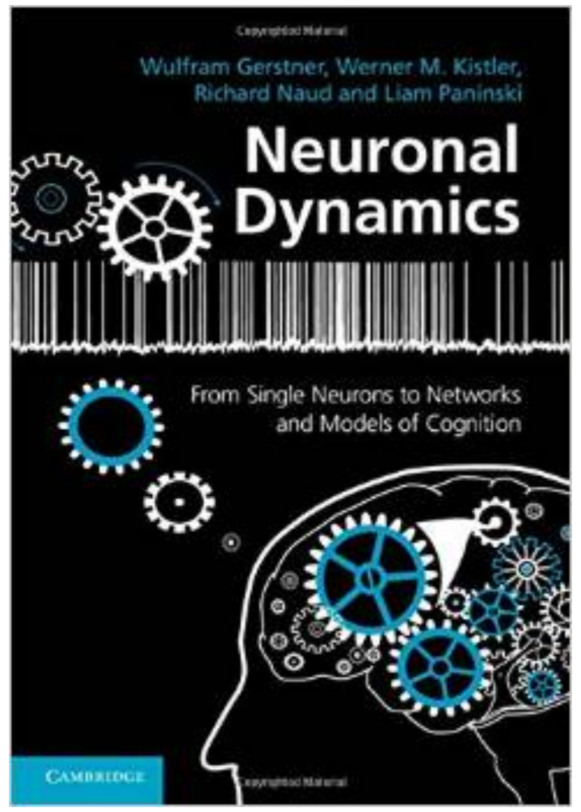
Computational Neuroscience: Neuronal Dynamics

Week 4

Reducing detail:

Analysis of 2D models

Reading for week 4: NEURONAL DYNAMICS - Ch. 4.4 – 4.7



Cambridge Univ. Press

√ 3.1 **From Hodgkin-Huxley to 2D**

√ 3.2 **Phase Plane Analysis**

√ 3.3 **Analysis of a 2D Neuron Model**

4.1 Separation of time scales

4.2 Type I and II Neuron Models
- limit cycles: constant input

4.3 Pulse input
- where is the firing threshold?

4.4. Nonlinear integrate-and-fire
- further reductin to 1 dimension

Lecture 4 of video series (last 60 minutes)

https://lcnwww.epfl.ch/gerstner/NeuronalDynamics-MOOCall.html

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EPFL

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Week 14: Neural Manifolds and low-rank networks

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How to best use the time in the inverted classroom

[ ] prof should spend more time on the Quiz questions
[ ] prof should spend less time on the Quiz questions

[ ] prof should spend more time on repetition of contents
[ ] prof should spend less time on repetition of contents

[ ] prof should be more explicit in answering posted questions
[ ] prof should be more concise in answering posted questions

[ ] it would be great if my classmates asked more questions
[ ] it would be great if my classmates asked fewer questions

[ ] Overall, the exercises should start after a max of 25 minutes
[ ] Overall timing OK as is

Before I start, are there any questions?

Are there any comments?

3.2. Nullclines of reduced HH model

stimulus

u-nullcline

w-nullcline

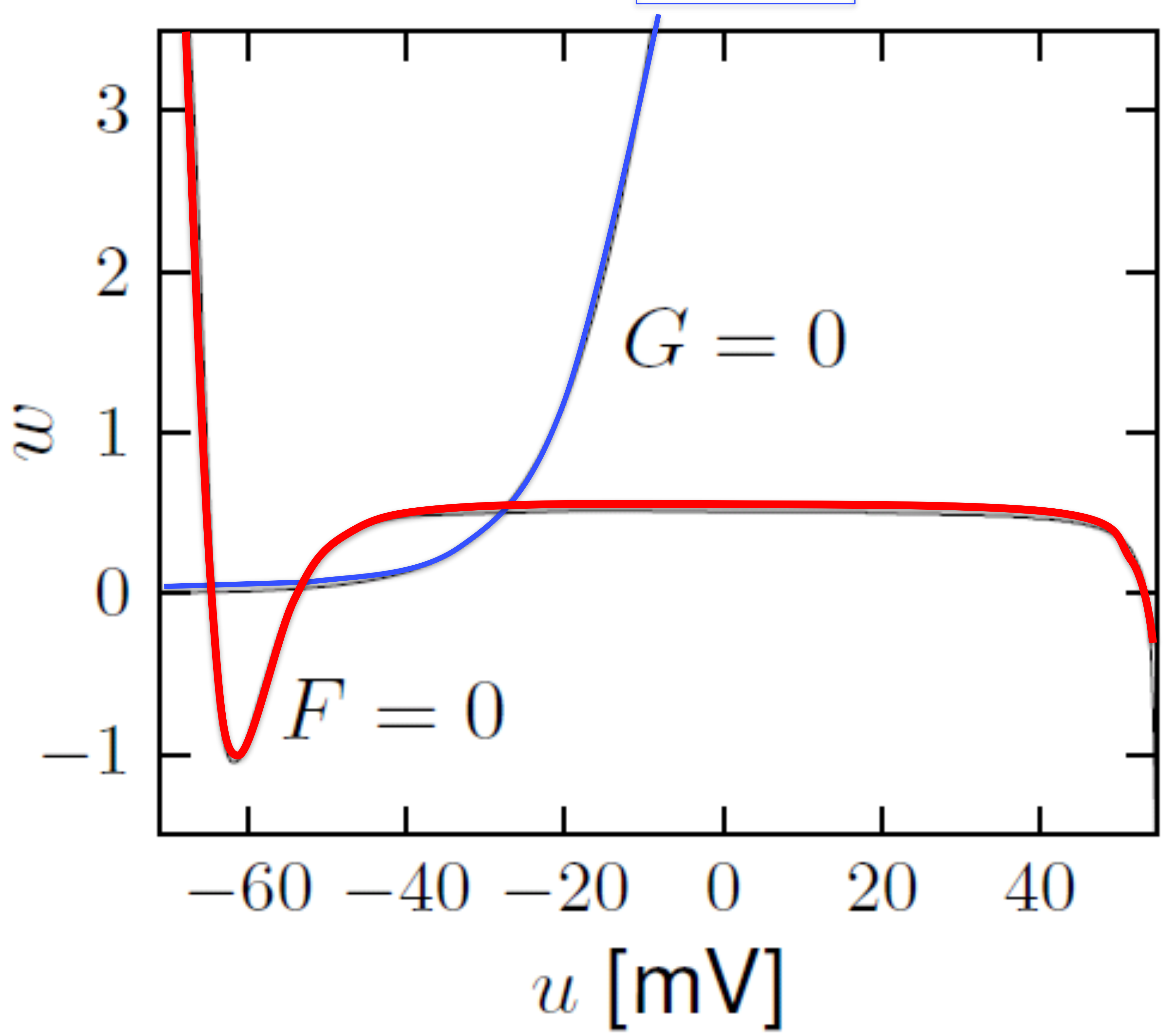

Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

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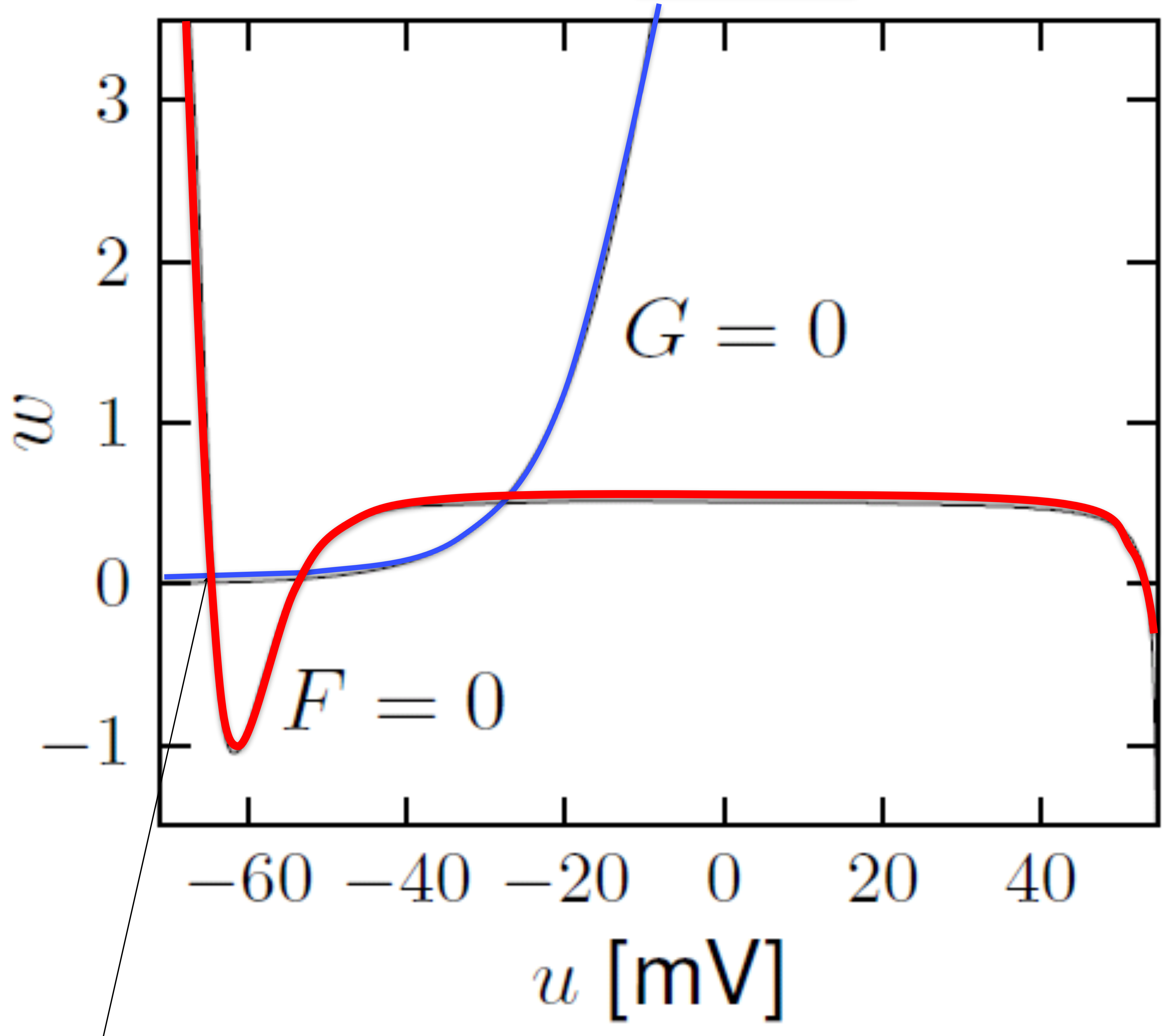


