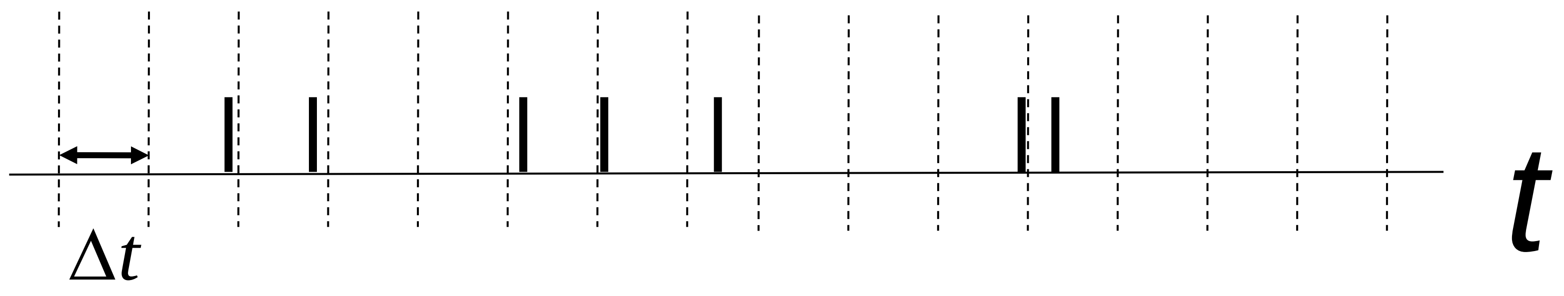
[2] V**. Schmutz** (2022), Mean-field limit of age and leaky memory dependent Hawkes processes. Stochastic Process. Appl., 149:39-59
https://doi.org/10.1016/j.spa.2022.03.006

[3] N. Fournier and E. **Löcherbach** (2016), On a toy model of interacting neurons. Annals Inst. H. Poincare, 52: 1844-1876
DOI: 10.1214/15-AIHP701

[4] J. Chevallier, Mean-field limit of generalized Hawkes processes (2017), Stochastic Process. Appl., 127:3870--3912. http://dx.doi.org/10.1016/j.spa.2017.02.012

Poisson Process (PP): 2 constructive procedures

classic procedure



Probability of generating an event

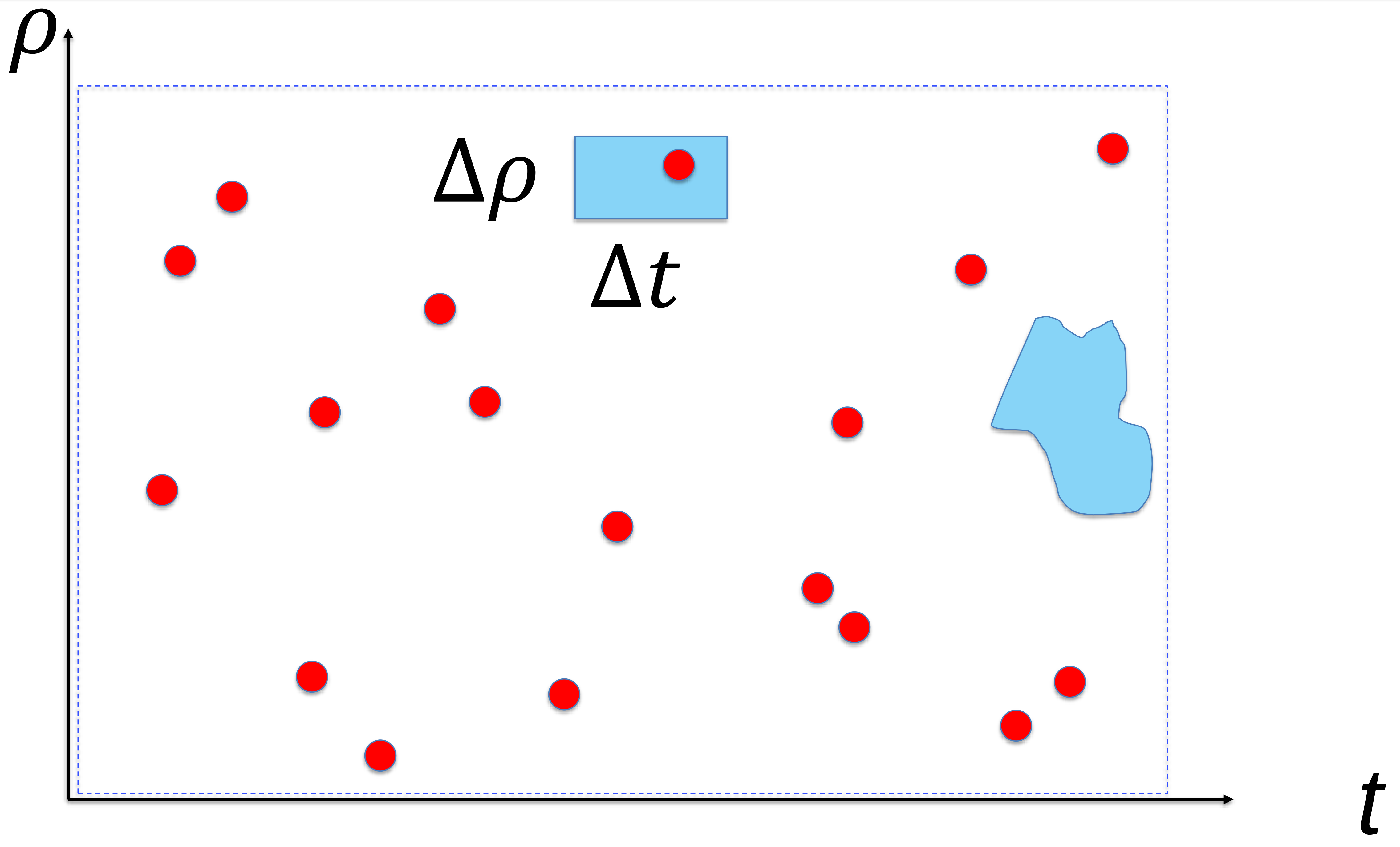
$$P_F = \rho_0 \Delta t$$

time step $\Delta t \rightarrow 0$

Inhomogeneous PP

$$P_F = \rho(t)\Delta t$$

call random number every time step



Probability of generating event: uniform in 2 dimensions:

E(number of events | area) = area

- choose number of events | total area
- call random number twice per event

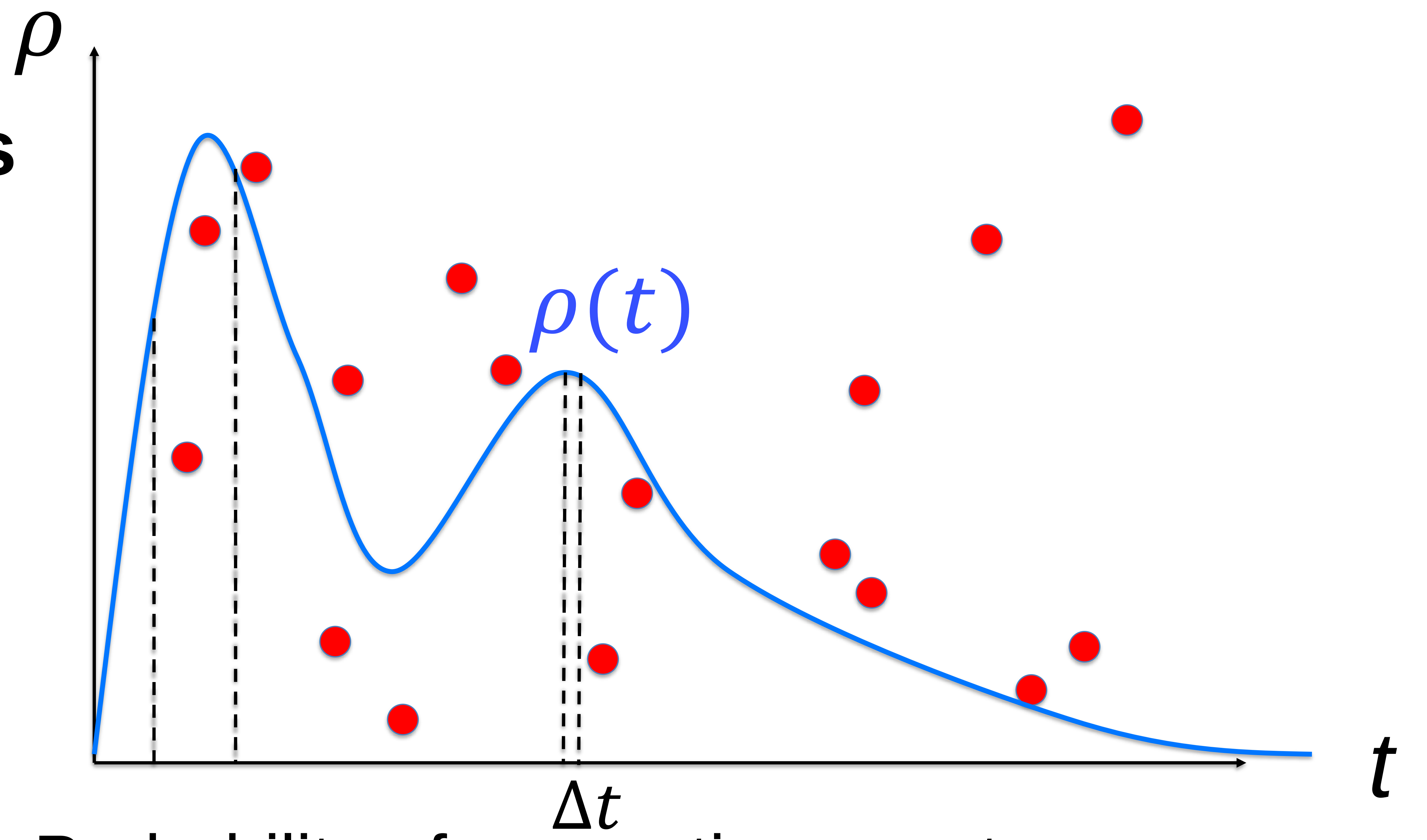
Efficient constructive procedure

Example:
inhomogeneous Poisson Process

rate (stochastic intensity):

$\rho(t) \quad = f(h(t))$

(i) create events in total area
(ii) visualize $\rho(t)$
(iii) project 'events below line' to *t*-axis
(iv) read-off event times
[(v) you may discretize]



Probability of generating event: uniform in 2 dimensions:

E(number of events | area) = area

- choose number of events | total area
- call random number twice per event

From 2 dimensions to spike trains $S(t)$ and counts $N(t)$

points in time yield:

(i) a pulse train

$$S_k(t) = \sum_f \delta(t - t_k^f)$$

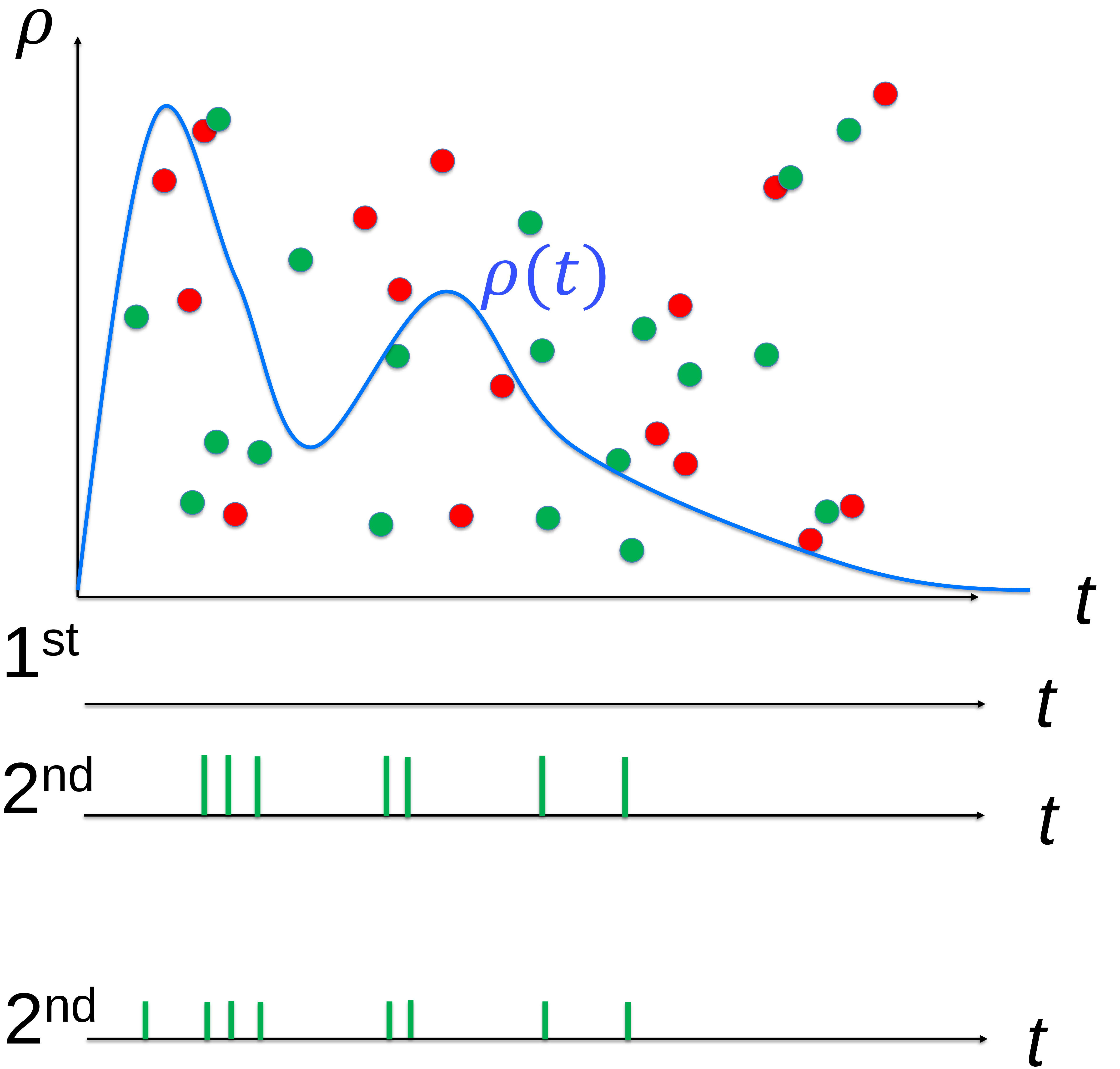
(ii) a counting process

$$N_k(t) = \int_0^t S_k(t')\, dt'$$

Expectation at time t:

$$\langle N(t) \rangle = E_k[N_k] = \int_0^t \rho(t')dt'$$

$$\langle S(t) \rangle = E_k\,[S_k(t)] \quad = \rho(t)$$



Poisson process: a modern view

- Realization can be generated **before** the start of the simulation
- Realization = points in 2 dimension:

  area of 2-dim surface = expected number of events
- Advantage:

  no need to know the time-dependent intensity $\rho(t)$ beforehand

  → could depend on what happens in other parts of an interacting network
- Number of actual events in interval $[t_0, t_1]$ generated by this realization = 'points below the curve $\rho(t)$'

$$E\left[events\ in\ [t_0, t_1]\right] = \int_{t_0}^{t_1} \int_0^{\rho(t)} dz\ dt$$

- in particular

$$\langle S(t) \rangle := E[S(t)] = \rho(t) = \left\langle \sum_f \delta(t - t^f) \right\rangle$$

Poisson Processes:
A modern approach

Interested in using this method?
Please cite for applications in the neurosciences

[1] V. Schmutz (2022), Mean-field limit of age and leaky memory dependent Hawkes processes.
Stochastic Processes and their Applications, 149:39-59
https://doi.org/10.1016/j.spa.2022.03.006

[2] N Fournier and E. Löcherbach (2016), On a toy model of interacting neurons.
Annals Inst. H. Poincare, 52: 1844-1876
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The classical reference for the 2-dimensional approach is

[3] A.W. Lewis, G.S. Shedler (1979),
Simulation of nonhomogeneous Poisson processes by thinning,
Naval Res. Logist. Q. 26: 403–413.

