Solution Series 1

Exercise 1: Irradiation & solar spectrum

a) What is the difference between AM1.5G and AM1.5D?

Solution:

The solar spectrum at the earth's surface contains several components (see Fig. 1):

- the direct radiation coming straight from the sun is defined as AM1.5D
- the diffuse radiation coming from scattering on particles in the atmosphere
- the radiation reflected from the surroundings (the amount of reflected radiation depends on the local reflectivity of the ground surface (albedo)).

The total radiation on the ground is the sum of these three components, is called *global* radiation and defined as $AMmG$ spectra.
The solar spectrum that reaches the ground is also affected by selective absorption in the near infrared by water vapor, carbon dioxide... and by wavelength dependent scattering mechanisms (as Rayleigh scattering).
For a typical cloudless atmosphere in summer and for zero zenith angle, the $1367\,\mathrm{W\,m^{-2}}$ reaching the outer atmosphere are reduced to $1050\,\mathrm{W\,m^{-2}}$ direct beam radiation, and $1120\,\mathrm{W\,m^{-2}}$ global radiation on a horizontal surface at ground level.

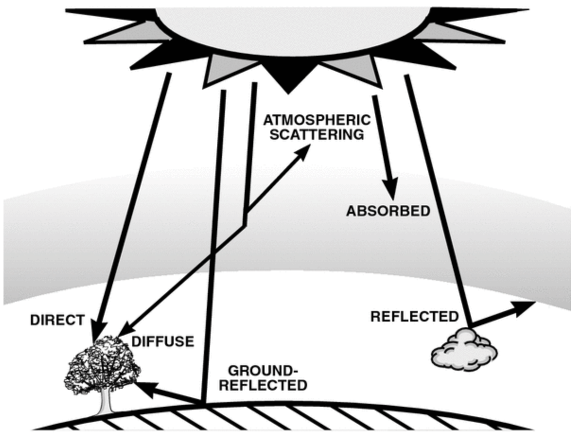


Figure 1: The total global radiation on the ground has direct, scattered and reflective components (taken from http://www.newport.com).

b) What does m indicate in the expression AMm? What is AM0?

Solution:

m is defined as:

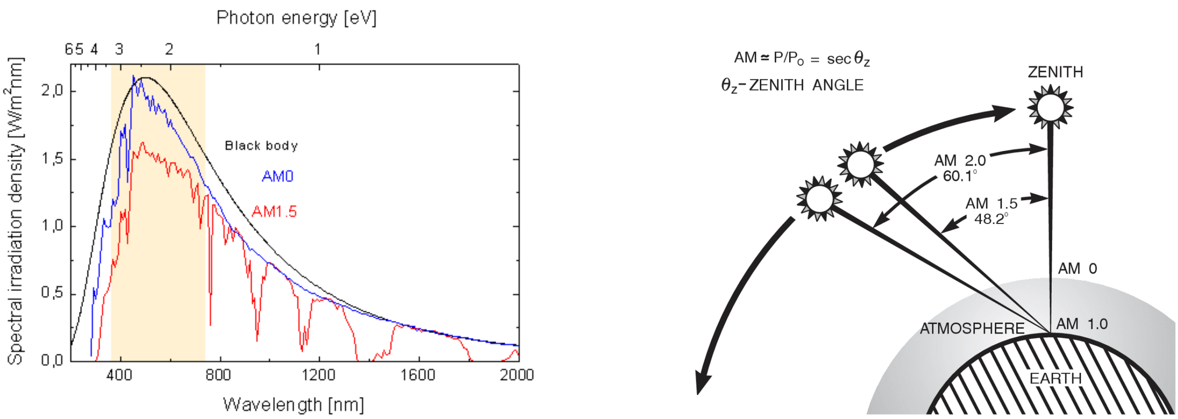
$$m = 1/\cos(\theta) \tag{1}$$

where θ is the angle between the Sun position and the Zenith.
The AM0 spectrum is the spectrum of the solar radiation arriving just outside the Earth's atmosphere.

c) How does AM0 differ from the idealized black body spectrum at 5800 K (Sun's surface temperature)? How does AM1.5G differ from AM0?

