Solution 12

Exercise 1: Particle detectors

a) **Direct detection**

I) **No space charge in the intrinsic layer of the diode (i.e. uniform field):**
Our intuition tells us: All electrons move with the same velocity towards the collection point. Therefore, the number of collected electrons per unit time (the current) should be constant.
However, this is wrong.
According to Ramo's theorem [1] as soon as one electron starts to drift, it induces an electron to flow in the external circuit. This effect can be compared to a charging capacitor: The more electrons gather on one plate, the more holes accumulates on the other plate, until equilibrium is reached.
Therefore, coming back to our detector, immediately after the charge generation throughout the sensing element, the electrons movement induce a pulse. This pulse is immediately at its maximum and eventually it decreases in intensity as more and more electrons are collected at the n-layer.

i. As the photon velocity in silicon is much larger than the electron drift velocity, $c_{\text{Si}}^{\lambda} >> v_{\text{drift}}^{\text{el}}$, the generation of the e^-/h^+-pairs can be considered to be instantaneous all over the diode. Therefore, the maximum pulse height $I_{\max}$ is measured immediately after the X-ray beam crossing when all generated electrons are moving. With $n = 100\,\text{pair}/\mu\text{m}$ the number of generated e^-/h^+-pairs per μm, $d = 10\,\mu\text{m}$ the diode thickness, Q the generated charge, $V_{\text{bias}} = 100\,\text{V}$, $v_{\text{drift}}^{\text{el}}$ the drift velocity and $\mu_{drift}^{\text{el}} = 1\,\text{cm}^2\,\text{V}^{-1}\,\text{s}^{-1}$ the mobility of electrons, it is

$$I_{\max} = q \cdot n \cdot v_{\text{drift}}^{\text{el}} = q \cdot n \cdot \mu_{drift}^{\text{el}} \cdot E = q \cdot n \cdot \mu_{drift}^{\text{el}} \cdot \frac{V_{\text{bias}}}{d} \quad (1)$$

$$= 1.6 \times 10^{-19}\,\text{C} \cdot 100\,\text{el}/\mu\text{m} \cdot 1\,\text{cm}^2\,\text{V}^{-1}\,\text{s}^{-1} \cdot \frac{100\,\text{V}}{10\,\mu\text{m}} = 16\,\text{nA} \quad (2)$$

ii. Neglecting the hole contribution (which generates another pulse smaller in intensity but longer in time), the pulse length T_{pulse} is determined by the electrons generated near the n-i interface that have to travel through the whole intrinsic layer:

$$T_{\text{pulse}} = \frac{d}{v_{\text{drift}}^{\text{el}}} = \frac{d}{\mu_{drift}^{\text{el}} \cdot E} = \frac{d}{\mu_{drift}^{\text{el}} \cdot \frac{V_{\text{bias}}}{d}} \quad (3)$$

$$= \frac{(10\,\mu\text{m})^2}{1\,\text{cm}^2\,\text{V}^{-1}\,\text{s}^{-1} \cdot 100\,\text{V}} = 10\,\text{ns} \quad (4)$$

[1] S. Ramo, "Currents Induced by Electron Motion". Proc. I.R.E, 1939, vol. 27, p. 584.

iii. From the considerations above ($v_{\text{drift}}^{\text{el}} = const$) it is clear that the current decreases linearly within T_{pulse} from $I_{\max}$ to 0.

iv. The X-ray beam has enough energy to cross the detector and generates

$$N = n \cdot L = 100\,\text{pairs/µm} \cdot 10\,\text{µm} = 1000\,\text{pairs}. \tag{5}$$

If all the electrons are collected, the totally collected charge is

$$Q = -\int_0^{T_{\text{pulse}}} I\,\mathrm{d}t = -80 \times 10^{-18}\,\text{C}. \tag{6}$$

Another way to calculate Q is

$$Q = -\frac{1}{2} \cdot q \cdot N = -\frac{1}{2} \cdot 1.6 \times 10^{-19}\,\text{C/el} \cdot 1000\,\text{el} = -80 \times 10^{-18}\,\text{C} \tag{7}$$

Where does the factor $\frac{1}{2}$ come from? We are considering here only the pulse caused by electrons. In fact, the second pulse much weaker and longer from the holes is superposed to the fast electron one. If one consideres the charges collected by both pulses, the factor $\frac{1}{2}$ vanishes. Another way to explain the same physical behaviour is that the electrons move in average only through half the detector, therefore, only the charge of half an electron is measured in average for each electron.

