SOLUTION SERIES 7

Exercise 1: Generalities on recombination mechanisms

a) Name the three main bulk recombination mechanism ocurring in a semiconductor and give a brief description. Illustrate each of these mechanisms with a band diagram.

1. Radiative (or band to band) recombination
2. Auger recombination
3. Shockley-Read-Hall recombination (SRH), also called trap-assisted recombination

Solution:

Description:

1. The radiative recombination is the inverse mechanism of the generation (or photon absorption) mechanism. In this case, an electron in the conduction band looses its energy and recombines with a hole in the valence band. During the process, the excess of energy is released in the form of a photon with energy approximately equal to the bandgap energy. It is called band to band recombination because the process involves only carriers present in the conduction (e.g. electrons) and in the valance band (e.g. holes).
2. The SRH recombination involves instead 3 energy bands: the valence band, the conduction band and an energy level present in the bandgap. These energy states can be present near the band edges (shallow levels, called traps) or near the middle of the gap (deep levels, acting like recombination centres). They originate from imperfections and impurities present in the semiconductor lattice. In this scenario, recombination can occur in two different ways:
   - Two steps electron relaxation: CB $->$ energy state $->$ VB, or CB $->$ energy state $->$ returns to CB (in the las case, it is not a problem since they do not recombine)
   - Recombination in the energy state if it catches both one electron and one hole. More probable if the energy states is near middle bandgap.
3. The Auger recombination involves at least 3 particles: 2 electrons and 1 hole. The 2 electrons in the conduction band collide and exchange momentum. One electron gains energy (then lost by thermalization), the second electron loses energy and falls down in the valence band. Here, it recombines with a hole.

See Fig. 1 for the band diagrams representation.

b) Explain how carrier concentration affects the recombinations rates.

Reminder:

- carrier concentration $n = n_0 + \Delta n$ (same with p) is the sum of carrier concenration in the equilibrium state n_0 (generally the doping concentration at 25°C) and carrier injection Δn (out of equilibrium concentration, coming from generation, carrier diffusion, etc.). When we study recombinations, we study the **minority** carrier concentration, n or p.

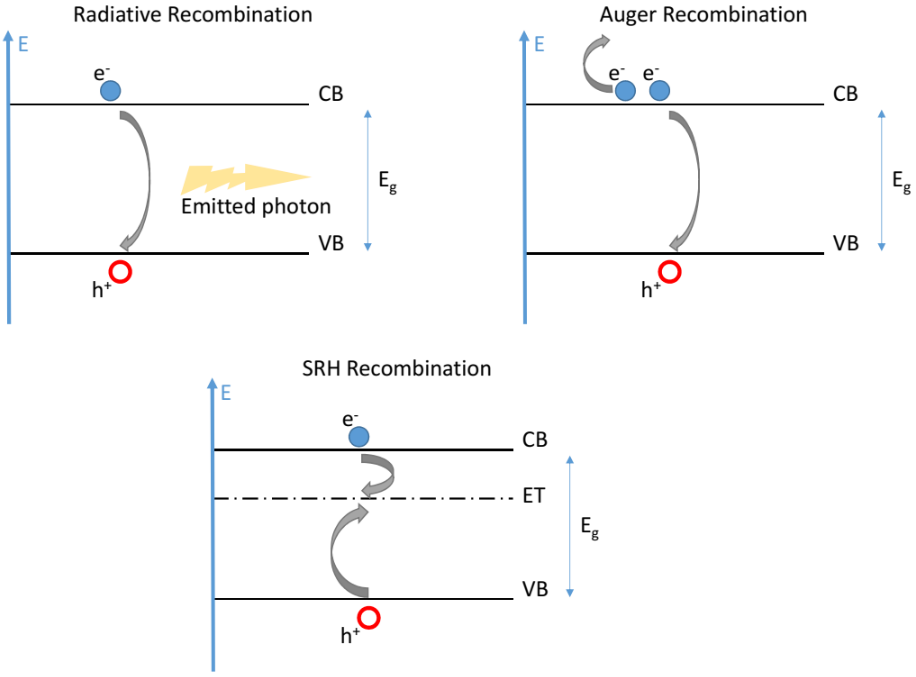

Figure 1: Band diagrams representing the three main bulk recombination mechanisms occuring in a semiconductor

- for ”low injection” condition, we have: $n_0 \gg \Delta n$, thus $n \approx n_0$. For ”high injection conditions, we have $n_0 \ll \Delta n$, thus $n \approx \Delta n$.