Solution:

The differences between a black body radiation spectrum and AM0 is mainly due to absorption and scattering occurring in the cool peripheral solar gas (H, Fe, Ca . . .).
In contrast, the AM1.5G spectrum specifies what actually arrives at the earth's surface, assuming the sun standing an angle of 48.19° with respect to a flat surface on earth at midday. The AM 1.5G spectrum differs from the AM0 spectrum as it has to pass through the atmosphere and suffers from absorptions (water vapor, ozone, dust . . .) and Rayleigh scattering.



(a) (b)

Figure 2: (a) Solar spectrum outside the atmosphere AM0 and on earth's surface AM1.5 compared to the black body radiation spectrum at 5800 K (taken from www.superstrate.net). The shaded area indicates the visible range. (b) Geometry for the definition of m in (AMm). For an incidence angle $\theta \neq 0\,°C$ the optical light path through the atmosphere is longer than for normal incidence (taken from www.newport.com).

d) Calculate the area (A) of the sun's surface that is required to obtain 1 GW (i.e. the power delivered by a large nuclear power plant) of emitted radiation? Use the Stefan-Boltzmann law, $P_{\mathrm{em}} = A\,\sigma\,T_{\mathrm{sun}}^4$. Assume a surface temperature of $T_{\mathrm{sun}} = 5800\,\mathrm{K}$ and $\sigma = 5.67 \cdot 10^{-8}$ $\mathrm{W\,m^{-2}\,K^{-4}}$.

Solution:

The Stefan-Boltzmann law defines the emitted power by the black body with respect to its temperature: $P = 1\,\mathrm{GW} = A \cdot \sigma \cdot T_{\mathrm{sun}}^4$. Considering the sun's temperature to be $T_{\mathrm{sun}} = 5800\,\mathrm{K}$ and with Stefan-Boltzmann constant, we find that the required area is $\sim 15.6\,\mathrm{m}^2$.

e) Suppose that the integrated radiation power density impinging on the earth's surface is $P = 1000\,\mathrm{W\,m^{-2}}$, and that 100 % of this power is absorbed. Calculate the total power absorbed by Earth. Earth's radius $R = 6370\,\mathrm{km}$.

Solution:

The area in question is the area projected on the slice plane through the middle of the earth directly facing the sun (normal vector parallel to the direction of the light rays). The area is $A_2 = \pi R^2 = 1.27 \times 10^{14}\,\mathrm{m^2}$, which implies that the total power absorbed by the earth is $P_{\text{total}} \approx 127\,500\,\mathrm{TW}$.

f) In order to establish thermal equilibrium with the sun, the power absorbed by the earth must be equal to the power emitted by the earth in all spatial directions. Using the total power from the result of part e), calculate the earth's surface temperature using Stefan-Boltzmann's law suitable also for the radiation leaving the earth. Why does this calculation yield a temperature well below the mean earth surface temperature of 288 K?

Solution: In order to calculate the earth's temperature T_{earth}, we use the Stefan-Boltzmann law again. As the earth is in thermal equilibrium with the sun, the absorbed power (calculated in exercise d)) and the emitted power have to be equal. Using the earth's total surface area, we obtain the following: $A_{\text{total}} = 4 \cdot \pi \cdot R^2$.

$$P_{\text{total}} = A \cdot \sigma \cdot T_{\text{earth}}^4 \tag{2}$$

$$T_{\text{earth}} = \sqrt[4]{\frac{P_{\text{total}}}{A_{\text{total}} \cdot \sigma}} = \sqrt[4]{\frac{1000 \cdot \pi \cdot R^2}{4 \cdot \pi \cdot R^2 \cdot \sigma}} \approx 257\,\mathrm{K} \tag{3}$$

The result is lower than the earth's mean temperature of 288 K. This is due to the fact, that we did not take into account processes happening within the atmosphere. The power that is emitted from the earth's surface, will be absorbed and re-emitted from gas particles within the atmosphere, this is known as the *greenhouse effect*. This underlines the importance of the latter for life on earth.