Week 5 – part 4 :Stochastic spike arrival

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Neuronal Dynamics:
Computational Neuroscience of Single Neurons

Week 5 – Variability and Noise:
The question of the neural code

Wulfram Gerstner
EPFL, Lausanne, Switzerland

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- experiments

√ **5.2 Sources of Variability?**
- Is variability equal to noise?

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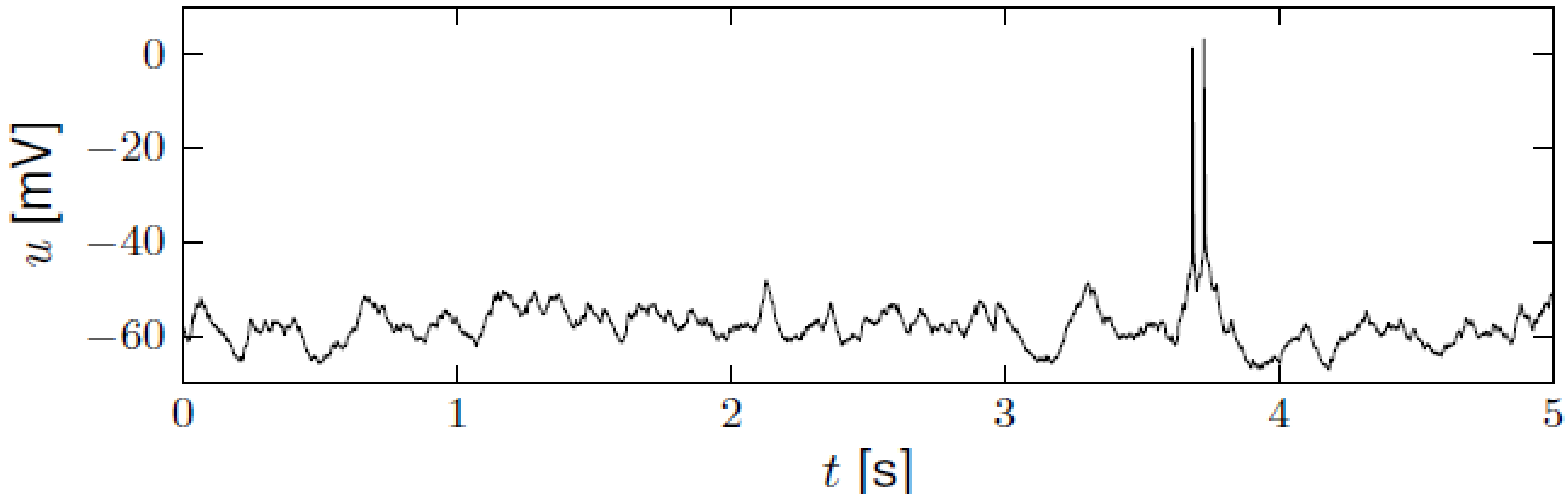
- Membrane potential fluctuations

5.5. Stochastic spike firing
- subthreshold and superthreshold

Spontaneous activity *in vivo*

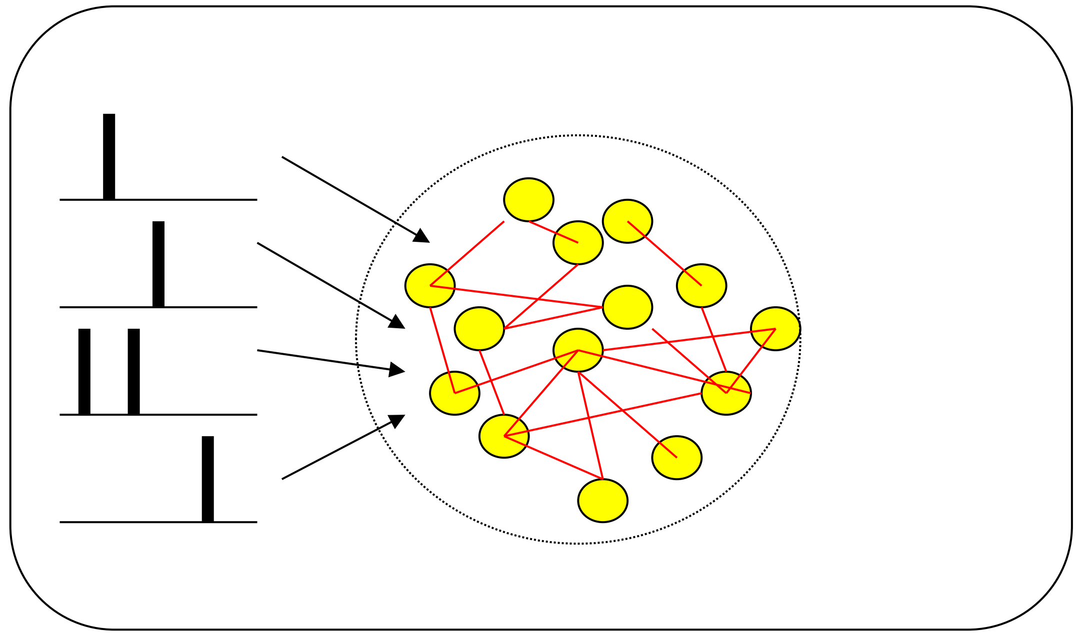
Variability
of membrane potential?

awake mouse, freely whisking,

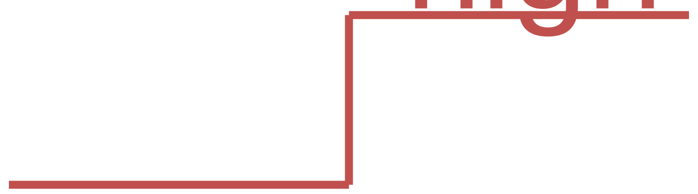


Crochet et al., 2011

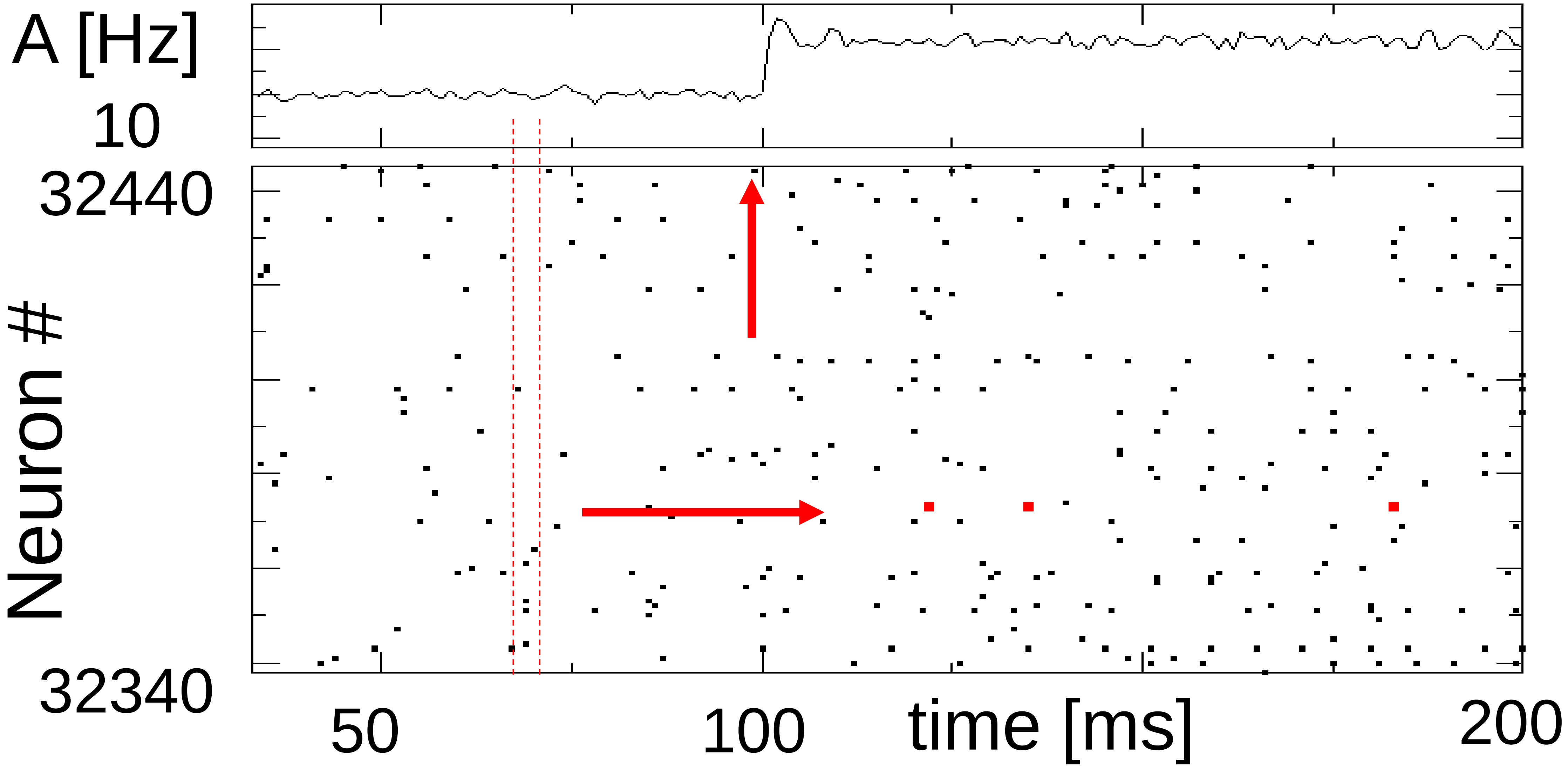
Random firing in a population of LIF neurons

