NX-414
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Exam Revision - Part 1

Fisher Information

2023, Exercise 1.1

Write down the definition of the Fisher Information. [max 3 lines]

2023, Exercise 1.2

Explain the meaning of the Fisher information in the context of neuronal coding. What does it tell us? [max 6 lines]

2023, Exercise 1.3

Consider a Poisson neuron with tuning curve corresponding to the one-dimensional Von Mises function $\Omega(x)$, defined as:

$$\Omega(x) = f_{max}\tau \exp\left(\frac{\cos(x-\varphi)-1}{\sigma^2}\right), \tag{1}$$

with preferred phase φ, peak firing rate f_{max}, time interval τ and tuning width σ.
The number of spikes K emitted by the neuron in a fixed interval τ follows a Poisson probability distribution:

$$P(k|x) = \frac{(\Omega(x))^k}{k!}\exp(-\Omega(x)). \tag{2}$$

Show that the Fisher information at location x is given by:

$$I(x) = \frac{\sin^2(x-\varphi)}{\sigma^4}\Omega(x). \tag{3}$$

2023, Exercise 1.4

Discuss the expression of the Fisher information for a Poisson neuron with Von Mises tuning curve (Eq. 3), with a focus on its dependency on the parameter σ and on the preferred phase φ.
*N*ote: this can be done fully independent of Exercise 1.3.

2024, Exercise 1.1

Define what a tuning curve of a neuron is and provide one example (from the literature). [max 5 lines]

2024, Exercise 1.2

Consider a neuron whose observed firing rate is the sum of the stimulus response $r(\theta) : \mathbb{R} \to \mathbb{R}$ (θ indicates a generic stimulus) and a white noise component $H(\theta)$, which is distributed according to a Gaussian distribution with mean $\mu = 0$ and variance $\sigma^2 = \alpha r(\theta) + \beta$, where $\beta \in \mathbb{R}$ is a constant background component, while $\alpha \in \mathbb{R}$ is the weight of a noise component proportional to the mean response $r(\theta)$. Write the probability density function of the random variable $X = r(\theta) + H$, representing the observed firing rate.

2024, Exercise 1.3

Show that the Fisher information about the stimulus response r contained in the random variable X has the following expression:

$$I(r) = \frac{1}{(\alpha r + \beta)^2} \left(\alpha r + \beta + \frac{\alpha^2}{2} \right). \tag{4}$$

For the moment, consider r as a fixed parameter and ignore its dependency on the stimulus θ.

2024, Exercise 1.4

Let us now assume that the neuron presents a Gaussian tuning curve to the stimulus θ:

$$r(\theta) = \exp\left(-\frac{(\theta - m)^2}{2s^2} \right), \tag{5}$$

characterized by the tuning parameters $m \in \mathbb{R}$ and $s^2 \in \mathbb{R}$. Compute the Fisher information $I(\theta)$ about θ contained in the random variable X. Recommendation: Build on the solution from the previous exercise.

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Model Fitting

2024, Exercise 2.1

During a behavioral experiment, a mouse is performing a decision making task (“lick” or “not lick”) based on scalar input stimulus $x \in \mathbb{R}$. The activity of a certain neuron is observed to be correlated with the animal’s behavior following the stimulus. We represent the event ”firing” with a random variable Y, which can have value 0 (not firing) or 1 (firing). We model the probability density of Y with a logistic function, depending on the stimulus x:

$$p(y = 1|x) = \frac{1}{1 + e^{-(\beta_0+\beta_1 x)}},$$

where $\beta_0, \beta_1 \in \mathbb{R}$ are the parameters of the logistic function. In this way, defining whether the neuron fires or not given a stimulus x can be seen as a binary classification problem. Derive the equation for the decision boundary as a function of the stimulus variable x and the parameters β_0 and β_1. Assume the decision threshold to be 0.5. Discuss how the decision boundary is influenced by the parameters β_0 and β_1.

2024, Exercise 2.2

You are provided with a dataset of stimuli and firing events $\{(x_i, y_i)\}_{i=1}^{n}$, where $y_i \in \{0, 1\}, i = 1, ..., N$ represent the binary outcomes, $x_i \in \mathbb{R}, i = 1, ..., N$ represent the stimuli. Write the expression of the log-likelihood of the observed firing events $y_i, i = 1, ..., N$ given the stimuli $x_i, i = 1, ..., N$, as a function of the parameters β_0 and β_1.

2024, Exercise 2.3

Use the expression of the log-likelihood derived in Exercise 2.2 to find an implicit expression for the parameters β_0 and β_1. Incorporate a L2 regularization term in the expression of the log-likelihood found in Exercise 2.2, modulated by the regularization factor λ. Derive an implicit expression for the parameters β_0 and β_1 which maximize the regularized log-likelihood. Discuss qualitatively how the regularization parameter λ affects the parameter estimation.