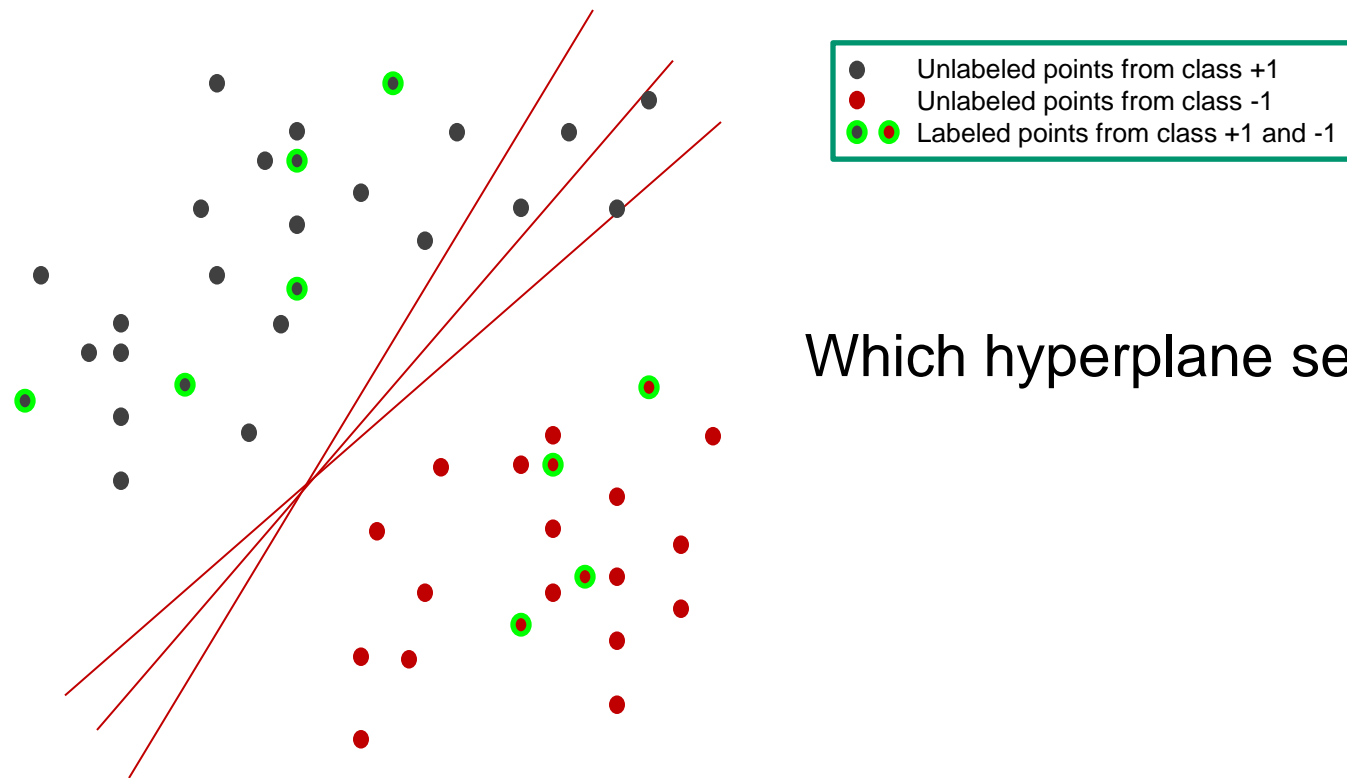# Transductive SVM
(semi-supervised clustering)

see supplement (Joachims, Thorsten. "Transductive support vector machines."

Chapelle et al.(2006) (2006): 105-118.)

Transductive Support Vector Machine

Transductive SVM: learn from partially labeled datapoints
Inductive SVM: learn from fully labeled datapoints (regular SVM)

Unlabeled points from class +1
Unlabeled points from class -1
Labeled points from class +1 and -1

Which hyperplane separates best?