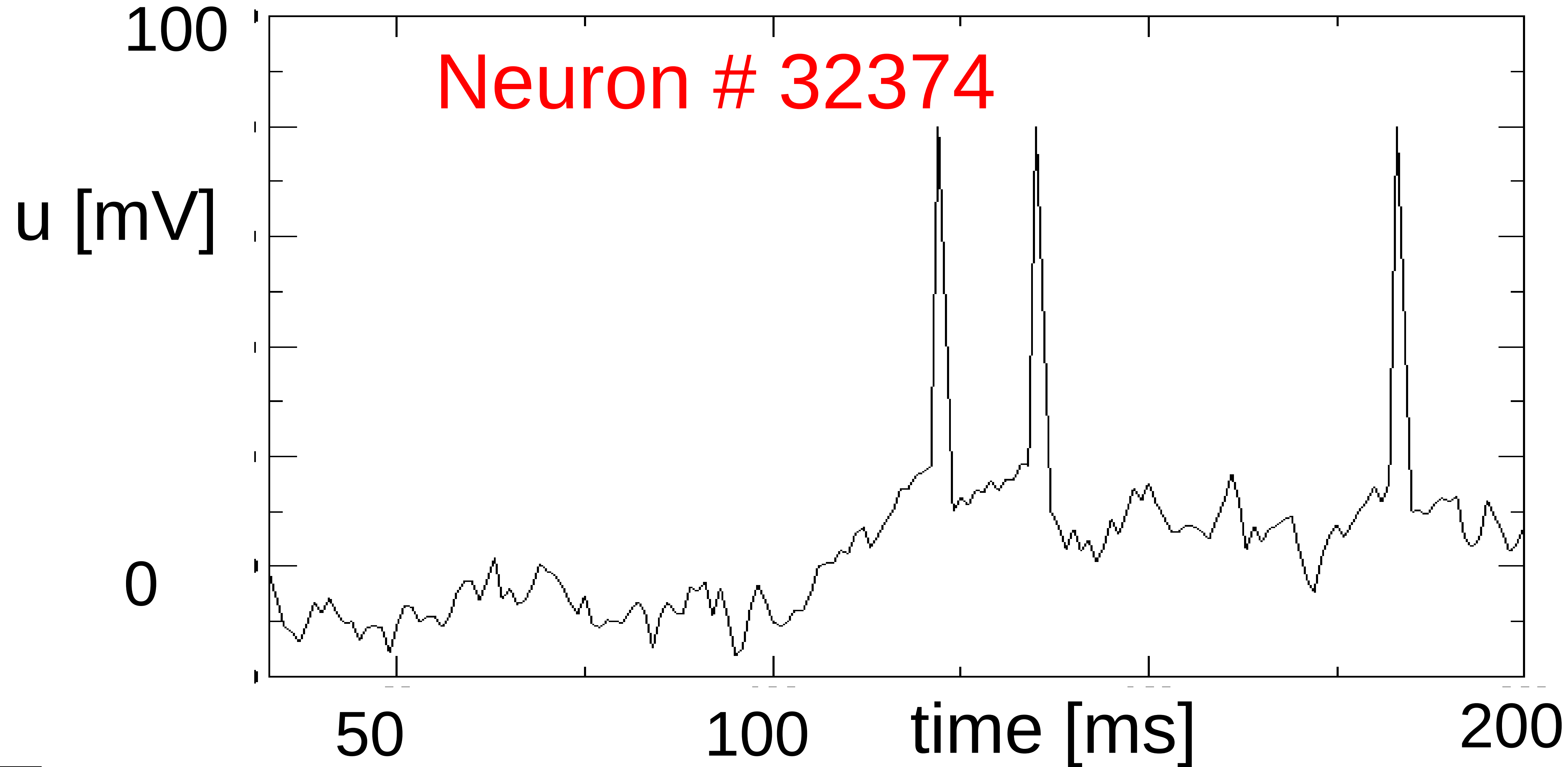


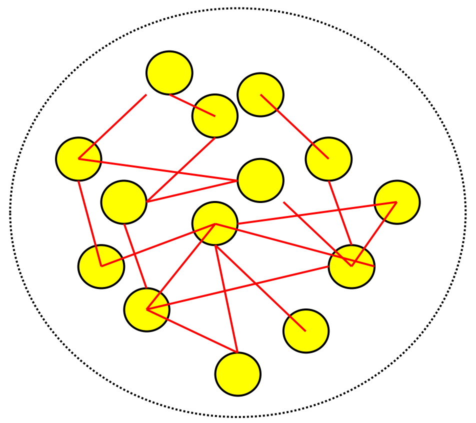
input {low rate / high rate



Population
- 50 000 neurons
- 20 percent inhibitory
- **randomly connected**



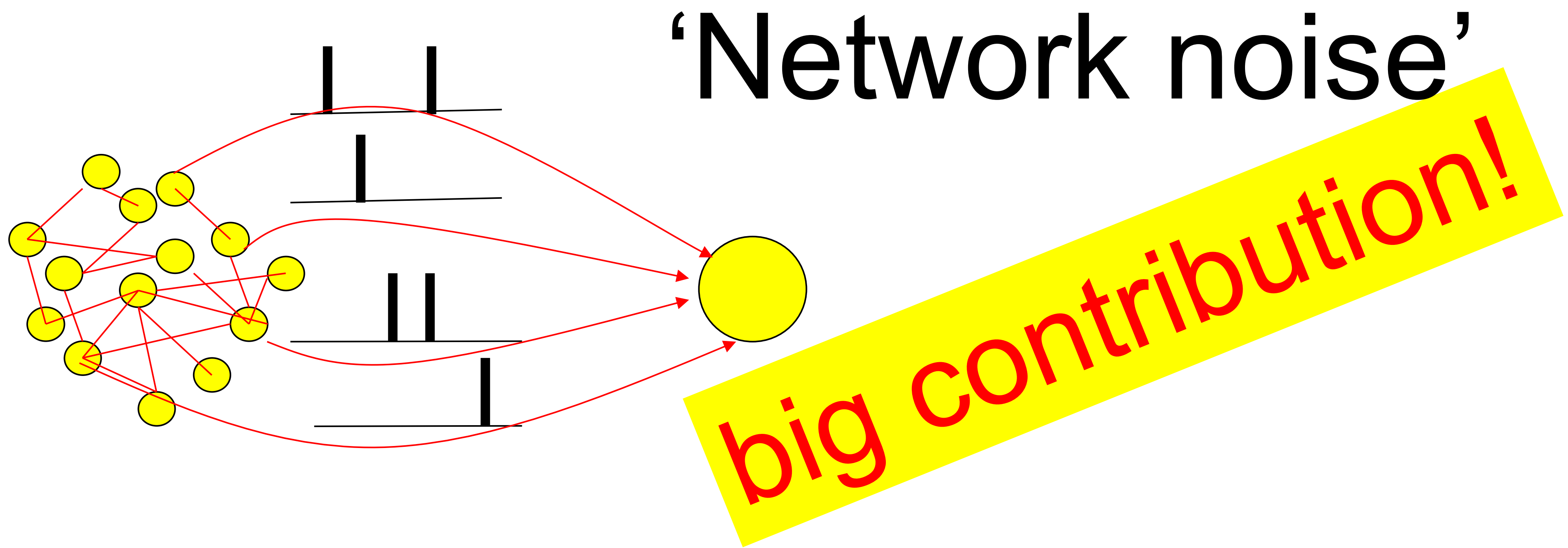


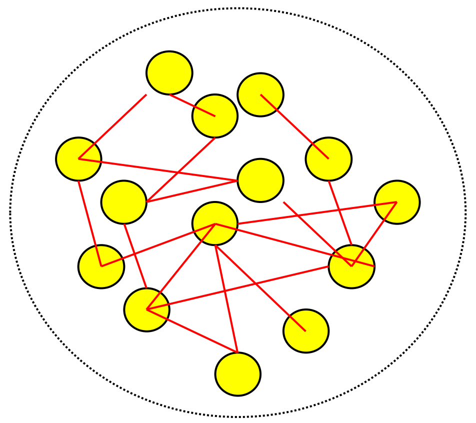


from neuron's point of view:

stochastic spike arrival

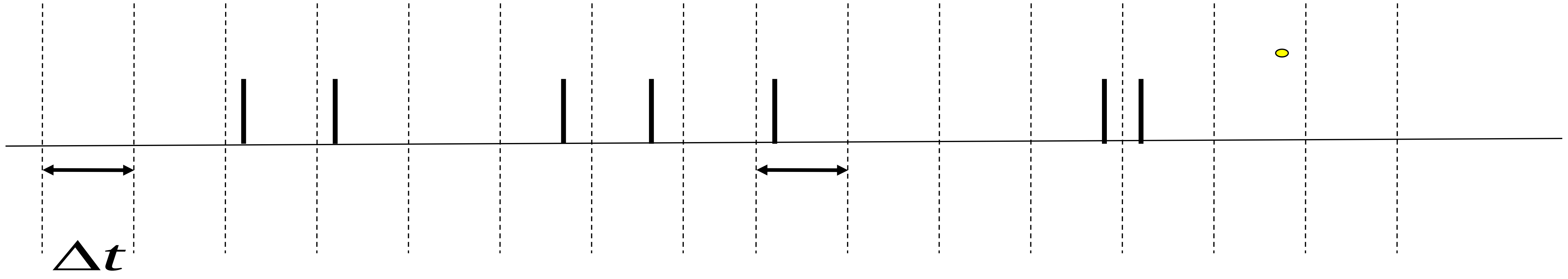
Pull out one neuron





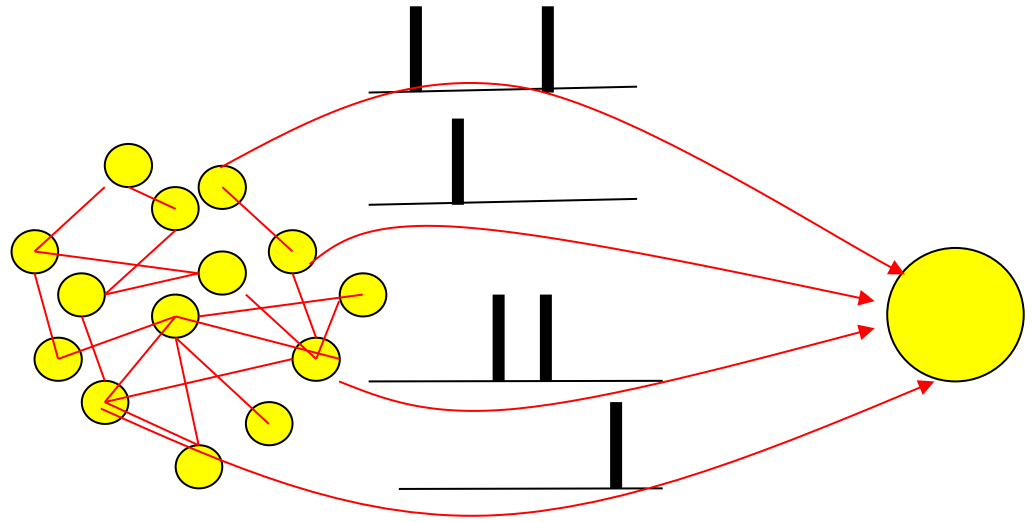
math detour now!

Total spike train of K presynaptic neurons



spike train

Pull out one neuron



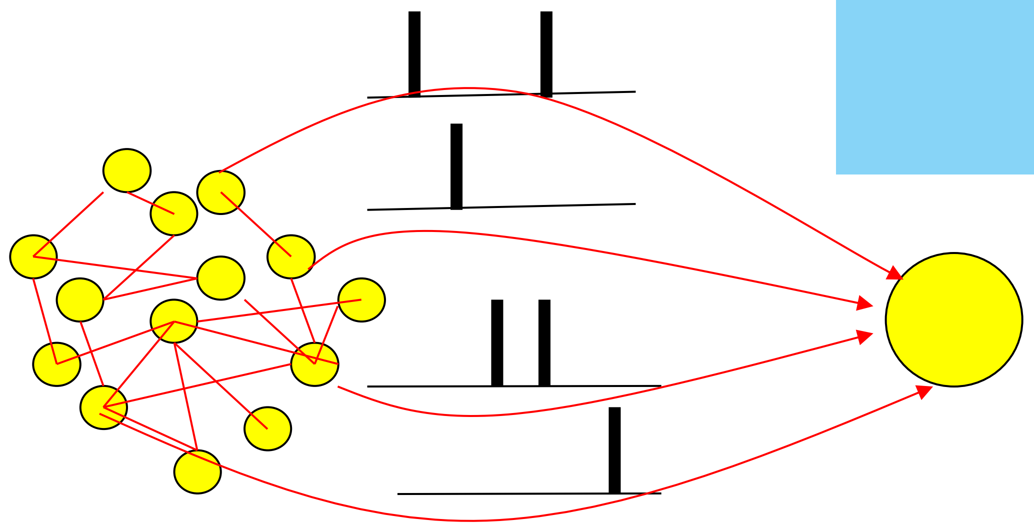
Probability of spike arrival:

$$P_F = K \rho_0 \, \Delta t$$

Take $\Delta t \to 0$ *expectation*

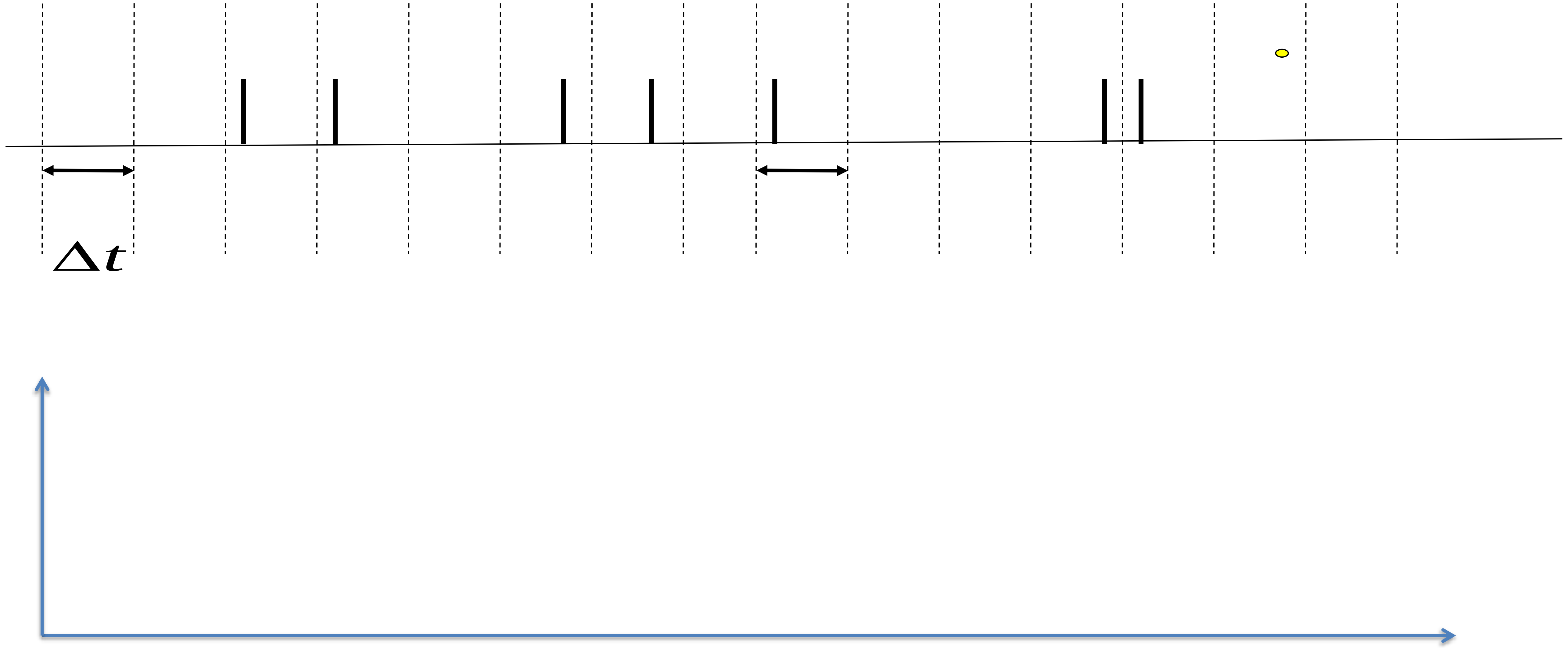
$$S(t) = \sum_{k=1}^{K} \sum_{f} \delta(t - t_k^f)$$

Neuronal Dynamics – 5.4. Fluctuation of input current

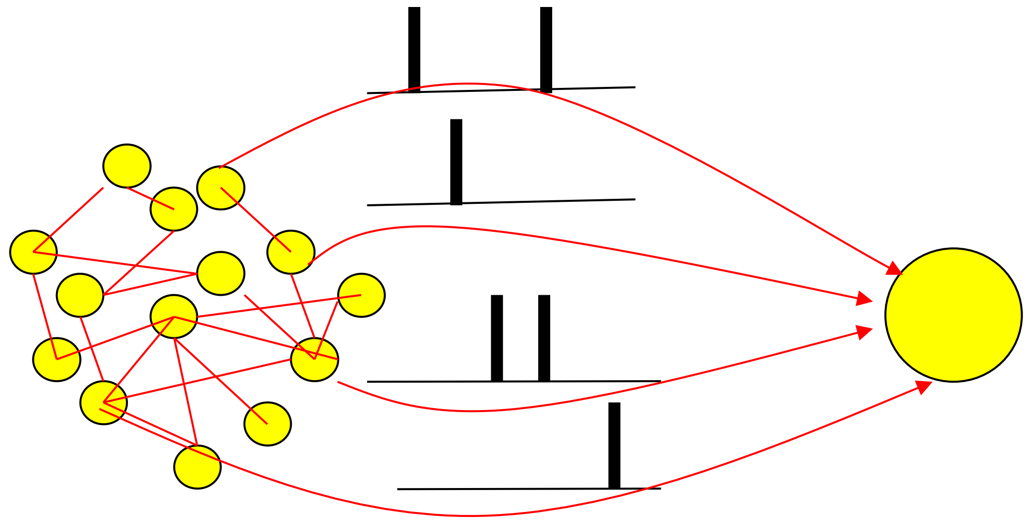


math detour now!