II) **Uniform defect density of $N_{\text{db}} = 2 \times 10^{16}\,\text{cm}^{-3}$ with all dangling bonds polarized in the space charge region:**

In contrast to the case discussed above, the electric field is no more constant within the intrinsic layer with charged dangling bonds. Poisson's equation

$$\frac{\partial E}{\partial x} = \frac{\rho}{\epsilon_0 \epsilon_{\text{r}}} = \frac{\pm q \cdot N_{\text{db}}}{\epsilon_{\text{r}} \epsilon_0} = \frac{\pm 1.6 \times 10^{-19}\,\text{C} \cdot 2 \times 10^{16}\,\text{cm}^{-3}}{10 \cdot 8.6 \times 10^{-12}\,\text{A}\,\text{s}\,\text{V}^{-1}\,\text{m}^{-1}} \approx \pm 3.7 \times 10^5\,\text{V}\,\text{cm}^{-1}\,\text{µm}^{-1}, \tag{8}$$

tells us that the constant electric field $E_0 = \frac{100\,\text{V}}{10\,\text{µm}} = 10^5\,\text{V}\,\text{cm}^{-1}$ of exercise a) is reduced (in absolute values from the p/i and i/n interface) by $3.7 \times 10^5\,\text{V}\,\text{cm}^{-1}\,\text{µm}^{-1}$ or

$$E_{\text{shield}}(\Delta x) = \int_{x_1}^{x_2} \frac{\partial E}{\partial \xi}\,\mathrm{d}\xi. \tag{9}$$

Therefore, the total electric field within the charged regions is

$$E(x) = E_0 - E_{\text{shield}}(\Delta x) = E_0 - \left| \int_{x_1}^{x_2} \frac{\partial E}{\partial \xi}\,\mathrm{d}\xi \right|. \tag{10}$$

Under the given assumptions, $E_0 = E_{\text{shield}}$ or $E = 0$ at a distance L_{shield} from the i-layer borders:

$$\begin{aligned} E(L_{\text{shield}}) &= E_0 - \int_0^{L_{\text{shield}}} \frac{\partial E}{\partial \xi}\,\mathrm{d}\xi && (11) \\ &= 10^5\,\text{V}\,\text{cm}^{-1} - L_{\text{shield}} \cdot 3.7 \times 10^5\,\text{V}\,\text{cm}^{-1}\,\text{µm}^{-1} && (12) \\ &\overset{!}{=} 0 && (13) \\ \Rightarrow L_{\text{shield}} &= 0.27\,\text{µm}. && (14) \end{aligned}$$

Therefore, the electric field is

$$E(x) = \begin{cases} 10^5\,\mathrm{V\,cm^{-1}} - x \cdot 3.7 \times 10^5\,\mathrm{V\,cm^{-1}\,\mu m^{-1}} & \text{for } 0 < x < L_{\text{shield}} \\ 0 & \text{for } L_{\text{shield}} < x < d - L_{\text{shield}} \\ (x + L_{\text{shield}} - d) \cdot 3.7 \times 10^5\,\mathrm{V\,cm^{-1}\,\mu m^{-1}} & \text{for } d - L_{\text{shield}} < x < d. \end{cases} \tag{15}$$

with a charge distribution

$$\rho(x) = \begin{cases} q \cdot N_{\text{db}} = 3.2\,\mathrm{\mu C\,cm^{-3}} & \text{for } 0 < x < L_{\text{shield}} \\ 0 & \text{for } L_{\text{shield}} < x < d - L_{\text{shield}} \\ -q \cdot N_{\text{db}} = -3.2\,\mathrm{\mu C\,cm^{-3}} & \text{for } d - L_{\text{shield}} < x < d. \end{cases} \tag{16}$$

For all further equations only electrons that are closer to the n-layer than $d - L_{\text{shield}}$ must be considered, as all other electrons recombine directly within the bulk of the diode and do not contribute to a pulse.

i. With the same argumentation as in I), the maximum pulse is measured immediately after the x-ray passing through the diode. However, with the x dependence of E, the x dependence of v has to be considered too. As only the electrons generated between $d - L_{\text{shield}}$ and d contribute to the pulse it is