Exercise 2: Temperature effect, STC & NOCT

Consider a PV array operating under standard test condition (STC, i.e. AM1.5G normalized to $1000\,\mathrm{W\,m^{-2}}$, ambient temperature (T_a) 25 °C). In these conditions, a cell without any encapsulation would receive an input power of $1000\,\mathrm{W\,m^{-2}}$ and operate at a temperature of 25 °C. However, when considering a PV array operating in the field, the cells are encapsulated in glass and polymers to protect them from external atmosphere (we call this entire system a "module"). Due to this encapsulation, cells generally operate at higher temperatures than the standard 25 °C, depending on the illumination they receive. To calculate the cells output power in these conditions, it is important to establish a new standard for cell temperature estimation in the module, called NOCT (Nominal Operating Cell Temperature), defined as the temperature reached by open circuited (OC) cells in a module under the conditions: irradiance on cell surface = $800\,\mathrm{W\,m^{-2}}$, air temperature = 20°C, wind velocity = 1 m/s, mounting = open back side. A module's nominal operating cell temperature (NOCT) is around 47 °C.
We consider here an incident power density P_in of $0.9\,\mathrm{kW\,m^{-2}}$.

a) Knowing that there is a 3 % array loss due to mismatched modules, dirt loss of 4 %, module power conversion efficiency (PCE) of 18 %, and the efficiency of the inverter is 95 %, calculate the output power (i.e. after the inverter) for $1\,\mathrm{m^2}$ of modules at 25 °C. This power is the module output power with "AC" losses only, without taking into account the higher cell temperature, that we will call P_AC.

Solution:

The power produced at 25 °C (i.e. under STC) for $1\,\mathrm{m^2}$ with all the "AC" losses (mismatch, dirt, and inverter) can be calculated to:

$$P_\mathrm{AC} = \underbrace{0.9}_{P_\mathrm{in}}\,\mathrm{kW}\cdot\underbrace{0.18}_{PCE}\cdot\underbrace{0.97}_{-3\,\%}\cdot\underbrace{0.96}_{-4\,\%}\cdot\underbrace{0.95}_{-5\,\%} = 0.143\,\mathrm{kW} \tag{4}$$

b) Using the graphs plotted Fig. 3 which follow the Ross law: $T_\mathrm{cell} - T_\mathrm{air} = \mathrm{m}\cdot P_\mathrm{in}$ (with $T_\mathrm{air} = T_\mathrm{a}$ here), estimate the cell temperature in the module, with $T_\mathrm{a} = 22\,°\mathrm{C}$.

Solution:

However, the module is not operating at STC but at $T_\mathrm{a} = 22\,°\mathrm{C}$. The module operating temperature (which is not the same than NOTC) can be obtained using the graph given (Fig. 3), which shows the temperature difference between air and module as a function of the illumination. An approximate expression for calculating the cell temperature is given by (Ross, see footnote):

$$\Delta T = T_\mathrm{cell} - T_\mathrm{air} = \mathrm{m}\cdot P_\mathrm{in} \tag{5}$$

In order to determine the slope, we use two points in Fig. 3, on the black graph since it represents typical values. One point is for instance given for $P_\mathrm{in} = 0\,\mathrm{W\,m^{-2}}$, where $\Delta T_\mathrm{Pin=0} = 0\,°\mathrm{C}$ since we have a linear relation, (and $T_\mathrm{cell} = T_\mathrm{air}$ at that point). A possible second point is at $P_\mathrm{NOCT} = 80\,\mathrm{mW\,cm^{-2}}$, where $T_\mathrm{air} = 20\,°\mathrm{C}$. With the cell temperature at $T_\mathrm{cell} = 47\,°\mathrm{C}$, we get $\Delta T_\mathrm{NOCT} \approx 27\,°\mathrm{C}$. Thus for the slope we obtain:

$$m = \Delta T_\mathrm{NOCT}/80 = 0.3375 \quad \text{(mind the units)} \tag{6}$$

Considering that the cooling of the array only happens by conduction through the encapsulation, the temperature of the cell is estimated by a linear law:

$$T_{\text{cell}} = T_{\text{air}} + m \cdot P_{\text{in}} = 52.38\,^{\circ}\text{C} \tag{7}$$

c) Knowing that the output power at the maximum power point (MPP) drops by 0.5 %/°C for cells temperatures above the STC temperature, estimate the real outpout power P_{out} of the module for $T_{\text{a}} = 22\,^{\circ}\text{C}$. What happens if T_{a} 40 °C?