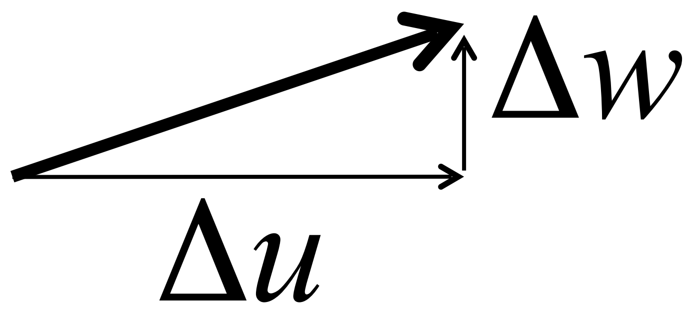
Stable fixed point

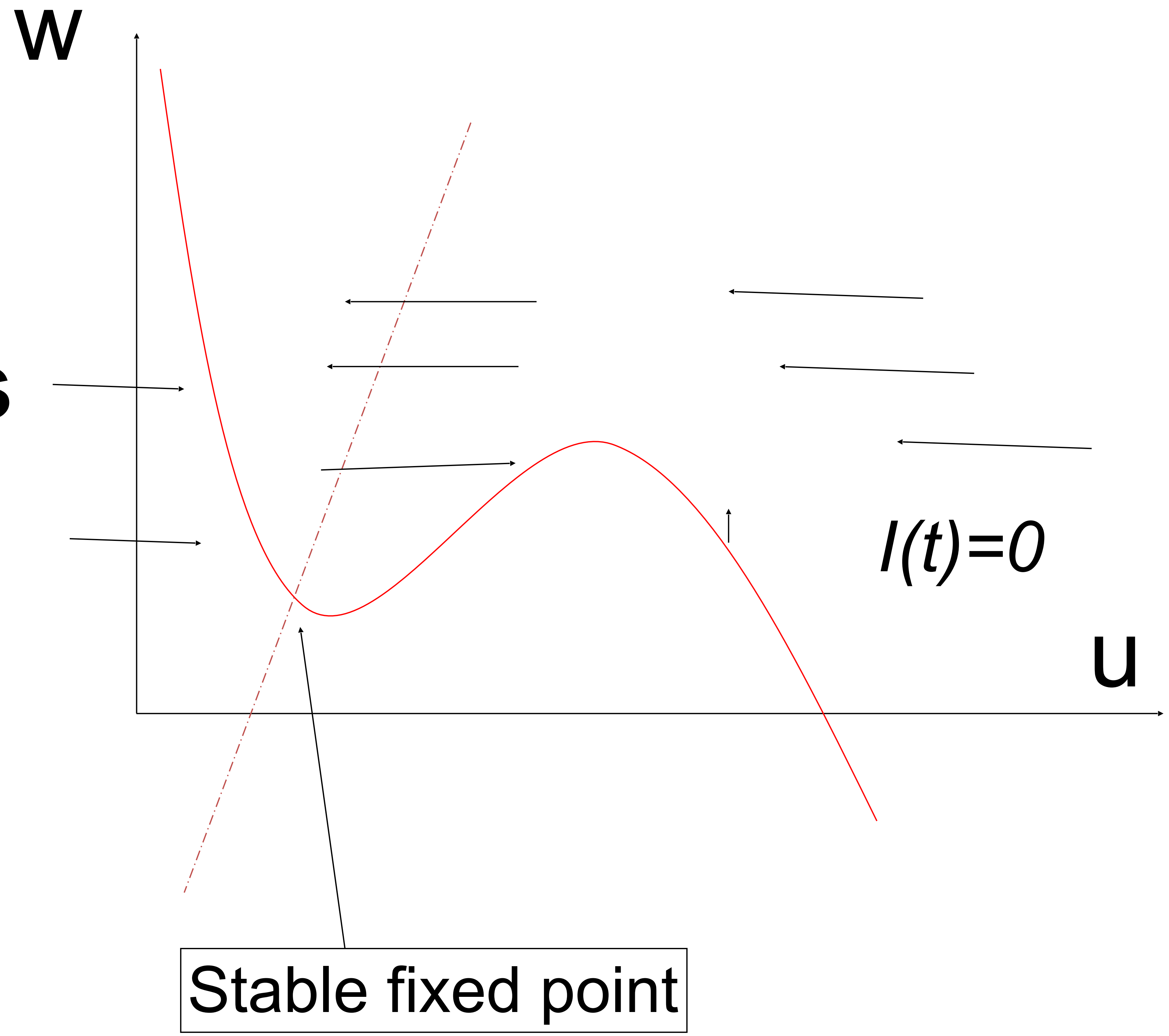
Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

4.1. Second Separation of time scales

stimulus

Separation of time scales





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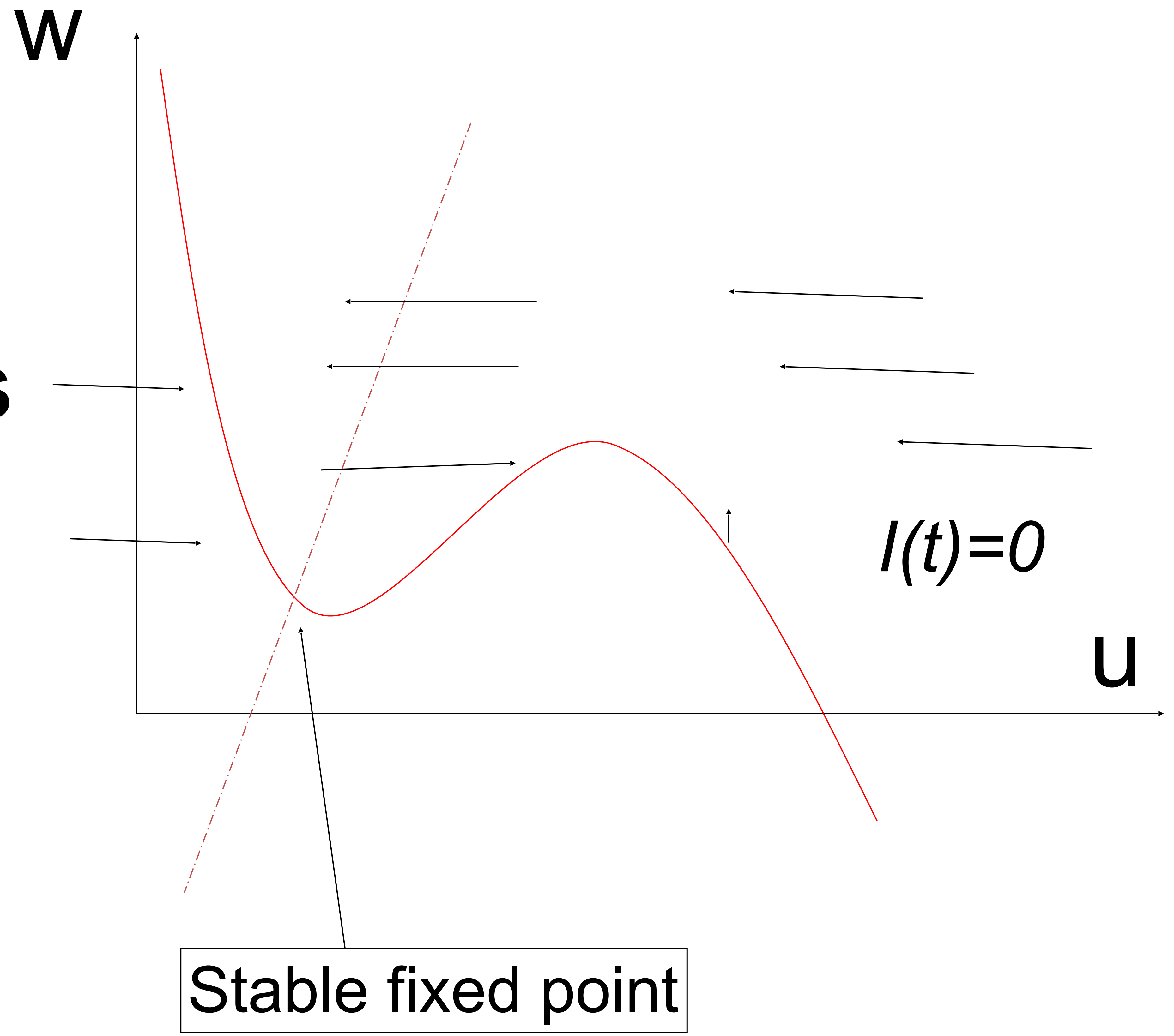
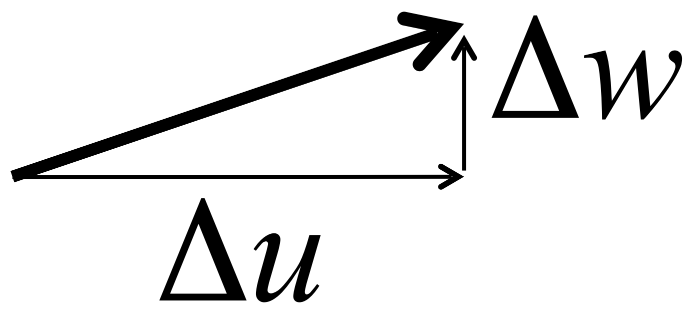


Separation of time scales



$\longrightarrow \Delta w << \Delta u$

Unless close to nullcline

4.1. Summary: Separation of time scales

We have seen a first separation of time scales last week to remove the *m*-variable. Today I have introduced a second separation of time scale: the *w*-variable is (in reality only a bit) slower than the voltage variable.
For mathematical reasons we considered the limit where *w* is MUCH slower than the voltage variable.
In this limit, the flow arrows are all horizontal – except in the region very close to the *u*-nullcline.
This condition can be exploited for two interesting stimuli:

(i) A constant stimulus strong enough to evoke a limit cycle. In this case the trajectory either jumps or follows the *u*-nullcline.
(ii) A pulse stimulus. In this case, the voltage either goes rapidly back to the fixed point or it takes a detour.

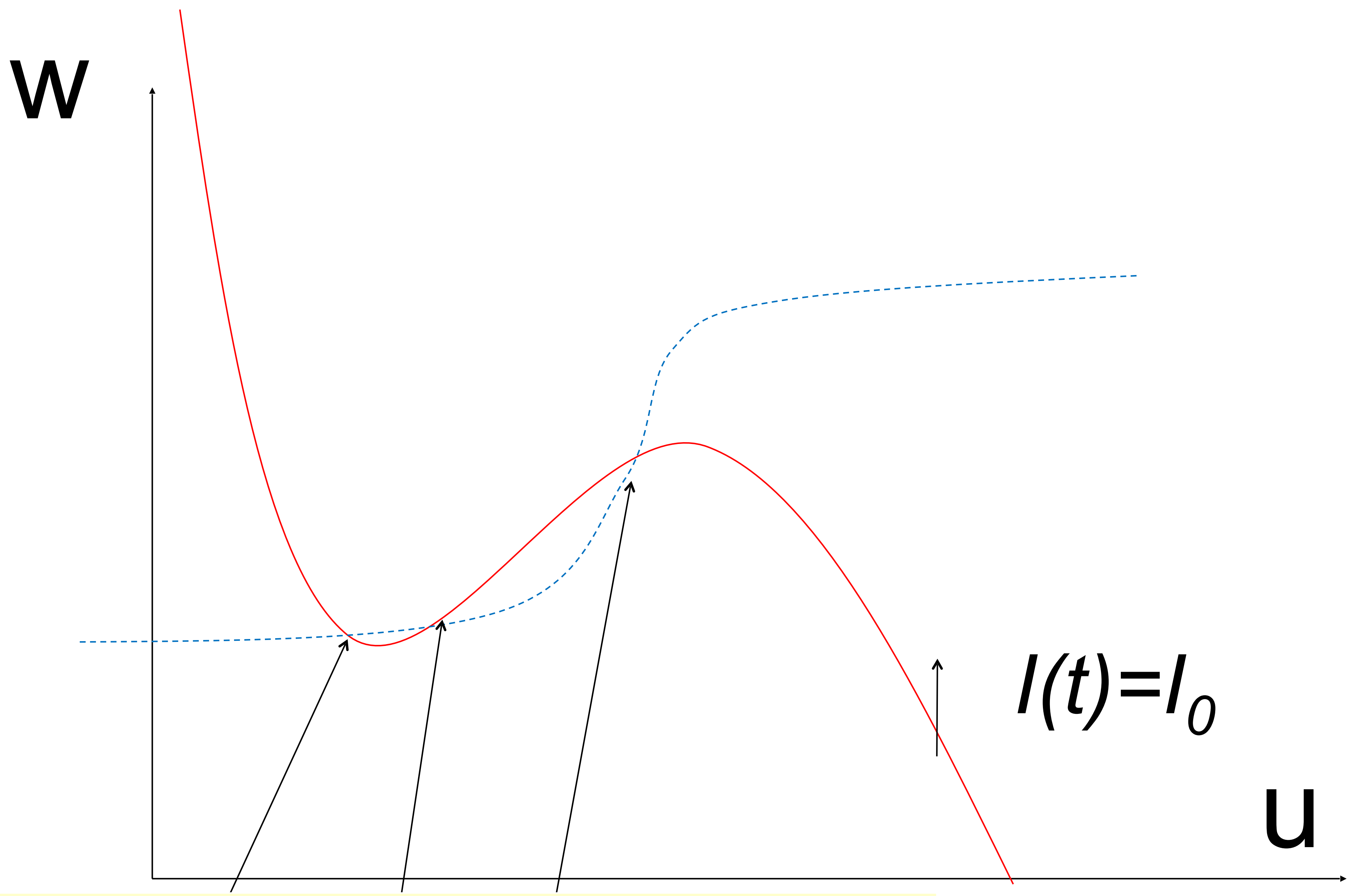
We look at both stimulation paradigms again throughout the lecture.

