Crack tip plasticity

Mode I crack of length a in a solid with a far-field average stress σ and no work hardening.

First order approximate of plastic zone size with truncation

$$r_y = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$

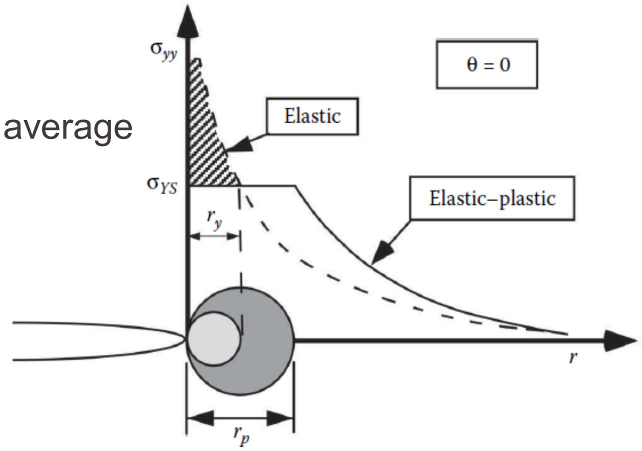


FIGURE 2.29
First-order and second-order estimates of plastic zone size (r_y and r_p, respectively). The cross-hatched area represents the load that must be redistributed, resulting in a larger plastic zone.

Second order approximate of plastic zone size with stress redistribution

$$\sigma_{YS} r_p = \int_0^{r_y} \sigma_{yy}\, dr = \int_0^{r_y} \frac{K_I}{\sqrt{2\pi r}}\, dr$$

$$r_p = \frac{1}{\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$

From T.L. Anderson, chap2 LEFM

Crack tip plasticity

First order approximate of plastic zone size with truncation $r_y = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$

The reduction in stress in the yield zone must be compensated by an increase in K (small plastic zone so the stresses at the crack tip are controlled by K). In other words, **the specimen should behave as if the crack length has increased on yielding by some distance that we shall call Δa.**

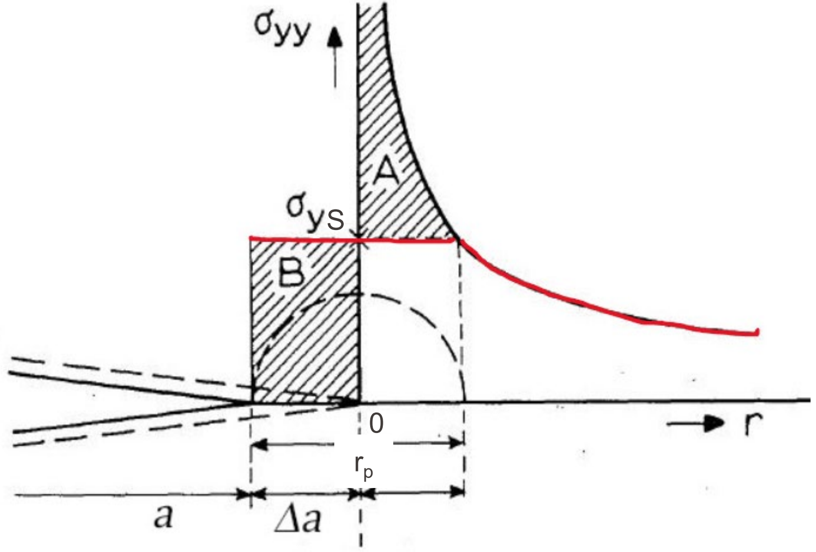
How to calculate Δa ? by iterations .....

Let's suppose that the specimen behaves as if it contains a virtual crack of length a + Δa with a plastic zone of length r_p - Δa ahead of its virtual tip.

KI = KI(a + Δa) where a is the true crack length.

Therefore, area A must equal area B

$$\Delta a \sigma_{YS} = \int_0^{r_p - \Delta a} \frac{K_I[a+\Delta a]}{\sqrt{2\pi r}} dr - \sigma_{YS}(r_p - \Delta a)$$



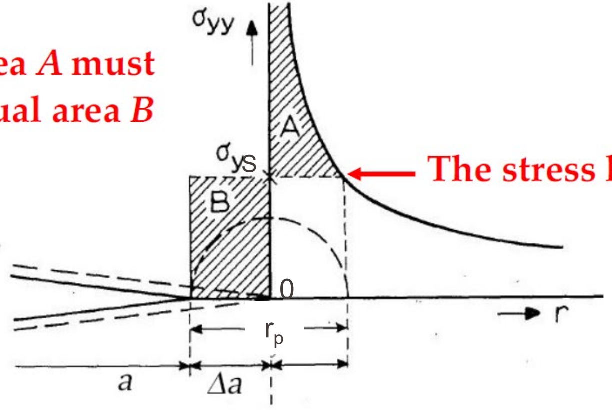
Crack tip plasticity

In an all-elastic solution, the normal stress ahead of the tip of this virtual crack will be given by

$$\sigma_{yy} = \frac{K_I[a+\Delta a]}{\sqrt{2\pi r}}$$ taking the origin of r to be at the virtual crack tip.