Total spike train of K presynaptic neurons



Neuronal Dynamics – 5.4. Fluctuation of current/potential



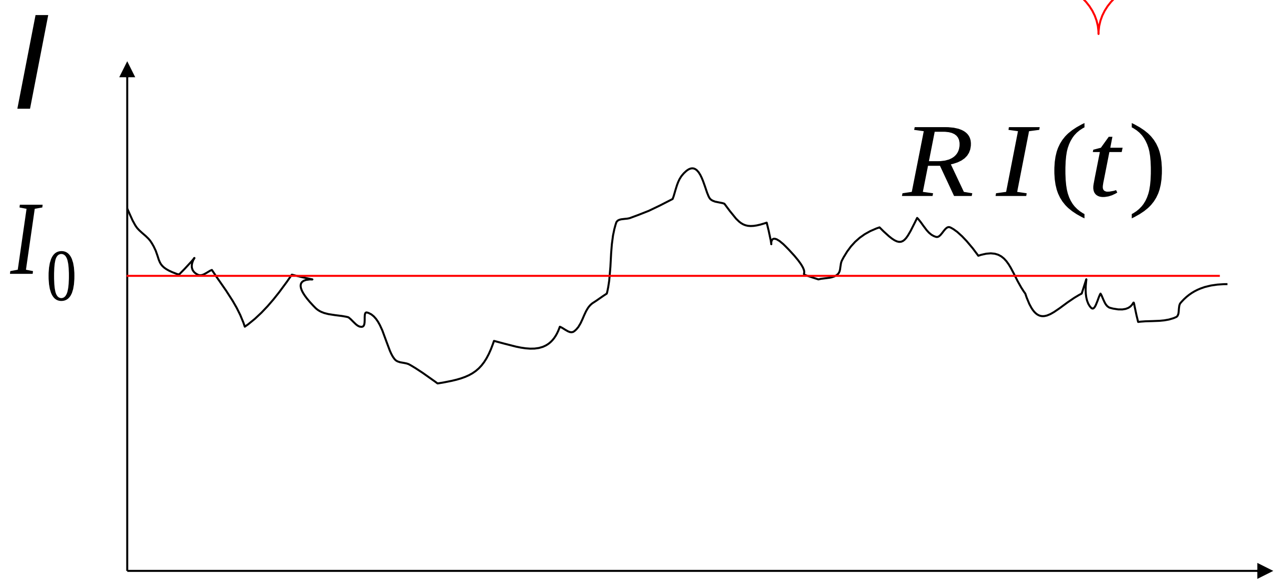
Synaptic current pulses of shape α

$$RI^{syn}(t) = \sum_k w_k \sum_f \alpha(t - t_k^f)$$

EPSC

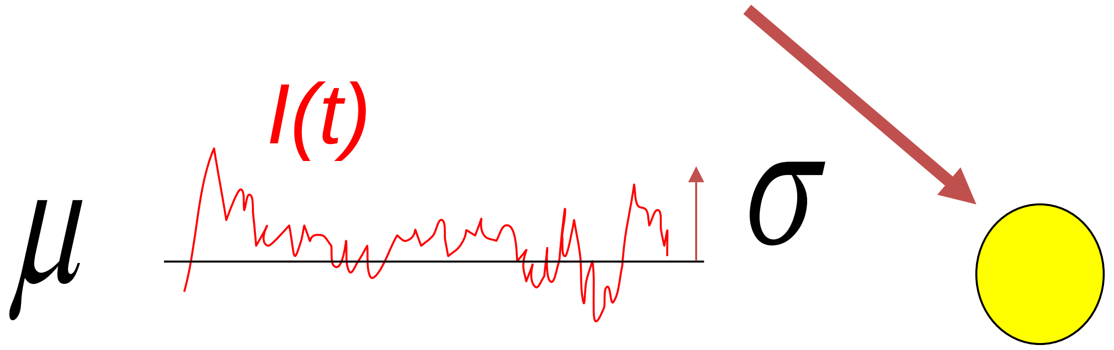
Passive membrane

$$\tau \frac{d}{dt}u = -(u - u_{rest}) \quad + R\,I^{syn}(t)$$



$$I^{syn}(t) = I_0 + I^{fluct}(t)$$

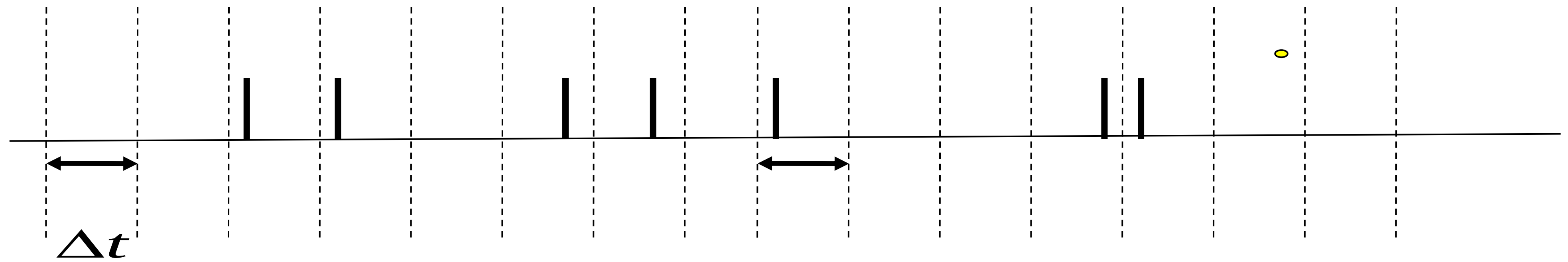
→ Fluctuating potential



Fluctuating input current

Neuronal Dynamics – 5.4. Calculating the mean

$$RI^{syn}(t) = \sum_k w_k \sum_f \alpha(t - t_k^f)$$



$$I^{syn}(t) = \frac{1}{R} \sum_k w_k \sum_f \int dt' \alpha(t-t') \delta(t' - t_k^f)$$

$$x(t) = \sum_f \int dt' f(t-t') \delta(t' - t_k^f)$$

mean: assume Poisson process

$$I_0 = \langle I^{syn}(t) \rangle = \frac{1}{R} \sum_k w_k \int dt' \alpha(t-t') \langle \sum$$

use for assignment/ homework!

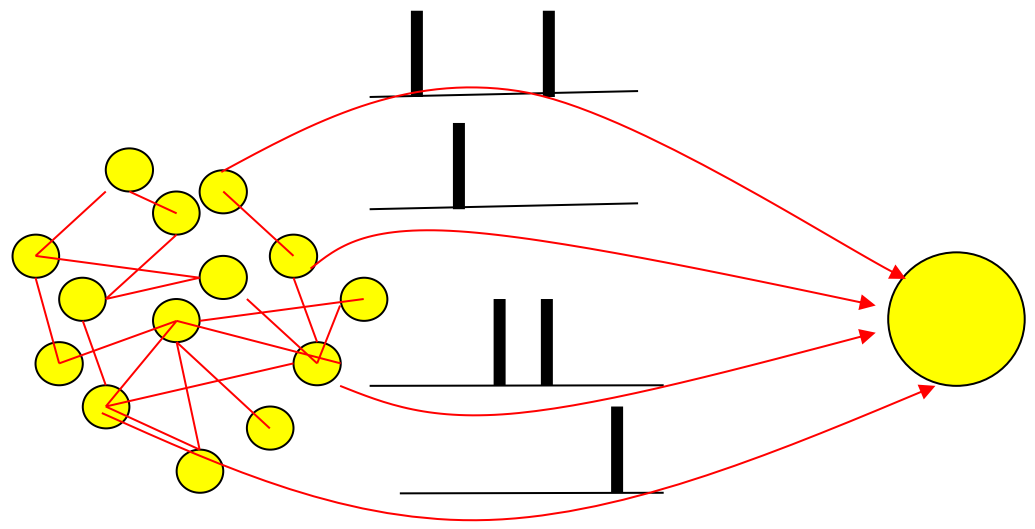
$$\langle x(t) \rangle = \int dt' f(t-t') \left\langle \sum_f \delta(t' - t_k^f) \right\rangle$$

$$I_0 = \frac{1}{R} \sum_k w_k \int dt' \alpha(t-t') \; \nu_k$$

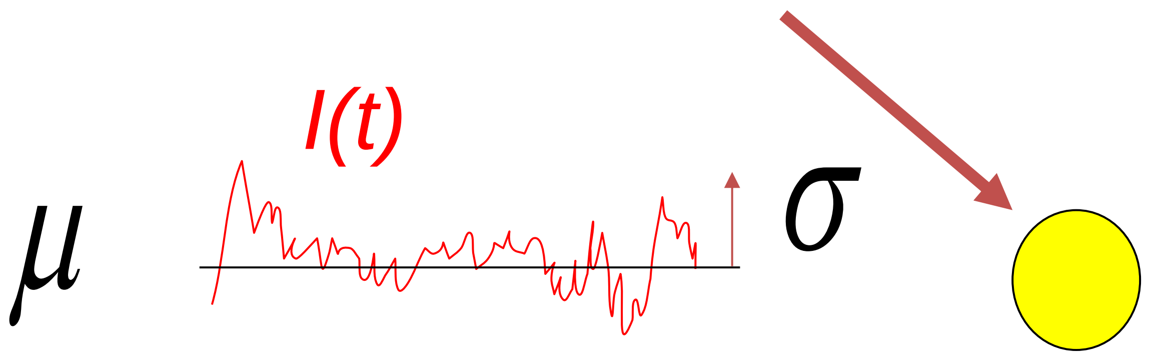
$$\langle x(t) \rangle = \int dt' f(t-t') \; \rho(t')$$

rate of inhomogeneous Poisson process

Neuronal Dynamics – 5.4. Fluctuation of current/potential

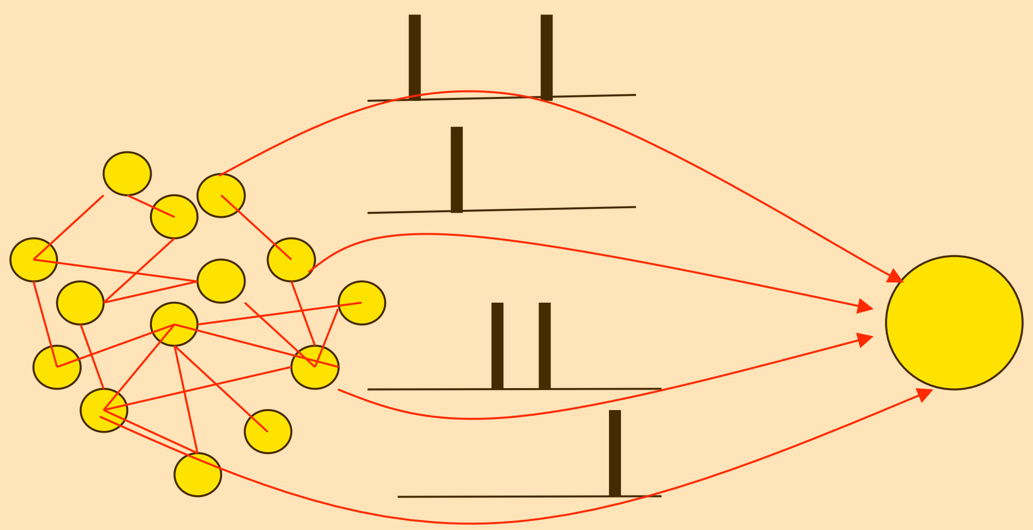


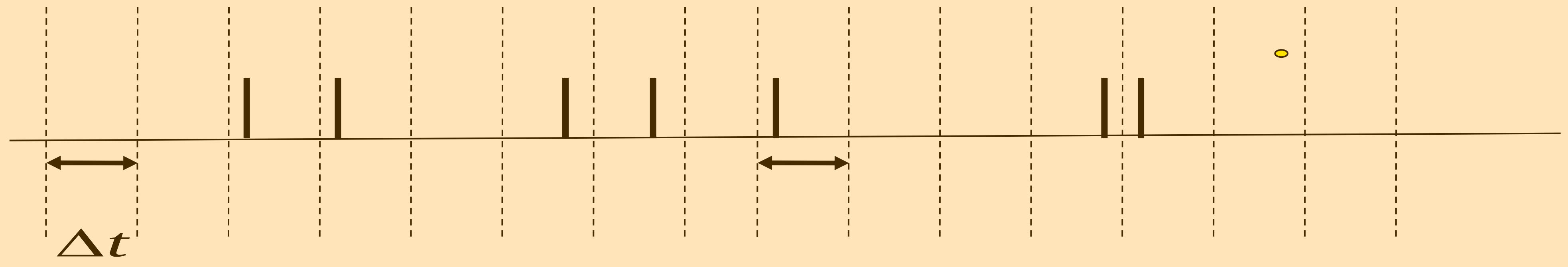
fluctuating input current



fluctuating potential

Neuronal Dynamics – Assignment/homework





$$u(t) = \sum_f \int dt' \, f(t-t') \, \delta(t'-t_k^f)$$

A leaky integrate-and-fire neuron receives stochastic spike arrival, described as a homogeneous Poisson process.