Solution:

- Radiative recombination rate is proportional to n (or p).
- Auger recombination rate is proportional to the square concentration n^2 (or p^2).
- At very low concentrations (generally around $10^{12}\,\mathrm{cm}^{-3}$), and high concentrations (above $10^{17}\,\mathrm{cm}^{-3}$), SRH recombination rate does not depend on carrier concentration.

c) What is the predominant recombination mechanism occuring in low doped (or low injection) Si? What happens if the carrier concentration increases?

Solution:

Fig. 2 schemes typical log-log curves of the carriers lifetime as a function of carrier concentration in Si. It is extracted from simulation of a Si wafer in which Fe impurities were added intentionnaly (see the lecture slides). Still, the curves trend here is similar to more general cases.

The predominant recombination mechanism is the one which limits minority carrier (n or p) lifetime the most.

For low carrier concentration in Si (= low doping, low injection):

- Radiative recombination is not dependant on carrier concentration. Since Si is an indirect semiconductors, this recombination mechanism is generally not the predominant

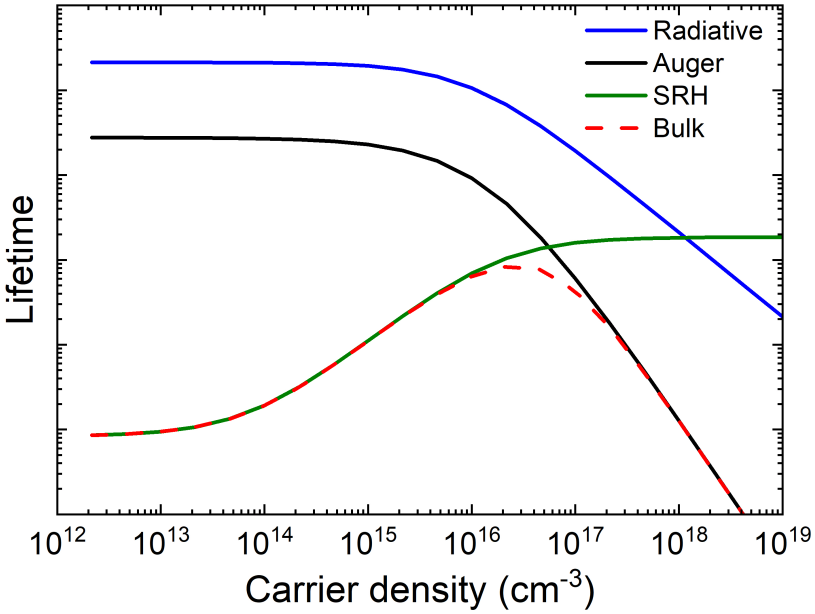

Figure 2: Carrier lifetime as a function of carrier concentration in a typical Si wafer.

recombination mechanism (indirect band-gap: weak probability of radiative recombination since it implies the interaction with a phonon).

- Auger recombination is not dependant on carrier concentration neither. Since this mechanism requires interaction with three particles, it is also generally not the predominant recombination mechanism at low carrier concentration, but in silicon it is usually more likely than radiative recombination.
- SRH recombination comes from defect states induced in the band gaps due to impurities of the Si material. It is also not dependant on carrier concentration at very low injection. Even if Si wafer are highly purified, it remains often the most probable recombination mechanism at low carrier concentration (in the graph shown here, you can note that SRH lifetime is very low compared to the other two mechanisms, due to Fe impurities intentionally added in Si here. In a more general case, this value at low injection should be higher, but the curve trend remains the same).

For high carrier concentration in Si (= high doping or high injection):

- Radiative recombination lifetime begins to decrease around $10^{16}\,\mathrm{cm}^{-3}$ and we know that it depends on $1/\Delta n$ (or $1/\Delta p$) .
- Auger recombination lifetime begins to decrease around $10^{16}\,\mathrm{cm}^{-3}$ too, and we know that it depends on $1/(\Delta n)^2$ (or $1/(\Delta p)^2$), which is a stronger carrier concentraton dependance compared to radiative recombination. Above $10^{17}\,\mathrm{cm}^{-3}$, it is generally the predominant recombination mechanism over all the other bulk recombination mechanisms.
- SRH recombination lifetime is also constant at high injection, but with an other value (see the lcture slides) than the low injection one.