4.2. Type I Neuron Models: saddle-node bifurcation

stimulus

$$+RI(t)$$

constant input



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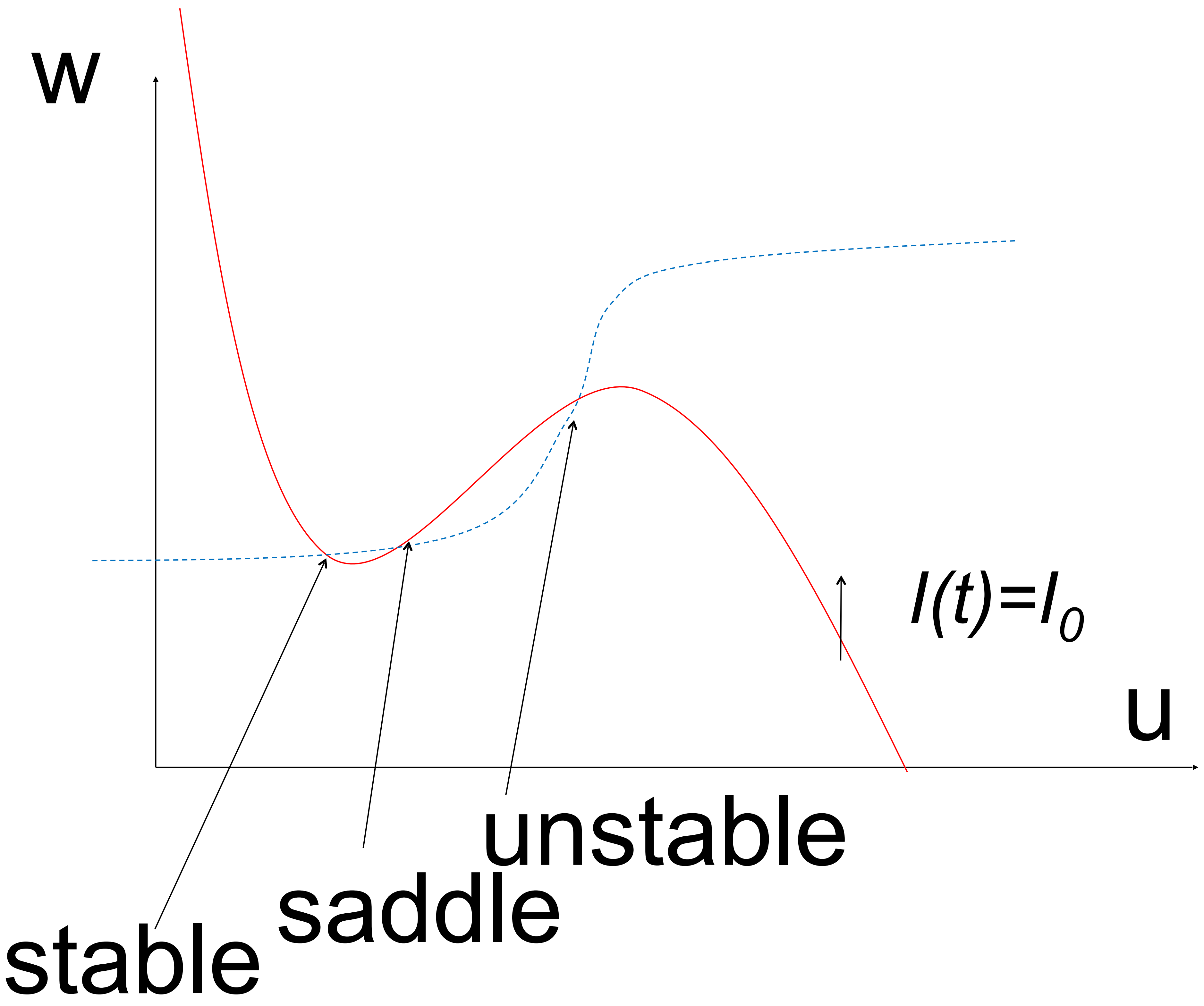

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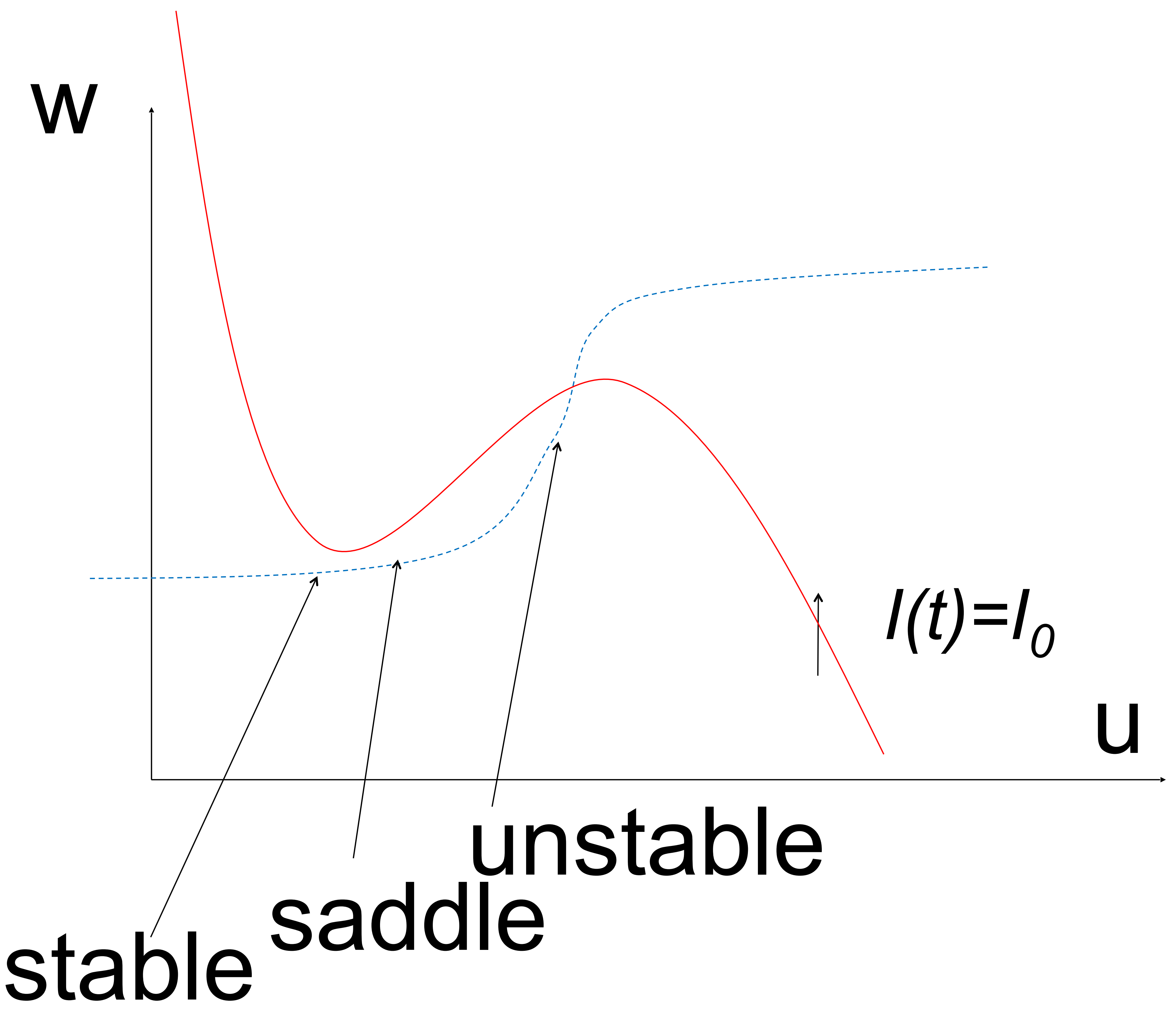

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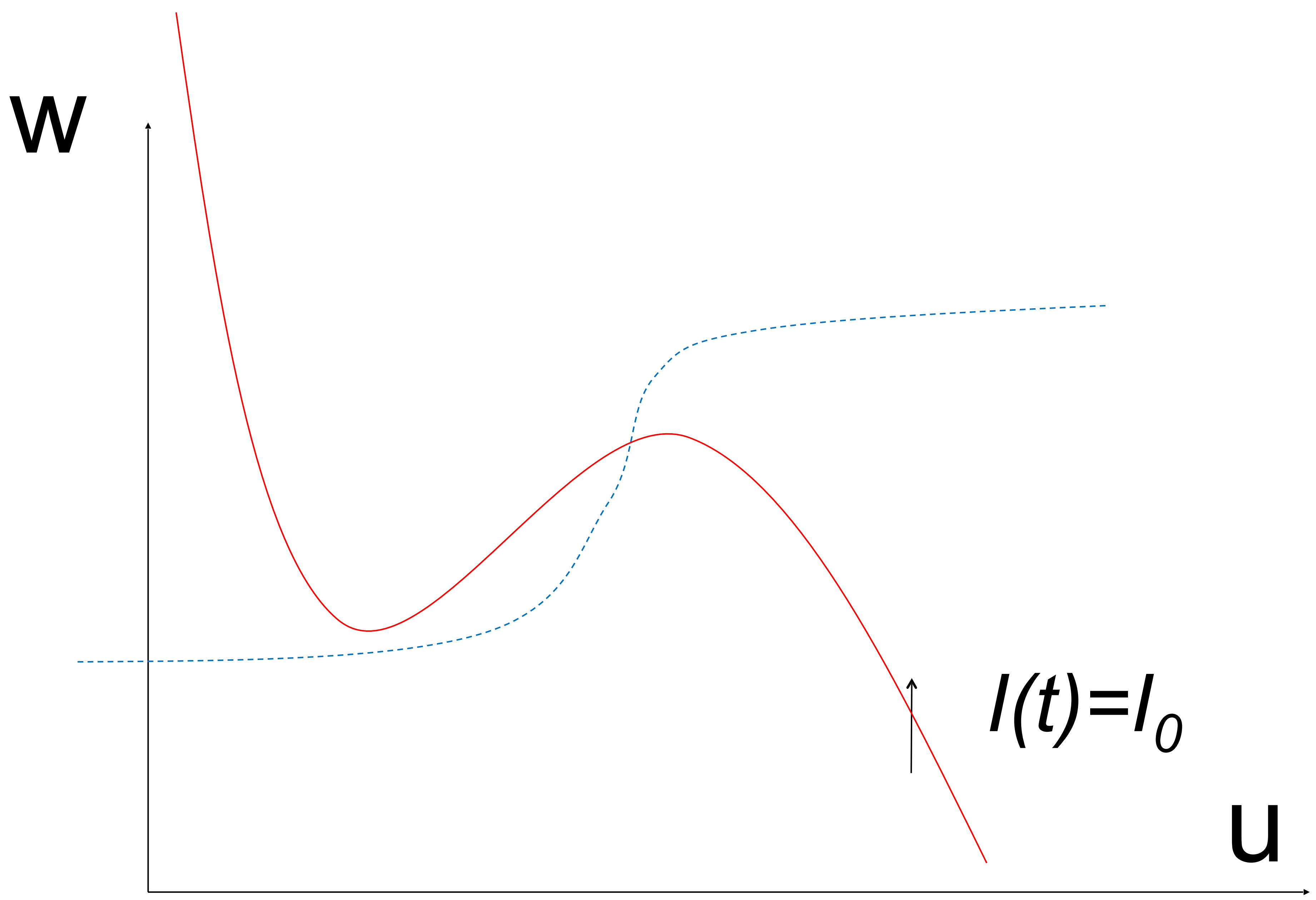

unstable

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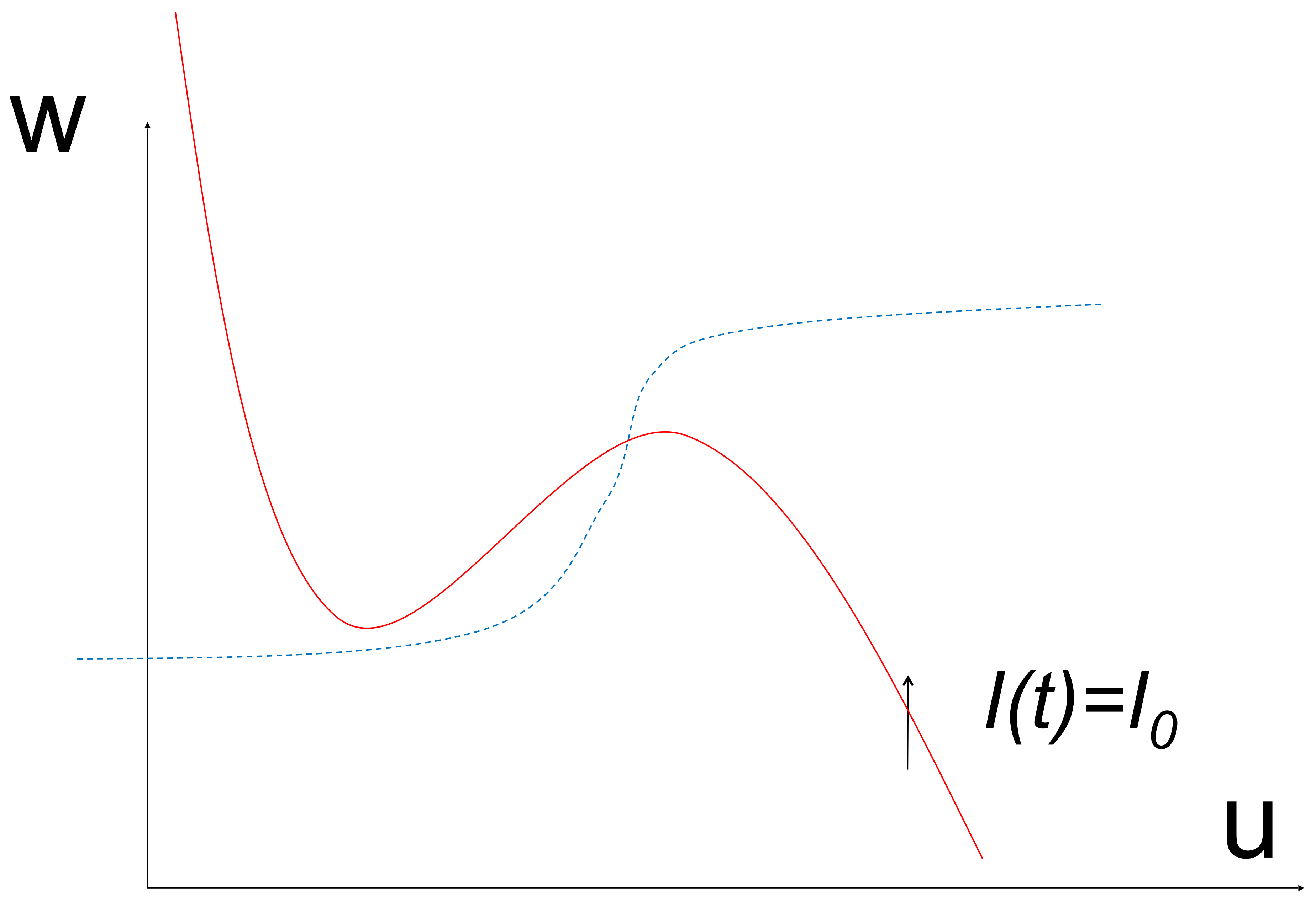