Calculate the mean membrane potential. To do so, use the above formula.

Week 5 – part 4b : Membrane potential fluctuations

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The question of the neural code

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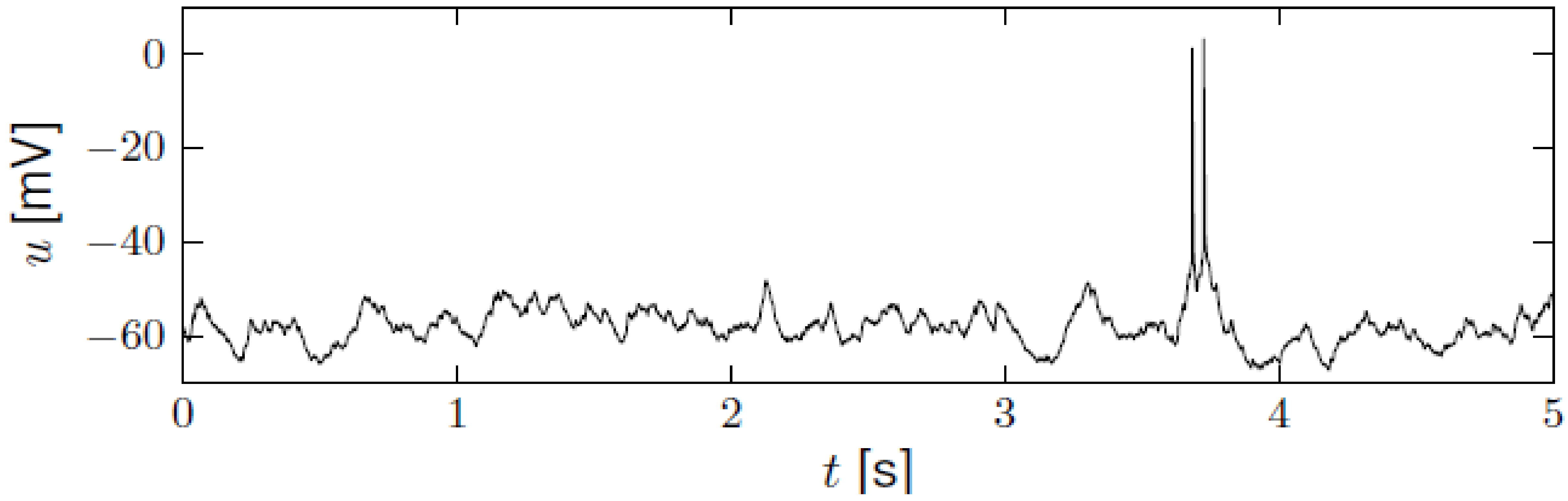
Spontaneous activity *in vivo*

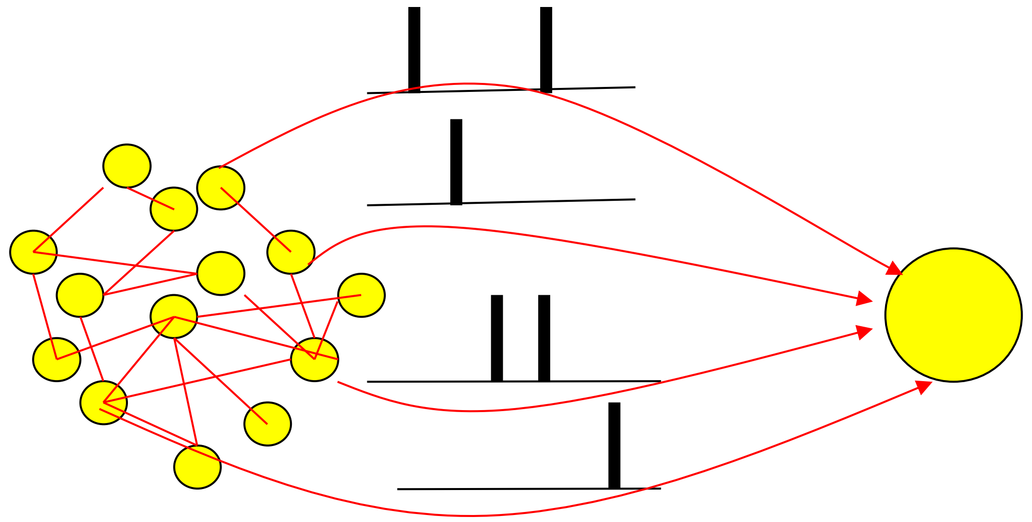
Variability
of membrane potential?

awake mouse, freely whisking,



Crochet et al., 2011

Neuronal Dynamics – 5.4b. Fluctuations of potential



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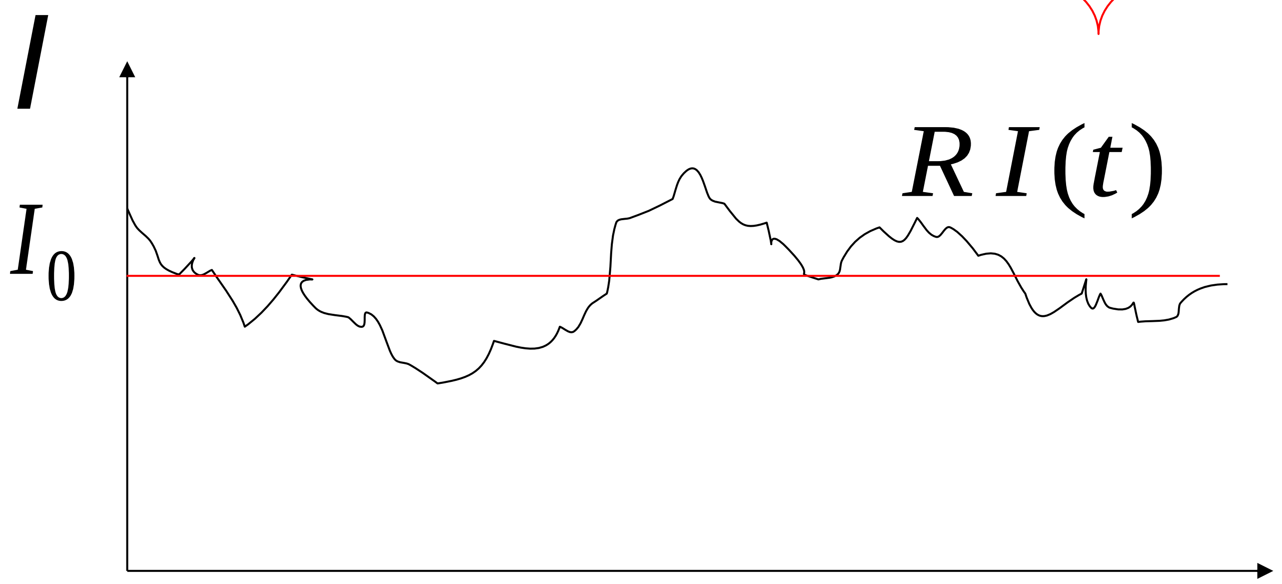
$$RI^{syn}(t) = \sum_k w_k \sum_f \alpha(t - t_k^f)$$

EPSC

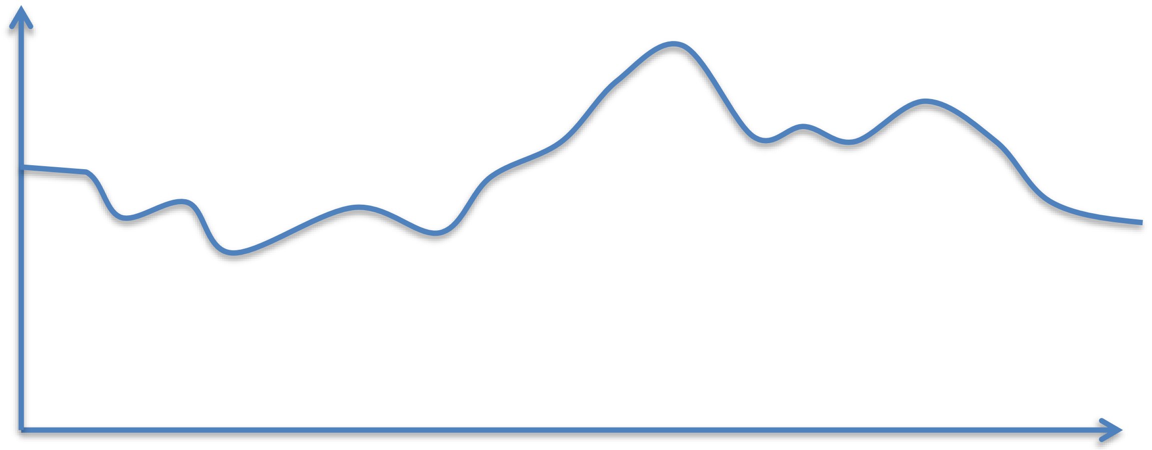
Passive membrane

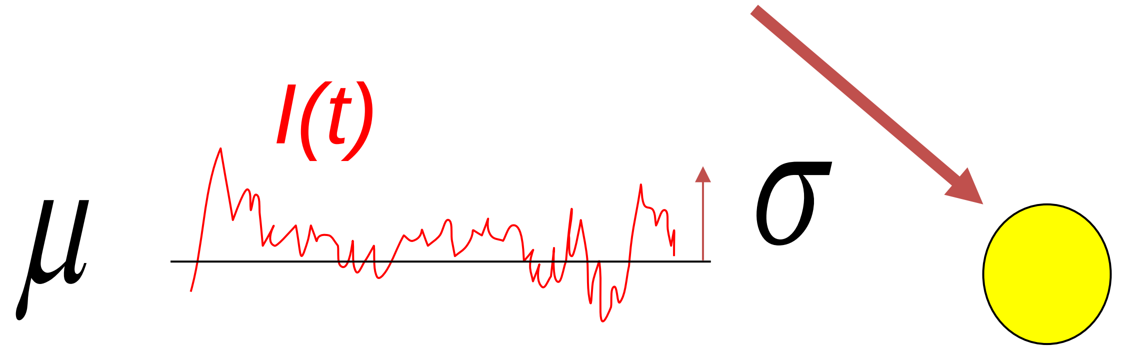
$$\tau \frac{d}{dt} u = -(u - u_{rest}) \quad + R\,I^{syn}(t)$$



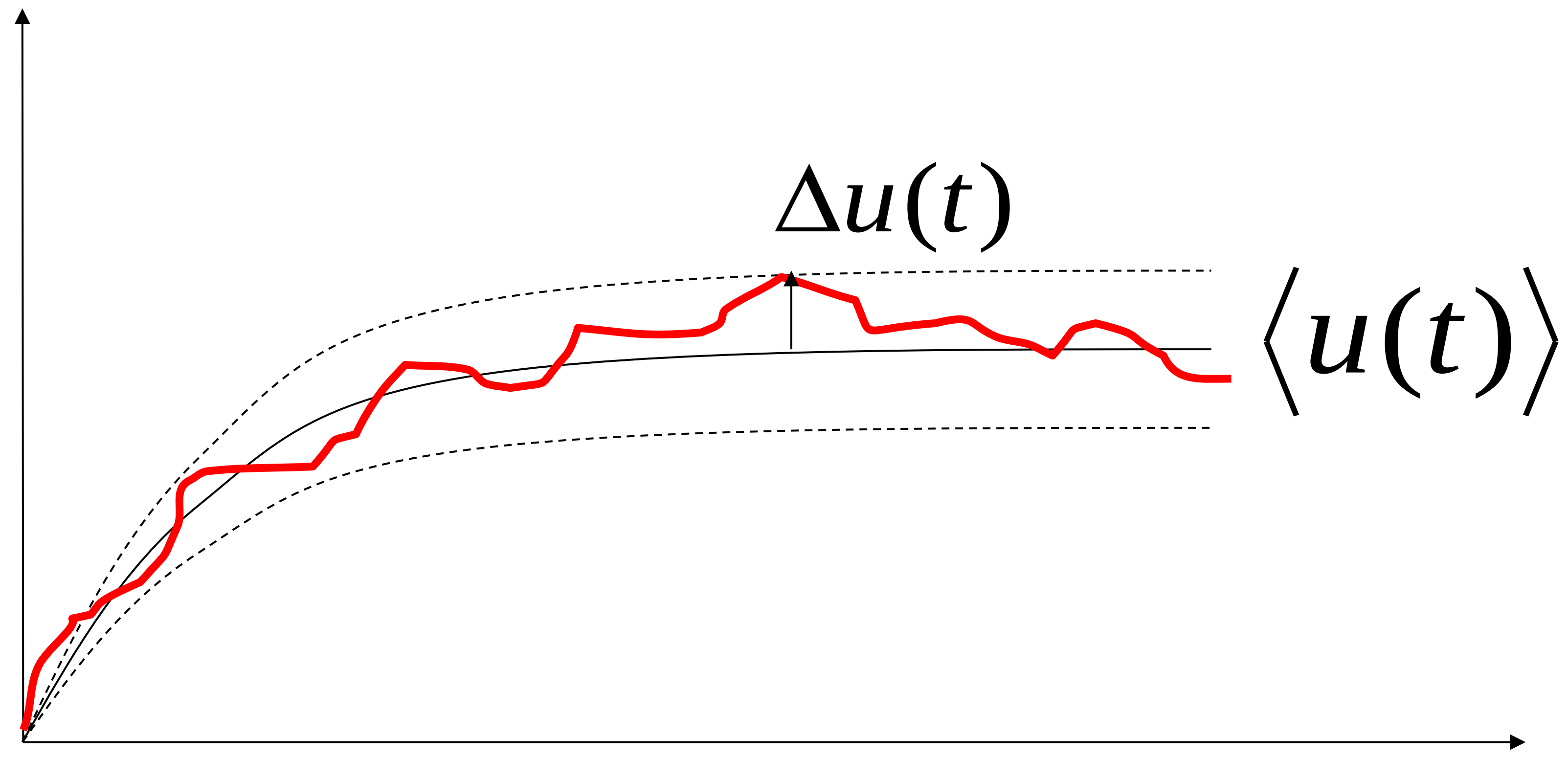
$$I^{syn}(t) = I_0 + I^{fluct}(t)$$

→ Fluctuating potential



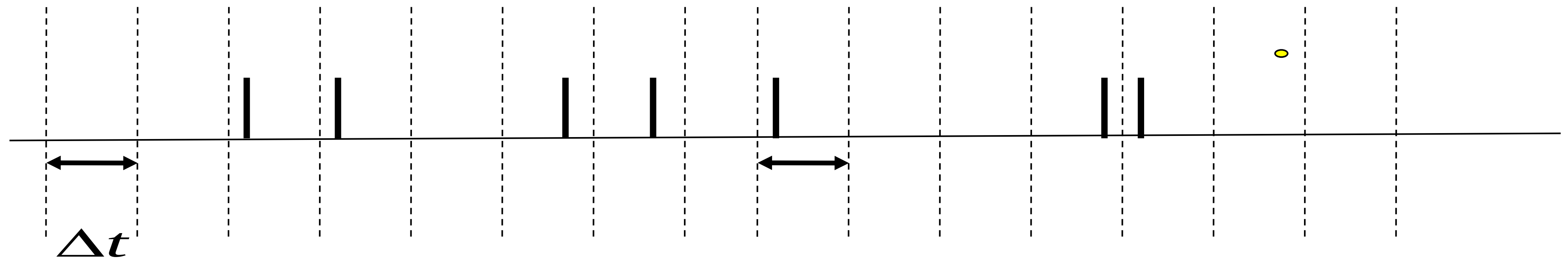


Fluctuating input current



$$\langle \Delta u(t) \Delta u(t) \rangle = \langle u(t) u(t) \rangle - \langle u(t) \rangle^2 =$$