Transductive Support Vector Machine

Transductive SVM: learn from partially labeled datapoints
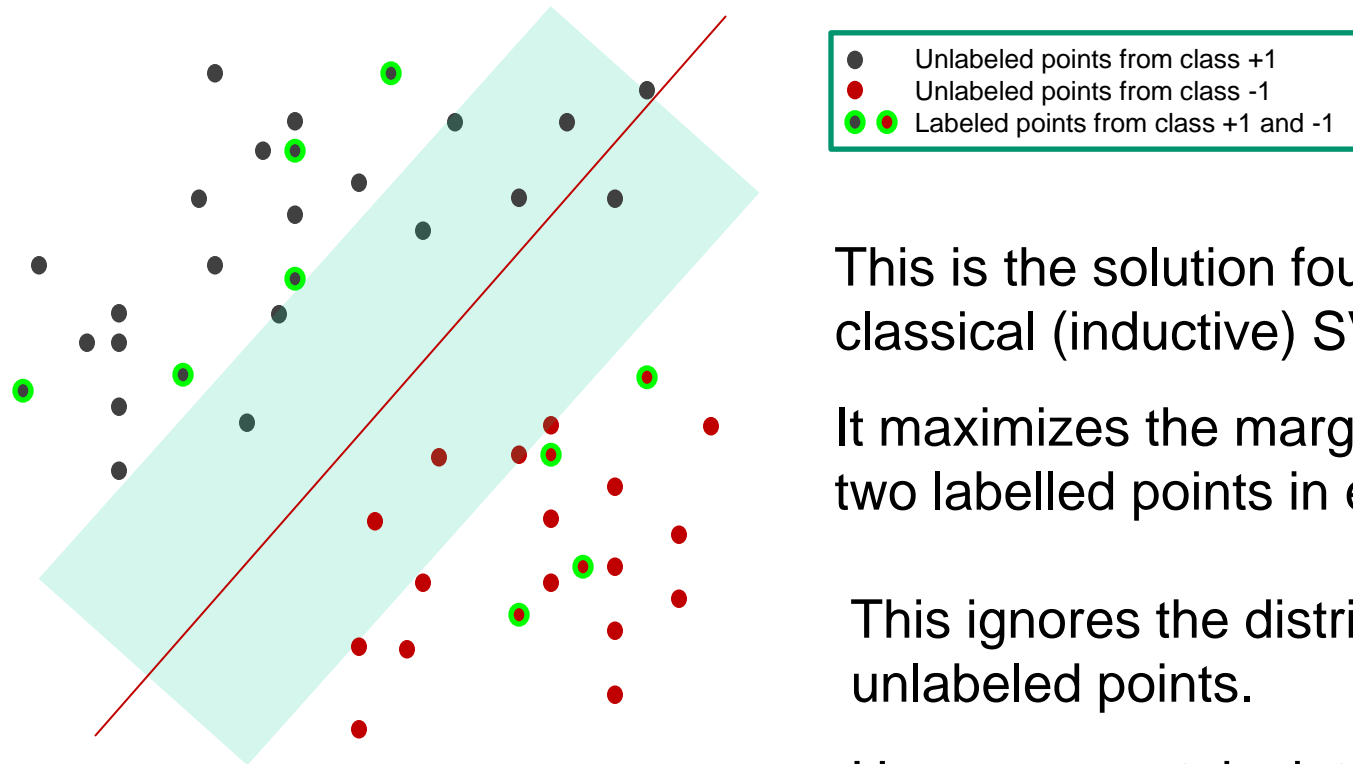Inductive SVM: learn from fully labeled datapoints (regular SVM)

●   Unlabeled points from class +1
●   Unlabeled points from class -1
◉ ◉   Labeled points from class +1 and -1

This is the solution found by classical (inductive) SVM.

It maximizes the margin between the two labelled points in each class.
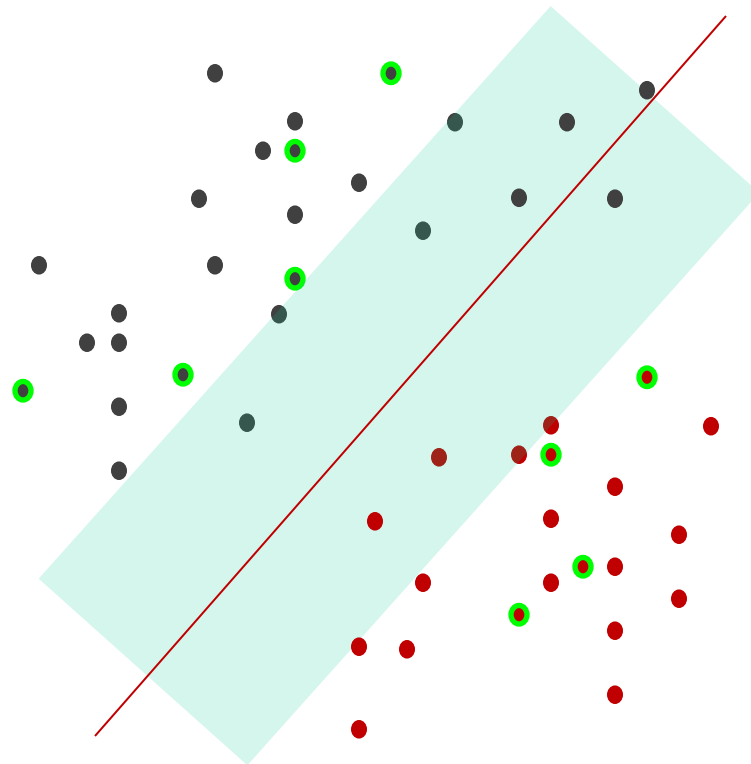
This ignores the distribution of unlabeled points.

