Exercise 2

Exercise 1:

Dielectric function of materials with free carriers
The dielectric function of a material with bound electrons was briefly presented in the lecture:

$$\epsilon(\omega) = \epsilon_\infty + \frac{e^2 N}{\epsilon_0 m V} \cdot \frac{1}{\omega_0^2 - 2i\beta\omega - \omega^2}$$

where ϵ_∞ is the relative permittivity of silicon ($\epsilon_\infty = 11.68$), e is the electron charge, N/V is the carrier density, m is the electron mass, ϵ_0 is the vacuum permittivity, ω_0 is the resonance frequency, ω is the angular frequency and β is the damping term which takes into account that an electron excited by an impulsive electric field does not oscillate for ever.
In some materials the electrons are no longer bound to individual cores but move freely within the volume (c.f. Drude's model for metals). Charge carriers can flow with an applied field, there is no restoring force. Consequently, the resonance frequency ω_0 becomes equal to zero.
To do:

a. Separate the resulting dielectric function into real and imaginary part, making use of the following definitions:

- Plasma frequency:

$$\omega_p = \sqrt{\frac{e^2 N}{\epsilon_0 m V}}$$

- Relaxation time:

$$\tau = \frac{1}{2\beta}$$

Your result should resemble the following one:

$$\epsilon(\omega) = \underbrace{\epsilon_\infty - \frac{\omega_p^2 \tau^2}{1 + \omega^2 \tau^2}}_{\epsilon'} + i \cdot \underbrace{\frac{1}{\omega\tau} \cdot \frac{\omega_p^2 \tau^2}{1 + \omega^2 \tau^2}}_{\epsilon''}$$

b. The relation between dielectric function and refractive index is given by:

$$\epsilon' + i\epsilon'' = (n + i\kappa)^2$$

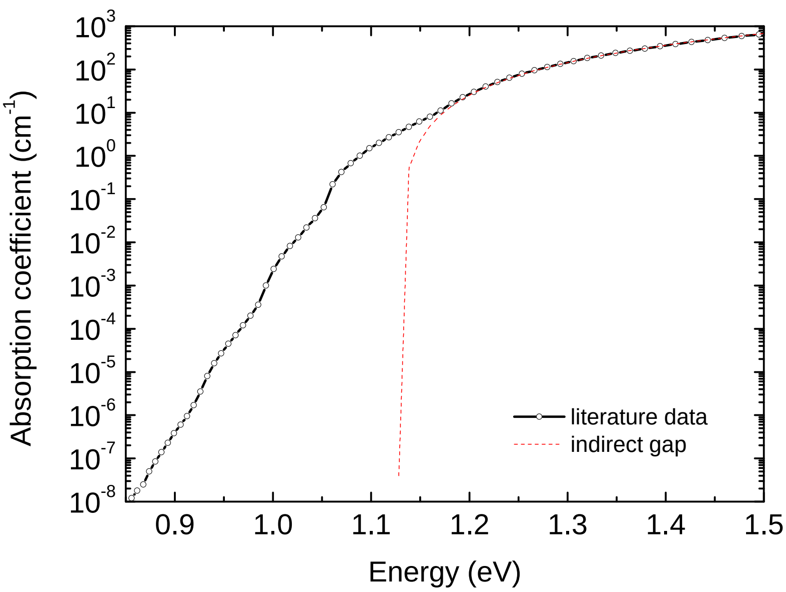
For weakly absorbing media, $\epsilon \approx n^2$; show by binomial expansion that the extincion coefficient κ is approximately given by:

$$\kappa \approx \frac{\epsilon''}{2\sqrt{\epsilon'}}$$

c. Calculate the absorption coefficient α , defined as $\alpha = 4\pi\kappa/\lambda$, versus the photon energy (starting at 1 eV) as it was done in Daub's paper[1] for strongly boron-doped silicon. You need to use the formulas previously obtained and a hole density of $p = 5 \times 10^{18}\,\mathrm{cm}^{-3}$. For free carriers in semiconductors, the relaxation time τ can be related to the carrier mobilty μ via:

$$\tau = \frac{\mu m^*}{e}$$

The hole mobility of intrinsic and weakly doped silicon at RT is $480\,\mathrm{cm\,V^{-1}\,s^{-1}}$, but in case of high carrier concentration, this value is reduced to about $75\,\mathrm{cm\,V^{-1}\,s^{-1}}$ by ionized impurity scattering [2]. Moreover, light holes with an effective mass m^* equal to $0.16\ m_e$ will dominate the optical properties.
Compare your results with the α published by Green without free carrier absorption:



[1] E. Daub *et al.*, J. Appl. Phys. 80(9), p. 5325 (1996)
[2] S. S. Li, Solid State Electron. 21, p. 1109 (1978), Masetti IEEE Trans. Electron Dev. 30(7), p. 764 (1983)