unstable

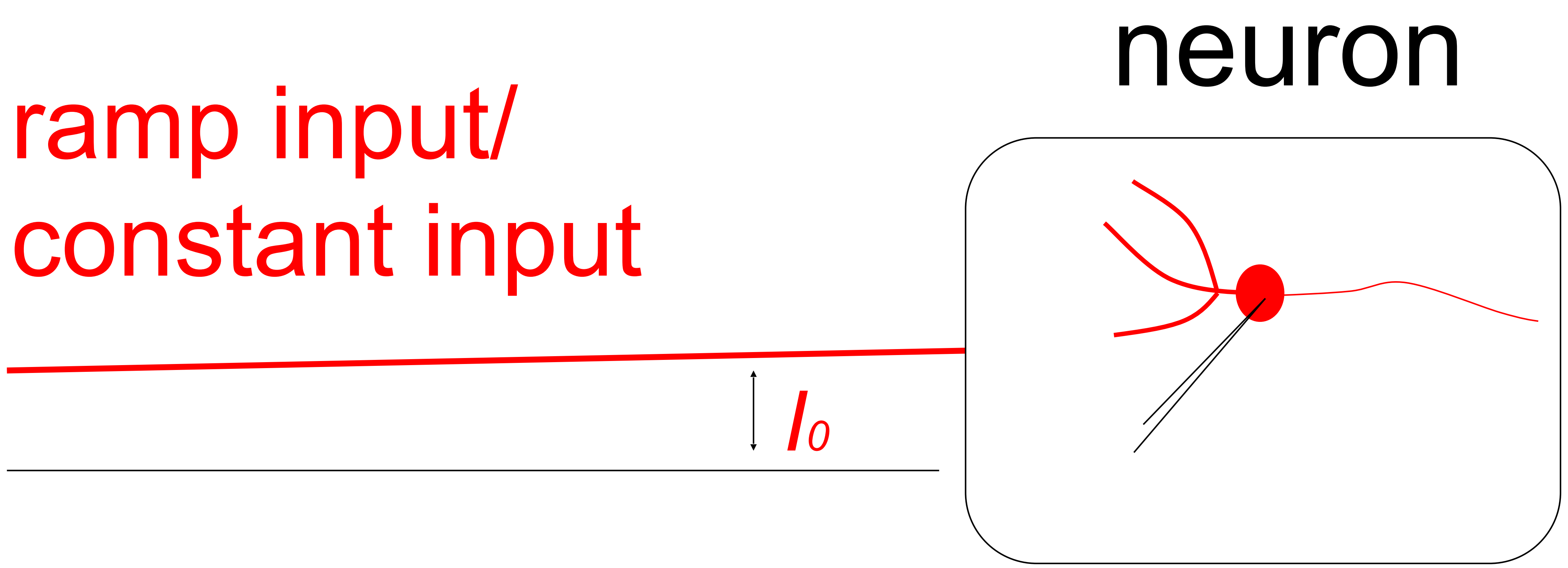
- flow arrows,
- ghost/ruins

4.2. Type I and II Neuron Models

2-dimensional equation

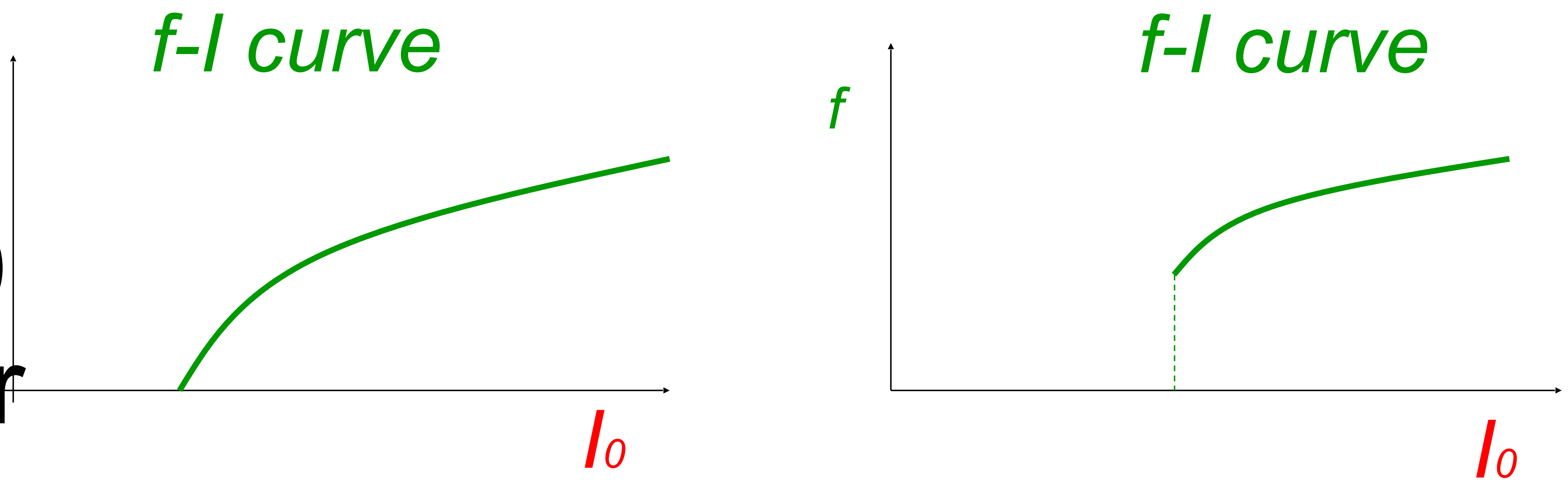
stimulus

↓



Enables graphical analysis!

Type I and type II models



Constant input

→ repetitive firing (or not)

→ limit cycle (or not)

Neuronal Dynamics – Quiz 4.1.

A. 2-dimensional neuron model with (supercritical) saddle-node-onto-limit cycle bifurcation

[ ] The neuron model is of type II, because there is a jump in the f-I curve

[ ] The neuron model is of type I, because the f-I curve is continuous

[ ] The neuron model is of type I, if the limit cycle passes through a regime where the flow is very slow.

B. Threshold in a 2-dimensional neuron model with subcritical Hopf bifurcation

[ ] The neuron model is of type II, because there is a jump in the f-I curve

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[ ] in the regime below the Hopf bifurcation, bistability between regular firing and rest state is possible.

Neuronal Dynamics – Quiz 4.1.

A. 2-dimensional neuron model with (supercritical) saddle-node-onto-limit cycle bifurcation

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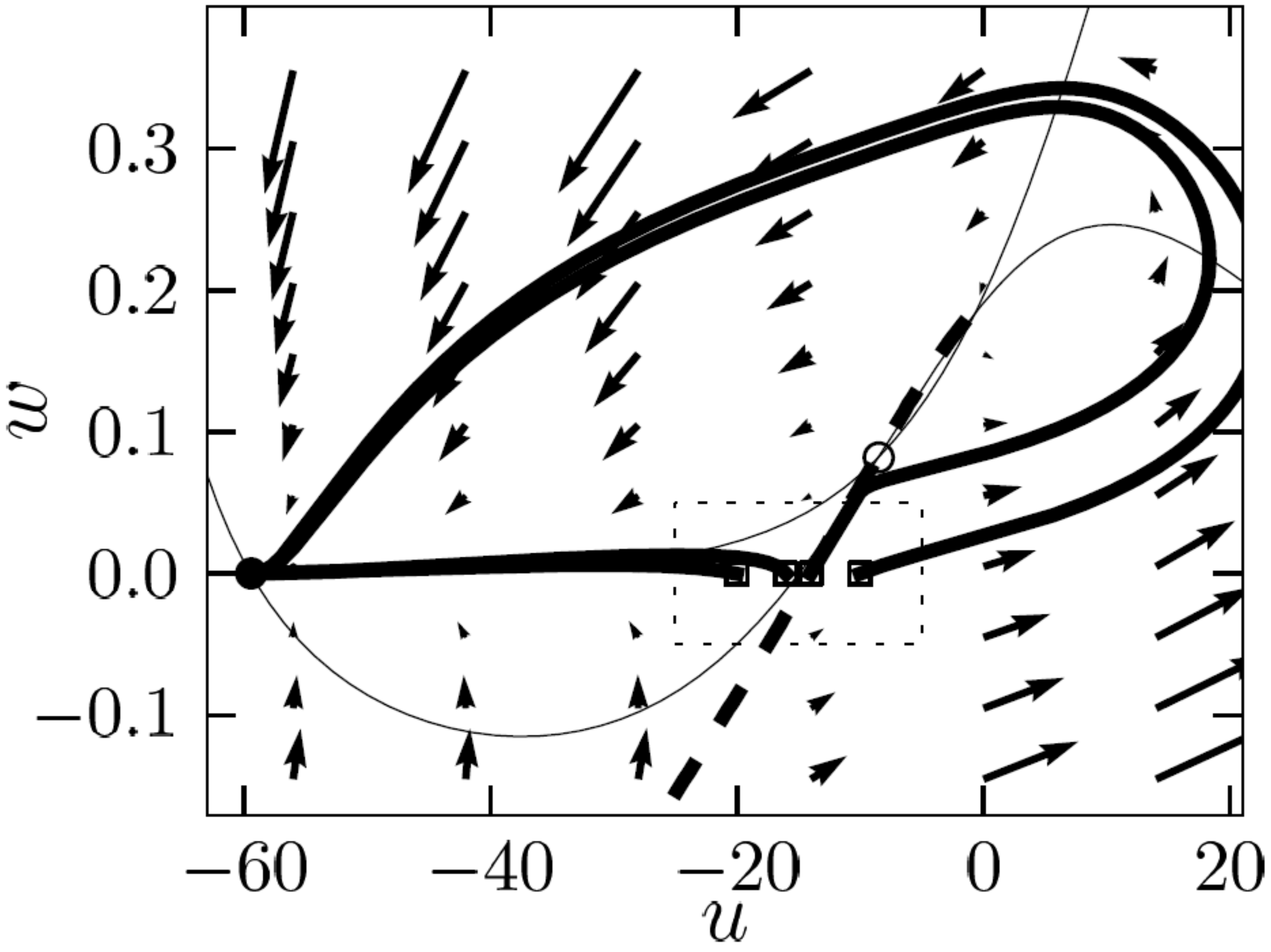
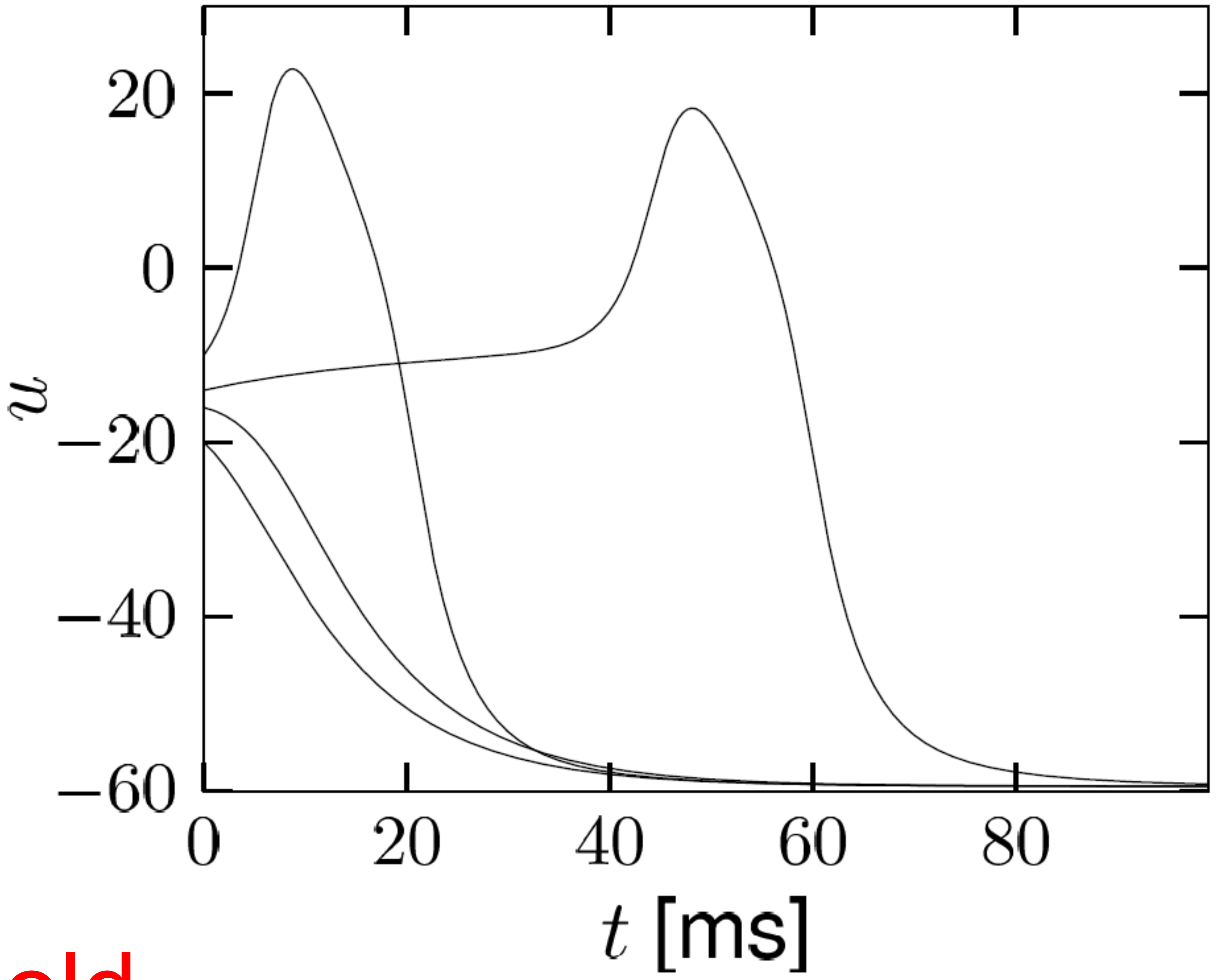
[X] [ ] in the regime below the Hopf bifurcation, bistability between regular firing and rest state is possible.