$$\begin{aligned}
I_{\max} &= \int_0^Q \frac{\mathrm{d}Q'}{L_{\text{shield}}} v_{\text{drift}}^{\text{el}}(x) && (17)\\
&= \int_{d-L_{\text{shield}}}^{d} \frac{nq}{L_{\text{shield}}} v_{\text{drift}}^{\text{el}}(x)\,\mathrm{d}x \quad \text{as} \quad \mathrm{d}Q' = nq\,\mathrm{d}x && (18)\\
&= \int_{d-L_{\text{shield}}}^{d} \frac{nq}{L_{\text{shield}}} \mu_{drift}^{\text{el}} \cdot E(x)\,\mathrm{d}x && (19)\\
&= \frac{nq\mu_{drift}^{\text{el}}}{L_{\text{shield}}} \int_{d-L_{\text{shield}}}^{d} E(x)\,\mathrm{d}x && (20)\\
&= \frac{nq\mu_{drift}^{\text{el}}}{L_{\text{shield}}} \int_{d-L_{\text{shield}}}^{d} (x + L_{\text{shield}} - d) \cdot 3.7 \times 10^5\,\mathrm{V\,cm^{-1}\,\mu m^{-1}}\,\mathrm{d}x && (21)\\
&= \frac{nq\mu_{drift}^{\text{el}}}{L_{\text{shield}}} \cdot 3.7 \times 10^5\,\mathrm{V\,cm^{-1}\,\mu m^{-1}} \int_0^{L_{\text{shield}}} t \cdot dt && (22)\\
&= \frac{nq\mu_{drift}^{\text{el}}}{L_{\text{shield}}} \cdot \frac{1}{2} \cdot L_{\text{shield}}^2 \cdot 3.7 \times 10^5\,\mathrm{V\,cm^{-1}\,\mu m^{-1}} && (23)\\
&= \mu_{drift}^{\text{el}} nq \cdot 0.27\,\mathrm{\mu m} \cdot 3.7 \times 10^5\,\mathrm{V\,cm^{-1}\,\mu m^{-1}} && (24)\\
&= 1\,\mathrm{cm^2\,V^{-1}\,s^{-1}} \cdot 100\,\mathrm{el}/\mathrm{\mu m} \cdot 1.6 \times 10^{-19}\,\mathrm{C/el}\ \cdot && (25)\\
&\ \frac{1}{2} \cdot 0.27\,\mathrm{\mu m} \cdot 3.7 \times 10^5\,\mathrm{V\,cm^{-1}\,\mu m^{-1}} && (26)\\
&= 8\,\mathrm{nA} && (27)
\end{aligned}$$

(Intuitively it is clear that the average velocity of the electrons is just half the maximum velocity with constant spatial distribution and linearly increasing velocity from 0.)

ii. In analogy to I), the pulse length is given by

$$\begin{aligned}
T_{\text{pulse}} &= \int_{d-L_{\text{shield}}}^{d} \frac{1}{v_{\text{drift}}^{\text{el}}(x)}\,\mathrm{d}x && (28)\\
&= \frac{1}{\mu_{drift}^{\text{el}}} \int_{d-L_{\text{shield}}}^{d} \frac{1}{E(x)}\,\mathrm{d}x && (29)\\
&= \frac{1}{\mu_{drift}^{\text{el}}} \int_{d-L_{\text{shield}}}^{d} \frac{1}{(x+L_{\text{shield}}-d)\cdot 3.7\times10^{5}\,\mathrm{V\,cm^{-1}\,\mu m^{-1}}}\,\mathrm{d}x && (30)\\
&= \frac{1}{\mu_{drift}^{\text{el}}\cdot 3.7\times10^{5}\,\mathrm{V\,cm^{-1}\,\mu m^{-1}}} \int_{0}^{L_{\text{shield}}} \frac{1}{t}\,\mathrm{d}t && (31)\\
&= \frac{1}{\mu_{drift}^{\text{el}}\cdot 3.7\times10^{5}\,\mathrm{V\,cm^{-1}\,\mu m^{-1}}} [\ln(L_{\text{shield}}) - \ln(0)] && (32)\\
&\to \infty && (33)
\end{aligned}$$

The pulse seems to have no end. This is not physical and it comes from the approximated approach we adopted. Anyway from point iii it comes out that:

iii. Due to $I \propto (d-x)^2$ (see e.g. equation (23)), the current decreases quadratically.

iv. In analogy to I) it is

$$N = n \cdot L_{\text{shield}} = 100\,\text{pair}/\mu\text{m} \cdot 0.27\,\mu\text{m} = 27\,\text{pairs} \qquad (34)$$

If all the electrons are collected, the total charge

$$Q = -\frac{1}{2}\cdot q \cdot N = -\frac{1}{2}\cdot 1.6\times10^{-19}\,\text{C/el}\cdot 27\,\text{el} = -2.15\times10^{-18}\,\text{C} \qquad (35)$$

can be measured.