Solution:

With a power loss of 0.5 % per degree above 25 °C, the output power at an ambient temperature of $T_{\text{air}} = 22\,^{\circ}\text{C}$ and the $T_{\text{cell}} = 52.38\,^{\circ}\text{C}$ can be calculated to:

$$P_{\text{out}} = 0.143\,\text{kW} \cdot \underbrace{[1 - 0.005(52.38 - 25)]}_{\text{loss to the DC power}} = 0.123\,\text{kW} \tag{8}$$

Increasing the ambient temperature to $T_{\text{air}} = 40\,^{\circ}\text{C}$ results in a different cell temperature, namely $T_{\text{cell}} = 70.38\,^{\circ}\text{C}$. Considering the temperature coefficient of 0.5 % per degree above 25 °C, the resulting AC power is:

$$P_{\text{out}} = 0.143\,\text{kW} \cdot [1 - 0.005(70.38 - 25)] = 0.111\,\text{kW} \tag{9}$$

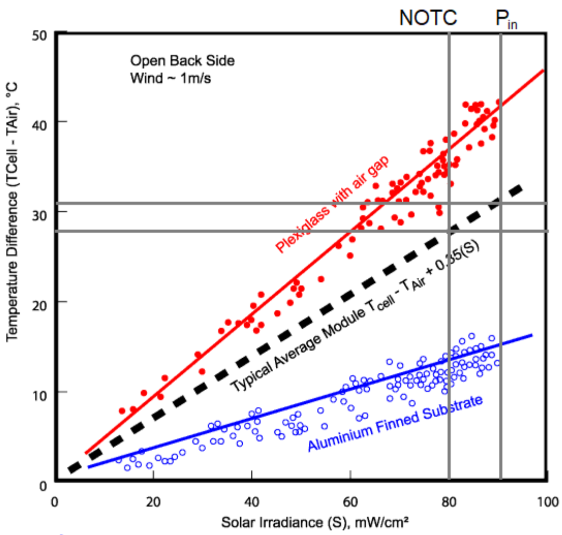


Figure 3: Temperature increases, above ambient levels, with increasing solar irradiance for different module types. [2]

[0]Ross, R.G. Jnr. and Smokler, M.I. (1986), Flat-Plate Solar Array Project Final Report, Volume VI: Engineering Sciences and Reliability, Jet Propulsion Laboratory Publication 86-31.

Exercise 3: Key figure in Switzerland & the world

Assumptions: In Switzerland a solar panel of 1 W_p produces every year 1 kW h (i.e. 2 W_p produce 2 kW h). In North Africa a panel of 1 W_p produces 2.4 kW h every year (4.8 kW h for a 2 W_p panel).

- The world's annual energy consumption is 1.4×10^5 TW h
- Switzerland's annual energy consumption is 250 TW h
- The world's annual electricity consumption is 1.6×10^4 TW h
- Switzerland's annual electricity consumption is 58.8 TW h

Solution:

Before elucidating the solution we would like to draw your attention to the fact that the sources of energy are **NOT** equivalent. Indeed when gasoline is used to fuel the engine of a car, around 75 % of the stored energy is lost due to waste heat and other losses. In contrast to gasoline, electricity has a much higher conversion efficiency depending on how you transform it in energy. Assuming an efficiency of 93 % for charging the batteries of an electrical engine and assuming 93 % as well for the conversion efficiency from electricity to kinetic energy (movement of the car), the total efficiency of the engine is 86 %.

Nowadays 80 % to 90 % of the energy used in the world, is consumed on petrol derivatives using engines with a low conversion efficiency. As a matter of fact, if the final energy consumed was produced by electricity, the primary total energy consumption would be far less.
As you can learn from Fig. 4, the consumption of energy in Switzerland depends on the energy sources. The upper pie chart shows the gross consumption before the conversion into the final form of energy, which is actually consumed by the final consumer. However these graphs are misleading as they underestimate the real part of electricity. For the electricity it takes into account the kW h that are consumed (useful energy) but not the energy that is needed to supply this amount of energy (kW h) to the final consumer. Transferring this thought to the consumption of gasoline, the graph should take into account the energy that is actually transmitted to the wheels (useful energy) and not the litres of gasoline that is used to fuel the engines (primary energy). For gasoline driven engines the useful energy is much smaller than the primary energy. The best efficiency conversion is obtained from electricity.

a) The Sahara's surface is approximately 9 000 000 km^2. How much of this surface would have to be covered by solar arrays, that measure 1 m^2 and produce *(1)* 100 W_p (10 % efficiency) or *(2)* 200 W_p (20 % efficiency), to cover the world need in electricity, in energy?

