

## CERAMIC AND COLLOIDAL PROCESSING - EXERCISES

Prof. Paul Bowen  
Dr. Andrea Testino  
2021

### Exercises 7

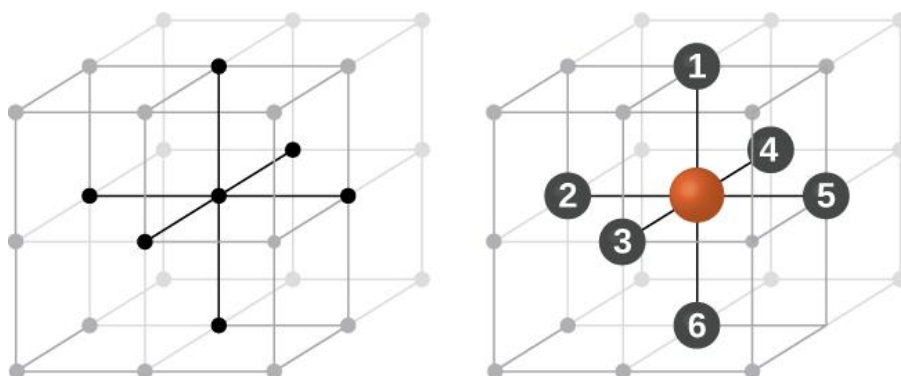
**1. Calculate the surface tension of a liquid at temperature,  $T$  knowing its molar mass  $MM$ , its density  $\rho$  as well as the values of  $W_{AA}$  and  $Z_b$ . Example: perform the calculation for  $CCl_4$  ( $MM = 153.82$  g/mole) at  $25^\circ\text{C}$ ,**

**With,  $\rho = 1.6 \times 10^3$  [kg/m<sup>3</sup>],  $W_{AA} = -1.78 \times 10^{-20}$  [J/bond] and  $Z_b = 6$ .**

**Note: Assume for simplicity that the  $CCl_4$  molecules form a simple cubic structure in the liquid phase. (length of unit cell = side of cube =  $c = ((MM/\rho)/(N_A))^{1/3}$ )**

#### **Solution.**

By transferring a liquid molecule from the bulk to the surface, its coordination changes from  $Z_b = 6$  to  $Z_s = 5$ . i.e. central orange atom below as bulk  $Z_b = 6$  and atom 1 for surface,  $Z_s = 5$ .



Knowing the density and assuming for simplicity a simple cubic arrangement of molecules in the liquid, we calculate the side of the cube

$$c = [(153.82 \cdot 10^{-3} / 1.6 \cdot 10^3) / (6.0 \cdot 10^{23})]^{1/3} = 5.42 \cdot 10^{-10} \text{ [m]} = 0.542 \text{ [nm]}$$

So the area  $A$  of a face of the cube is:

$$A = c^2 = 2.94 \cdot 10^{-19} \text{ [m}^2\text{]} = 0.294 \text{ [nm}^2\text{]}$$

From Eq. (2.1.3) we calculate :

$$E_s/A = 0.5 \times (-1.78 \cdot 10^{-20}) \times (5-6) / (2.94 \cdot 10^{-19}) = 3.02 \cdot 10^{-2} \text{ [J/m}^2\text{]} = 30.2 \text{ [mN/m]}$$

**2 Given the respective surface tensions of water and benzene (in air) are :**

$$\gamma_{\text{water}} = 72.8 \text{ [mN / m]}, \gamma_{\text{benzene}} = 28.9 \text{ [mN / m]},$$

**and the interfacial tension for the water / benzene interface is**

$$\gamma_{\text{water / benzene}} = 35.0 \text{ [mN / m]},$$

**calculate the initial water – benzene spreading coefficient.**

**After equilibration, a little benzene dissolves in the water which brings its surface tension to  $\gamma_{\text{water}} = 62.4$  [mN / m]. What happens to the water-benzene spreading coefficient?**

**Solution.**

Using Equation 2.1.15b –

$$S_{SL} = -\Delta G_{SL} = (\gamma_{SV} - \gamma_{SL}) - \gamma_{LV}$$

$$S(\text{init.}) = 72.8 - (28.9 + 35.0) = 8.9 \text{ [mN/m]} > 0 : \text{benzene spread on water ;}$$

$$S(\text{eq.}) = 62.4 - (28.9 + 35.0) = -1.5 \text{ [mN/m]} < 0 : \text{benzene no longer spreads on water.}$$

### **3. Capillary Rise**

**Determine the height  $h$  [cm] that a liquid rises in a capillary tube, knowing  $\gamma_{LV}$ , the surface tension of the liquid,  $\rho_L$ , the density of the liquid,  $r$ , the internal radius of the tube, and  $\theta$ , the wetting angle of the tube wall by the liquid.**

**Example: the liquid is water ( $\gamma_{\text{eau}} = 72.8$  [mN / m],  $MM = 18.02$  [g / mole],  $\rho = 1'000$  [kg / m<sup>3</sup>]), the wetting angle  $\theta = 20^\circ$ , and the radius of the capillary  $r = 0.10$  [mm].**

**Solution.**

Equation (2.2.4) gives, after transformation:

$$h = 2\gamma_{LV} \cos\theta / \Delta\rho g r ;$$

$$\Delta\rho = \rho_L - \rho_V \cong \rho_L \Rightarrow h = 2\gamma_{LV} \cos\theta / \rho_L g r$$

thus :

$$h = 2\gamma_{LV} \cos\theta / \rho_L g r = 2 \times 72.8 \times 10^{-3} \cos(20^\circ) / 1'000 \times 9.81 \times 1.0 \times 10^{-5} = 0.148 \times 0.940 \\ = 0.139 \text{ [m]} = 13.9 \text{ [cm]}$$

### **4. What is the Hamaker constant and its role in the attractive forces between 2 particles?**

**Solution.**

With Van der Waals attractive forces, the intensity of the interaction energy is

$$V_A(h) = -A \cdot H$$

Where  $h$  is the separation distance between particles,  $H$  is a geometric factor and  $A$  is the Hamaker constant. This constant depends on the dielectric properties of the particles and of the medium separating them and determines the magnitude of the attractive VdW force.