SOLUTION 6

The aim of this exercise was to understand the following equations:

$$\begin{aligned} \mathrm{d}r_\mathrm{a} &= c(E_\mathrm{t}) \cdot g(E) f(E) \cdot N_\mathrm{t} \left[1 - f(E_\mathrm{t})\right] \mathrm{d}E && (1) \\ \mathrm{d}r_\mathrm{b} &= e(E_\mathrm{t}) \cdot N_\mathrm{t} f(E_\mathrm{t}) \cdot g(E) \left[1 - f(E)\right] \mathrm{d}E \quad \text{with} && (2) \\ f(E) &= \frac{1}{1 + e^{\frac{E - E_\mathrm{F}}{kT}}} && (3) \end{aligned}$$

Please be aware that we consider the system to be in thermal equilibrium here!

a)
- E_t is the energy level of the traps – in the SRH-model all traps are considered to be at the same energy level.
- E_F is the energy of the Fermi level.
- E ist the energy of electron states in the conduction band, i.e. it can lie in the range between the conduction band edge E_c and infinity.
- g is the density of electron states of the conduction band, i.e. the number of electron states per volume and energy.
- f is the occupation function which describes the probability that an electron state is occupied.
- N_t is the constant trap- or recombination center density.
- c and e are capture and emission coefficients, respectively: The whole physics of the capture and emission processes are considered therein. Multiplied with a density they give a capture and emission rate. The relationship between the capture cross section σ_c and c may also help for the understanding: $c = v_\mathrm{th} \cdot \sigma_\mathrm{c}$ with v_th the thermal velocity of electrons.
- r_a / r_b are the rates of electrons that are captured in / emitted from traps of the conduction band tail and contribute therefore to the Shockley-Read-Hall recombination. They depend on the (fixed) energy of the traps. To get the net SRH-recombination rate one has to integrate $\mathrm{d}r_\mathrm{a} - \mathrm{d}r_\mathrm{b}$ over the whole energy range of the conduction band.

b) The units of the parameters of the equations (1) to (3) are the following (not in SI system, but as commonly used in photovoltaics):

$$[r_\mathrm{a}] = [r_\mathrm{b}] = \mathrm{cm^{-3}\,s^{-1}} \tag{4}$$

$$[E] = [E_\mathrm{t}] = [E_\mathrm{F}] = \mathrm{eV} \tag{5}$$

$$[c] = [e] = \mathrm{cm^{3}\,s^{-1}} \tag{6}$$

$$[g] = \mathrm{cm^{-3}\,eV^{-1}} \tag{7}$$

$$[f] = 1 \tag{8}$$

$$[N_\mathrm{t}] = \mathrm{cm^{-3}} \tag{9}$$

c) As the net electron capture rate in equilibrium must vanish, it is

$$\begin{aligned}
0 &\overset{!}{=} \mathrm{d}r_\mathrm{a} - \mathrm{d}r_\mathrm{b} && (10)\\
&= g(E)N_\mathrm{t} \cdot \big\{c(E_\mathrm{t})f(E)\,[1-f(E_\mathrm{t})] - && (11)\\
&\qquad\qquad e(E_\mathrm{t})f(E_\mathrm{t})\,[1-f(E)]\big\}\,\mathrm{d}E && (12)\\
\Rightarrow \quad c(E_\mathrm{t})f(E)\,[1-f(E_\mathrm{t})] &= e(E_\mathrm{t})f(E_\mathrm{t})\,[1-f(E)] \quad \forall E && (13)\\
\Rightarrow \quad \frac{e(E_\mathrm{t})}{c(E_\mathrm{t})} &= \frac{f(E)\cdot[1-f(E_\mathrm{t})]}{f(E_\mathrm{t})\cdot[1-f(E)]} && (14)\\
&\overset{(*)}{=} \frac{f(E)\cdot e^{\frac{E_\mathrm{t}-E_\mathrm{F}}{kT}}\cdot f(E_\mathrm{t})}{f(E_\mathrm{t})\cdot e^{\frac{E-E_\mathrm{F}}{kT}}\cdot f(E)} && (15)\\
&= e^{\frac{E_\mathrm{t}-E_\mathrm{F}-E+E_\mathrm{F}}{kT}} && (16)\\
&= e^{\frac{E_\mathrm{t}-E}{kT}}. && (17)
\end{aligned}$$

where in (∗) has been used that

$$f(E) = \frac{1}{1+e^{\frac{E-E_\mathrm{F}}{kT}}} \quad \Rightarrow \quad 1-f(E) = \frac{1+e^{\frac{E-E_\mathrm{F}}{kT}}-1}{1+e^{\frac{E-E_\mathrm{F}}{kT}}} = e^{\frac{E-E_\mathrm{F}}{kT}}\cdot f(E). \qquad (18)$$