Solution: With the statement, stating that in North Africa a panel of 1 W_p produces 2.4 kW h every year, we can deduce that there are 2400 h of sunshine during 1 year in North Africa (2400 W h/ 1 W_p = 2400 h). Then, a 100 (200)W_p module of 1 m^2 installed in the Sahara, produces 240 (480)kW h m^{-2} yr^{-1}. Thus, to cover the world's annual energy needs:

$$\frac{1.4 \cdot 10^5 \text{ TWh}}{240\ (480)\ \text{kWh}/(\text{m}^2\ \text{year})} = 5.8 \cdot 10^5\ \text{km}^2\ (2.9 \cdot 10^5\ \text{km}^2) \qquad (10)$$

This corresponds to 6 % (3 %) of the Sahara's total surface area. To cover the world's electricity consumption you need:

$$\frac{1.6 \cdot 10^4\,\text{TWh}}{240\ (480)\,\text{kWh}/(\text{m}^2\,\text{year})} = 6.7 \cdot 10^4\,\text{km}^2\ (3.3 \cdot 10^4 \text{km}^2) \tag{11}$$

Which corresponds to 0.74 % (0.37 %) of the surface area.

b) Calculate the total energy impinging on the earth's surface from the sun each day. Assume a power density of $1000\,\text{W}\,\text{m}^{-2}$ and 24 h of illumination. Compare this figure with the world's annual energy and electricity needs.

Solution:

The total energy impinging on the earth each day is $\pi \cdot R^2 \cdot 1000 \cdot 24 = 3 \times 10^6\,\text{TW h}$. This figure is by far more than the annual world electricity consumption and far more than the annual world energy consumption.

c) How many m^2 of solar arrays with modules of $1\,\text{m}^2$ and *(1)* $100\,\text{W}_\text{p}$ (10 % efficiency) or *(2)* $200\,\text{W}_\text{p}$ 20 % efficiency) each do we need in Switzerland to replace the Gösgen nuclear power plant that has an annual power output of 8000 GW h?

Solution:

Assuming that $1\,\text{W}_\text{p}$ installed in Switzerland produces 1 kW h per year, these modules of $1\,\text{m}^2$ each produce $100\,\text{kW}\,\text{h}\,\text{m}^{-2}$ ($200\,\text{kW}\,\text{h}\,\text{m}^{-2}$). Thus:

$$8000\,\text{GWh}/100\,\text{kWh}/\text{m}^2 (8000\,\text{GWh}/200\,\text{kWh}/\text{m}^2) = 80\,\text{km}^2 (40\,\text{km}^2) \tag{12}$$

are needed to be able to compensate the output power of the Gösgen nuclear power plant.

d) Hydro power plants cover 55 % of the annual electricity needs and 11 % of the annual energy needs in Switzerland. If you subtract this amount from the total annual consumption (electricity and energy, given above), how many % of Switzerland's surface area would you have to be covered with solar panels, measuring $1\,\text{m}^2$ each and an output power of *(1)* $100\,\text{W}_\text{p}$ (10 % efficiency) or $200\,\text{W}_\text{p}$ 20 % efficiency), in order to cover the remaining amount of electricity and energy (Switzerland's surface area: $41\,285\,\text{km}^2$)? Do you think there are enough spots in Switzerland where these panels could be installed?

Solution:

After substracting the hydropower production from total, the following values remain:

- 89 % of the annual energy consumption: $2.22 \times 10^2\,\text{TW h}$
- 45 % of the annual electricity consumption: 26.4 TW h

Each year modules of 100 W_p (200 W_p) produce 100 kW h m^{-2} (200 kW h m^{-2}). The number of m^2 needed to cover the (remaining) energy needs in Switzerland are then given by:

$$\frac{2.22 \cdot 10^2 \,\text{TWh}}{100\,\text{kWh/m}^2 \ (200\ \text{kWh/m}^2)} \approx 2.2 \cdot 10^3\,\text{km}^2 \ (1.1 \cdot 10^3\,\text{km}^2) \tag{13}$$

