Exercise 1 Rolling Dice

(a) Write an expression for the probability of obtaining the same face value (of your choice) m times out of a roll of n dice. (*Hint: use the binomial distribution*).

(b) Roll n dice a few times and collect data on how often you observe m occurrences of your chosen face value. Make a rough histogram. How does it compare with the theoretical predictions? (*Hint: you can also combine your results with those of other groups to get better convergence!*)

(c) Repating this experiment by hand is quite tiresome, but thankfully, we have computers who can roll dice for us! Use the Mathematica interactive computable document "lab1_rolling_dice.cdf" and observe how the two interpretations of probability concepts (frequentist and inductivistic) coincide when the number of trials goes to infinity.

(d) What is the probability of obtaining *at least* m dice with the same face value of your choice out of a roll of n dice? Write down a mathematical expression (you don't need to simplify). Think about which probability axioms you have to invoke to make this operation.

(e) Open a Mathematica notebook and write functions that can compute these quantities. You can use the "lab1_example_notebook.nb" file as a starting point for your experiments.

Exercise 2 Let's play Perudo

Rules of the game Perudo is a dice game, also called liar's dice. The objective of Perudo is to be the last player with one or more dice. Perudo is played in rounds. At the beginning of a game, each player receives a cup and five dice. Each round begins with all players shaking all their dice in the cup, after which all players turn the cups over on a table top, so that the dice are rolled and under the cups. Each player may peek in his own cup.

Players take turns bidding: to bid, the player announces a number and a face value of dice, for example "three 5s" (meaning that they guess there are three 5s on the table). Play continues round the table to the left, with the next player either raising the bid (either the number of dice, or the face value of the dice, or both: for example, if player 1 bids three 2s, then player 2 could bid three 3s, three 4s, four 2s, ten 6s, etc.), or doubting the previous bid if the player believes the previous player has over-estimated: to do so, the player says "Dudo", which means "I doubt" in Spanish.

If a player calls Dudo then all players must show their dice, to verify whether the bid was indeed too high. If there as many dice of that number as bid, or more, then the player who called Dudo must place a die in the discard pile. Otherwise, the player who made the last bid must place a die in the discard pile. In either case, a new round begins. The player who lost a die in the last round is the first player in the new round. If the player lost his last die, then the player to his left plays first instead.

Start with five dice each, familiarizing yourself with the rules of the game and getting a feeling of what are reasonable bids in the different phases of the game. Then consider the following questions. You may want to experiment with Mathematica to evaluate the different quantities you are asked to compute.

(a) Roll all the dice without peeking at the outcome of yours. Make your bid and write the associated probability that your chosen bid is satisfied (*Hint: start from the expression you got in Ex. 1(a)*).

(b) Use the Mathematica notebook "lab1_perudo.cdf" to check the result you got in the previous point. Remember to set the variable *No. of My Dice* equal to zero. Was your bid the best among all possible ones? Assuming that the number of players is fixed at your lab group's size, What bidding strategy would you suggest?

(c) Roll the dice again. This time you can have a look at your dice. Make your bid and write the related probability. Specifically, express the probability of a successful bid for M dice of a specified face value out of N total dice, *conditional* on the fact that you have $n \leqslant N$ dice in your hand, m of which match that chosen value, i.e. $P(N, M|n, m)$. *Hint: can you write* $P(N, M, n, m) = P(N, M)P(n, m)$*? Use your intuition to determine* $P(N, M|n, m)$ *without using Bayes' rule.*

(d) Use the notebook to check your result and get an idea of the probabilities of all the possible bids. Is there any difference compared to point (b)? Is the knowledge of your hand affecting your way of bidding?

(e) Based on knowledge of your dice, can you tell something about what the other players have rolled?

The actual game contains an additional feature. The "1" counts as a wildcard, which is counted together with the face value called in the bid. For instance, if in a certain roll there are 5 dice with a "3" and 4 dice with a "1", a bid for 9 "3"s would be successful. Also, when bidding for the face value "1" explicitly, the value of the previous bid can be halved. For instance, a bid of 3 "1"s is allowed to follow a bid of 6 "4"s.

(f) Roll the dice again. What are the new expressions for the probability of some bid considering that this time the face-value "1" counts as a *wildcard*?

(g) Compute the new probabilities using the provided notebook. This time remember to check the *wildcard* check-box. How does the introduction of the *wildcard* affect the results?

(h) As a final mathematical question, consider the expression you got in point (c), i.e. $P(N, M|n, m)$. Choose a face value. Use Bayes' theorem to get the joint probability $P(N, M, n, m)$ of observing a roll in which there are N total dice, n dice in your hand, of which M and m with the chosen face value on the table and in your hand respectively. Why would it be difficult to directly evaluate this joint probability?

The End.