Solution Series 5

Exercise 1: EQE, spectral response, ideal solar cell efficiency under illumination

Three single junction solar cells are made of three different semiconductors materials, characterized by different band gap energies E_g (at 273 K):

- Germanium $E_g = 0.8\,\text{eV}$
- Crystalline silicon $E_g = 1.12\,\text{eV}$
- Gallium Arsenide $E_g = 1.43\,\text{eV}$

Let's assume two hypotheses for this exercise:

- Each solar cell behaves like an ideal solar cell (one diode model), i.e. reflection at the front layers, recombination and other loss mechanisms are neglected.
- Photons with $h\nu < E_g$ are not absorbed, whereas photons with $h\nu \geq E_g$ are absorbed and create one electron-hole pair. These free carriers are collected with a quantum efficiency of one.

a) Draw a sketch of the EQE curve of each cell between 300 - 1600 nm

Solution:

As we consider ideal solar cells, the EQE = 1 when $h\nu \geq E_g$ and EQE = 0 when h$\nu < E_g$. It is thus a rectangular function from 300 nm to the transition wavelength corresponding to the band gap. The transition wavelength, $\lambda_{transition}(nm)$ is equal to $\frac{1240}{E_g(eV)}$.

SC bulk	Ge	c-Si	GaAs
$\lambda_{transition}(nm)$	1550	1107	867

The resulting EQE are shown in Fig. 1.

b) Define the Spectral Response (SR) and sketch the SR of a typical solar cell. Does it depend on the solar spectrum? Based on the EQE sketches in part a) and on the the solar irradiance spectrum, explain how you can obtain the short-circuit current density of the cells. The solar irradiance sprectrum outside the atmosphere and on the earth are shown in Fig. 2.
(data: http://pveducation.org/pvcdrom/appendices/standard-solar-spectra

Solution:

The spectral response definition is the following:

$$SR(\lambda) = \frac{Measured\,Current_{short-circuit}(\lambda)}{power_{incident}(\lambda)} \tag{1}$$

