# Solution Series 8

**Exercise 1: P-N junction**

a) Consider a p-n-junction with equal doping concentrations in both the p-doped and n-doped part and no bias voltage applied. Sketch the *charge densities*, the *electrical field* and *electrical potential* within the space charge region and explain how they are related to one another. Sketch also the *band diagram* for the same region.

## *Solution:*

The relation between the carrier concentration, the electrical field and the electrostatic potential is determined by the *Poisson's equation*[1]. Under the depletion approximation, it reads:

$$-\frac{\partial^2 \varphi}{\partial x^2} = \frac{\partial E}{\partial x} = \frac{\rho}{\epsilon} = \frac{q}{\epsilon}\left(p_0 - n_0 + N_\mathrm{D} - N_\mathrm{A}\right) \tag{1}$$

Starting from the charge density $\rho$, the electric field is obtained by integrating the charge density once and twice for the electrostatic potential. The relation is shown in Fig. 1 [2].

Note that the model used to determine the graphs in Fig. 1 is available on the Moodle website and was made with PC1D software (we will have workshop session to learn how to use it). We encourage you to play with the parameters, e.g. doping concentration, mobilities etc. to get a feeling for the p-n junction. Furthermore you can have a look on `http://pveducation.org/pvcdrom/bias-of-pn-junctions` website to get more information on the p-n junction.

b) Consider now a junction with a bias voltage applied.

(a) Sketch the carrier concentration for electrons and holes that will arise throughout the junction for forward and reverse bias (direct or reverse polarization).

(b) What will happen to the electric field across the junction, the width of the depletion region and the energy bands (CB and VB) for each case?

## *Solution:*

(a) The sketch of the p-n junction in forward and reverse bias is given in Fig. 2.

(b) For the other parameters:

**Forward bias:** The applied voltage will counteract the built-in electrical field, i.e. the bands on both sides will bend less. According to the square-root relation between the width of the depletion region and the amount of band bending, the depletion region will shrink.

[1] The variables refer to $\varphi$: electrostatic potential, $E$: electrical field, $\rho$: total charge density, $q$: elementary charge, $\epsilon$: permittivity, $n_0$ and $p_0$: electron and hole density in equilibrium, and $N_\mathrm{A}$ and $N_\mathrm{D}$: acceptor and donor density which is equal to the fixed charges in the space charge region.

[2] More detailed information can be found e.g. in *SEMICONDUCTOR DEVICES - Physics and Technology* by S. M. Sze (p. 88 and following)

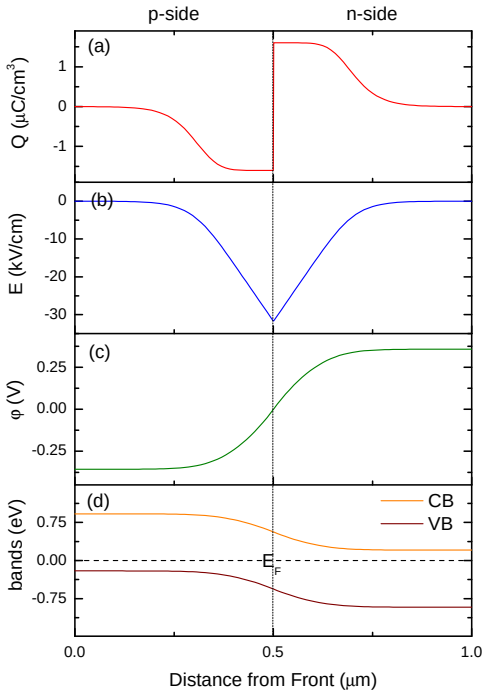


Figure 1: Example of a p-n junction in c-Si for a doping concentration of $1 \times 10^{16}\,\mathrm{cm}^{-3}$ and 0.5 µm thickness for both sides. (a) Shows the charge concentration, (b) the electrical field distribution, (c) the electrostatic potential and (d) the bands.

**Reverse bias:** In this case the bias supports the built-in electrical field. Consequently, the bands will bend more and the width of the depletion will increase.

c) Explain in your own words how the current flows across the junction in equilibrium, forward bias, and reverse bias. *Hint:* Consider drift and diffusion currents.