4.2. Summary: Limit cycles and neuron models

1) In 2 dimensions we have a powerful theorem: if we can find a bounding box around an unstable fixed such that all flow arrows point inside the box, then there must be a limit cycle.
2) We can change the stability of the fixed point(s) by a constant input.
3) The limit cycle MAY appear at the moment when the fixed point looses stability. In this case it would often be a limit cycle of small amplitude in the neighborhood of the fixed point.
4) But we can also observe bistability between the stable fixed point and a limit cycle.
5) Neuron models can be classified according to the bifurcation type that makes a limit cycle appear. Type 1 neuron models have a smooth f-I curve and are always linked to a saddle-node-onto limit cycle bifurcation.
6) Type 2 models can have various origins; an example is the subcritical Hopf-bifurcation

4.3 Type I model: Delayed spike initation for Pulse input



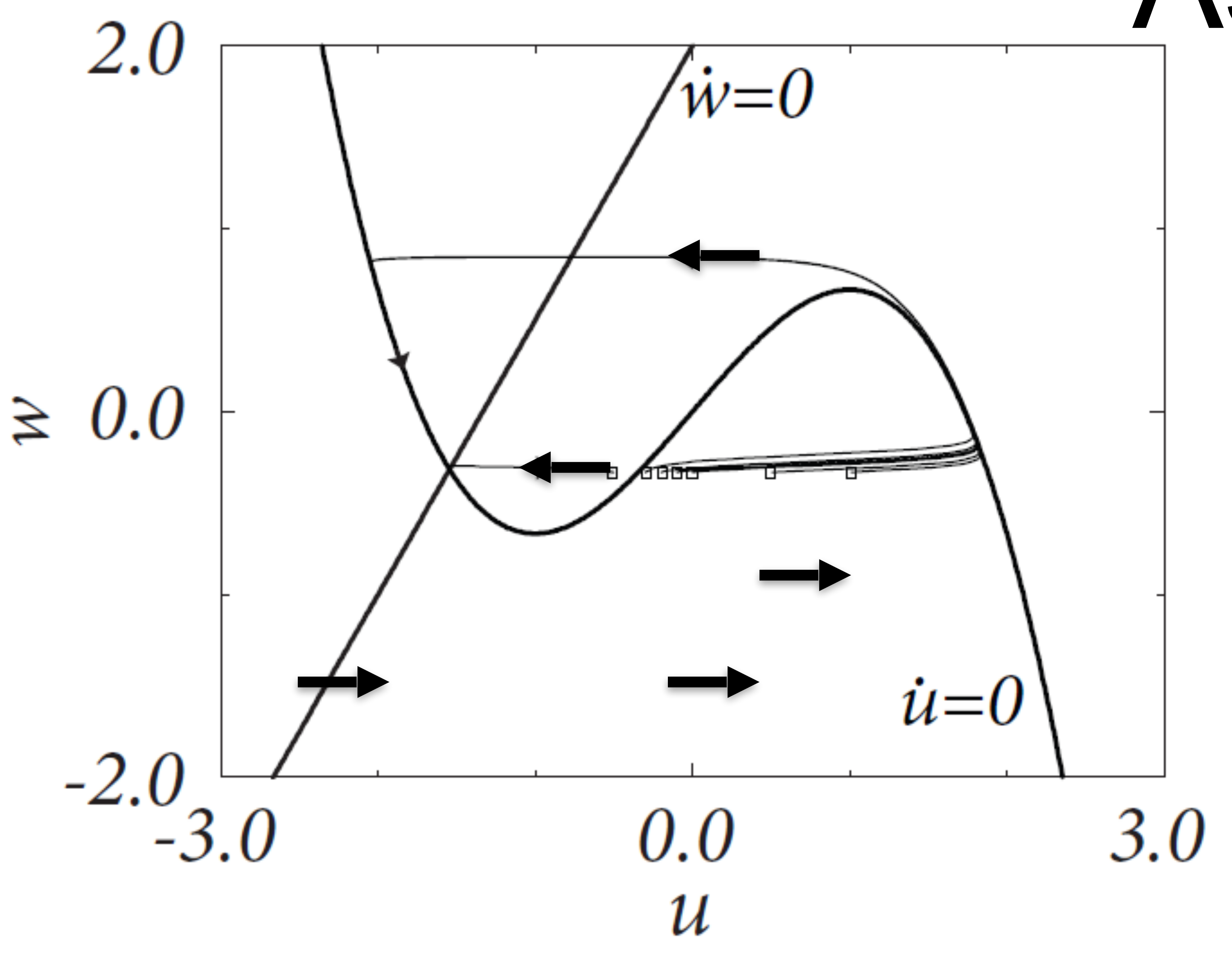


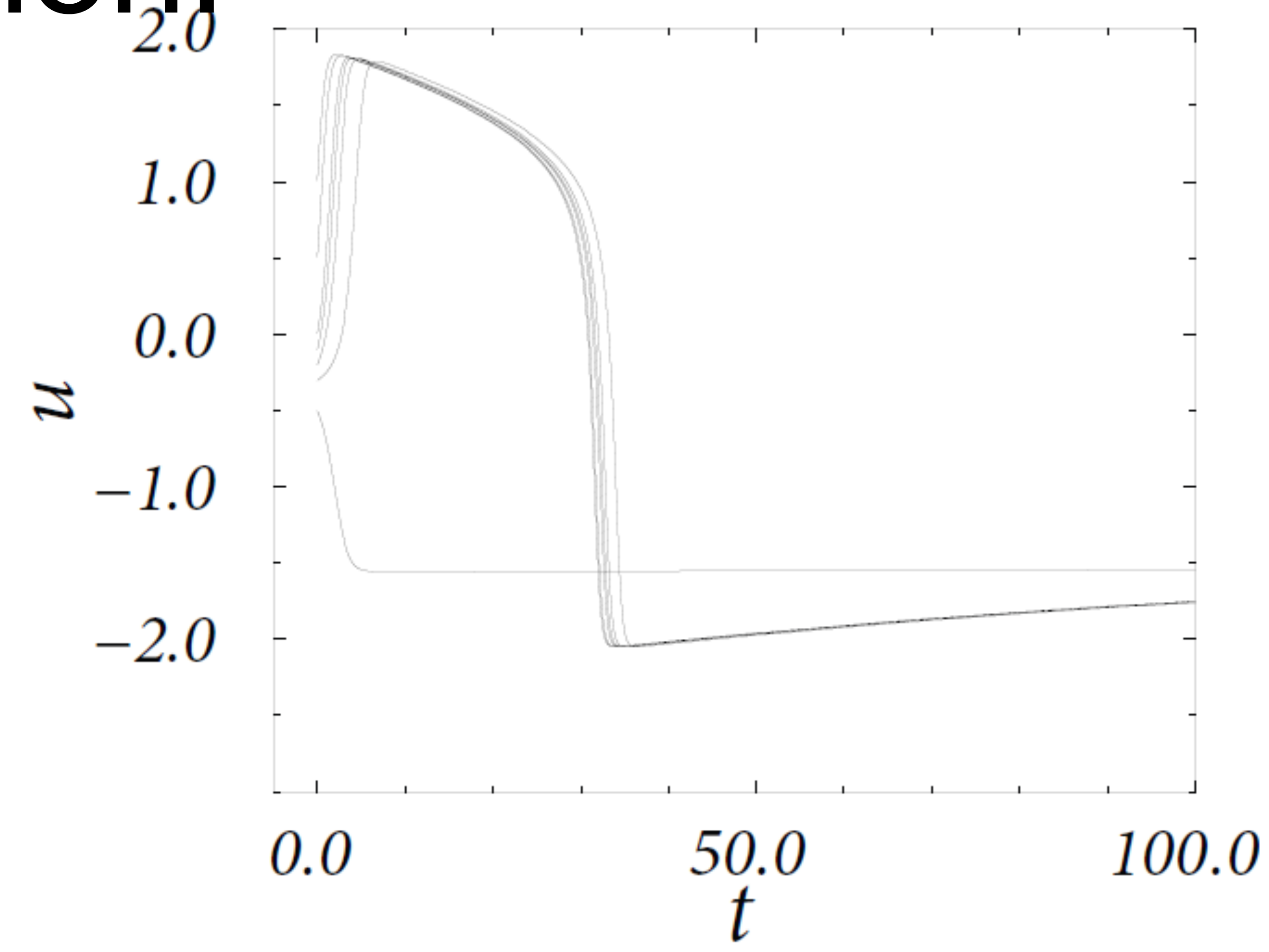
- Stable manifold plays role of threshold for pulse input
- Delayed spike initiation close to ‘Threshold’ (for pulse input)

Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

4.3 FitzHugh-Nagumo model: Threshold for Pulse input

Assumption:





Middle branch of u-nullcline plays role of 'Threshold' (for pulse input)

Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

Week 4– Quiz 4.2.

A. Threshold in a 2-dimensional neuron model with saddle-node bifurcation

[ ] The voltage threshold for repetitive firing is always the same as the voltage threshold for pulse input.

[ ] in the regime below the saddle-node bifurcation, the voltage threshold for **repetitive firing** is given by the stable manifold of the saddle.

[ ] in the regime below the saddle-node bifurcation, the voltage threshold for **action potential firing in response to a short pulse input** is given by the stable manifold of the saddle point.

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Week 4– Quiz 4.3.

B. Threshold in a 2-dimensional neuron model with subcritical Hopf bifurcation

[ ] in the regime below the Hopf bifurcation, the voltage threshold for firing of an isolated action potential firing in response to a short pulse input is the middle branch of the u-nullcline.