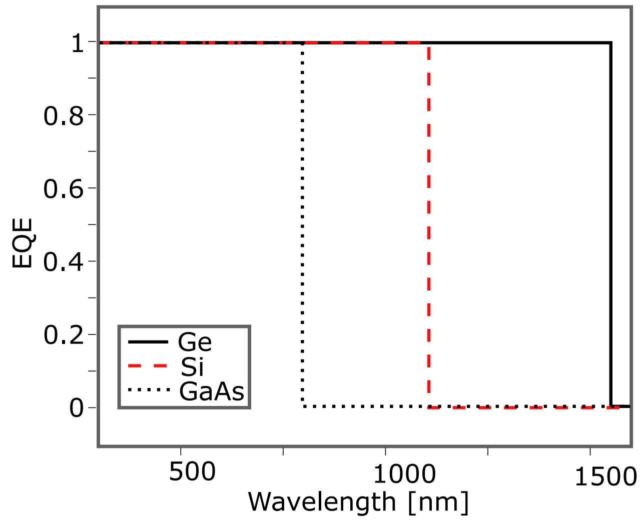

Figure 1: External Quantum efficiency of Ge, c-Si, GaAs bulk solar cells

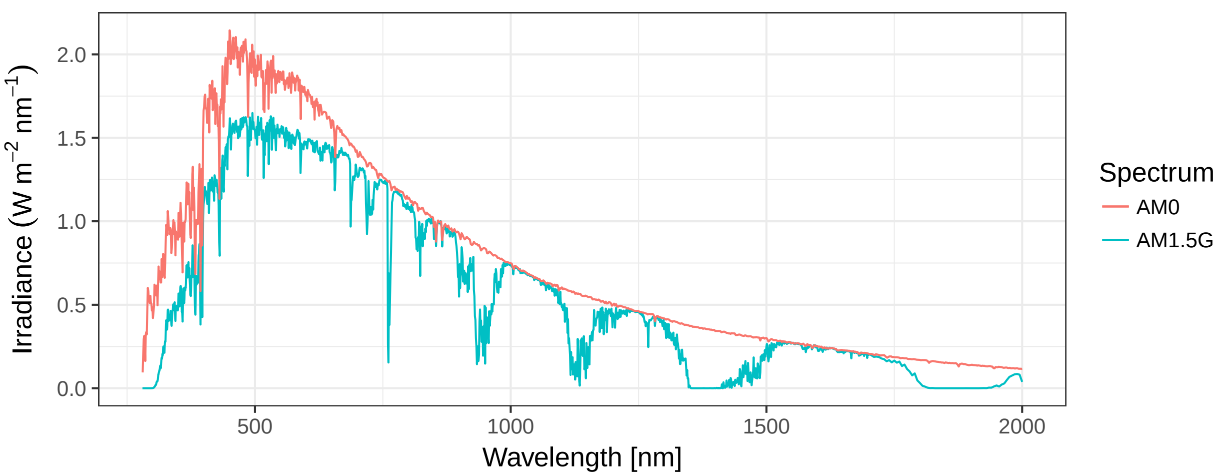

Figure 2: AM0 and AM1.5G spectrum.

We can deduce the SR graph with the formula of the EQE below :

$$SR(\lambda) = EQE(\lambda)\frac{q}{E(\lambda)} \quad (2)$$

As one can see, the density of incident photons ($photons/cm^{-2}/nm$) cancels out and thus the spectral response does not depend on the solar spectrum.

Remark : The quantum efficiency is the ratio of the number of electrons collected from the the solar cell compared to the number of photons incident on the cell. The spectral response is the ratio of the current generated by the solar cell and the power incident on the device.

The short-circuit current density J_{sc} is given by the formula below :

$$J_{cell}(mA \cdot cm^{-2}) = \int_{300}^{1600} Spectrum(\lambda) SR(\lambda) d\lambda \quad (3)$$

Thus the J_{sc} values are obtained by summing the solar spectrum irradiance value multiplied by the spectral response signal for each given wavelength over the wavelength range 300 - 1600 nm.

Thus resulting J_{sc} values are given in the table below :

SC bulk	Ge	c-Si	GaAs
Jsc_{AM0}(mA cm^{-2})	71.7	52.9	38.6
$Jsc_{AM1.5g}$(mA cm^{-2})	54.5	43.8*	31.9

*Note that in solar cells based on indirect semiconductors (like crystalline Si), the light absorption is mediated by phonon absorption from the lattice structure; this results in an effective narrowing of the bandgap. The narrowing leads to more photon absorption (in the IR of spectrum) in the semiconductor and thus, a higher current of up to 44 mA cm^{-2} can be achieved.

c) Compare the results obtained in part b) and given in Table 1. with the data provided in the Solar cells efficiency table - Version 58, available on Moodle. Explain the differences.

semiconducting bulk material	Ge	c-Si	GaAs
$J_{\text{sc, AM0}}$ (mA cm^{-2})	71.7	52.9	38.6
$J_{\text{sc, AM1.5g}}$ (mA cm^{-2})	54.5	43.8	31.9

Table 1: Short-circuit current of ideal single junctions solar cells

Solution:

The current densities calculated from part b) are higher than the empirical values achieved in laboratories and companies. For example the typical empirical value of c-Si current density is about 32 mA cm^{-2} under AM1.5g illumination. This difference is 25% lower than the calculated value of Table 1. It is due to the rectangular shape of EQE seen in question 1.a), where we assumed an ideal solar cell. Figure 3 shows the ideal spectral response for each cell whereas Fig. 3 shows the typical empirical EQE and spectral response of a solar cell. We see in this figure that the shape of the EQE curve of a real solar cell is not rectangular and does not reach 1 over the wavelength range. The reason is that optical losses occur in the cell such as the first reflection of the light at the air/front TCO interface, secondary reflections after the light reaching the back reflector and the parasite absorption of TCOs, back reflector and doped layers.

Check the quantum efficiency page of pveducation for more info:
http://www.pveducation.org/pvcdrom/quantum-efficiency