How can one take into account the unlabeled points?

Transductive Support Vector Machine

Transductive SVM: learn from partially labeled datapoints
Inductive SVM: learn from fully labeled datapoints (regular SVM)

●	Unlabeled points from class +1
●	Unlabeled points from class -1
● ●	Labeled points from class +1 and -1

We want:
1) zero error on labeled points
2) unlabeled points well separated

Mix between:
1) classification problem
2) clustering problem

→ use labels in 1 to guide 2.

Transductive Support Vector Machine

Consider the dataset composed of:

$\left\{ x^i, y^i \right\}$: set of labeled datapoints

$\left\{ \tilde{x}^i, \tilde{y}^i \right\}$: set of un-labeled datapoints, $\quad \tilde{y}^i$: unknown!
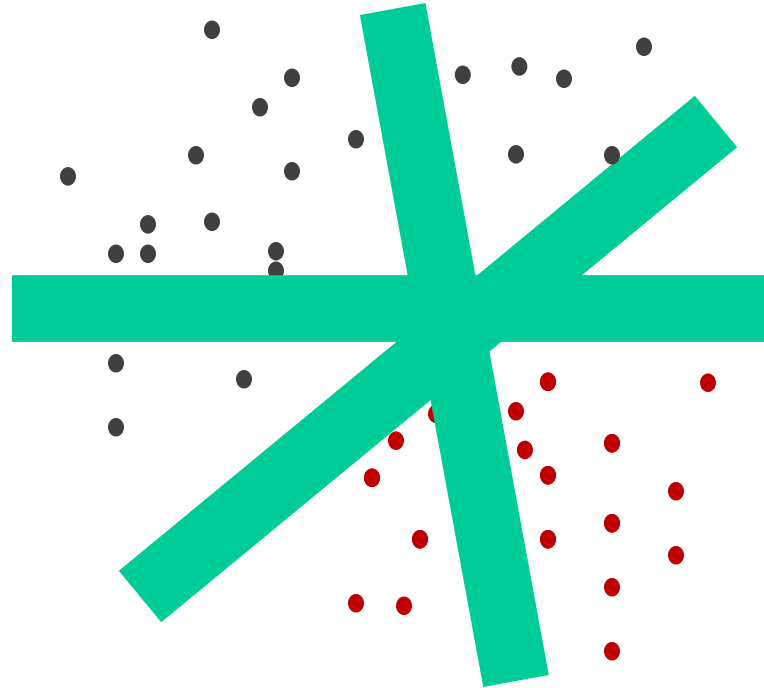
Formulates the constrained optimization problem:

$\min \; \|w\|^2 \quad$ with the following constraints:

$\forall_{i=1}^{M} \begin{cases} y^i \left\langle w, x^i \right\rangle \geq 1 \\ \\ \tilde{y}^i \left\langle w, \tilde{x}^i \right\rangle \geq 1 \\ \\ \tilde{y}^i \in \left\{ -1; +1 \right\} \end{cases}$