## *Solution:*

**No bias:** In equilibrium the *drift current* generated by the built-in electrical field (created by the uncompensated dopants near the junction) and the *diffusion current* generated by the gradient of carriers concentration will cancel each other out and the total current in the junction is zero.

$$J^{n,p}_{\mathrm{drift}} = J^{n,p}_{\mathrm{diff}} \tag{2}$$

**Forward:** We assume that the transport of majority carriers takes place with a negligible voltage drop. The external forward bias is thus applied directly to edges of the space charge region where it will reduce the built-in field. Due to the applied forward bias, majority carriers get injected into the space charge region. As they cross the space charge region, they become minority carriers when they cross the junction at $x = 0$. Eventually they reach far side of the space charge region, resulting in a concentration of minority carriers

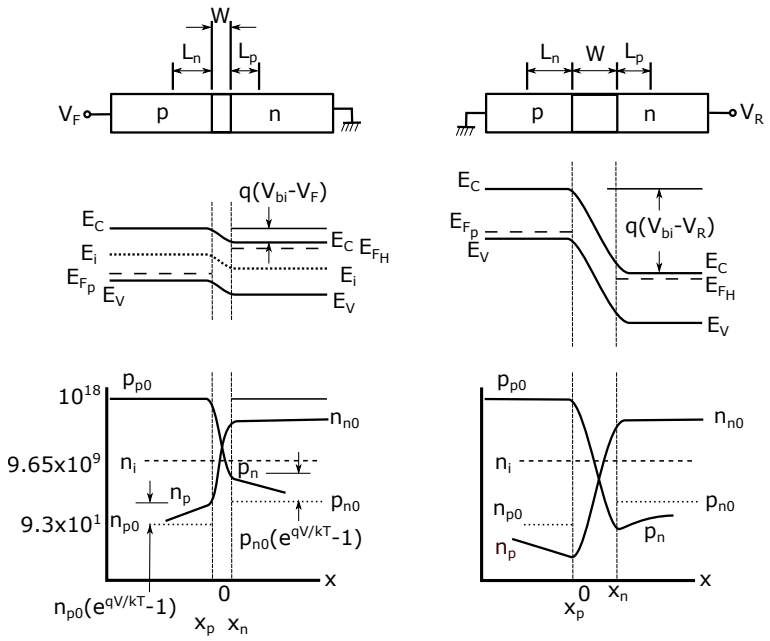


Figure 2: P-N junction in (a) forward bias (b) and reverse bias. Taken from *SEMICONDUCTOR DEVICES - Physics and Technology* by S. M. Sze

that is higher than the equilibrium value. Consequently, they are injected as minority carriers into the adjacent neutral region and they can propagate about one diffusion length until they recombine.

**Reverse:** Once again we assume that there is negligible voltage drop across the neutral regions, thus the reverse bias enhances the built-in field of the junction. In the presence of the enhanced field, minority carries at the edge of neutral region are attracted to the space charge region and swept across. Different from the case of forward bias where we can inject almost any amount of minority carriers, here we can only extract those minority carriers that are generated by thermal excitation. Moreover, we cannot collect all minority carriers, but only those that are generated within a diffusion length away from the edge of the space charge region. Increasing the reverse bias does not change the generation volume, therefore the reverse current saturates quickly.

**Conclusion:** In both cases, the properties of minority carriers control the current flow through the junction.

### Exercise 2: Lifetime, surface recombination velocity and saturation current

In this exercise we will consider a solar cell with an infinite base and determine the dominating contribution to the saturation current density $J_0$. It is the sum of two parts: the base $J_{0B}$ and the emitter $J_{0E}$ saturation current density. $J_0$ is detrimental to the cell performance as it influences the $V_{oc}$ and thus the overall cell efficiency.

We consider a solar cell based on p-type crystalline silicon wafer with an infinite base, i.e the thickness of the latter is much higher than the diffusion length of the electrons. The emitter is

1 $\mu$m thick ($x_j$=1 $\mu$m) and homogeneously $n^{++}$ doped ($N_S = 2 \cdot 10^{19}\,cm^{-3}$). After passivation of the emitter, the surface recombination velocity at the front surface $S_p$ is $10^4\,cm\,s^{-1}$. The doping concentration in the base is $N_A = 1 \cdot 10^{16}\,cm^{-3}$. Fig. 3 shows the relationship between $J_{0E}$, the surface recombination velocity $S_p$ and the surface concentration $N_S$.

