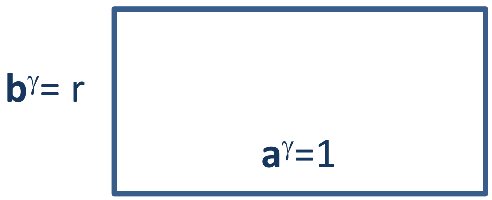
Exercice Série 12

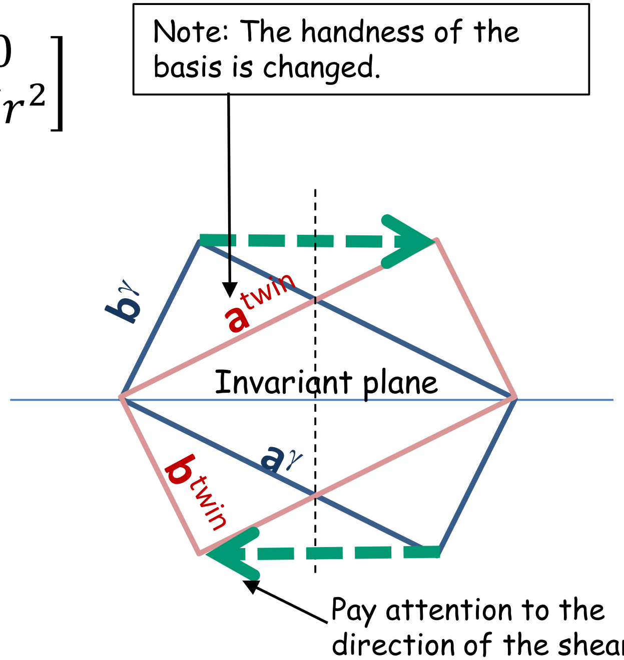
Tenseurs métriques



$$\mathbf{G} = [\mathbb{B}* \to \mathbb{B}] = \begin{bmatrix} 1 & 0 \\ 0 & r^2 \end{bmatrix}$$

$$\mathbf{G}^* = \mathbf{G}^{-1} = [\mathbb{B} \to \mathbb{B}^*] = \begin{bmatrix} 1 & 0 \\ 0 & 1/r^2 \end{bmatrix}$$

Calcul de s par la formule de Bevis&Crocker



$$s^2 = \mathrm{Tr}(\mathbf{C}^{\mathrm{t}}\mathbf{GCG}^{-1}) - 2$$

avec

$$\boxed{\begin{aligned} \mathbf{a}^{\gamma} &\to \mathbf{a}^{\gamma\prime} = \mathbf{b}^{twin} \\ \mathbf{b}^{\gamma} &\to \mathbf{b}^{\gamma\prime} = \mathbf{a}^{twin} \end{aligned}}$$

$$\Rightarrow \mathbf{C} = \mathbf{C}^{twin \to \gamma} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow s^2 = Tr\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & r^2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1/r^2 \end{bmatrix}\right) - 2 = \frac{(r^2-1)^2}{r^2}$$

$$\Rightarrow \boxed{s = \pm\left(r - \frac{1}{r}\right)}$$

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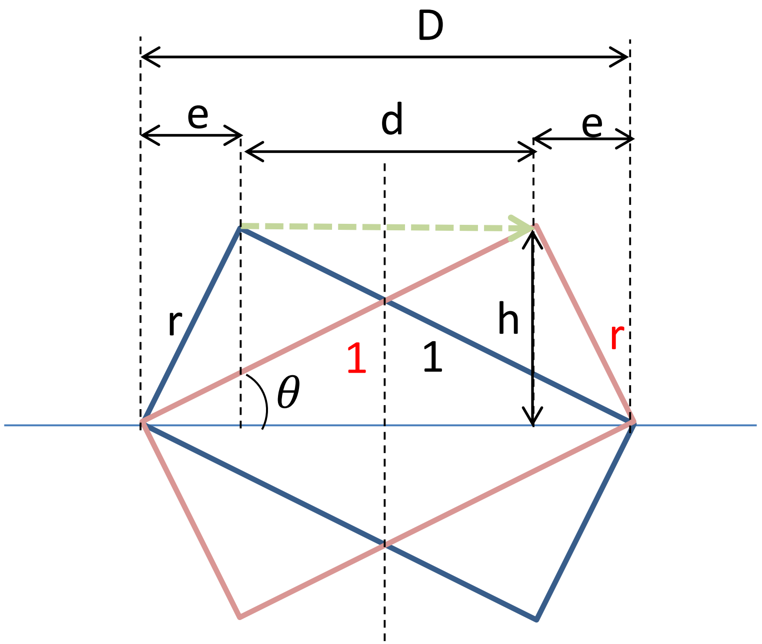
Calcul de s par géométrie

$$\boxed{s = \frac{d}{h}}$$



avec

$$D = \sqrt{1 + r^2}$$

$$hD = r \text{ (surface du rectangle)}$$

$$d = D - 2e$$

$$tan\theta = r = \frac{e}{h}$$

$$\Rightarrow \quad s = \frac{d}{h} = \frac{D - 2e}{h} = \frac{\sqrt{1 + r^2}}{h} - 2r = \frac{1 + r^2}{r} - 2r = \frac{1}{r} - r$$

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Direction **d** et plan **p**

Avant normalisation $\mathbf{d} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\quad \|\mathbf{d}\|^2 = \mathbf{d}^{\mathrm{t}}\, \mathbf{G}\, \mathbf{d} = 1 + r^2$

⇨ Après normalisation $\tilde{\mathbf{d}} = \frac{1}{\sqrt{1+r^2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Avant normalisation $\mathbf{p} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\quad \|\mathbf{p}\|^2 = \mathbf{p}^{\mathrm{t}}\, \mathbf{G}^{-1} \mathbf{p} = \dfrac{1 + r^2}{r^2}$

⇨ Après normalisation $\tilde{\mathbf{p}} = \frac{r}{\sqrt{1+r^2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

⇨ $\boxed{\mathbf{F} = \mathbf{I} + s\, \tilde{\mathbf{d}}.\tilde{\mathbf{p}}^{\mathrm{t}} = \frac{1}{1+r^2} \begin{bmatrix} 2r^2 & 1 - r^2 \\ r^2 - 1 & 2 \end{bmatrix}}$

<u>Note:</u> si vous voulez calculer la matrice dans Mathematica il faut utiliser $\mathbf{d}.\mathbf{p}^{\mathrm{t}} = Outer(\mathbf{d}, \mathbf{p})$

Det(**F**) = 1, Tr(**F**) = 2,

F $\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

F* = **F**$^{-t}$ qui se calcule facilement car on sait que Det(**F**) = 1, **F**$^{-t}$ = $\frac{1}{1+r^2} \begin{bmatrix} 2r^2 & r^2 - 1 \\ 1 - r^2 & 2 \end{bmatrix}$

F* $\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$