Requests perfect classification on labeled points

Searches for the labels $\tilde{y}^i$ such that unlabeled points live on either side of the margin.
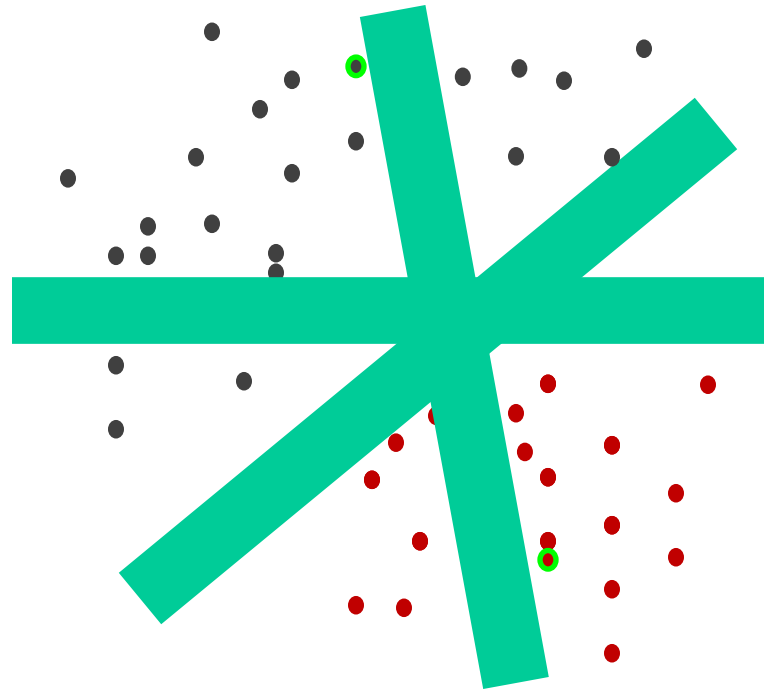
Transductive Support Vector Machine

Which hyperplanes satisfy these constraints?

$$\forall_{i=1}^{M} \begin{cases} \tilde{y}^i \left\langle w, \tilde{x}^i \right\rangle \geq 1 \\ \\ \tilde{y}^i \in \left\{ -1; +1 \right\} \end{cases}$$

Searches for the labels $\tilde{y}^i$ such that unlabeled points live on either side of the margin.
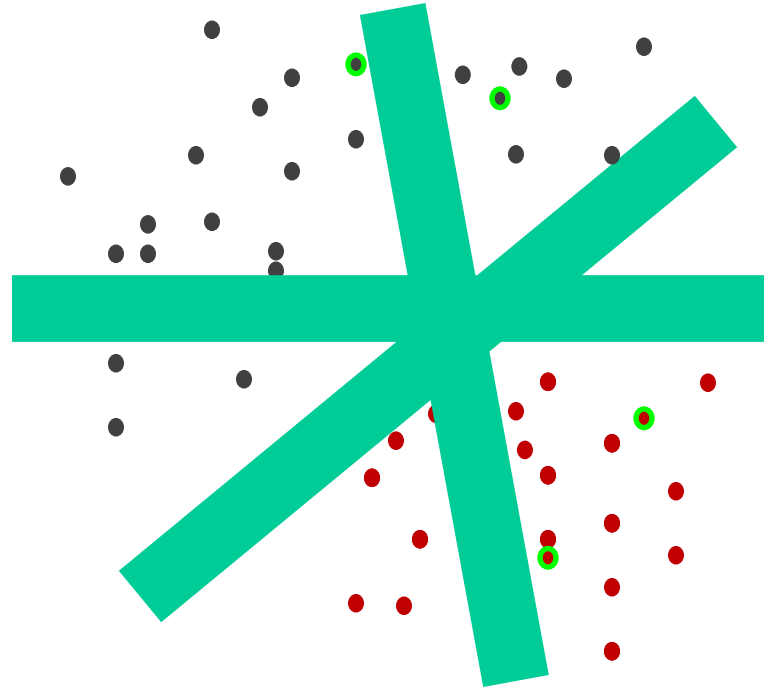
Transductive Support Vector Machine

Adding labeled points
reduces the possibilities

$$\forall_{i=1}^{M} \begin{cases} y^i \left\langle w, x^i \right\rangle \geq 1 \\ \tilde{y}^i \left\langle w, \tilde{x}^i \right\rangle \geq 1 \\ \tilde{y}^i \in \left\{ -1; +1 \right\} \end{cases}$$