Now assume that this stress distribution is the same as the stress distribution at the tip of the real crack of length a with a plastic zone of length rp, when r > rp - Δa, i.e. outside the plastic zone.

Area *A* must equal area *B*



The stress here must be $\sigma_{YS} = \frac{K_I[a+\Delta a]}{\sqrt{2\pi(r_p - \Delta a)}}$

$$\Delta a \sigma_{YS} = \int_0^{r_p-\Delta a} \frac{K_I[a+\Delta a]}{\sqrt{2\pi r}} dr - \sigma_{YS}(r_p - \Delta a)$$

$$r_p \sigma_{YS} = \int_0^{r_p-\Delta a} \frac{K_I[a+\Delta a]}{\sqrt{2\pi r}} dr = \frac{K_I[a+\Delta a]}{\sqrt{2\pi}} 2\sqrt{r_p - \Delta a}$$

$$r_p \sigma_{YS} = r_p \frac{K_I[a+\Delta a]}{\sqrt{2\pi(r_p - \Delta a)}} = 2\frac{K_I[a+\Delta a]}{\sqrt{2\pi}}\sqrt{r_p - \Delta a}$$

$$r_p = 2(r_p - \Delta a) \text{ i.e. } r_p = 2\Delta a \text{ and } \Delta a = r_p/2$$

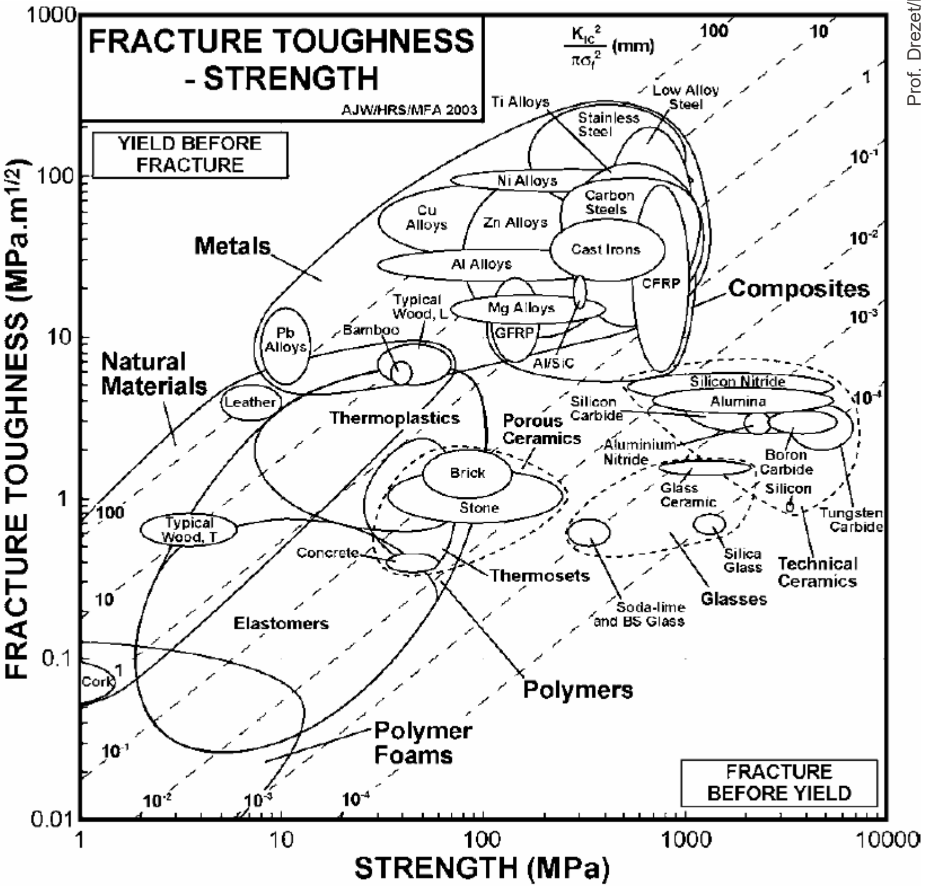
Fracture toughness, yield strength and process zone

Diameter of the process zone :

$$r_p = \frac{1}{\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$

can be larger than 100 mm !