This amounts to 5.3 % (2.7 %) of Switzerland's surface area. In order to cover the electricity needs the result is the following:

$$\frac{26.4\,\text{TWh}}{100\,\text{kWh/m}^2 \ (200\,\text{kWh/m}^2)} = 264\,\text{km}^2 \ (132\,\text{km}^2) \tag{14}$$

so 0.6 % (0.3 %) of the surface area. In Switzerland approximately 4000 km^2 (9 %) of the territory is covered with buildings. Out of these 4000 km^2, 3 % (130 km^2) are well oriented roofs where photovoltaic arrays could be installed. So by installing solar panels (20 % efficiency) on the roofs of homes (and industry buildings) you could almost cover Switzerland's remaining electricity needs (after subtracting the hydro contribution). Other options can be seen in Germany, where photovoltaic arrays are installed along the highways. They can even be used as sound barriers (see the Fig. 5).

In Switzerland there are 1500 km of highways and 18 300 km of national roads. Covering one side of one third of these roads with a 10 m wide band of photovoltaic arrays would result in an additional 66 km^2. So to sum up, installing photovoltaic arrays of 20 % conversion efficiency on the well oriented roofs as well as alongside the roads (as discussed before) and using the already installed hydraulic plants Switzerland's entire electricity demand could be covered.

e) Assuming you mount a 1 m^2 module of 200 W_p (a temperature coefficient of 0 %/°C) on a tracker, and install it at the equator in a sunny plain (no shadows). How many kW h annual electricity per m^2 can you expect? *(1)* If the tracker perfectly follows the sun? *(2)* This corresponds to how many litres of oil (diesel generator has an efficiency of 25 % and 1 L = 10 kW h thermal)? *(3)* If the module remains still?

Solution:

(1) Using a tracker won't increase the number of sunlight hours per day but will ensure constant maximum irradiance (1000 W m^{-2}) to reach the solar panel during daylight hours. At the equator, there are more daylight hours per year compared to Switzerland, where you have 1000 h yr^{-1}. So assuming 12 h of sun par day at 1000 W m^{-2} and neglecting the air mass variation, you obtain:

$$E = 12\,\text{h}\cdot 365\,\text{days}\cdot 200\,\text{Wp} = 876\,\text{kWh/(m}^2\,\text{year)!}$$

(2) 876 kW h corresponds to 87.6 L of oil. Unfortunately to obtain 87.6 L of "useful" energy with a standard generator we need 4 times more oil (25 % of efficiency) so 350.4 L.
Actually Qatar is using a lot of diesel generators as oil is cheap there. According to a recent report, they would gain money by selling their oil instead of using it for personal electricity

production and using that money to buy solar panels (sunny country) to produce electricity.

(3) If the tracker remains still the area of the module that is perpendicular to the sun decreases by a factor (see Figure 6):

$$\frac{\int_0^\pi \sin\theta \mathrm{d}\theta}{\int_0^\pi \sin(90) \mathrm{d}\theta} = \frac{2}{\pi} = 0.64 \tag{15}$$

Thus, as illustrated in Fig.6, without a tracker you loose 36 % of your electricity production and obtain only