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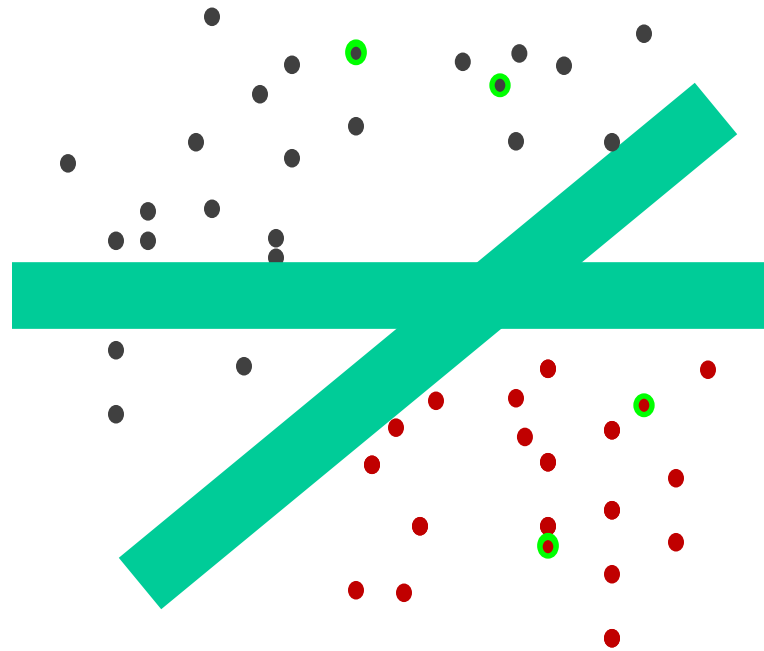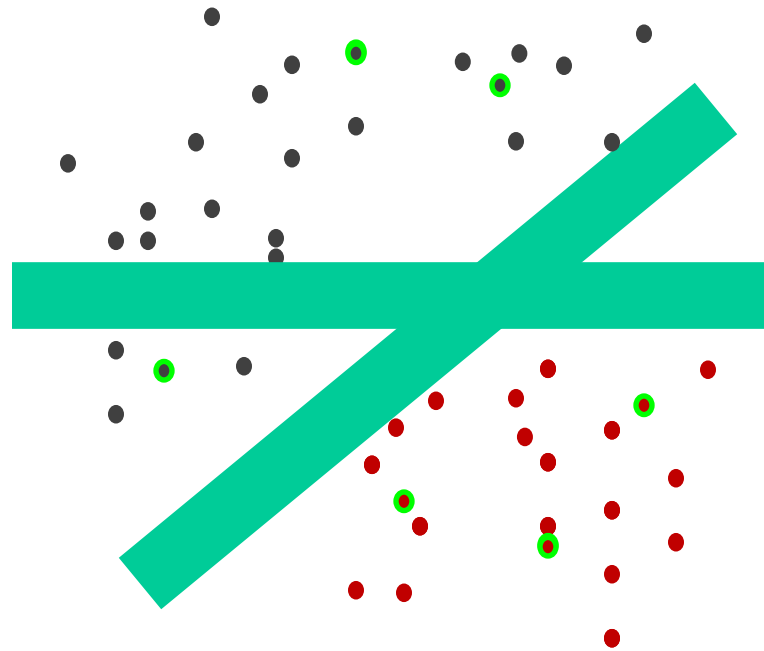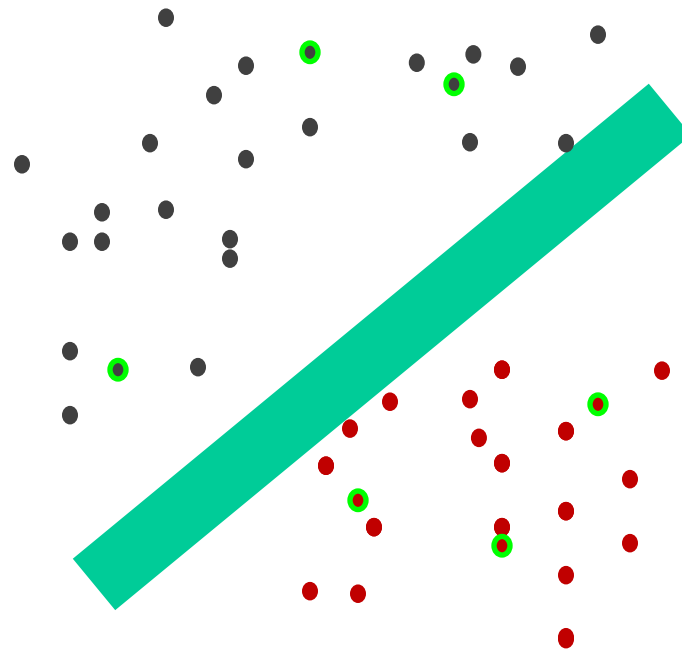# Transductive Support Vector Machine

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