The resulting lifetime curve for carriers in Si (taking into account all the bulk recombination mechanisms) is the "Bulk" curve on the graph, in red. The inverse bulk lifetime is calculated by adding inverse lifetimes of each mechanism (see the lecture). Then the bulk lifetime is simply obtained taking the inverse of the inverse bulk lifetime.

d) Why there is recombination at the surface of a semicondctor? How do we call this type of recombination? Does it have the same dependance on carrier concentration as a bulk recombination?

Solution:

There is recombination at the surface of a semiconductor because dangling bonds (see Fig. 3) induce recombination centers (like for SRH recombination, they induce energy levels in the band gap which can trap holes and electrons).

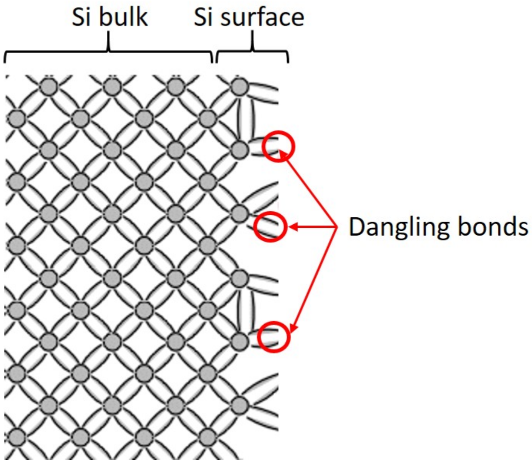

Figure 3: Carrier lifetime as a function of carrier concentration in a typical Si wafer.

We call this recombination "surface recombination", and it does not depend on the bulk carrier density Δn but on the surface carrier density Δn_s, and the "surface recombination velocity" (SRV) S. It can be limited by passivating the semiconductor surface or interface.

Exercise 2: Shockley-Read-Hall recombination

As has been discussed in the lecture, the Shockley-Read-Hall recombination model describes recombination in the bulk as a two-step process through a defect level situated within the bandgap at energy E_T. The recombination rate U_{SRH} is given by the formula below:

$$U_{\mathrm{SRH}} = \frac{v_{\mathrm{th}} N_{\mathrm{t}} (n \cdot p - n_{\mathrm{i}}^2)}{\frac{1}{\sigma_{\mathrm{p}}}(n + n_1) + \frac{1}{\sigma_{\mathrm{n}}}(p + p_1)} \tag{1}$$

with v_{th} [$\mathrm{cm\,s^{-1}}$] being the thermal velocity, N_{t} [$\mathrm{cm^{-3}}$] the trap or defect level density, σ_{n} and σ_{p} [$\mathrm{cm^{-2}}$] being the electron and hole capture cross-section respectively and n_1 and p_1 are the carrier concentration when $E_F = E_T$.

a) Show that at low minority carrier injection the Shockley-Read-Hall recombination rate U_{SRH} in an n-type silicon wafer is given by:

$$U_{\mathrm{SRH}} = \frac{\Delta p\, v_{\mathrm{th}}\, N_{\mathrm{t}}\, \sigma}{1 + \frac{2n_{\mathrm{i}}}{N_{\mathrm{D}}} \cosh(\frac{E_{\mathrm{T}} - E_{\mathrm{F}}^{\mathrm{i}}}{kT})} \tag{2}$$

Use the following hints:

- *(i) Assume equal capture cross-sections for electrons and holes:* $\sigma_{\mathrm{n}} = \sigma_{\mathrm{p}} = \sigma$
- *(ii) For low injection the following relation holds:* $p_0 \ll \Delta n = \Delta p \ll n_0$
- *(iii) And* $n \approx N_{\mathrm{D}}$

Solution:

From the general equation:

$$U_{\mathrm{SRH}} = \frac{v_{\mathrm{th}} N_{\mathrm{t}} (n \cdot p - n_{\mathrm{i}}^2)}{\frac{1}{\sigma_{\mathrm{p}}}(n + n_1) + \frac{1}{\sigma_{\mathrm{n}}}(p + p_1)} \tag{3}$$