$$\mathrm{E} = 560.6\,\mathrm{kWh/(m^2*year)}$$

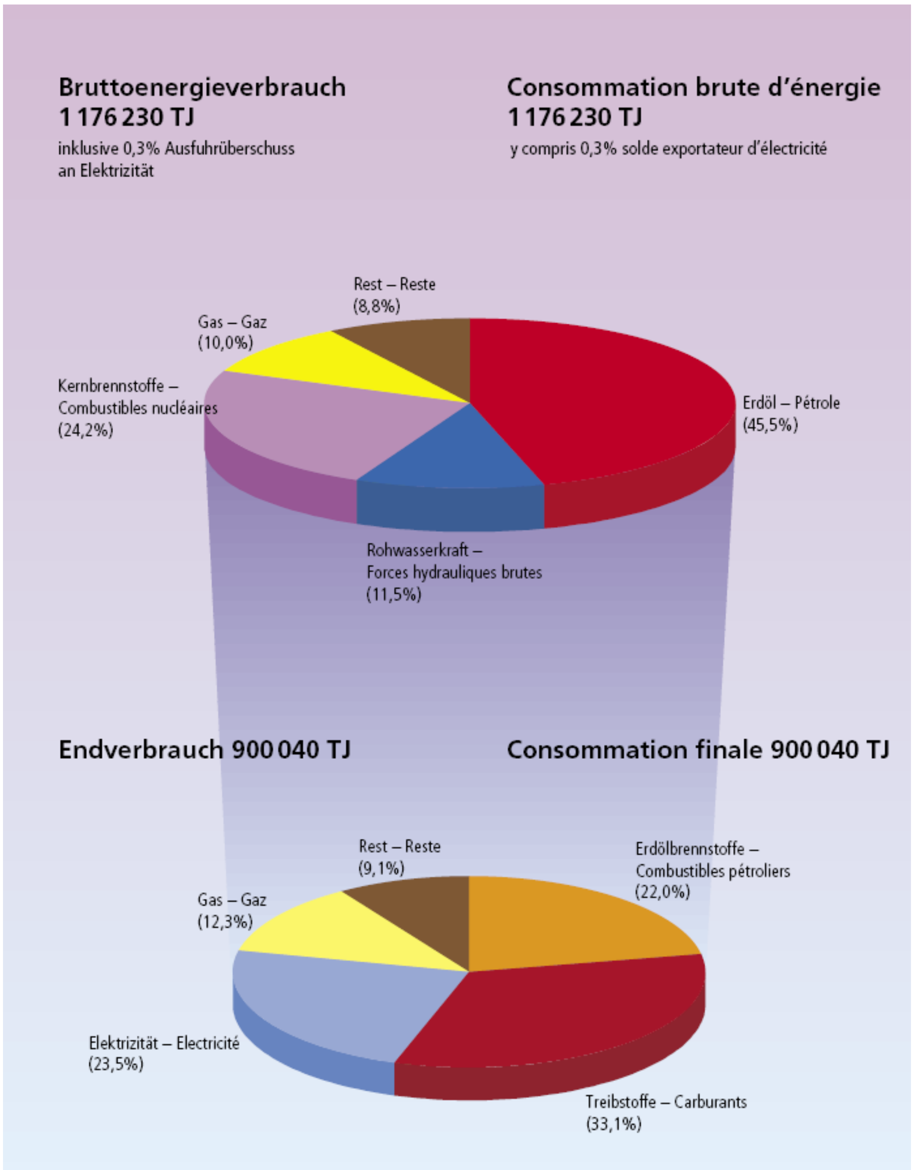


Figure 4: Comparison of the gross energy consumption and the final consumption in Switzerland.

Figure 5: Solar panels alongside the highway as sound barrier.

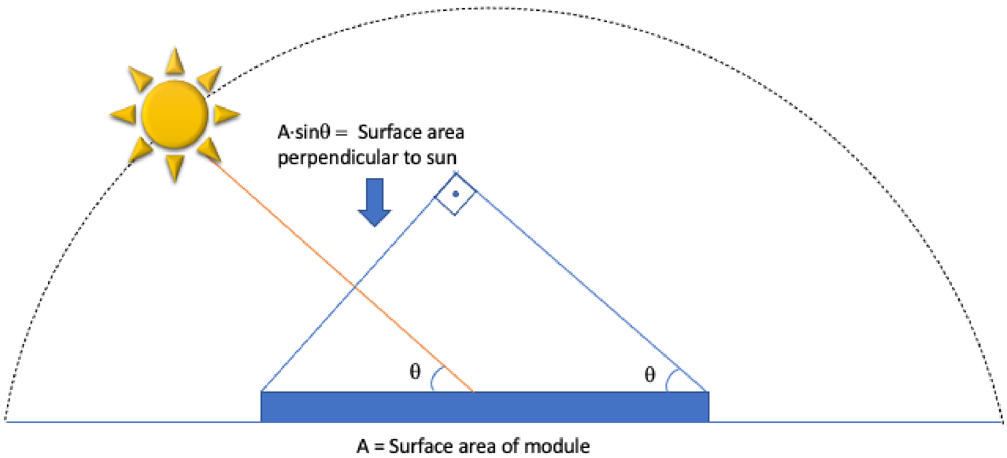


Figure 6: Illustration of Exercise 3 e) part 3.