Figure 3.4: Fracture toughness (plane strain), K_{IC}, against failure strength, σ_f. Failure strength is defined as the *tensile elastic limit* (usually yield stress) for all materials other than ceramics, for which it is the *compressive strength*. The contours show $K_{IC}^2 / \pi\sigma_f^2$, which is approximately the diameter of the process zone at a crack tip. Valid application of linear elastic fracture mechanics using K requires that the specimen and crack dimensions are large compared to this process zone. The design guide-lines are used in selecting materials for damage tolerant design. (Data courtesy of Granta Design Ltd)



Elastic stress field in mode I

Stress components along xx and yy

TABLE 2.1

Stress Fields Ahead of a Crack Tip for Modes I Material

	Mode I
σ_{xx}	$\frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1-\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]$
σ_{yy}	$\frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1+\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]$
τ_{xy}	$\frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right)$
σ_{zz}	0 (Plane Stress) $\nu(\sigma_{xx}+\sigma_{yy})$ (Plane Strain)
τ_{xz}, τ_{yz}	0

Note: ν is Poisson's ratio.

From T.L. Anderson, chap2 LEFM

3 principal stress components

$$\sigma_1 = \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1+\sin\left(\frac{\theta}{2}\right)\right] \quad (2.94)$$

$$\sigma_2 = \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1-\sin\left(\frac{\theta}{2}\right)\right] \quad (2.95)$$

$$\sigma_3 = 0 \quad \text{(plane stress)}$$
$$= \frac{2\nu K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right) \quad \text{(plane strain)} \quad (2.96)$$

Equivalent Von Mises stress:

$$\sigma_e = \frac{1}{\sqrt{2}}\left[(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2\right]^{1/2}$$

Plastic zone shape

Locus $r_y(\theta)$ where Von Mises is equal to σ_{YS} :

$$\sigma_{YS} = \frac{1}{\sqrt{2}}\left[(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2\right]^{1/2}$$

For plane stress:

$$r_y(\theta) = \frac{1}{4\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2\left[1+\cos\theta+\frac{3}{2}\sin^2\theta\right] \qquad (2.97)$$

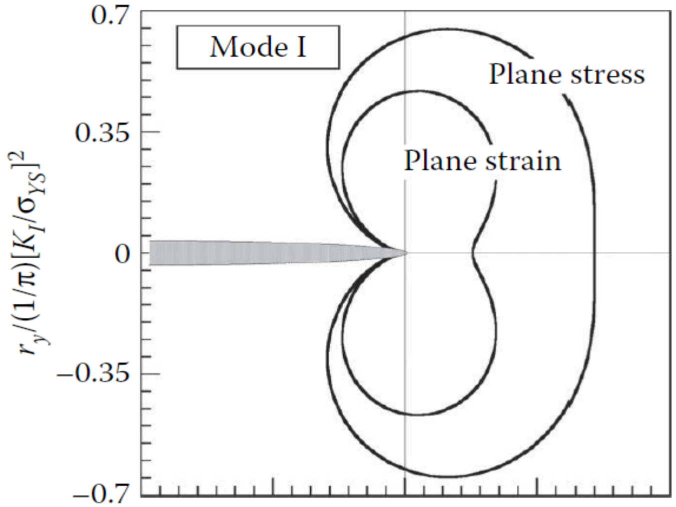
For plane strain:

$$r_y(\theta) = \frac{1}{4\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2\left[(1-2\nu)^2(1+\cos\theta)+\frac{3}{2}\sin^2\theta\right] \qquad (2.98)$$

With $\nu \approx 0.3$, $(1\text{-}2\nu)^2 \approx (1\text{-}0.6)^2 = 0.16$ so $(1\text{-}2\nu)^2\,(1+\cos\theta)$ varies between 0. and 0.32