Following hint (i) the denominator becomes:

$$\frac{1}{\sigma_{\mathrm{p}}}(n + n_1) + \frac{1}{\sigma_{\mathrm{n}}}(p + p_1) = \frac{1}{\sigma}(n + n_1 + p + p_1) \tag{4}$$

With the equations given in the lecture slides for n_1 and p_1 the following holds (using $\cosh(x) = (\exp(x) + \exp(-x))/2$):

$$\begin{aligned} n_1 + p_1 &= n_{\mathrm{i}} \exp\left(\frac{E_{\mathrm{T}} - E_{\mathrm{F}}^{\mathrm{i}}}{kT}\right) + n_{\mathrm{i}} \exp\left(\frac{E_{\mathrm{F}}^{\mathrm{i}} - E_{\mathrm{T}}}{kT}\right) \\ &= 2\, n_{\mathrm{i}} \left(\frac{1}{2} \exp\left(\frac{E_{\mathrm{T}} - E_{\mathrm{F}}^{\mathrm{i}}}{kT}\right) + \frac{1}{2} \exp\left(-\frac{E_{\mathrm{T}} - E_{\mathrm{F}}^{\mathrm{i}}}{kT}\right)\right) \\ &= 2\, n_{\mathrm{i}} \cdot \cosh\left(\frac{E_{\mathrm{T}} - E_{\mathrm{F}}^{\mathrm{i}}}{kT}\right) \end{aligned}$$

Using hint (ii) and (iii) the denominator becomes:

$$\begin{aligned} & \frac{1}{\sigma}\left(n + p + 2\, n_{\mathrm{i}} \cdot \cosh\left(\frac{E_{\mathrm{T}} - E_{\mathrm{F}}^{\mathrm{i}}}{kT}\right)\right) \\ =\ & \frac{1}{\sigma}\left(N_{\mathrm{D}} + 0 + 2\, n_{\mathrm{i}} \cdot \cosh\left(\frac{E_{\mathrm{T}} - E_{\mathrm{F}}^{\mathrm{i}}}{kT}\right)\right) \\ =\ & \frac{N_{\mathrm{D}}}{\sigma}\left(1 + \frac{2\, n_{\mathrm{i}}}{N_{\mathrm{D}}} \cdot \cosh\left(\frac{E_{\mathrm{T}} - E_{\mathrm{F}}^{\mathrm{i}}}{kT}\right)\right) \end{aligned} \tag{5}$$

By using the hint number (ii) the numerator becomes:

$$
\begin{aligned}
\left(n \cdot p - n_i^2\right) &= \left((n_0 + \Delta n)(p_0 + \Delta p) - n_i^2\right) \\
&= \underbrace{n_0\, p_0}_{=n_i^2} + \underbrace{n_0}_{\approx N_D}\, \Delta p + \underbrace{p_0}_{\ll n_0}\, \Delta n + \underbrace{\Delta n\, \Delta p}_{\ll n_0\, \Delta p} - n_i^2 \\
&= N_D\, \Delta p
\end{aligned}
$$

Inserting all this into Eq. 3 leads to the expression of U_{SRH} in n-type silicon wafer at low injection:

$$
U_{\mathrm{SRH}} = \frac{\Delta p\, v_{\mathrm{th}}\, N_{\mathrm{t}}\, \sigma}{1 + \frac{2n_i}{N_D} \cosh\left(\frac{E_T - E_F^i}{kT}\right)} \tag{6}
$$

b) Determine the expression for Shockley-Read-Hall lifetime τ_{SRH} in a n-type silicon wafer using the result from a). The result is shown in Fig. 4.

