Neural Electrodes - Exercise set A

Exercice 1. Electrical stimulation and safety.

a. Charge density is q=25 μ·C·cm^{-2} and the area A=0.03 cm^2. Assume k=1.75.

1. Medtech safety limit is 30 μ·C·cm^{-2}. Since **30 μ·C·cm^{-2} > q(=20 μ·C·cm^{-2})**, this electrode can be used to stimulate a specific circular region of the brain.

2. Shannon's Law indicates that:

$$\log(q) < k - \log(Q_i)$$

$$\text{k=1.75}$$

$$\text{Charge } Q = q \cdot A = 0.75\ \mu \cdot C$$

$$\log(25) < 1.75 - \log(0.75)$$

$$\mathbf{1.39 < 1.87}$$

The stimulation protocol is within the safety limits of Shannon's Law.

b. The waveform is cathodic-first, biphasic, symmetric pulse with pulse width (PW) = 0.6 ms and t_{delay}=0.01 ms.

$$Charge\ Q = q \cdot A = 0.75\ \mu \cdot C$$

$$Current\ Amplitude\ (I) = \frac{Q}{PW} = \frac{0.75\ \mu \cdot C}{0.6\ ms} = 1.25\ mA$$

$$Va = I \cdot R_{series} + \eta_C$$

$Ignoring\ \eta_C$:

$$R_{series} = R_{track} + R_{spread}$$

$$R_{track} = R_{sheet} \cdot \frac{L}{W} = 5\ \Omega \cdot \frac{21\ mm}{0.5\ mm} = 210\ \Omega$$

$$R_{spread} = \frac{\rho}{4 \cdot r} = \frac{\rho}{4 \cdot \sqrt{\frac{A}{\pi}}} = \frac{300\ \Omega \cdot cm}{4 \cdot \sqrt{\frac{0.03\ cm^2}{\pi}}}$$

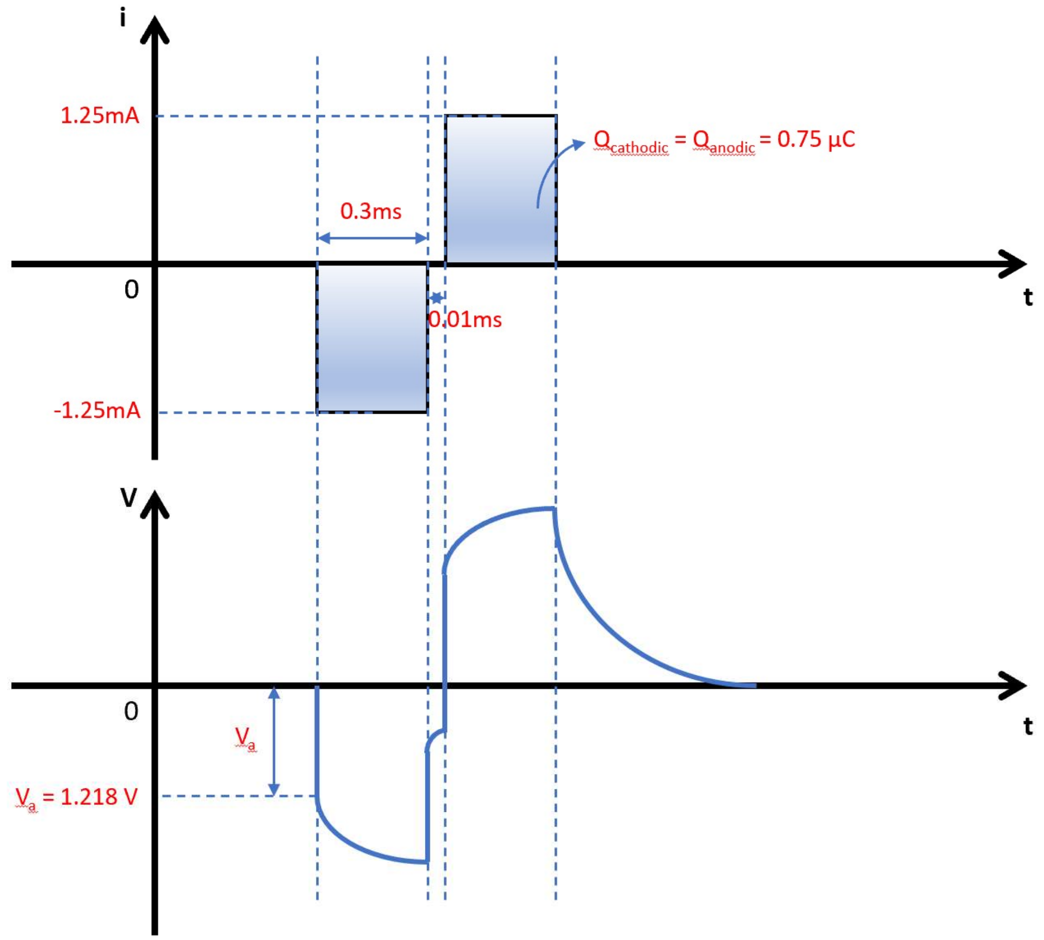
$$R_{spread} = \frac{300\ \Omega \cdot cm}{4 \cdot 0.098\ cm} = 765\ \Omega$$