Input: step + fluctuations



Autocorrelation

$$\langle x(t)x(t')\rangle =$$

$$x(t) = \sum_f \int dt' \, f(t-t')\,\delta(t'-t_k^f)$$

$$= \int dt' \, f(t-t')\,S(t')$$

Mean:

$$\langle x(t)\rangle = \int dt' \, f(t-t')\langle S(t')\rangle$$

$$\langle x(t)\rangle = \int ds \, f(s) \; \rho_0$$

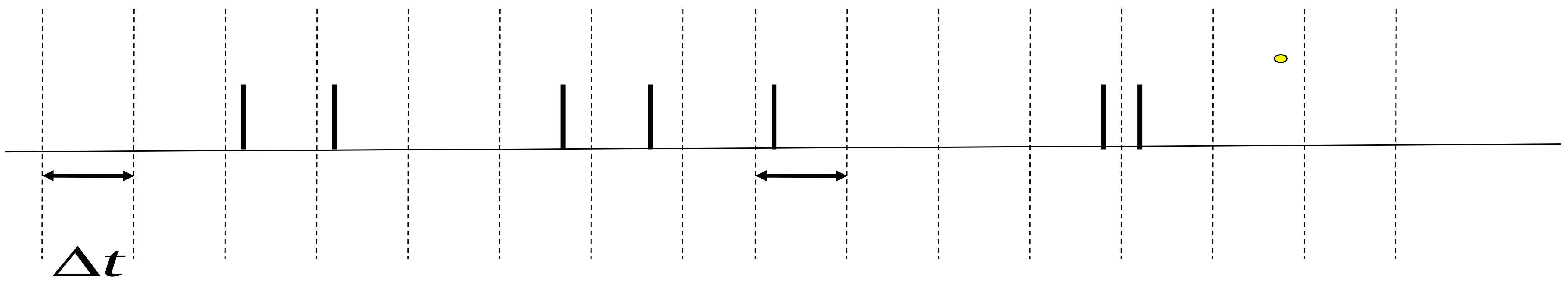
rate of homogeneous Poisson process

$$\langle x(t)x(\hat{t})\rangle = \int dt' \int dt'' f(t-t')\,f(\hat{t}-t'')\langle S(t')S(t'')\rangle$$

Neuronal Dynamics – 5.4b. Autocorrelation of Poisson

math detour now!

Probability of spike in step *n* **AND** step *k*



spike train

Probability of spike in time step:

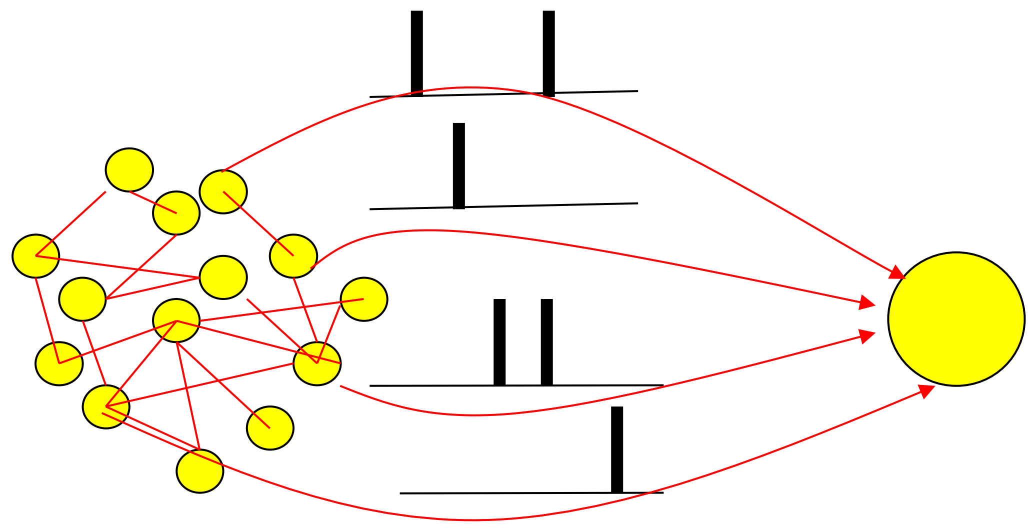
$$P_F = \rho_0 \, \Delta t$$

Autocorrelation (continuous time)

$$\langle S(t) S(t') \rangle = \rho_0 \, \delta(t - t') + [\rho_0]^2$$

Neuronal Dynamics – 5.4b. Fluctuation of potential

for a passive membrane, we can analytically predict the amplitude of membrane potential fluctuations



Passive membrane

$$u(t) = \sum_k w_k \sum_f \varepsilon(t' - t_k^f)$$

$$= \sum_k w_k \int dt' \varepsilon(t - t') S_k(t')$$

fluctuating potential

$$\langle \Delta u(t) \Delta u(t) \rangle = \langle [u(t)]^2 \rangle - \langle u(t) \rangle^2$$

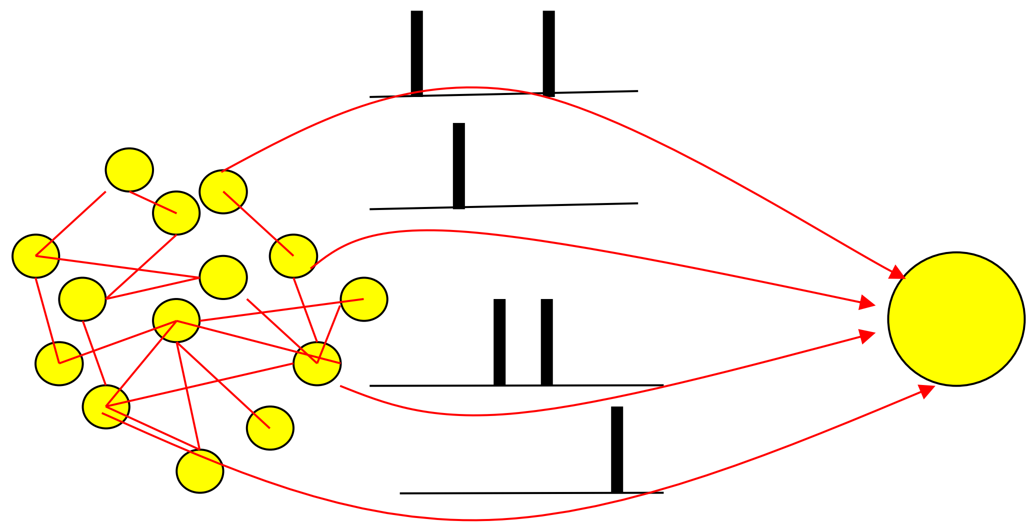
Leaky integrate-and-fire in subthreshold regime

Neuronal Dynamics – 5.4b. Fluctuation of potential

Stochastic spike arrival:

for a passive membrane, we can analytically predict the amplitude of membrane potential fluctuations

Leaky integrate-and-fire in subthreshold regime



Passive membrane

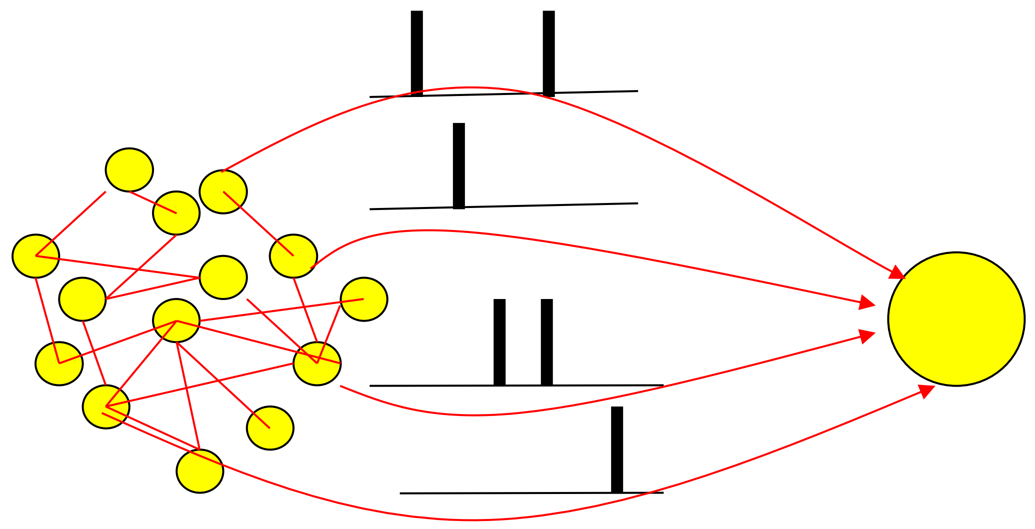
$$u(t) = \sum_k w_k \sum_f \varepsilon(t' - t_k^f)$$

$$= \sum_k w_k \int dt' \varepsilon(t - t') S_k(t')$$

fluctuating potential

$$\langle \Delta u(t) \Delta u(t) \rangle = \langle [u(t)]^2 \rangle - \langle u(t) \rangle^2$$

Neuronal Dynamics – 5.4b. Fluctuation of potential



Passive membrane

$$u(t) = \sum_k w_k \sum_f \varepsilon(t' - t_k^f)$$

$$= \sum_k w_k \int dt' \varepsilon(t - t') S_k(t')$$

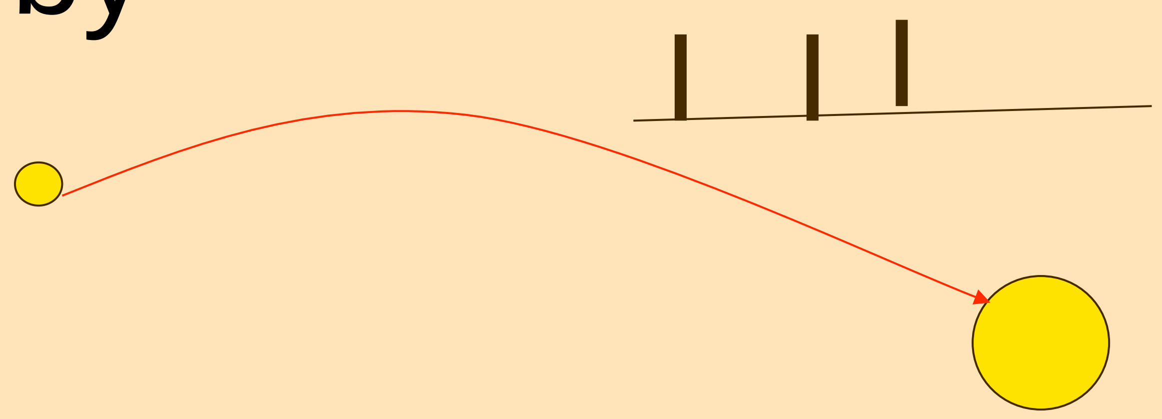
Fluctuations of potential

$$\langle [\Delta u(t)]^2 \rangle = \langle [u(t)]^2 \rangle - \langle u(t) \rangle^2$$

Neuronal Dynamics – Quiz 5.4

A linear (=passive) membrane has a potential given by

$$u(t) = \sum_f \int dt' \, f(t-t') \, \delta(t'-t_k^f) + a$$



Suppose the neuronal dynamics are given by

$$\tau \frac{d}{dt} u = -(u - u_{rest}) \quad + q \sum_f \delta(t - t^f)$$

[ ] the filter *f* is exponential with time constant τ

[ ] the constant *a* is equal to the time constant τ

[ ] the constant *a* is equal to u_{rest}

[ ] the amplitue of the filter *f* is *q*

[ ] the amplitue of the filter *f* is u_{rest}

Week 5 – part 5 : Stochastic spike firing in integrate-and-fire models

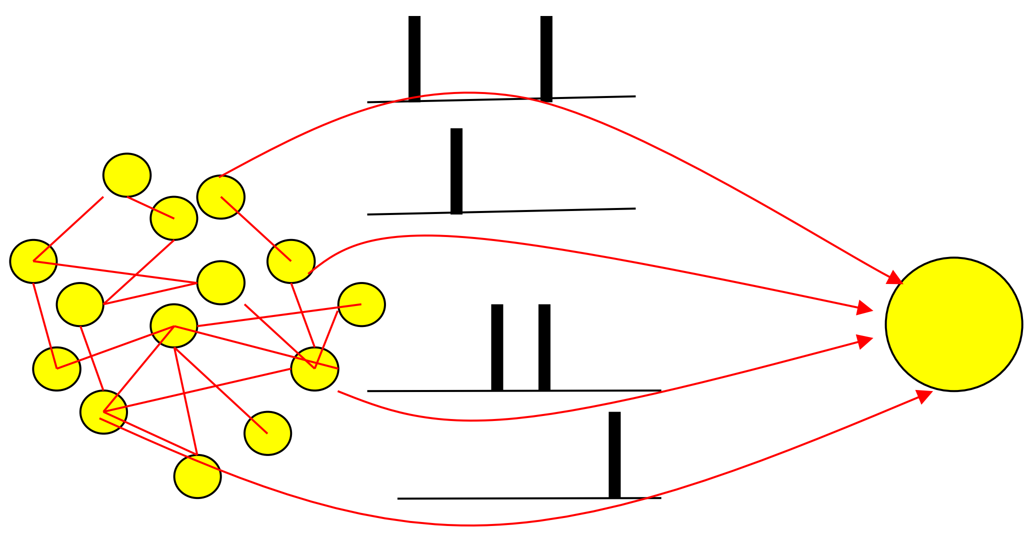
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