[ ] in the regime below the bifurcation, a voltage threshold for firing of an isolated action potential f in response to a short pulse input exists only if

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B. Threshold in a 2-dimensional neuron model with subcritical Hopf bifurcation

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[x] [ ] in the regime below the bifurcation, a voltage threshold for firing of an isolated action potential f in response to a short pulse input exists only if

4.3. Summary: Pulse input and thresholds

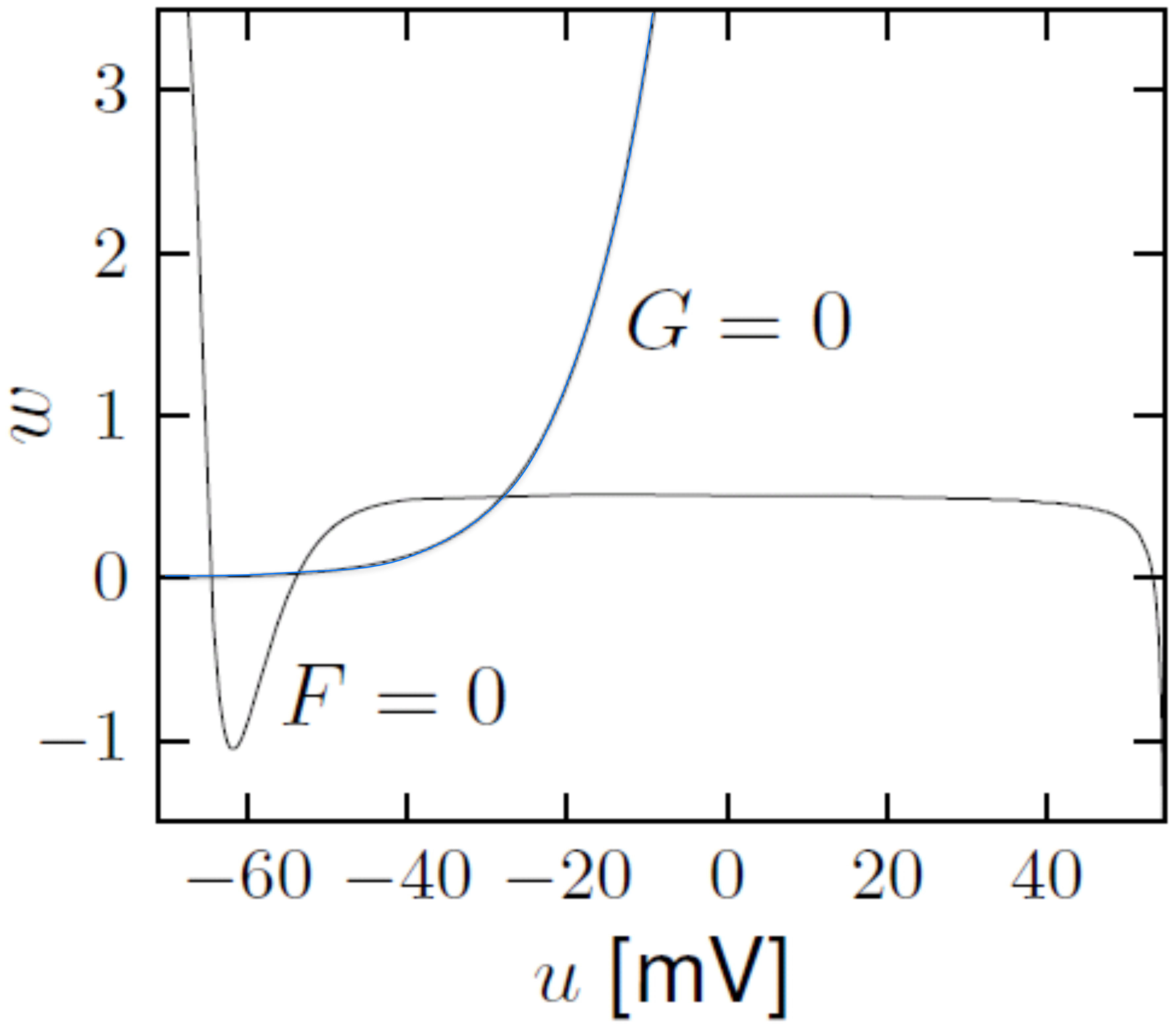
Neuron models with Saddle-node-onto limit cycle bifurcation have

- a smooth f-I curve
- a well-defined threshold for pulse input: either an AP occurs or not.
- Transition from subthreshold to superthreshold happens via an AP with very large delay.

Neuron models with subcritical Hopf-bifurcation have

- a non-smooth f-I curve
- not a well-defined threshold: there is a small regime where an AP transforms smoothly into non-AP
- However, together with a separation of time scale, the middle branch of the *u*-nullcline acts as a voltage threshold.

4.4. 2D model, after spike initiation

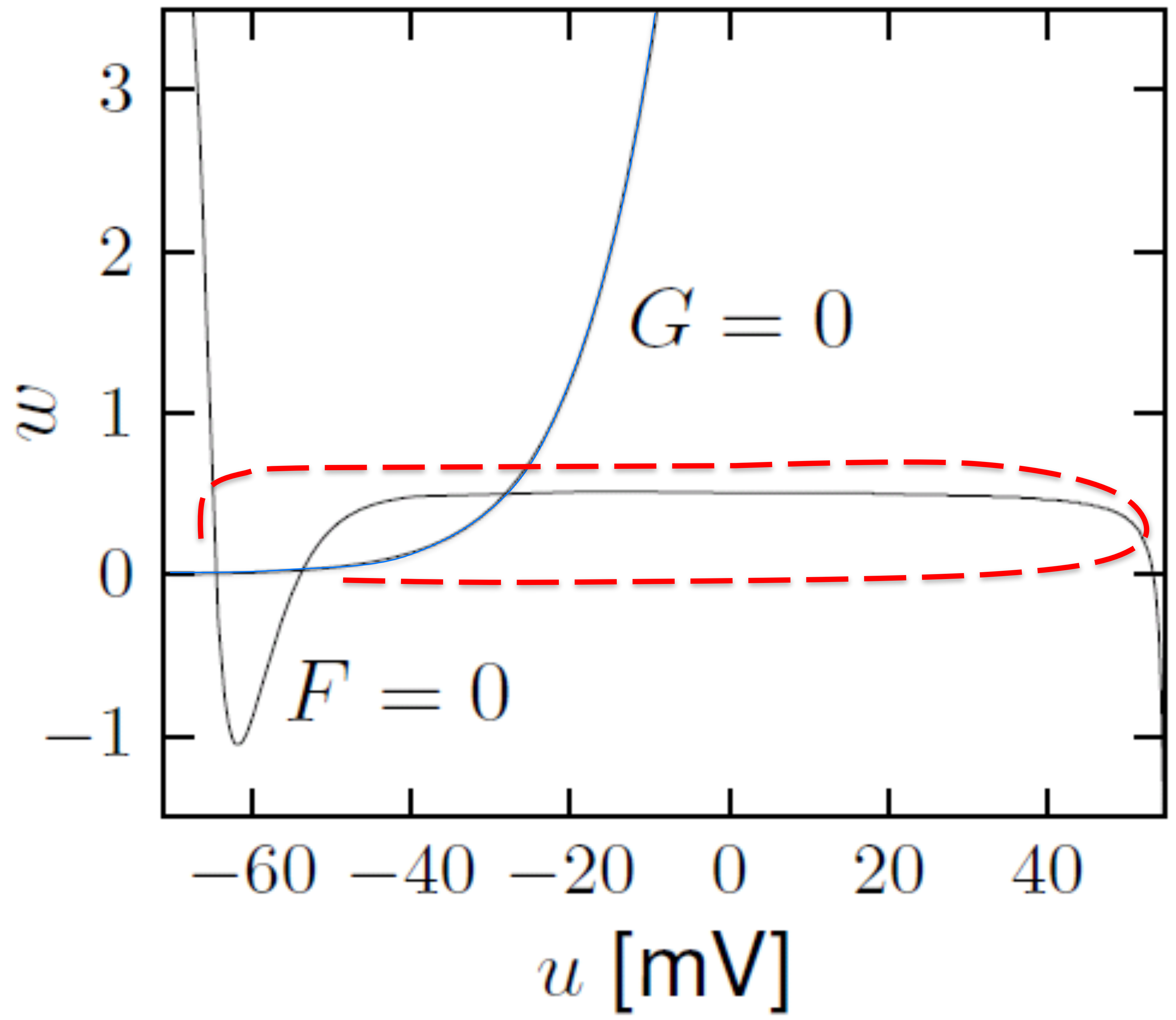


2-dimensional equation

Separation of time scales:

w is constant (if not firing)

4.4. 2D model, after spike initiation



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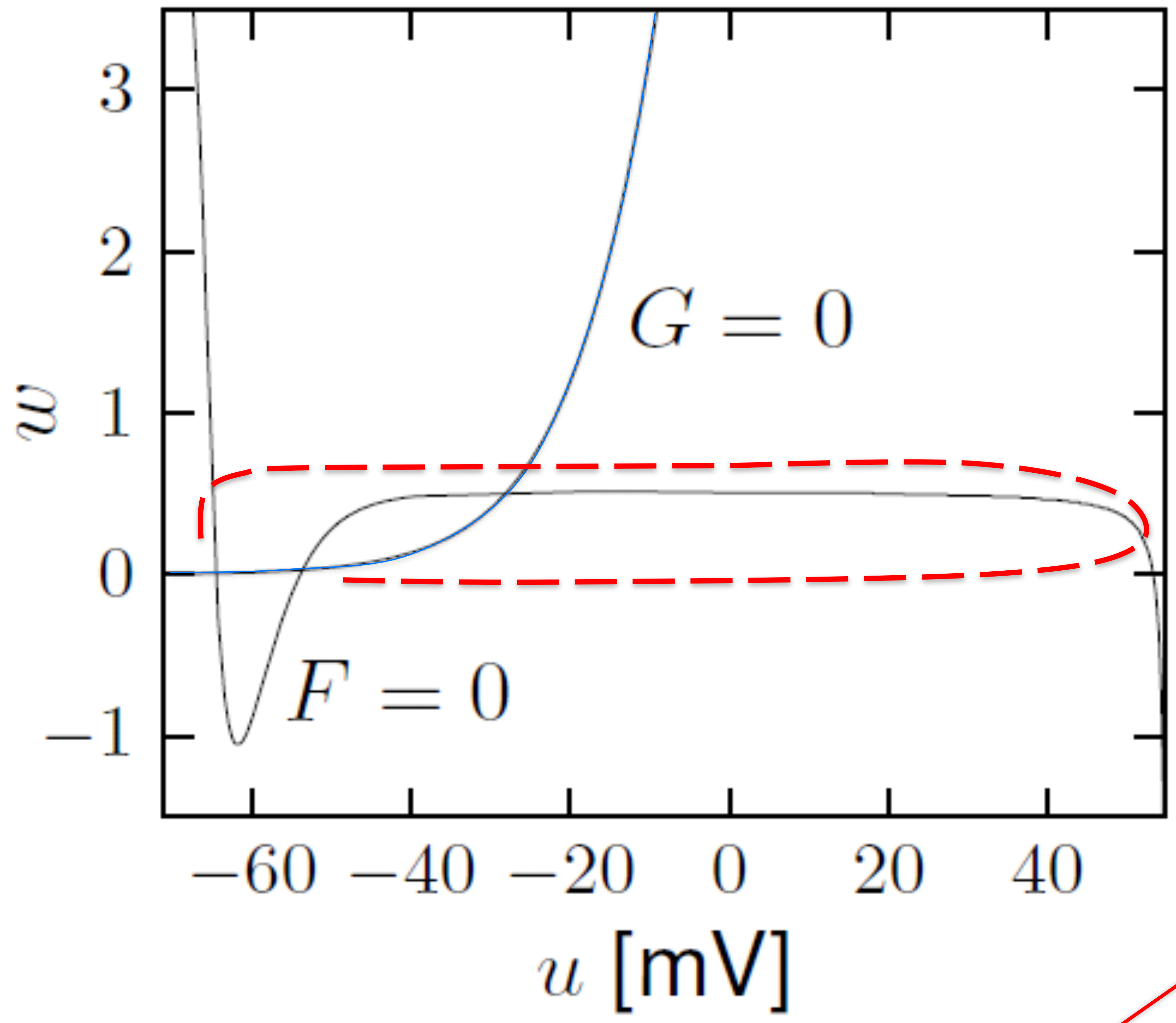