When compared to I), it is clear that charged dangling bonds may cause severe problems to particle detectors as explained in the lecture!

b) **Indirect detection**
With a light yield $\gamma = 50\,000\,\text{ph/MeV}$ and a $E_\gamma = 50\,\text{keV}$-photon, the scintillator irradiates $N_{\text{irr}} = \gamma \cdot E_\gamma$, which $N = 20\,\% \cdot \gamma \cdot E_\gamma = 500$ photons are absorbed in the diode. Each of these secondary photons is supposed to generate a e^-/h^+-pair in the bulk of the diode.

I) From here on, this exercise is exactly the same as a) but with other numerical values:

i.

$$I_{\max} = \frac{Q}{d}\cdot v_{\text{drift}}^{\text{el}} = \frac{Q}{d}\cdot \mu_{drift}^{\text{el}} \cdot \frac{V_{\text{bias}}}{d} = \frac{8\times10^{-17}\,\text{C}}{1\,\mu\text{m}}\cdot 1\,\text{cm}^2\,\text{V}^{-1}\,\text{s}^{-1}\cdot\frac{5\,\text{V}}{1\,\mu\text{m}} = 40\,\text{nA} \qquad (36)$$

ii.

$$T_{\text{pulse}} = \frac{d^2}{\mu^{\text{el}}_{drift} \cdot V_{\text{bias}}} = \frac{(1\,\mu\text{m})^2}{1\,\text{cm}^2\,\text{V}^{-1}\,\text{s}^{-1} \cdot 5\,\text{V}} = 2\,\text{ns} \tag{37}$$

iii. the current decreases linearly within T_{pulse} from I_{max} to 0.

iv. $Q = -q \cdot N = 8 \times 10^{-17}\,\text{C}$. Such a detector would be very nice, but ...

II) ... more realistically the scintillator irradiates the secondary photons very slowly and spread over a time interval of typically $T_{\text{irr}} \approx 1\,\mu\text{s}$.

i. As the collection within the diode is much faster than the irradiation of the secondary photons, $T_{\text{coll}} << T_{\text{irr}} = 1\,\mu\text{s}$, the electrons can be considered to be collected instantaneously and the pulse length $T_{\text{pulse}} \approx T_{\text{irr}} = 1\,\mu\text{s}$. The secondary photons are homogeneously distributed over T_{irr} which means with a frequency of $\nu = \frac{N}{T_{\text{irr}}} = \frac{500\,\text{ph}}{1\,\mu\text{s}} = 0.5\,\text{ns}^{-1}$ the photons arrive in the diode where the photoelectron goes through within 2 ns . Therefore, only one photoelectron passes the diode at once and

$$\begin{aligned} I_{\text{max}} &= I & (38)\\ &= \frac{Q}{d} \cdot v^{\text{el}}_{\text{drift}} & (39)\\ &= q \cdot n \cdot \mu^{\text{el}}_{drift} \cdot \frac{V_{\text{bias}}}{d} & (40)\\ &= 1.6 \times 10^{-19}\,\text{C/el} \cdot 1\,\text{el}/\mu\text{m} \cdot 1\,\text{cm}^2\,\text{V}^{-1}\,\text{s}^{-1} \cdot \frac{5\,\text{V}}{1\,\mu\text{m}} & (41)\\ &= 80\,\text{pA} & (42) \end{aligned}$$

which is hardly of any use. However, scintillator-coupled diodes are well suited for high particle fluxes and often used as they are cheap and easy to use.

ii. Solved in i. already: $T_{\text{pulse}} \approx T_{\text{irr}} = 1\,\mu\text{s}$.

iii. Solved in i. already: The pulse is constant.

iv. With an EQE of 100 %, it is $Q = -q \cdot N = -8 \times 10^{-17}\,\text{C}$.

Exercise 2: As the paper cited on slides 48 and 49 of the course (Fortunato et al., Proc. of the IEEE 93 (2005) 1281) seems to contain substantial mistakes, we do not consider the formula given there but do some calculations with simplier assumptions:
The device shown in the exercise shall be biased at $-1\,\text{V}$ and consist of a continuous perfect diode with no shunts ($R_{\text{p}} \to \infty$).
Therefore, one can use the diode equation

$$J = J_{\text{s}} \cdot \left[e^{\frac{qV}{nkT}} - 1 \right] \tag{43}$$

with $n = 2$ to calculate the (constant) current through the diode that leads to the fall-off of the laser-generated current towards the ends of the detector. Assuming a laser-line that generates a photocurrent of $1\,\mathrm{mA\,cm^{-1}}$ at x_0, a current