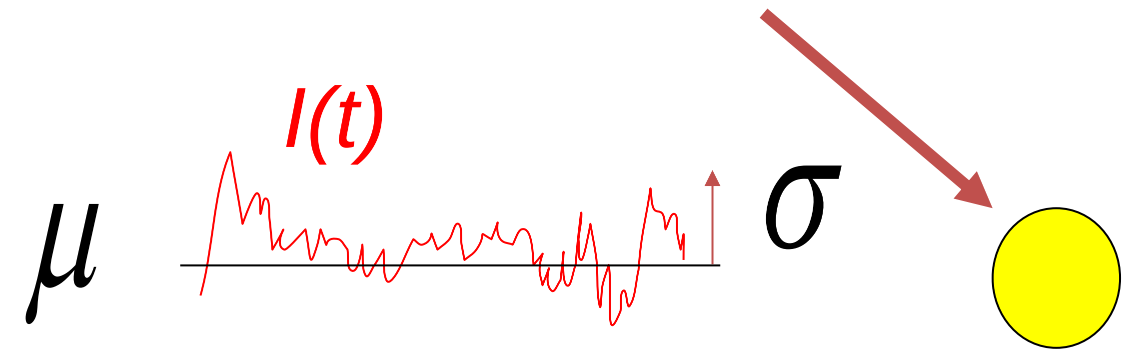
Neuronal Dynamics:
Computational Neuroscience of Single Neurons

Week 5 – Variability and Noise:
The question of the neural code

Wulfram Gerstner
EPFL, Lausanne, Switzerland

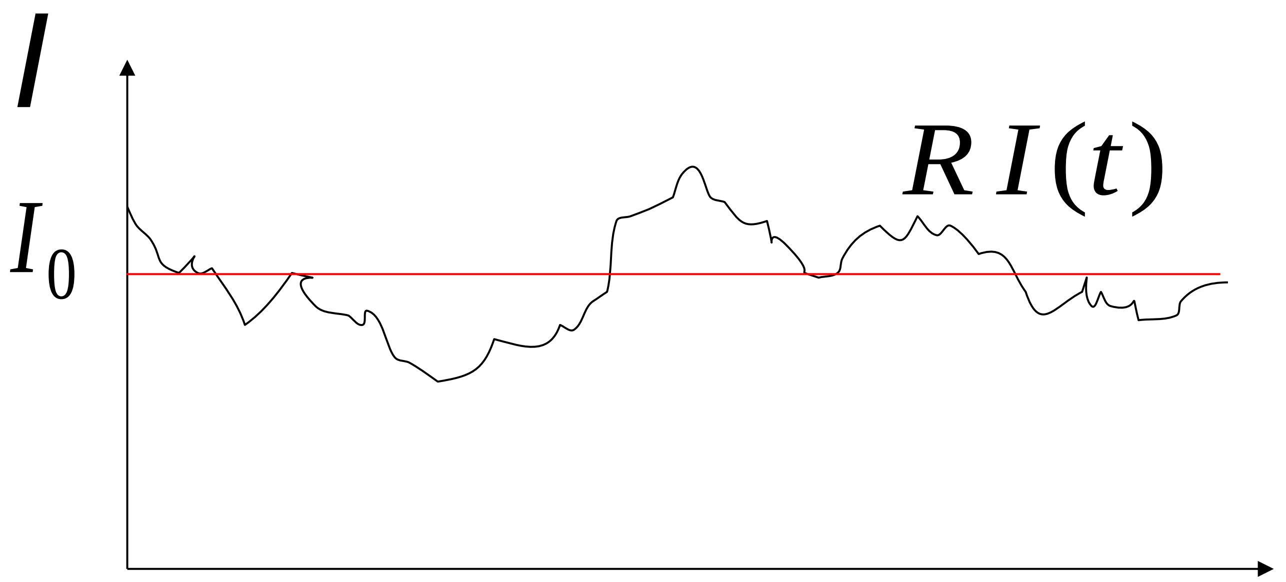
Neuronal Dynamics – review: Fluctuations of potential





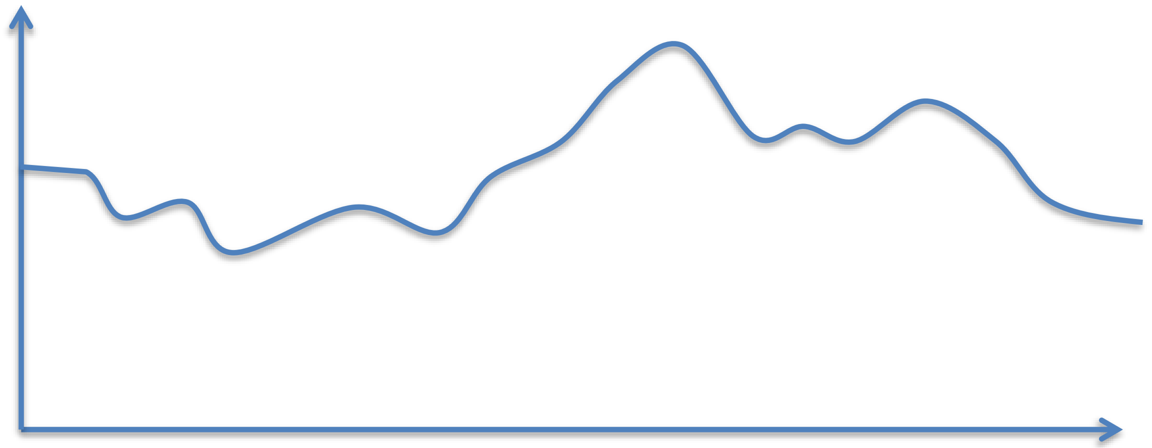
Passive membrane

$$\tau \frac{d}{dt}u = -(u - u_{rest}) \quad + R\,I(t)$$



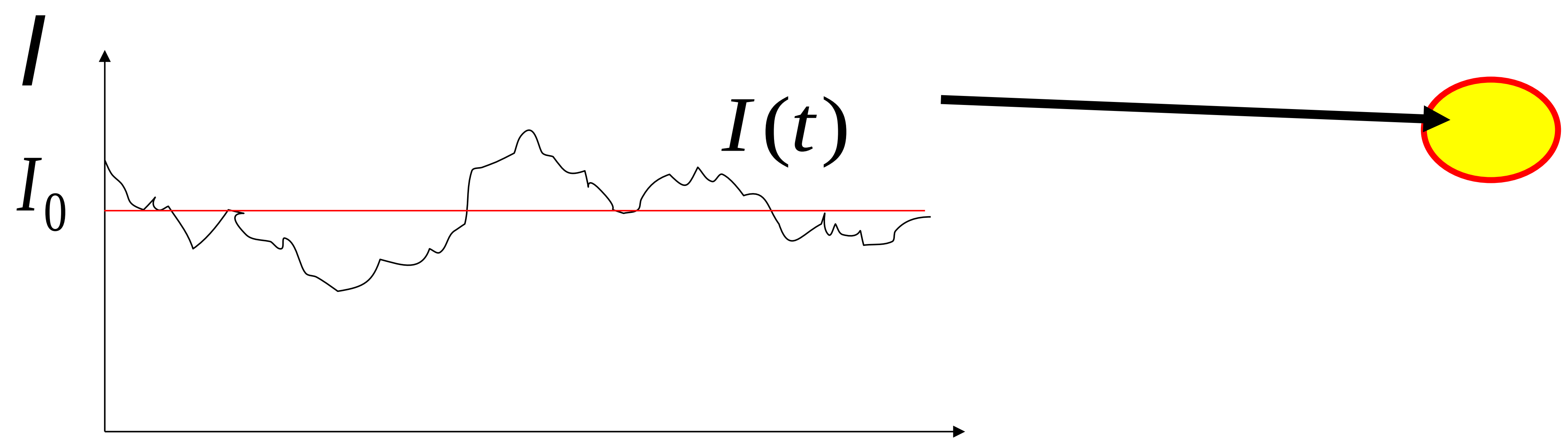
$$I^{syn}(t) = I_0 + I^{fluct}(t)$$

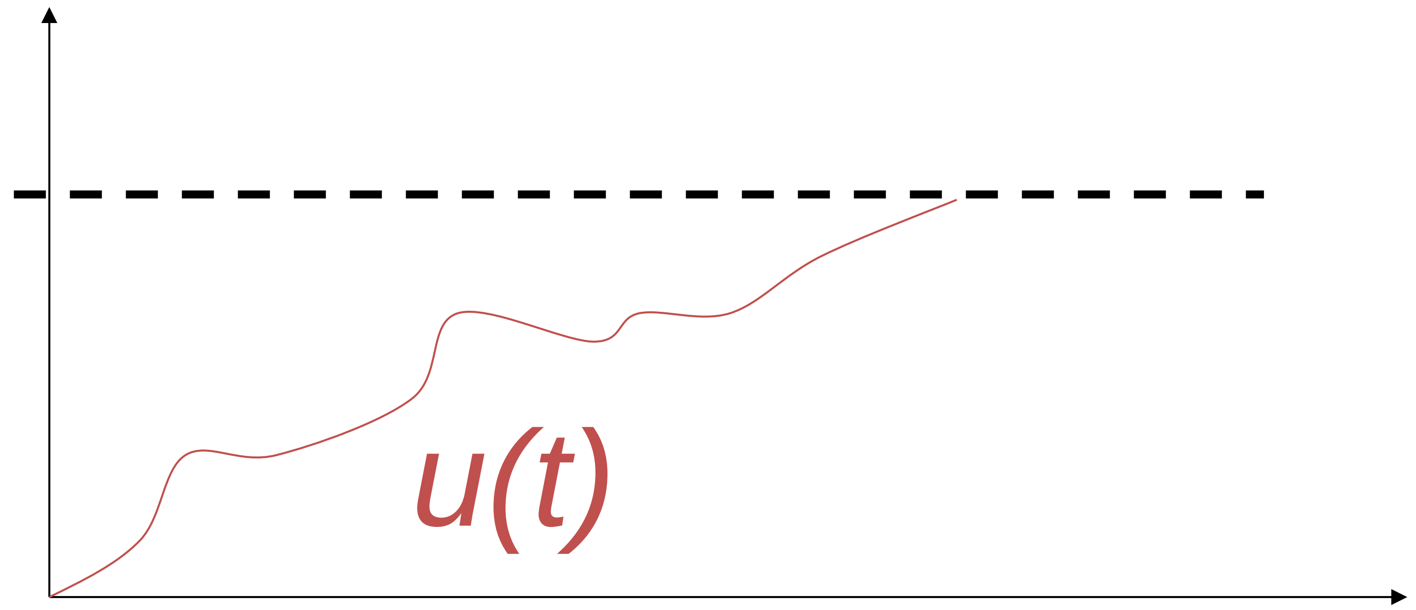
→ Fluctuating potential



Neuronal Dynamics – 5.5. Stochastic leaky integrate-and-fire

effective noise current





LIF

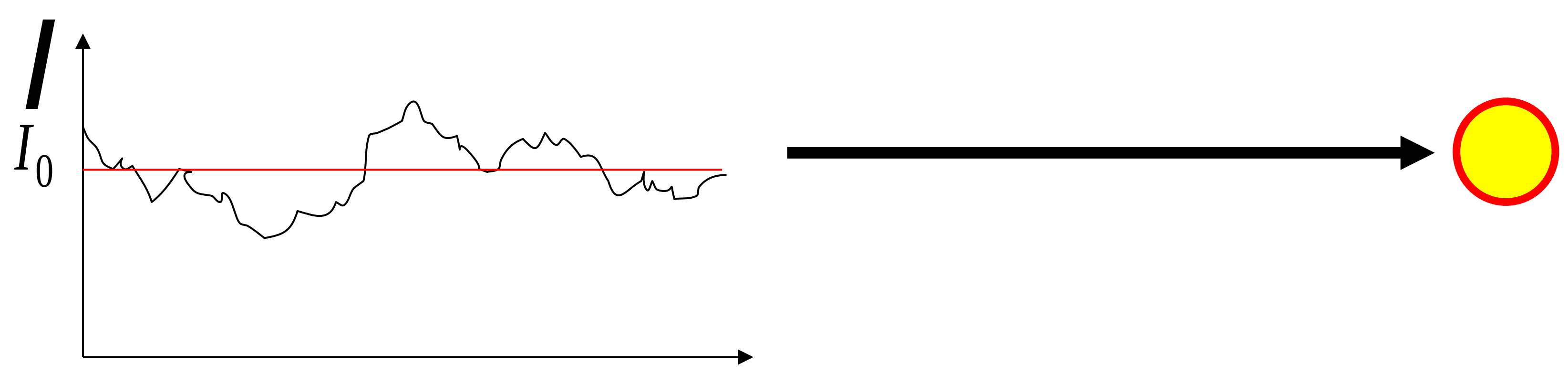
$$\tau \frac{d}{dt}u = -(u - u_{rest}) \quad + R\,I(t)$$