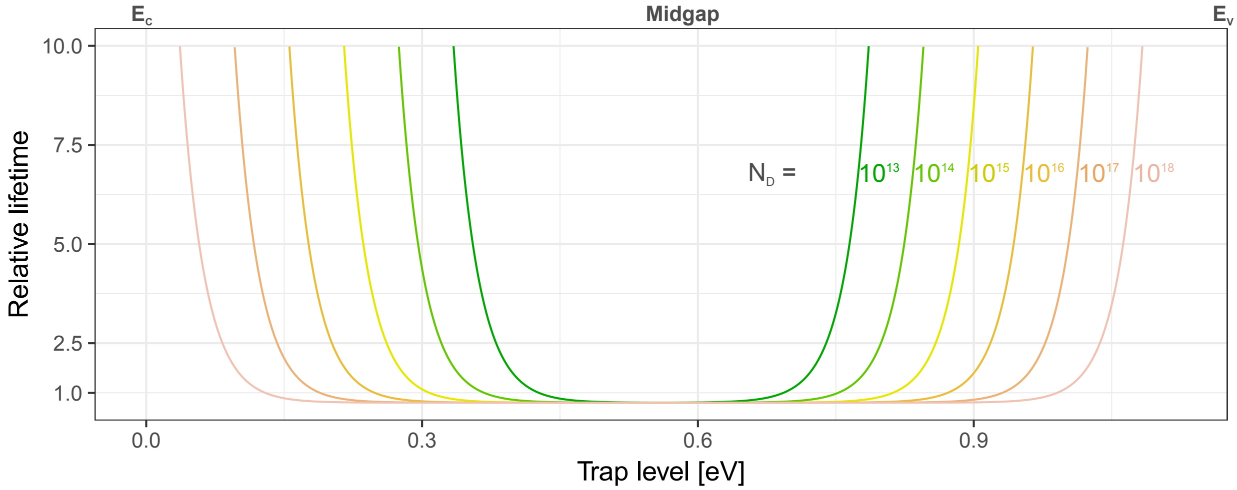

Figure 4: Carrier lifetime as a function of the position of the trap level in the bandgap of crystalline silicon, given for various doping levels. ["Crystalline Silicon Solar Cells", 1994, A. Goetzberger, J. Knobloch, and B. Voss].

Solution:

As the lifetime is given by $\tau_{\mathrm{SRH}} = \frac{\Delta n}{U_{\mathrm{SRH}}}$, the result is easily found using Eq. 6:

$$
\tau_{\mathrm{SRH}} = \frac{1 + \frac{2n_i}{N_D} \cosh\left(\frac{E_T - E_F^i}{kT}\right)}{v_{\mathrm{th}}\, N_{\mathrm{t}}\, \sigma} \tag{7}
$$

This result shows, that the lifetime strongly depends on whether the impurity energy level is close to the mid-gap or not. If $E_F^i = E_T$ the numerator is at its minimum, as $cosh(0) = 1$. So note that mid-gap defects are the most detrimental to the lifetime. For the case of a fixed doping level, the lifetime increases for a defect level that is closer to the band edges as the carriers are remitted into the bands. Another thing to note is the fact that increasing the doping level will lead to a broader range in which defect levels lead to a significant decrease in lifetime.

Exercise 3: Surface recombination

The Fig. 5 represents a silicon rod which is homogeneously illuminated with infrared light (i.e. homogeneous photogeneration). Surface recombination occurs only at the surface for $x = 0$ with a surface recombination velocity S_p.

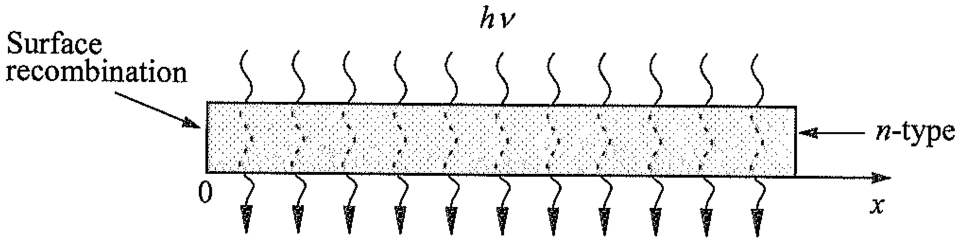

Figure 5: Silicon rod with a recombinant surface at $x = 0$. From S.M. Sze, Semiconductor Devices, Physics and Technology.

(a) Sketch the density of holes $p(x)$ (i.e. $p(x) = p_0 + \Delta p$) that you expect for $S_p \to 0$ and $S_p \to \infty$.

Solution:

If there is no surface recombination, $p(x)$ will be constant for all x.
If the surface recombination velocity increases to infinity, the excess carrier density near the surface will go to 0 and $p(0) = p_0$.
Fig. 6 represents $p(x)$ in these two limit conditions
Notice that if S_p has a finite value, at x = 0 the carrier concentration assumes a value larger than p_0.