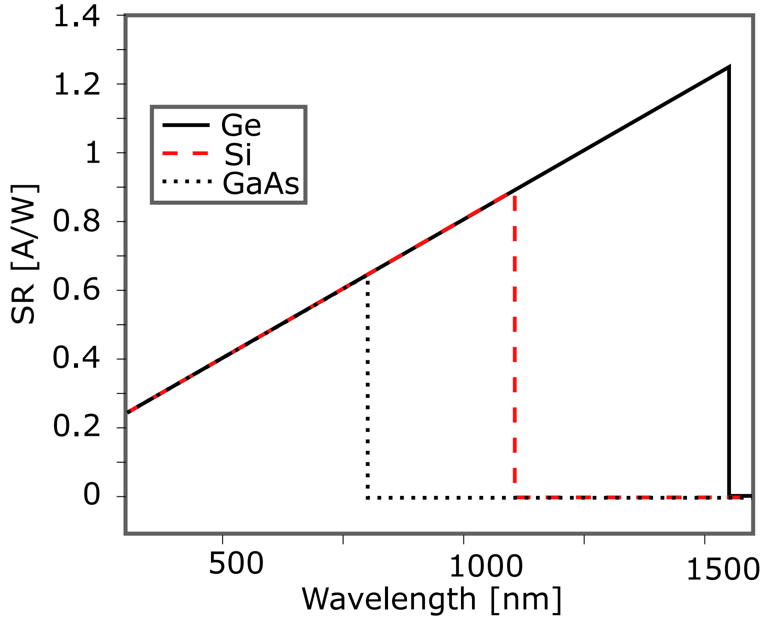

Figure 3: Ideal Spectral Responses of Ge, c-Si, GaAs bulk cells.

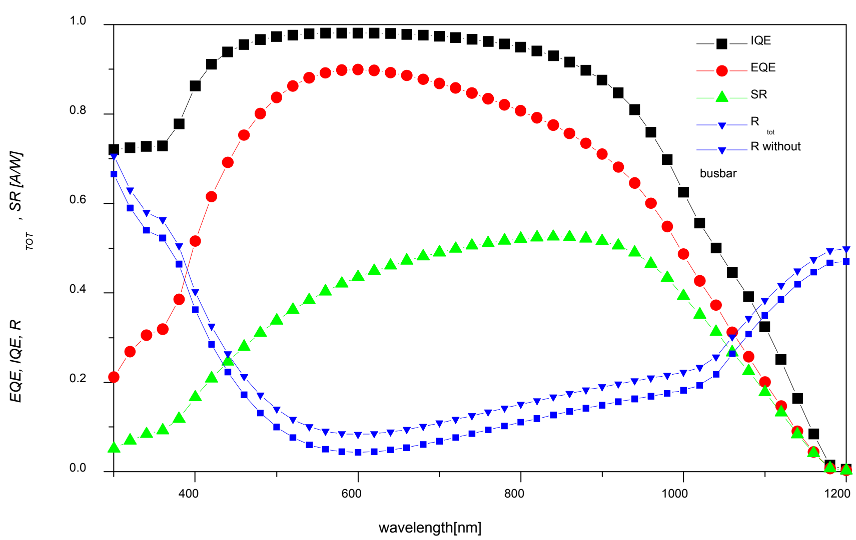

Figure 4: Real optical properties of a solar cell.

d) Calculate the open-circuit voltage V_{oc}, the fill factor FF and the efficiency η of each of the cells for the AM0 and AM1.5g spectrum (use the information given in Table 1). Keep in mind that the first approximation of the solar cell working point under illumination is based on the superposition principle of the generated current: See the 'Chapter I. General introduction to photovoltaics.

Solution:

Since one considers the ideal one diode model, the dark diode current I_{dark} is modeled by:

$$I_{dark} = I_0 \left(\exp \frac{qV}{k_b T} - 1 \right) \tag{4}$$

with I_0= 1.5 10^8 exp $\frac{-E_g}{k_b T}$.

Under illumination, the photogenerated current is modeled by a current generator I_L that is connected in parallel to the dark diode.

- I_{sc}:
  Thus one obtains:

  $$I(V) = I_0 \left(\exp \frac{qV}{k_b T} - 1 \right) - I_L \tag{5}$$

  For an ideal illuminated diode under short cicrcuit condition, we have I_{sc}= I_L and under open circuit condition (I=0), V= V_{oc}.

- V_{oc}:
  From equation 5 in open circuit condition, we have:

  $$I(V_{oc}) = 0 = I_0 \left(\exp \frac{-qV_{oc}}{k_b T} - 1 \right) - I_{sc}$$

  We can then extract an expression for the V_{oc}

  $$V_{oc} = \frac{k_b T}{q} \ln \left(\frac{I_{sc}}{I_0} + 1 \right) \tag{6}$$

- FF:
  The Fill Factor FF can be approximated by a function of the V_{oc} for 1 sun illumination:

  $$FF = \frac{v_{oc} - ln(v_{oc} + 0.72)}{v_{oc} + 1} \tag{7}$$

  with $v_{oc} = \frac{V_{oc}}{k_b T/q}$.