$$R_{series} = R_{track} + R_{spread} \cong 975\ \Omega$$

$$Va = I \cdot R_{series} + \eta_C \quad ignore\ \eta_C$$

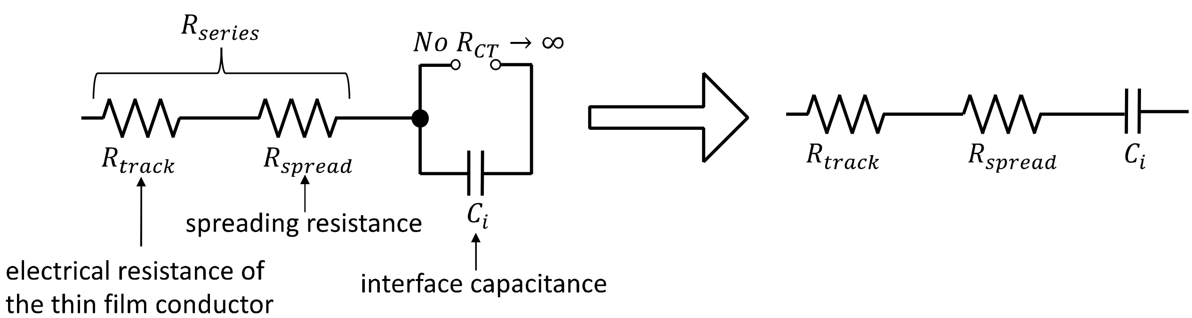
$$V_a = 1.25\ mA \cdot 975\ \Omega$$

$$\boldsymbol{V_a = 1.21875\ V}$$



Exercise 2 – Electrochemical impedance.

a.



b.

$$Z = R_{track} + R_{spread} + Z_{C_i}$$

$From\ the\ previous\ exercise: R_{series} = R_{track} + R_{spread} \cong 975\ \Omega\ and\ R_{spread}\ is\ \boldsymbol{on\ the\ tissue}.$

$$\boldsymbol{In\ saline}: R_{spread} = \frac{\rho}{4 \cdot r} = \frac{60\ \Omega \cdot cm}{4 \cdot 0.098\ cm} \cong 153\ \Omega$$

$$R_{series} = 210\ \Omega(track) + 153\ \Omega(spread) = 363\ \Omega$$

For the capacitor:

$$Z_{C_i} = \frac{1}{j \cdot \omega \cdot C_i} = -j \cdot \frac{1}{2 \cdot \pi \cdot f \cdot C_i}$$