$$\begin{aligned} i(x) &= i_0 - \left| \int_{x_0}^{x} J_\mathrm{s} \cdot \left[e^{\frac{qV}{nkT}} - 1 \right] \mathrm{d}\xi \right| && (44) \\ &= i_0 + J_\mathrm{s} \cdot \left[e^{\frac{qV}{nkT}} - 1 \right] \cdot |x - x_0| && (45) \\ &= 1\,\mathrm{mA\,cm^{-1}} + 10^{-7}\,\mathrm{mA\,cm^{-2}} \cdot \left[e^{\frac{1\,\mathrm{eV}}{2 \cdot 0.026\,\mathrm{eV}}} - 1 \right] \cdot |x - x_0| && (46) \\ &= 1\,\mathrm{mA\,cm^{-1}} - m \cdot |x - x_0| && (47) \end{aligned}$$

flows in the upper ZnO:Al-layer perpendicular to the laser-line. This means that the current decreases by $m \approx 10^{-7}\,\mathrm{mA\,cm^{-2}}$.
Continuing with the assumptions above, we try to find the maximum size L_max of a device such that the smallest measured voltage $V_\mathrm{ph/T}^\mathrm{min} \geq \frac{V_0}{100}$, where V_0 is the voltage between the p and n layer at the illumination point.
The x dependence of the current leads to a x dependence of the voltage drop too:

$$\mathrm{d}V = \mathrm{d}(R \cdot I(x)). \qquad (48)$$

Assuming a detector width of $w = 1\,\mathrm{cm}$ (length of laser line) we get a relative resistance

$$r = \frac{R}{|x - x_0|} = \frac{R_\square \cdot \frac{|x - x_0|}{w}}{|x - x_0|} = \frac{R_\square}{w} = \frac{1000\,\Omega\,\square^{-1}}{1\,\mathrm{cm}} = 1000\,\Omega\,\mathrm{cm^{-1}} \qquad (49)$$

and with that

$$\begin{aligned} \Delta V(|x - x_0|) &= \int_{x_0}^{x} I(x) \frac{\mathrm{d}R}{\mathrm{d}x} \mathrm{d}\xi = \frac{\mathrm{d}R}{\mathrm{d}x} \cdot \int_{x_0}^{x} I(x)\,\mathrm{d}\xi && (50) \\ &= r \cdot \int_{x_0}^{x} I(x)\,\mathrm{d}\xi = r \cdot \int_{x_0}^{x} i(x) \cdot w\,\mathrm{d}\xi && (51) \\ &= r \cdot \int_{x_0}^{x} \left\{ i_0 - \left| \int_{x_0}^{x} J_\mathrm{s} \cdot \left[e^{\frac{qV}{nkT}} - 1 \right] \right| \right\} \cdot w\,\mathrm{d}\xi && (52) \\ &= r \cdot \int_{x_0}^{x} \left\{ 1\,\mathrm{mA\,cm^{-1}} - m \cdot |x - x_0| \right\} \cdot 1\,\mathrm{cm}\,\mathrm{d}\xi && (53) \\ &= r \cdot 1\,\mathrm{mA\,cm^{-1}} \cdot |x - x_0| \cdot 1\,\mathrm{cm} - \frac{1}{2} \cdot r \cdot m \cdot |x - x_0|^2 \cdot 1\,\mathrm{cm} && (54) \\ &\approx 1000\,\Omega\,\mathrm{cm^{-1}} \cdot 1\,\mathrm{mA} \cdot |x - x_0| - \frac{1}{2} \cdot 1000\,\Omega\,\mathrm{cm^{-1}} \cdot 10^{-7}\,\mathrm{mA\,cm^{-2}} \cdot |x - x_0|^2 && (55) \\ &= 1\,\mathrm{V\,cm^{-1}} \cdot |x - x_0| - \frac{1}{2} \cdot 10^{-7}\,\mathrm{V\,cm^{-2}} \cdot |x - x_0|^2. && (56) \end{aligned}$$

Assuming a maximal photogenerated voltage $V_0 = 1\,\mathrm{V}$ we require $\Delta V = V_0 - V_\mathrm{ph/T}^\mathrm{min} \leq 0.99\,V_0 = 0.99\,\mathrm{V}$ which leads to the quadratic equation

$$\Delta V(|x - x_0|) = 1\,\mathrm{V\,cm^{-1}} \cdot |x - x_0| - \frac{1}{2} \cdot 10^{-7}\,\mathrm{V\,cm^{-2}} \cdot |x - x_0|^2 \leq 0.99\,V_0 = 0.99\,\mathrm{V}. \qquad (57)$$

This equation can numerically easily be solved and one gets a maximum detector length of

$$L_\mathrm{max} = |x - x_0| \leq 0.99\,\mathrm{cm}. \qquad (58)$$