Relevant during spike
and immediately
after downswing of AP

2-dimensional equation

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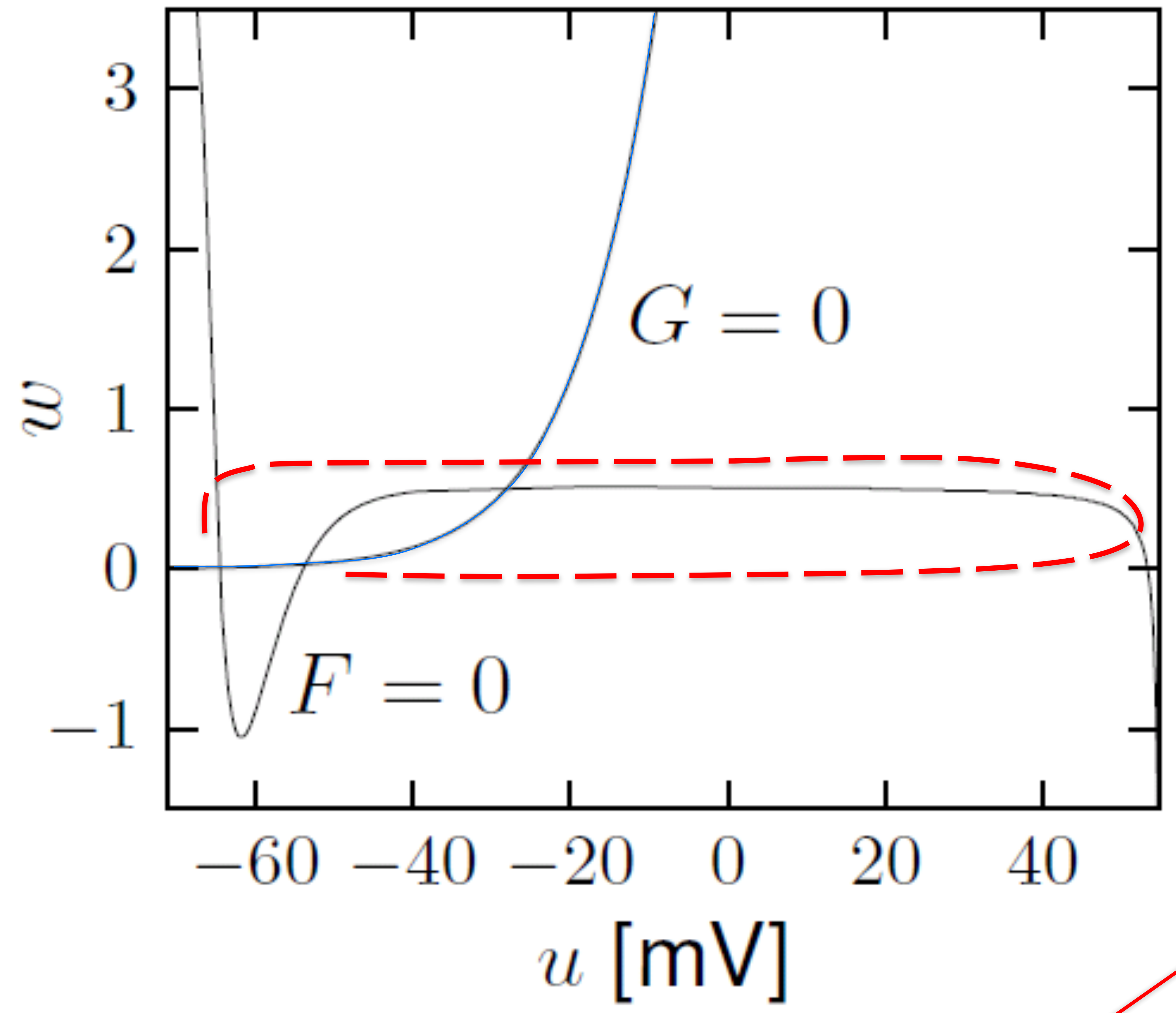

2-dimensional equation

Separation of time scales:
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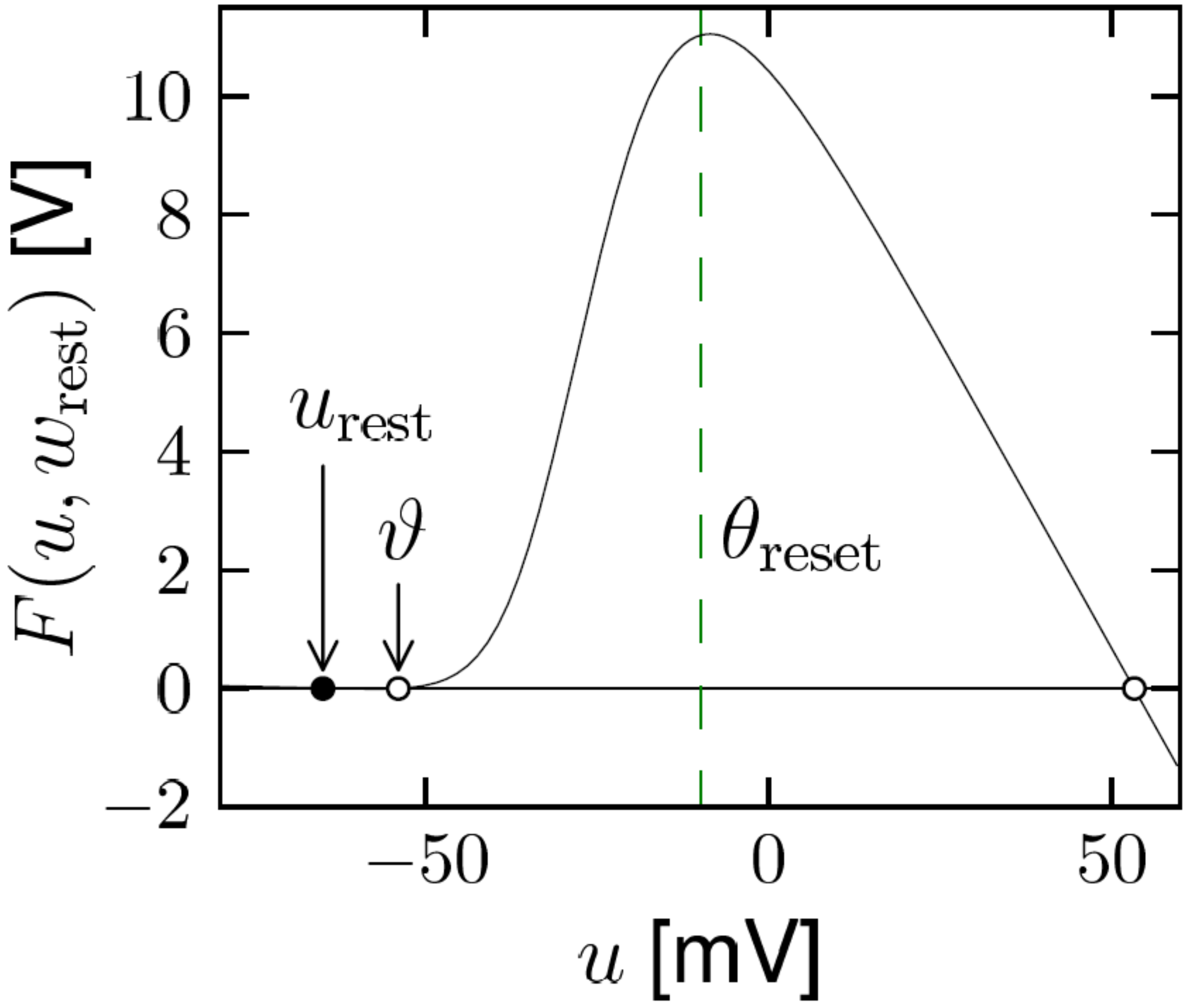
Relevant during spike
and immediately
after downswing of AP

Integrate-and-fire:
threshold+reset for AP

4.4. Spike initiation: Nonlinear Integrate-and-Fire Model



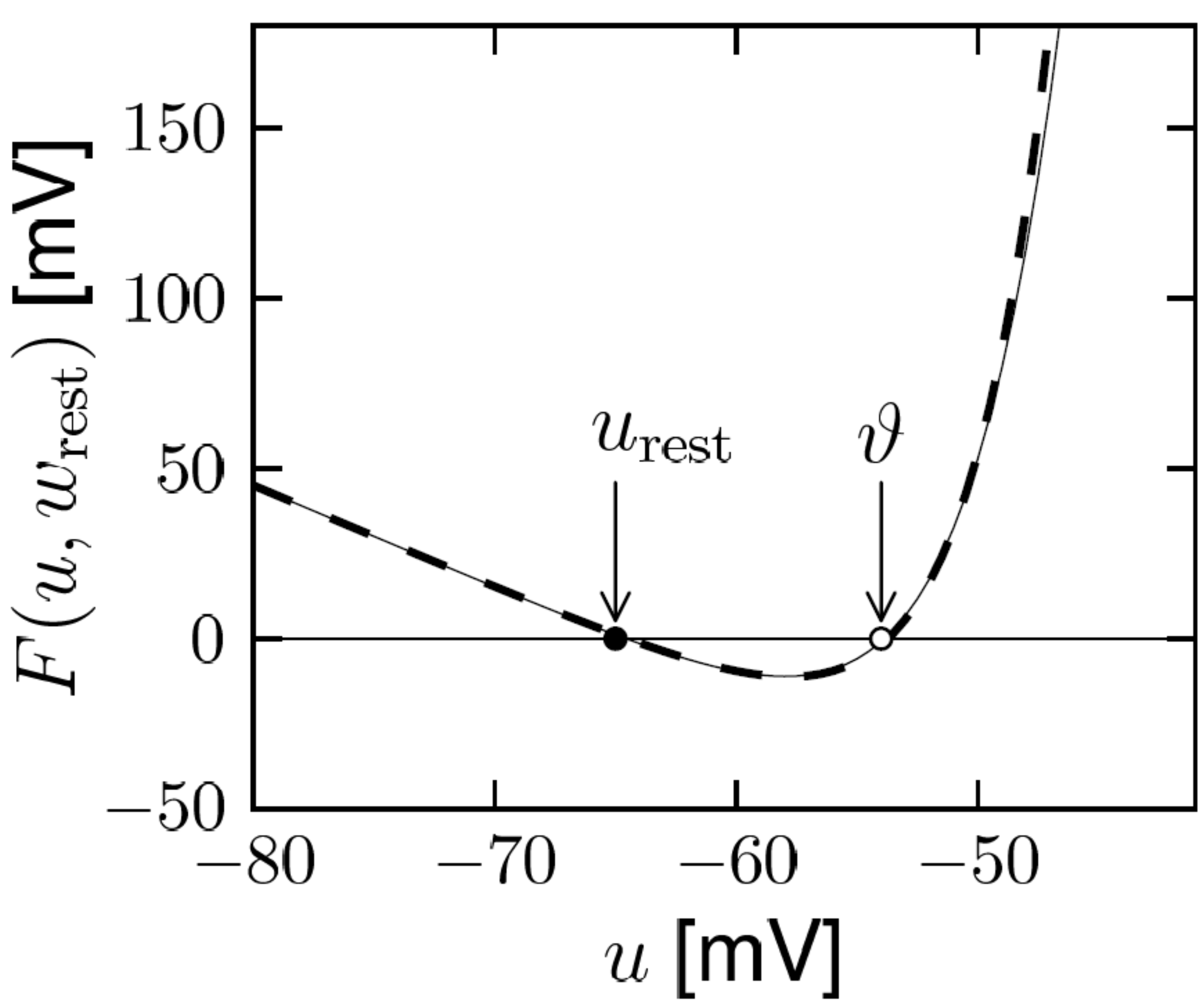


Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

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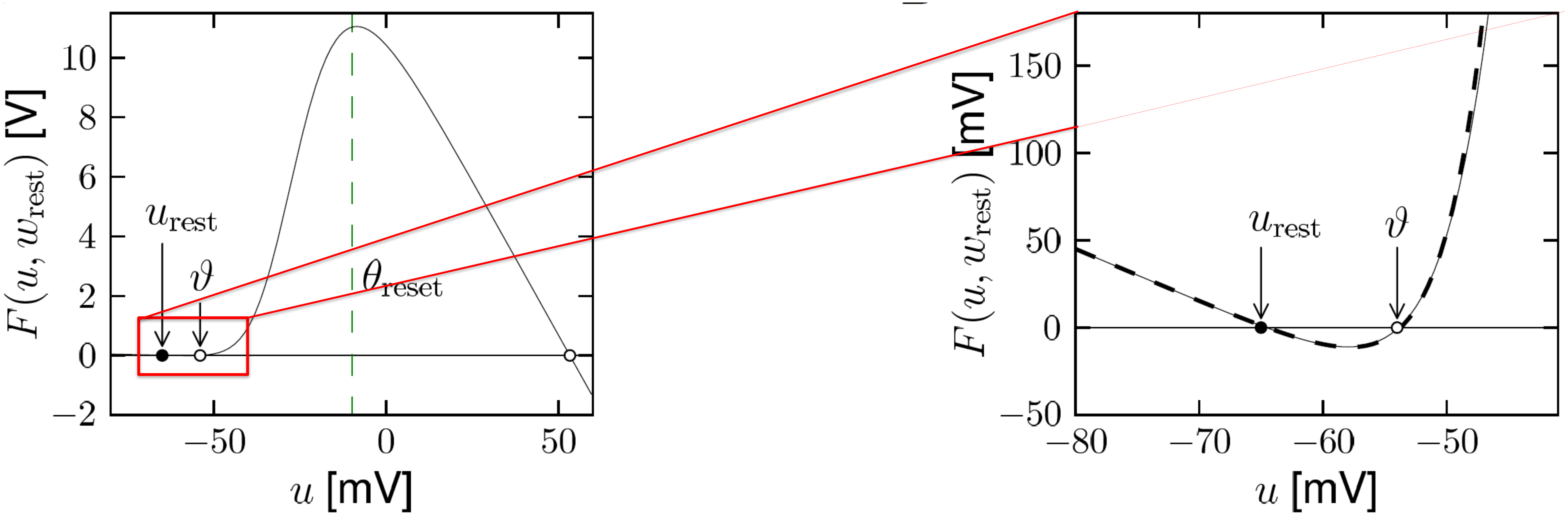