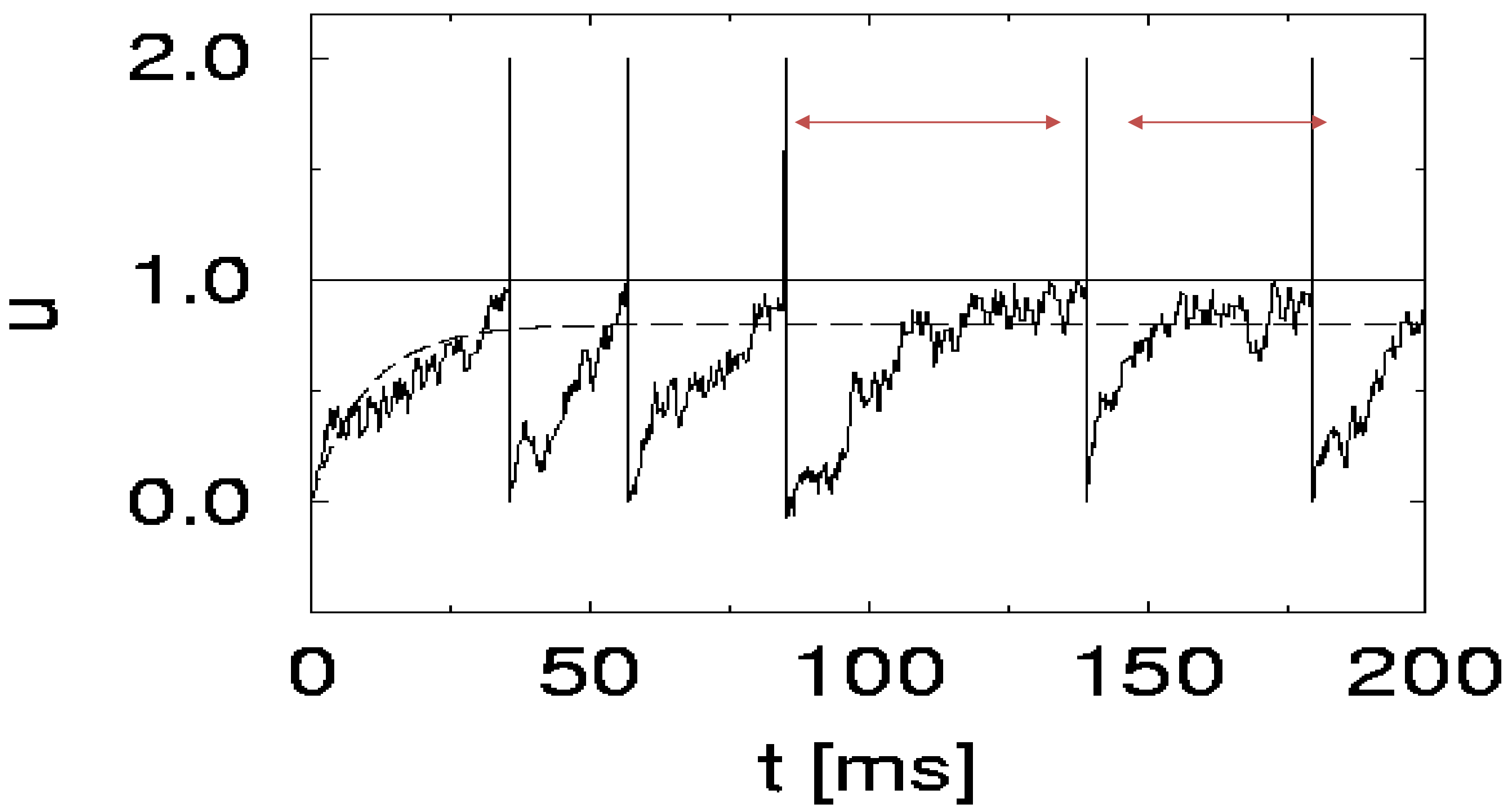
$$I(t) = [I_o + I_{noise}]$$

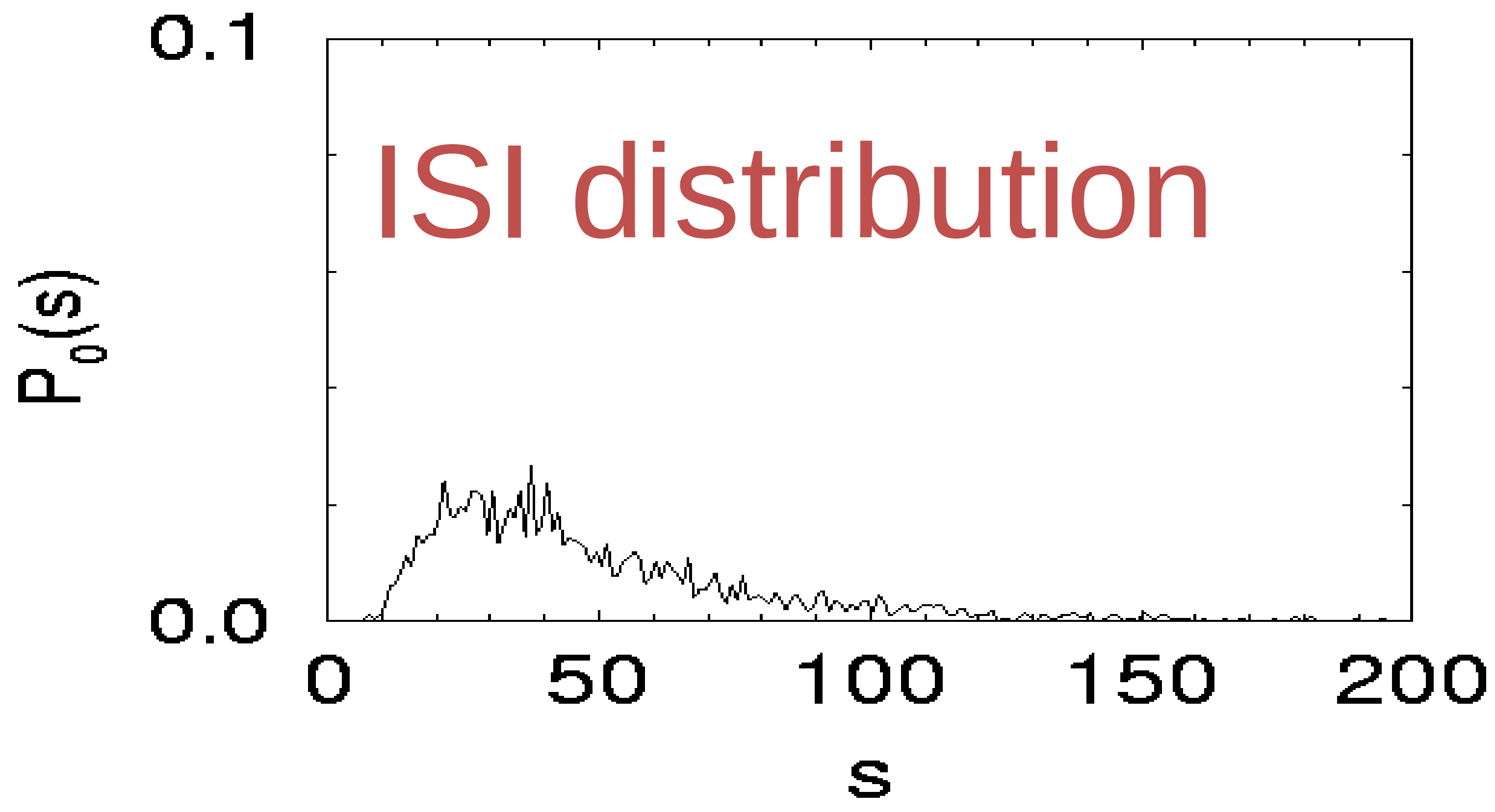
$$IF \;\; u(t) = \vartheta \;\; THEN \;\; u(t+\Delta) = u_r$$

noisy input/
diffusive noise/
stochastic spike
arrival

stochastic spike arrival in I&F – interspike intervals

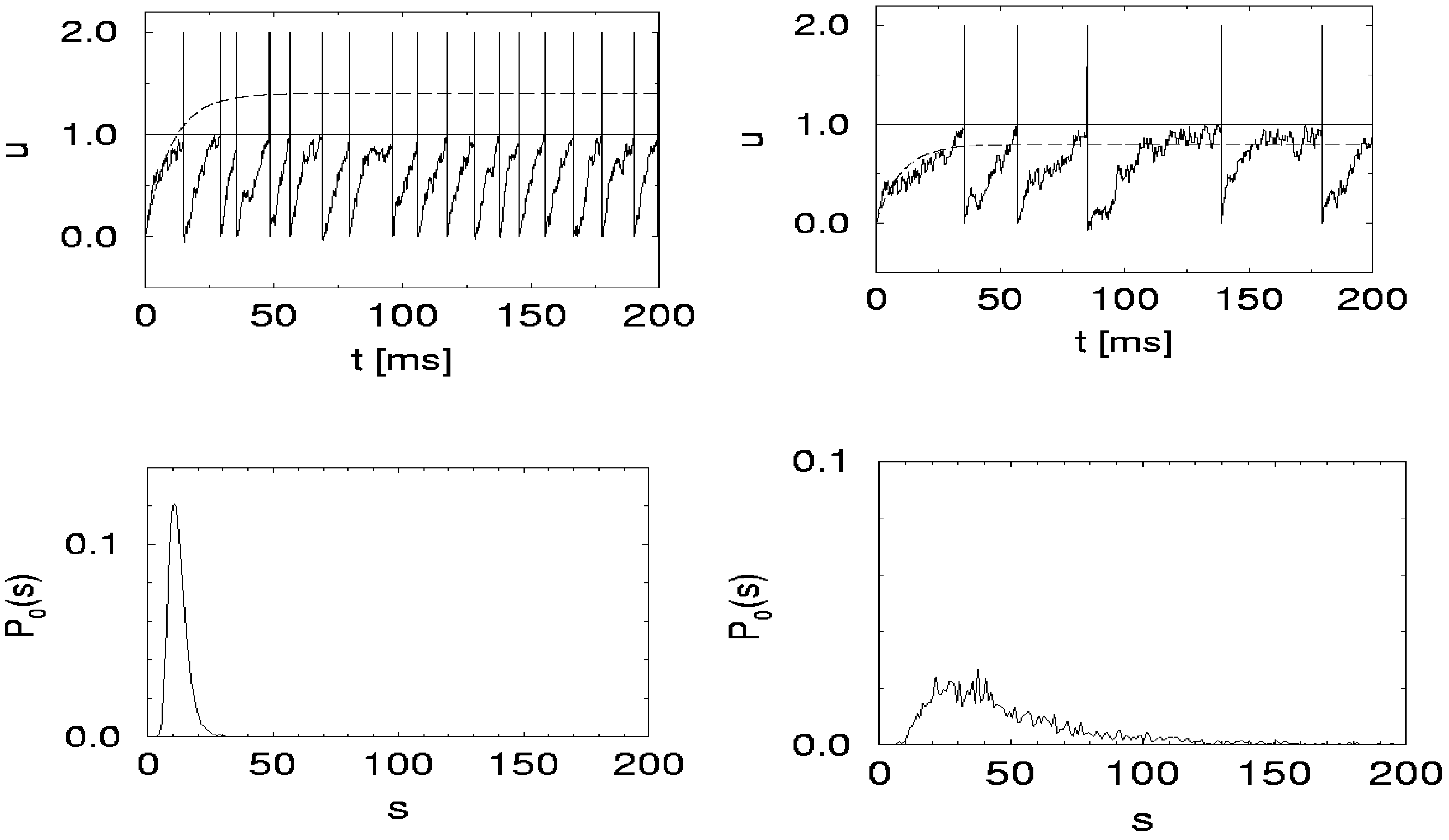




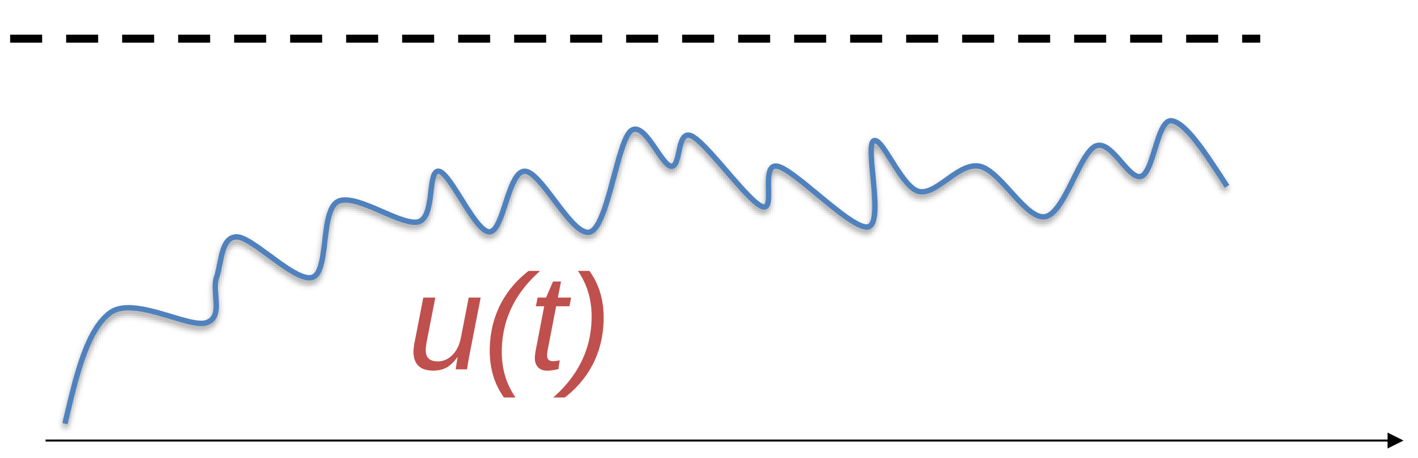


LIF with Diffusive noise (stochastic spike arrival)

Superthreshold vs. Subthreshold regime



noisy input/ diffusive noise/ stochastic spike arrival



subthreshold regime:

- firing driven by fluctuations
- broad ISI distribution
- *in vivo* like

Neuronal Dynamics week 5– References and Suggested Reading

Reading: W. Gerstner, W.M. Kistler, R. Naud and L. Paninski,
Neuronal Dynamics: from single neurons to networks and models of cognition. Ch. 7,8: Cambridge, 2014
OR W. Gerstner and W. M. Kistler, Spiking Neuron Models, Chapter 5, Cambridge, 2002

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-Siegert, A. (1951). On the first passage time probability problem. Phys. Rev., 81:617{623.
-Konig, P., et al. (1996). Integrator or coincidence detector? the role of the cortical neuron revisited. *Trends Neurosci,* 19(4):130-137.