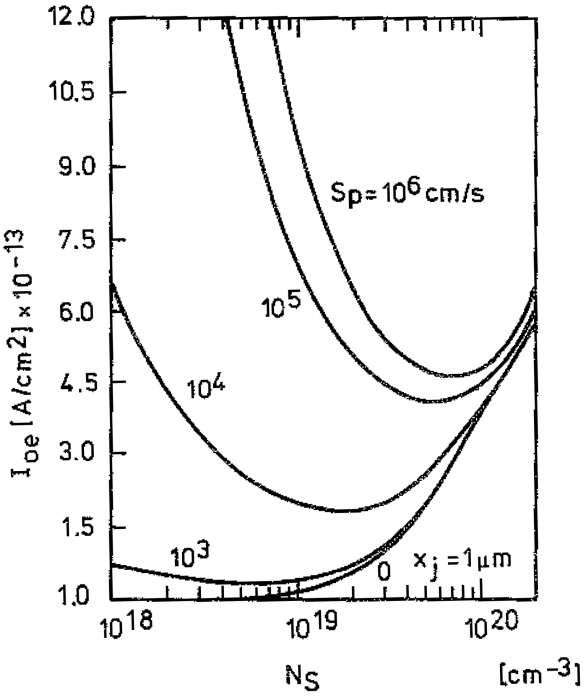


Figure 3: Saturation current of an emitter as a function of surface concentration, with $S_p$ being parameter. Taken from *Crystalline silicon solar cells* by A. Goetzberger, J. Knobloch, and B. Voss.

a) Is the saturation current density in the base $J_{0B}$ influenced by the surface recombination velocity in this specific case? Considering Fig. 3 and assuming a lifetime in the base of $\tau_n = 4\,ms$, which saturation current governs the solar cell efficiency? Is it the same if $\tau_n$ is equal to 10 µs?

## *Solution:*

The carrier recombination in the bulk and at the back surface contribute to the base saturation current density $J_{0B}$. However, as the electrons' diffusion length is lower than the base thickness, the recombination in the base does only depend on the bulk recombination and not on the surface recombination velocity as the electrons can not reach the rear surface. Thus, as a consequence $J_{0B}$ does not depend on the surface recombination velocity.

In order to determine the dominating contribution to the saturation current we will calculate the value for $J_{0B}$ and compare it to the value of $J_{0E}$ given in Fig. 3. From the graph it is obvious that the surface recombination velocity or passivation quality plays a more important role the lower the emitter doping concentration $N_S$ is. In order to reduce $J_{0E}$ in a lowly doped emitter (e.g. $10^{18}\,cm^{-3}$) the passivation has to be very good. In contrast to this, for a highly doped emitter (e.g. $10^{20}\,cm^{-3}$) even a perfect passivation won't change the $J_{0E}$ by much.

Thus it is possible to in a way neglect the passivation, use strong doping in the emitter and still obtain decent $J_{0E}$ values. This is the concept of the PERL cell, where the doping beneath the contacts is reinforced.

For this specific case the value is $J_{0E} \approx 2 \times 10^{-13}\,\mathrm{A\,cm^{-2}}$ (Fig. 3). The value for $J_{0B}$ can be calculated using the following formula [3]:

$$J_{0B} = \frac{q \cdot n_i^2 D_n}{N_A \cdot L_n} G_F \tag{3}$$

For a base doping of $N_A = 10^{16}\,\mathrm{cm^{-3}}$ and $\tau_n = 4\,\mathrm{ms}$ the diffusion coefficient for electron is around $D_n = 28\,\mathrm{cm^2\,s^{-1}}$ (Einstein's formula[4]) and the diffusion length $L_n \approx 0.33\,\mathrm{cm}$. Then we obtain:

$$J_{0B} \approx 1.35 \times 10^{-13}\,\mathrm{A\,cm^{-2}} \tag{4}$$

So for this case $J_{0E}$ will dominate the total saturation current and thus the cell's efficiency. Decreasing the lifetime $\tau_n = 10\,\mu\mathrm{s}$, the contribution $J_{0B\text{-bulk}}$ increases and $J_{0B} \approx 2.68 \times 10^{-12}\,\mathrm{A\,cm^{-2}}$ which is 10 times higher than $J_{0E}$. Therefore recombination in the base will determine the total saturation current. Note that the $V_{oc}$ and thus solar cell's efficiency are governed by the largest saturation current.

b) With the results of part a) in mind explain why the temperature coefficient for a-Si (0.2 %/°C) is lower than for c-Si (0.5 %/°C).

## *Solution:*

As we have seen in part a) $J_{0B}$ is proportional to $n_i^2$. Considering that $n_i^2 \propto \exp -E_g/kT$ it is obvious that the temperature dependence is less pronounced for higher band gap materials. This is why a-Si with a band gap of 1.7 eV has a lower temperature coefficient with respect to c-Si which has a band gap of 1.12 eV.

[3] The variables refer to $q$: elementary charge, $n_i^2$: intrinsic carrier concentration, $D_n$: diffusion coefficient, $N_A$: base doping level, $L_n$: diffusion length of electrons in the base, and $G_F$ is the geometric factor which is 1 for an infinite base.

[4] The value of the carrier mobility for a given doping concentration can be found using for example the following online simulator, which basically applies formulas similar to those given in slide 28 of the lecture of week 5: http://www.pveducation.org/pvcdrom/materials/general-properties-of-silicon