(b) Find the differential equation and the boundary condition at $x = 0$ to find $p(x)$
Hint : The boundary condition for $x = 0$ is found by equalizing the diffusion current at $x = 0$ with the current induced by the surface recombinations in steady-state condition.

Solution:

The differential equation is the continuity equation given in the course :

$$\frac{\partial}{\partial t}p(x) = -\frac{1}{q}\frac{\partial}{\partial x}J_p(x) - U + G$$

Where U is the bulk recombination rate and G is the photogeneration rate. The hole current J_p is only due to the diffusion process as no electric field is applied :

$$\frac{\partial}{\partial t}p(x) = D_p\frac{\partial^2}{\partial x^2}p(x) - \frac{p(x) - p_0}{\tau_p} + G$$

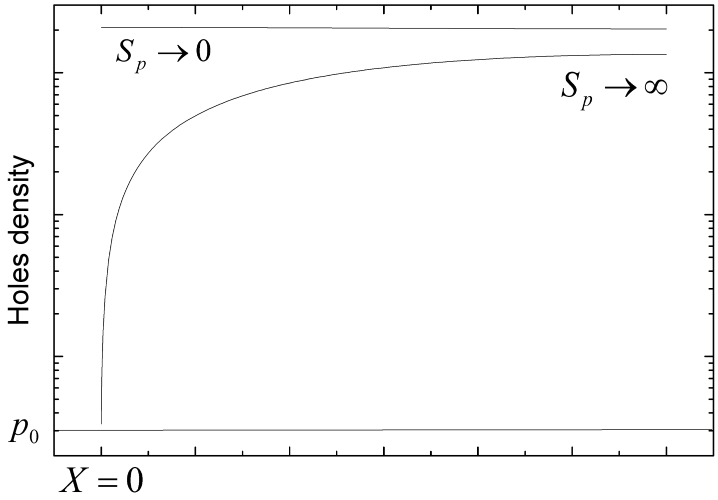

Figure 6: $p(x)$ for $S_p \to 0$ and $S_p \to \infty$

For the steady-state, $\frac{\partial}{\partial t}p(x) = 0$ the following holds :

$$D_p \frac{\partial^2}{\partial x^2} p(x) = \frac{p(x) - p_0}{\tau_p} - G$$

The boundary condition is found by equalizing the diffusion current at $x = 0$ and the current induced by the surface recombinations.

$$J_{diff\ p} = qD_p \frac{\partial}{\partial x} p(x)$$

$$qD_p \frac{\partial}{\partial x} p(x)|_{x=0} = qU_s = qS_P(p(x) - p_0)$$

(c) The solution of the differential equation found in b) is the following:

$$p(x) = p_0 + \tau_p G \left(1 - \frac{\tau_p S_p}{L_p + S_p \tau_p} e^{\frac{-x}{L_p}}\right)$$

What is happening if $S_p \to \infty$ and $S_p \to 0$? Does it fit with your findings at point a)?

Solution:

The solving of the differential equation is described in the bonus at the end of this correction.
For $S_p \to 0$:

$$p(x) = p_0 + \tau_p G \quad \forall x$$

For $S_p \to \infty$:

$$p(x) = p_0 + \tau_p G(1 - e^{\frac{-x}{L_p}}) \ \ \forall x$$

Therefore when x = 0,

$$p(x) \to p_0$$

(d) Calculate the injection level difference between a "perfect surfaces" case and $S_p = 100\,\mathrm{cm\,s^{-1}}$, in the middle of a 200 µm thick wafer. What is the loss in V_{oc} resulting from the bad surface passivation?
Input values: Doping level = $10^{16}\,\mathrm{cm^{-3}}$, intrinsic carrier concentration at 300 K = $10^{10}\,\mathrm{cm^{-3}}$, minority carrier lifetime = 10 ms, diffusion length = 0.35 cm, homogeneous generation rate = $10^{18}\,\mathrm{cm^{-3}\,s^{-1}}$.
Hint: the voltage can be deduced by the injection level[1]:

$$V_{oc} = \frac{K_B T}{q} \ln\left[\frac{(N_D + \Delta p)\Delta p}{n_i^2}\right]$$

Solution:

If we consider the minority carrier density very far from the interface (i.e. more than a few diffusion length), the carriers are not in the influence of surfaces. The holes density far from the surface is the following:

$$p(x \to \infty) = p_0 + \tau_p G \simeq \tau_p G = 10^{16}\,\mathrm{cm^{-3}}$$

Here we assume $p_0 << \tau_p G$. We can observe that this limit is similar to the case where $S_p = 0$, as in this case one can completely neglect the influence of the surface, the bulk case is found for any x. The V_{oc} corresponding to such an injection level is the following:

$$V_{oc} = \frac{K_B T}{q} \ln\left[\frac{(N_D + \Delta p)\Delta p}{n_i^2}\right] = 0.732\,\mathrm{V}$$

Using the relation given in point c and considering a symmetrical hole density, the hole density at the middle of the wafer (x = 100 µm) can be approximated as the following:

$$p(x) = p_0 + \tau_p G \left(1 - \frac{\tau_p S_p}{L_p + S_p \tau_p} e^{\frac{-x}{L_p}}\right) = 2.80 \times 10^{15}\,\mathrm{cm^{-3}}$$

The corresponding V_{oc} is 0.69 V.
We see that in this specific case, 44 mV are lost due to the effect of the surfaces.

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[1] This phenomenon will de described in the next lectures and referred as the quasi Fermi level splitting.

Exercise 4: Surface recombination vs Bulk recombination

Consider a 150 µm thick n-type wafer with well passivated surfaces (S on both side is equal to $100\,\mathrm{cm\,s^{-1}}$) at room temperature. This wafer contains $10^{13}\,\mathrm{cm^{-3}}$ nickel impurities that are situated at 0.23 eV above the valence band with a cross section $\sigma = 10^{-15}\,\mathrm{cm^{-2}}$.

(a) Express τ_{eff} as a function of the different recombination processes. Mention which recombination processes vary with the carrier density (i.e. doping or injection level).

Solution:

τ_{eff} is made up of the contribution of the surface recombinations and of the bulk recombinations. The bulk recombinations are radiative, Auger and SRH processes.

$$\frac{1}{\tau_{eff}} = \frac{1}{\tau_{surface}} + \frac{1}{\tau_{Bulk}} = \frac{1}{\tau_{surface}} + \frac{1}{\tau_{rad}} + \frac{1}{\tau_{auger}} + \frac{1}{\tau_{SRH}}$$

We can separate surface recombination from radiative, Auger and SRH since it is the only process which does not depend on the bulk carrier density.

(b) For an injection level of $10^{16}\,\mathrm{cm^{-3}}$, and two different doping levels, respectively of 10^{17} and $10^{14}\,\mathrm{cm^{-3}}$, calculate τ_{eff}.

Solution:

For both doping levels:

$$\tau_{surface} = \frac{W}{S_1 + S_2} = \frac{150\,\mathrm{\mu m}}{200\,\mathrm{cm\,s^{-1}}} = 75\,\mathrm{\mu s}$$

From the course, we know the radiative recombination coefficient for silicon $R_{ec} = 2 \times 10^{-15}\,\mathrm{cm^3\,s^{-1}}$. For $N_d = 10^{17}\,\mathrm{cm^{-3}}$ and $\Delta n = \Delta p = 10^{16}\,\mathrm{cm^{-3}}$ (low injection case), we calculate the other lifetime contributions :

$$\tau_{Rad} = \frac{1}{R_{ec} \cdot n_0} = \frac{1}{2 \times 10^{-15}\,\mathrm{cm^3\,s^{-1}} \cdot 10^{17}\,\mathrm{cm^{-3}}} = 5000\,\mathrm{\mu s}$$

From the course, we know the Auger coefficient $C_n = 4 \times 10^{-31}\,\mathrm{cm^6\,s^{-1}}$.

$$\tau_{Auger} = \frac{1}{C_n \cdot n_0^2} = \frac{1}{4 \times 10^{-31}\,\mathrm{cm^6\,s^{-1}} \cdot (10^{17}\,\mathrm{cm^{-3}})^2} = 250\,\mathrm{\mu s}$$

$$\tau_{SRH} = \frac{1 + \frac{2 \cdot n_i}{N_D} \cdot cosh(\frac{E_T - E_i}{kT})}{\sigma \cdot v_{th} \cdot N_T} = \frac{1 + \frac{2 \cdot 10^{10}}{10^{17}} \cdot cosh(\frac{0.23 - 0.56}{0.026})}{10^{-15}\,\mathrm{cm^2} \cdot 10^{7}\,\mathrm{cm\,s^{-1}} \cdot 10^{13}\,\mathrm{cm^{-3}}} = 10.3\,\mathrm{\mu s}$$

