Solution 2

Exercise 1:

a. Starting from the formula:

$$\epsilon(\omega) = \epsilon_\infty + \frac{e^2 N}{\epsilon_0 m V} \cdot \frac{1}{\omega_0^2 - 2i\beta\omega - \omega^2}$$

and with $\omega_0 = 0$, we get:

$$\begin{aligned}
\epsilon(\omega) &= \epsilon_\infty - \omega_p^2 \cdot \frac{1}{2i\beta\omega + \omega^2} \cdot \frac{2i\beta\omega - \omega^2}{2i\beta\omega - \omega^2} = \\
&= \epsilon_\infty - \omega_p^2 \cdot \frac{2i\beta\omega - \omega^2}{-4\beta^2\omega^2 - \omega^4} = \\
&= \epsilon_\infty + \omega_p^2 \cdot \frac{i\omega/\tau - \omega^2}{\omega/^2\tau^2 + \omega^4} = \\
&= \epsilon_\infty + \omega_p^2 \cdot \left(-\frac{\tau^2}{1 + \omega^2\tau^2} + \frac{i\tau}{\omega + \omega^3\tau^2} \right) = \\
&= \epsilon_\infty - \frac{\omega_p^2\tau^2}{1 + \omega^2\tau^2} + i \cdot \frac{1}{\omega\tau} \cdot \frac{\omega_p^2\tau^2}{1 + \omega^2\tau^2}
\end{aligned}$$

b. The relation between dielectric function and refractive index is given by:

$$\epsilon = \epsilon' + i\epsilon'' = (n + ik)^2 = n^2 - k^2 + 2ink$$

and by separating the real and imaginary terms, we get:

$$\begin{cases} n^2 - k^2 = \epsilon' \\ 2nk = \epsilon'' \end{cases}$$

In the case of weakly absorbing media ($\epsilon \approx n^2$), ϵ'' is small compared to ϵ', and k^2 is negligible compared to n^2. Thus, $\epsilon' \approx n^2$, and using $2nk = \epsilon''$ we get:

$$k = \frac{\epsilon''}{2n} \approx \frac{\epsilon''}{2\sqrt{\epsilon}} \approx \frac{\epsilon''}{2\sqrt{\epsilon'}}$$

c. We start to calculate the absorption coefficient for an energy of 1 eV.
The relaxation time is:

$$\tau = \frac{75 \times 10^{-4}\,\mathrm{m^2\,V^{-1}\,s^{-1}} \cdot 0.16 \cdot 9.11 \times 10^{-31}\,\mathrm{kg}}{1.6 \times 10^{-19}\,\mathrm{C}} = 6.8 \times 10^{-15}\,\mathrm{s}$$

The plasma frequency is:

$$\omega_p = \sqrt{\frac{(1.6 \times 10^{-19}\,\mathrm{C})^2 \cdot 5 \times 10^{24}\,\mathrm{m^{-3}}}{8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}} \cdot 0.16 \cdot 9.11 \times 10^{-31}\,\mathrm{kg}}} = 3.16 \times 10^{14}\,\mathrm{Hz}$$

The wavelength corresponding to the energy of 1 eV is given by:

$$\lambda(\mathrm{nm}) = \frac{1240\,\mathrm{nm\,eV^{-1}}}{1\,\mathrm{eV}} = 1.24\,\mu\mathrm{m}$$

and the angular frequency ω is:

$$\omega = 2\pi\nu = 2\pi \cdot \frac{3 \times 10^8\,\mathrm{m\,s^{-1}}}{1.24 \times 10^{-6}\,\mathrm{m}} = 1.5 \times 10^{15}\,\mathrm{s^{-1}}$$

The numeric value of ϵ' is:

$$\epsilon' = 11.68 - \frac{(3.16 \times 10^{14}\,\mathrm{Hz})^2 \cdot (6.8 \times 10^{-15}\,\mathrm{s})^2}{1 + (1.5 \times 10^{15}\,\mathrm{Hz})^2 \cdot (6.8 \times 10^{-15}\,\mathrm{s})^2} = 11.64$$