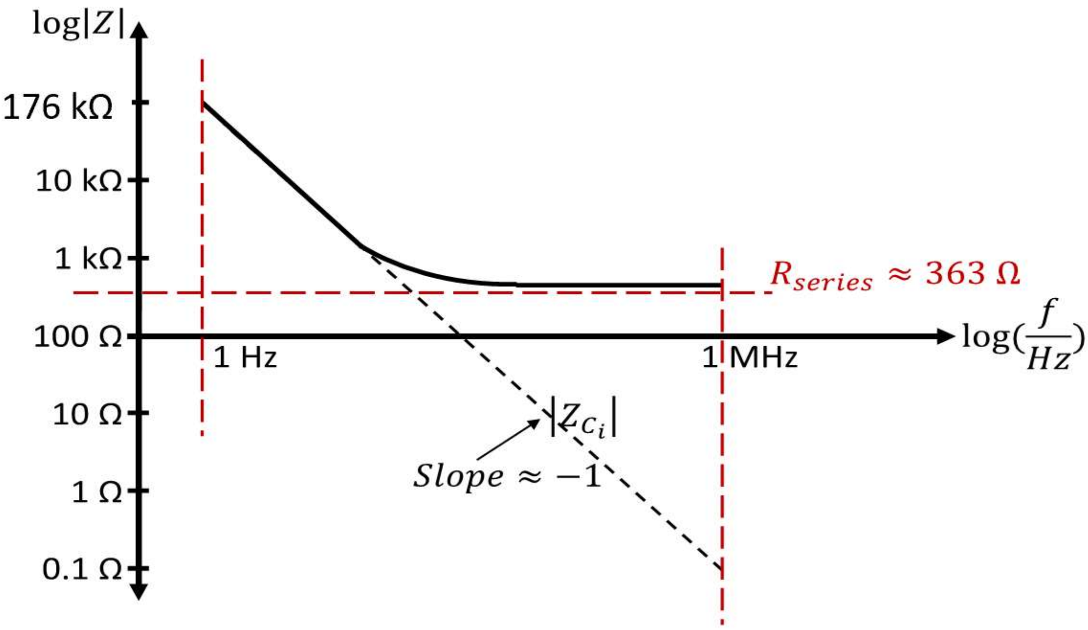
$$|Z_{C_i}| = \left|\frac{1}{2 \cdot \pi \cdot f \cdot C_i}\right|$$

$$\log|Z_{C_i}| = -\log f - \log 2 \cdot \pi \cdot C_i$$

Where Ci = Ci *ESA = 20 μ·F·cm^{-2} * 4.5 mm^2 = 0.9 μ·F

$$At\ 1\ Hz: |Z_{C_i}| = \left|\frac{1}{2 \cdot \pi \cdot C_i \cdot (1\ Hz)}\right| \cong 176.838\ k\Omega$$

$$At\ 1\ MHz: |Z_{C_i}| = \left|\frac{1}{2 \cdot \pi \cdot C_i \cdot (1\ MHz)}\right| \cong 0.1768\ \Omega$$



Capacitance is proportional to area.

For instance:

$$C = \frac{A \cdot \varepsilon_0 \cdot \varepsilon_R}{d}$$

$$C_i \to 30C_i \Rightarrow Z_{C_i} \to \frac{1}{30} Z_{C_i}$$

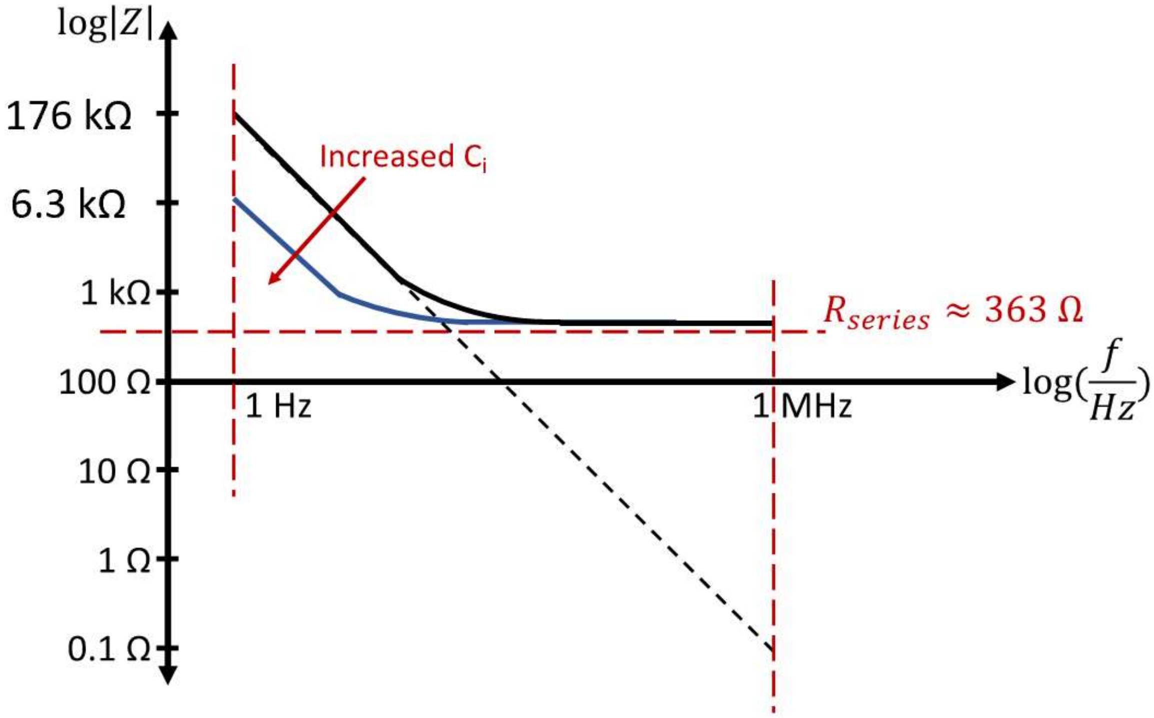
$At\ 1\ Hz$:

$$|Z_{initial}| = 363\ \Omega +\ 176.8\ k\Omega \approx 176.8\ k\Omega$$

$$|Z_{final}| = 363\ \Omega +\ 5.89\ k\Omega \approx 6.25\ k\Omega \quad (1/30X\ change)$$

$At\ 1\ HMz$:

$$|Z_{initial}| = 363\ \Omega +\ 0.1768\ \Omega \approx 363.1768\ \Omega$$
$$|Z_{final}| = 363\ \Omega +\ 0.00589\ \Omega \approx 363\ \Omega \quad (no\ change)$$



Exercise 3 - Subthreshold membrane phenomena

a. Nernst Equation: This equation defines the relation between the concentrations of an ion on either side of a membrane that it perfectly selective for that ion and the potential difference (voltage, E) that will be measured across that membrane under equilibrium conditions.

$$E_{ion} = \frac{RT}{zF} . ln \frac{[ion]_{out}}{[ion]_{in}}$$

[ ] = ionic concentration (inside or outside of the cell)

R = ideal gas constant = 8.314 J/K°/Mole

T = absolute temperature in K (37°C = 310K)

z = ion valence e.g. (Cl^- = 1, Ca^{2+} = +2, K^+ = +1 etc…)

F = Faraday's constant 96'500 C/Mole

b. Given: Permeabilities(p) Cl^- = 0.09, K^+ = 1.00 and Na^+ = 0.04

Ionic concentrations $[Cl^-]_i$=178, $[Cl^-]_o$=0.47, $[K^+]_i$=135, $[K^+]_o$=83, $[Na^+]_i$=0.05, $[Na^+]_o$=118; all in mmol/mm^3

T = 37 °C = 310 K

Gas constant R = 8.314 J/mol*K, Faraday constant: F = 9.649x104 C/mol]

Derived from Nernst equation and electrical model for neuronal membrane

$$V_m = \frac{g_{Na}E_{Na} + g_K E_K + g_{Cl}E_{Cl}}{g_{Na} + g_K + g_{Cl}}$$

We obtain Goldman-Hodgkin-Katz equation as:

$$V_m = \frac{RT}{F}\frac{p_K[K^+]_o + p_{Na}[Na^+]_o + p_{Cl}[Cl^-]_i}{p_K[K^+]_i + p_{Na}[Na^+]_i + p_{Cl}[Cl^-]_o}$$

Entering corresponding values, we obtain V_m = - 7.044 mV

And from Nernst equation, we obtain equilibrium potential of Cl^- ions E_{Cl-} = 158.59 mV

Thus, the electrochemical driving force acting on Cl^- would be $\Delta V_{Cl-} = V_m - E_{Cl-}$ = -165.63 mV

Thus, the driving force acting on Cl^- leads to Cl^- efflux from the cell.