We observe that the SRH recombination is the dominant term, then:

$$\tau_{eff} = 8.7\,\mu\text{s}$$

For $N_d = 10^{14}\,\text{cm}^{-3}$ and $\Delta n = \Delta p = 10^{16}\,\text{cm}^{-3}$ (high injection case), we calculate the other lifetime contributions :

$$\tau_{Rad} = \frac{1}{Rec \cdot \Delta n} = \frac{1}{2 \times 10^{-15}\,\text{cm}^3\,\text{s}^{-1} \cdot 10^{16}\,\text{cm}^{-3}} = 50\,000\,\mu\text{s}$$

$$\tau_{Auger} = \frac{1}{(Cn + Cp) \cdot (\Delta p)^2} = \frac{1}{8 \times 10^{-31}\,\text{cm}^6\,\text{s}^{-1} \cdot (10^{16}\,\text{cm}^{-3})^2} = 12\,500\,\mu\text{s}$$

$$\tau_{SRH} = \frac{2}{\sigma \cdot v_{th} \cdot N_T} = \frac{2}{10^{-15}\,\text{cm}^2 \cdot 10^{7}\,\text{cm}\,\text{s}^{-1} \cdot 10^{13}\,\text{cm}^{-3}} = 20\,\mu\text{s}$$

We observe that the SRH recombination is again the dominant term, then:

$$\tau_{eff} = 15.8\,\mu\text{s}$$

Bonus: differential equation solving

With the differential equation found in the exercise 1.b) and with our assumptions made in 1.a), we can deduce that $p(x)$ should be expressed as:

$$p(x) = A + B \cdot e^{-Cx}$$

Insert it into the differential equation leads to :

$$D_p B C^2 \cdot e^{-Cx} = \frac{A + B \cdot e^{-Cx} - p_0}{\tau_p} - G$$

For $x \to \infty$:

$$0 = \frac{A - p_0}{\tau_p} - G$$

$$A = \tau_p G + p_0$$

For $x \to 0$ we have $e^{Cx} = 1$:

$$D_p B C^2 = \frac{A + B - p_0}{\tau_p} - G$$

$$D_p B C^2 = \frac{A}{\tau_p} + \frac{B}{\tau_p} - \frac{p_0}{\tau_p} - G$$

With $A = \tau_p G + p_0$:

$$D_p B C^2 = \frac{B}{\tau_p}$$

$$D_p C^2 = \frac{1}{\tau_p}$$

$$C = \frac{1}{\sqrt{D_p \tau_p}} = \frac{1}{L_p}$$

To find the parameter B, we insert A and C into the boundary condition:

$$q D_p \frac{\partial}{\partial x} p(x)|_{x=0} = q U_s = q S_p(p(x) - p_0)$$

$$-D_p B C = S_p((A + B) - p_0)$$

$$B(-D_p C - S_p) = S_P(A - p_0)$$

$$B = \frac{S_p(A - p_0)}{-D_p C - S_p}$$

$$B = \frac{-S_p(\tau_p G)}{D_p C + S_p}$$

$$B = \frac{-S_p(\tau_p G)}{\frac{D_p}{Lp} + S_p}$$

Hence, $p(x)$ is:

$$p(x) = A + B \cdot e^{-Cx} = \tau_p G + p_0 + \frac{-S_p \tau_p G}{\frac{D_p}{Lp} + S_p} e^{\frac{-x}{Lp}}$$

$$p(x) = p_0 + \tau_p G(1 - \frac{\tau_p S_p}{L_p + S_p \tau_p} e^{\frac{-x}{Lp}})$$

Note the dependence on the diffusion length L_p that will strongly influence the shape of $p(x)$ with respect to how far the depletion region reaches into the silicon bulk. Indeed if you have bigger diffusion length, the depletion region caused by a poorly passivated surface extends deeper into the bulk.