The numeric value of ϵ'' is:

$$\epsilon'' = \frac{1}{1.5 \times 10^{15}\,\mathrm{Hz} \cdot 6.8 \times 10^{-15}\,\mathrm{s}} \cdot \frac{(3.16 \times 10^{14}\,\mathrm{Hz})^2 \cdot (6.8 \times 10^{-15}\,\mathrm{s})^2}{1 + (1.5 \times 10^{15}\,\mathrm{Hz})^2 \cdot (6.8 \times 10^{-15}\,\mathrm{s})^2} = 0.004$$

and the extinction coefficient is:

$$k \approx \frac{0.004}{2 \cdot \sqrt{11.64}} = 0.00059$$

So finally α is:

$$\alpha = 4\pi \cdot k/\lambda = 4\pi \cdot 0.00059/1.24 \times 10^{-6}\,\mathrm{m^{-1}} = 60\,\mathrm{cm^{-1}}$$

Using your favourite mathematical software, you can compute and plot the absorption curve within the energy range of 1 - 1.5 eV. When doing so, you should obtain a result similar to the one of Daub [1], shown in fig.1 below. When taking a closer look, one notices the absorption coefficient we calculated is a bit higher than Daub's one, but the most important conclusion is that below the bandgap energy α increases by a few orders of magnitudes when there is free carrier absorption (compared to Green's and Daub's results (curve D) without free carrier absorption).

[1] E. Daub *et al.*, J. Appl. Phys. 80(9), p. 5325 (1996)

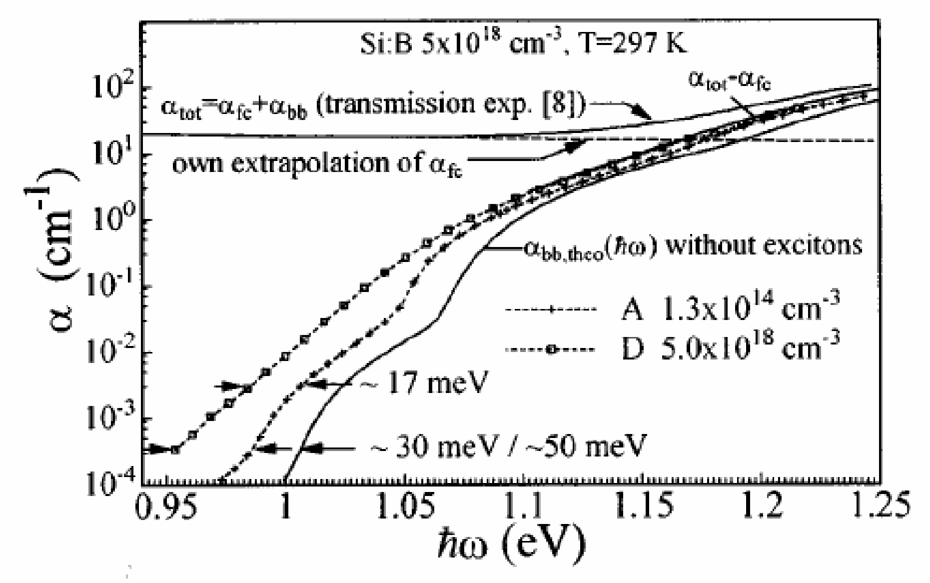

Figure 1: Comparison of the results for the determination of $\alpha_{bb}(\hbar\omega)$ from transmission and photoluminescence experiments (Si:B, $5 \times 10^{18}\,\mathrm{cm}^{-3}$). The values for $\alpha_{bb}(\hbar\omega)$ are nearly five orders of magnitude smaller than the absorption coefficient for free carrier absorption $\alpha_{fc}(\hbar\omega)$.