Plastic zone shape

Butterfly shapes



From plane stress at the surface to plane strain at the center…

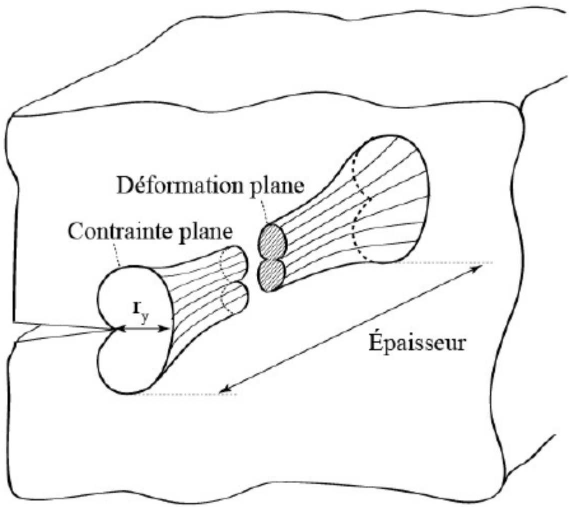
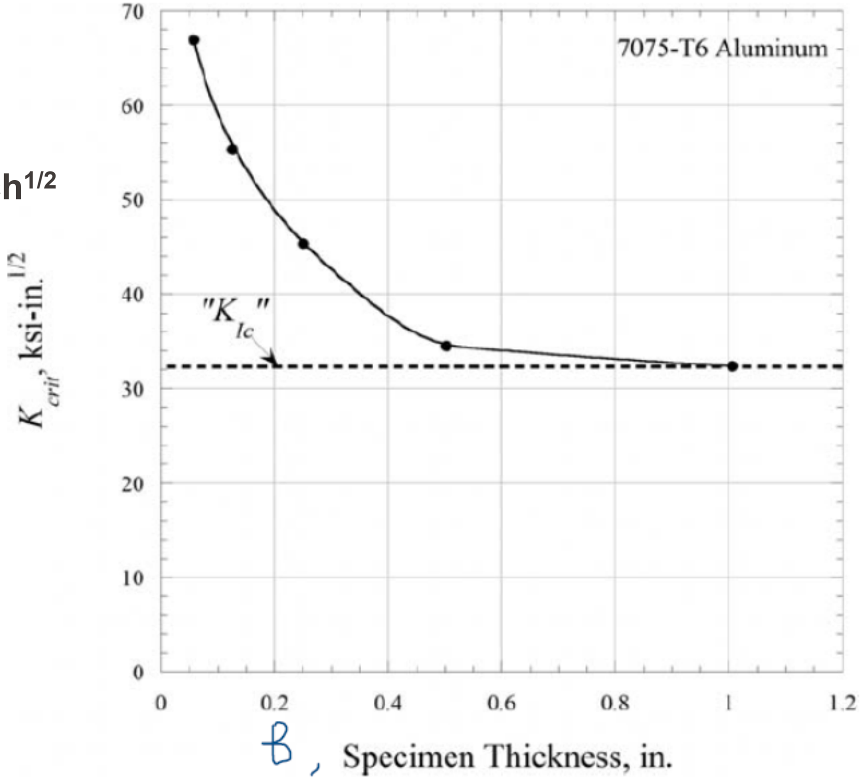


Figure 4-3. Sketch of the general shape of a Mode I crack tip plastic zone across a thick plate (from Janssen, Zuidema, Wanhill, Fracture Mechanics, 2nd Edition, CRC Press, London, 2014).

Influence of triaxiality

Mind the unit of KI ! Ksi-inch$^{1/2}$



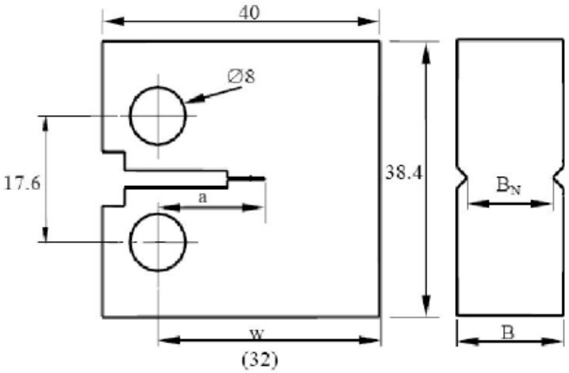
Variation of measured fracture toughness with specimen thickness for 7075-T6 Aluminium. (Adapted by Anderson from Barsom and Rolfe, Fracture and Fatigue Control in Structures. 2nd Ed., Prentice-Hall, Englewood Cliffs, NJ, 1987.

A large plastic zone, i.e. a thin specimen is expected to lead to more energy dissipation during crack advance and a higher G_C or K_C.

Prof. Drezet/Molinari

Conditions for valid K_{Ic} testing

When performing laboratory plane strain mode I K_{Ic} tests on standard specimens, the following empirical size requirements have been adopted to ensure reproducible results for different elastic-plastic materials:



$$a, B, (W-a) \geq 2.5\left(\frac{K_{Ic}}{\sigma_y}\right)^2$$

The minimum requirements for a and $W - a$ ensure that the plastic zone is sufficiently small for fracture to be *K*-controlled (20 to 50 times the plastic zone size). The requirement for B is intended to ensure plane strain conditions along the crack front, although it is often far more stringent than necessary.