- $Efficiency$:
  The efficiency η is calculated according to the formula:

  $$\eta = \frac{V_{oc} I_{sc} FF}{P_{AM1.5g}} \tag{8}$$

  with $P_{AM1.5g} = 100\,\mathrm{mW\,cm^{-2}}$ and $P_{AM1.0} = 137\,\mathrm{mW\,cm^{-2}}$.

Table 2: Characteristics of ideal single junctions solar cells under AM0 and AM1.5g

	SC bulk cells	**Ge**	**c-Si**	**GaAs**
	$J_0(\mathrm{mA\,cm^{-2}})$	5.5×10^{-6}	2.3×10^{-11}	1.4×10^{-16}
AM0	$Jsc_{AM0}(\mathrm{mA\,cm^{-2}})$	71.7	52.9	38.6
	Voc_{AM0} [V]	0.458	0.77	1.073
	$FF_{AM0}(\%)$	80.4	86.6	89.6
	$\eta_{AM0}(\%)$	19.3	25.8	27.1
AM1.5	$Jsc_{AM1.5}$ [$\mathrm{mA\,cm^{-2}}$]	54.5	43.8	31.9
	$Voc_{AM1.5}$ [V]	0.451	0.766	1.069
	$FF_{AM1.5g}$ [%]	80.2	86.6	89.6
	$\eta_{AM1.5g}$ [%]	19.7	29	30.5

Table 2 shows the characteristics of the three single junction cells under 2 illuminations.

e) Compute the values of J_{sc}, V_{oc}, FF and η in the case of a 1000 suns concentrated illumination for AM1.5g.

Hints: Use the following information for the calculations.

- Assume a temperature of $T = 273\,\mathrm{K}$.
- The dark saturation current density of the one diode model is given by:

$$J_0\ (\mathrm{mA/cm^{-2}}) = 1.5 \cdot 10^8 \exp\frac{-E_\mathrm{g}}{k_\mathrm{B}T} \tag{9}$$

- The fill factor FF dependence on the open-circuit voltage V_oc is given by:

$$FF = \frac{v_\mathrm{oc} - ln(v_\mathrm{oc} + 0.72)}{v_\mathrm{oc} + 1} \text{ with } v_\mathrm{oc} = \frac{V_\mathrm{oc}}{k_\mathrm{B}T/q} \tag{10}$$

  We assume that this approximation is valid for the AM1.5g spectrum as well as for the AM0 spectrum. In the case of the concentrated illumination of 1000 suns, we use the same FF value under the AM1.5g illumination because we assume that the conductivity of TCO is optimized so that no resistive losses occur.

Solution:

I_{sc} is proportional to the incident power. So we can simply multiply the values of I_{sc} under the AM1.5g illumination by a factor 1000. Due to the same dependency of the $P_{AM1.5g}$, the ratio $J_{sc}/P_{AM1.5g}$ does not lead to an increase in efficiency as highlighted in equ. 8 . However, the V_{oc} increases with illumination, as it depends logarithmically on the J_{sc} (equ. 6). As a result, the FF follows the same trend, **as** we assume no resistive losses in our devices. Thus, the overall efficiency increases. J_sc, $V_\mathrm{oc}$and efficiency values are showed in Table 3.
To conclude, higher efficiencies can be achieved with light concentration PV system. For instance, a record solar cell of 42.3 % efficiency fabricated by Spire Corporation was measured under 406 suns by the NREL group.

Table 3: Characteristics of ideal single junctions solar cells under a 1000 suns concentration at AM1.5g

SC bulk cells	**Ge**	**c-Si**	**GaAs**
$Jsc_{AM1.5}(\mathrm{mA\,cm^{-2}})$	54500	43800	31900
$J_0(\mathrm{mA\,cm^{-2}})$	$2.55\ 10^{-7}$	$3.16\ 10^{-13}$	$5.97\ 10^{-19}$
$Voc_{1000sun}(\mathrm{V})$	0.614	0.929	1.231
$FF_{AM1.5g}$ (%)	84.2	88.4	90.7
$\eta_{AM1.5g}$ (%)	29	36	35.6