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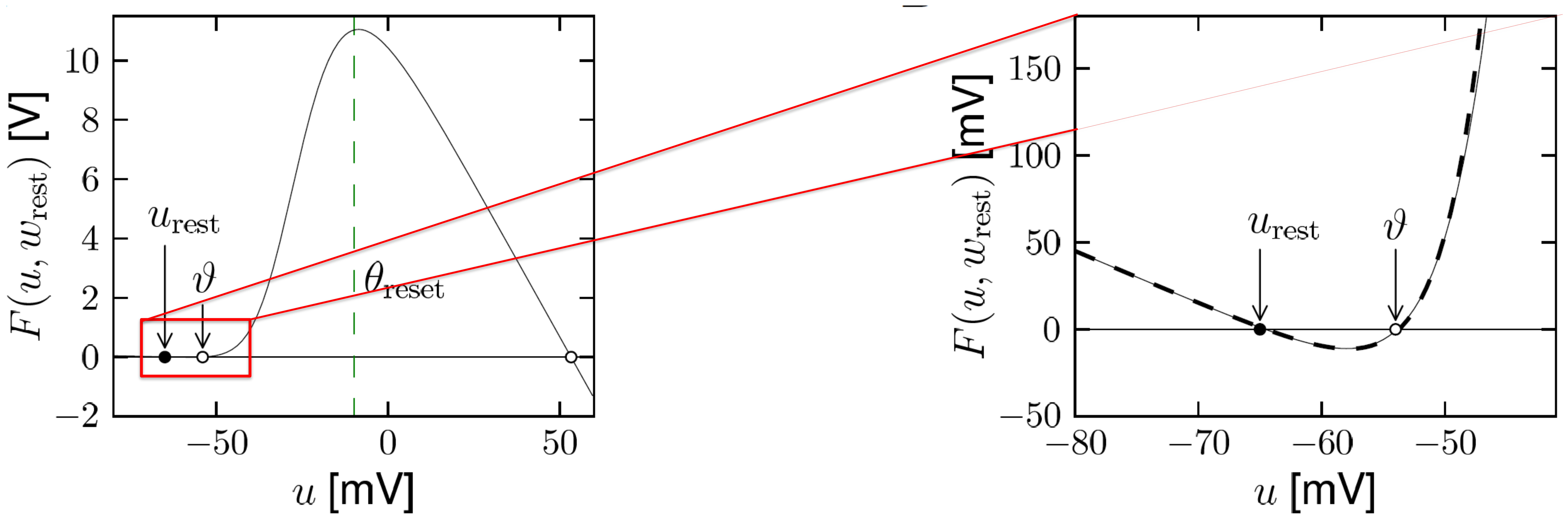

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→ Nonlinear I&F (see week 1!)

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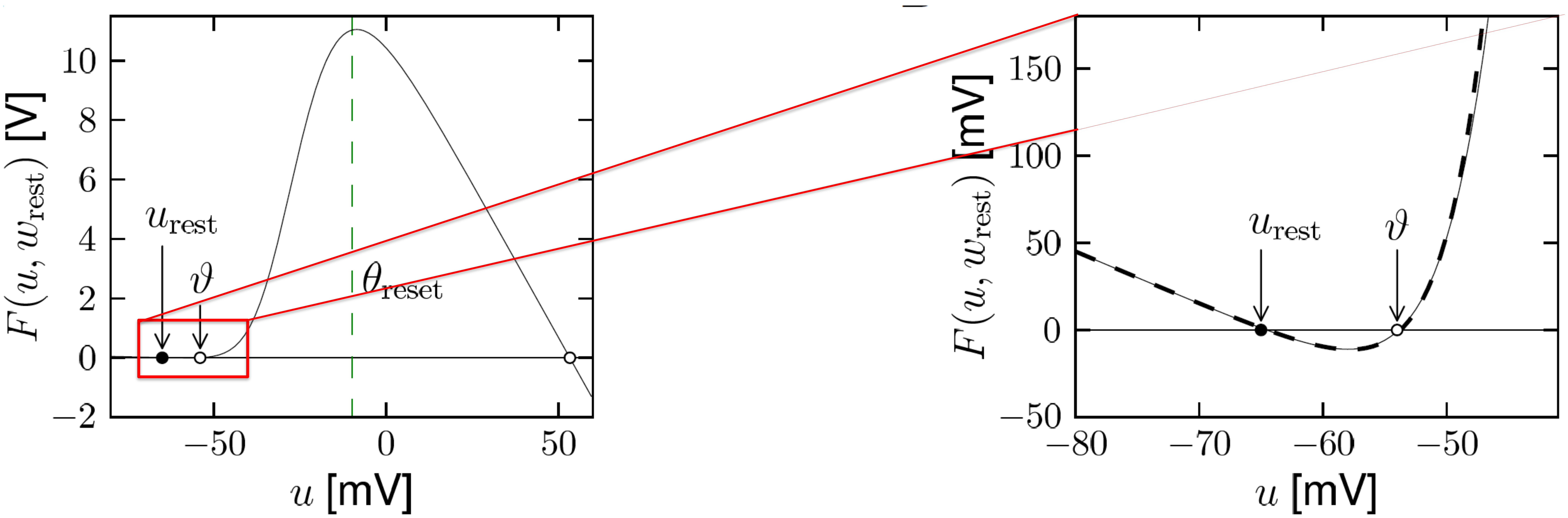

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→ Nonlinear I&F (see week 1!)

During spike initiation, the 2D models with separation of time scales can be reduced to a 1D model equivalent to nonlinear integrate-and-fire

4.3. Exponential Integrate-and-Fire Model

Exponential integrate-and-fire model (EIF)

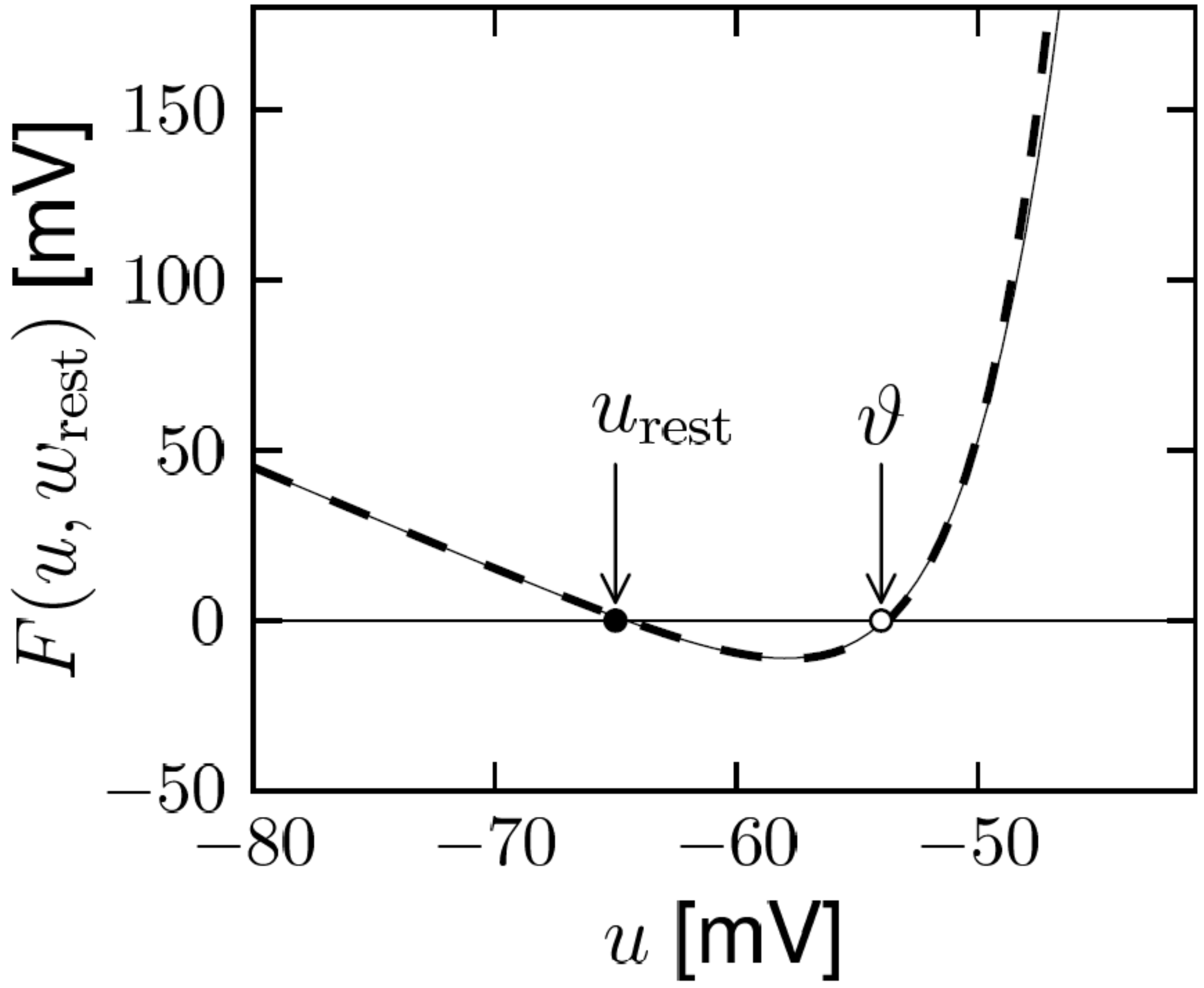

Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

→ Nonlinear I&F (see week 1!)

4.4 Extension to Adaptive Exponential I&F

Add adaptation variables:

$$+RI(t)$$

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after each spike
jumps by an amount

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SPIKE AND RESET

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SPIKE AND RESET

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jumps by an amount

AdEx model, Brette&Gerstner (2005):

4.4 Extension to Adaptive Exponential I&F

Add adaptation variables:

$+RI(t)$

Exponential I&F
+ **1 adaptation** var.
= AdEx

SPIKE AND RESET

after each spike
jumps by an amount

AdEx model, Brette&Gerstner (2005):

4.4. Summary: from HH to generalized integrate-and-fire

- The **reduction of the Hodgkin-Huxley (HH) model** from 4 to 2 dimensions generates nonlinear nullclines with several intersections.
- If we zoom in on the two left-most intersections the u-nullcline looks similar to a superposition of a linear and an exponential term
- Between (rare) spike events, the w-variable has always time to go back to resting potential. Hence during spike-initiation we can consider the w-variable as constant.
- This gives rise to the **exponential integrate-and-fire model**

- **Adaptation** means that for constant input the interspike intervals increase over time
- The standard HH-model shows no (or very little) adaptation
- More complicated Hodgklin-Huxley type models would have additional variables (describing other ion channels) that cause adaptation
- In integrate-and-fire models, these additional adaptation variables can often be approximated by a linear dynamics for new variables w_k

Computational Neuroscience: Neuronal Dynamics

Part I: Single Neurons, deterministic. Week 1-4

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Week 2: **Hodgkin-Huxley models and biophysical modeling**
Week 3: **Two-dimensional models and phase plane analysis**
Week 4: **Two-dimensional models, type I and type II models transition to IF models**

LEARNING OUTCOMES

- Solve linear one-dimensional differential equations
- Analyze two-dimensional models in the phase plane
- Develop a simplified model by separation of time scales
- Analyze connected networks in the mean-field limit
- Formulate stochastic models of biological phenomena
- Formalize biological facts into math
- Prove stability and convergence
- Apply model concepts in simulations
- Predict outcome of dynamics
- Describe neuronal phenomena

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