Solution Series 6

Exercise 1: Absorption

a) Which thickness of material do you need to absorb 90 % of the light with $\lambda = 500$ nm and 1000 nm in amorphous Si and in crystalline Si. Absorption coefficients are given in Tab. 1

Table 1: Absorption coefficients

$\lambda[nm]$	α for a-Si:H $[cm^{-1}]$	α for c-Si $[cm^{-1}]$
500	$2 \cdot 10^5$	10^4
1000	0.9	80

Solution:

From the Beer-Lambert law:

$$I(x) = I_0 e^{-\alpha x} [\frac{W}{m^2}]$$

We deduce

$$x = \frac{\ln[0.1]}{-\alpha} [cm]$$

The absorption depths at which 90 % of the light is absorbed, for the different materials and wavelengths are given in Tab. 2

Table 2: Absoption depths at which 90 % of light is absorbed

$\lambda[nm]$	x for a-Si:H [m]	x for c-Si [m]
500	1.12×10^{-7}	2.3×10^{-6}
1000	2.56×10^{-2}	2.88×10^{-4}

b) Draw the absorption profile for an intensity F_0 in a semi infinite wafer with a reflection of 10 % with $\lambda = 500$ nm and 1000 nm in amorphous Si and in crystalline Si.

Solution:

In Fig 1, the intensity profile is drawn for two wavelengths in a-Si:H and c-Si materials.

c) Taking the results of a) & b) into account, what are options to increase the absorption to more than 99% in the films without making them thicker than what you calculated in a) ?

Solution:

Finally there are only two ways to achieve this (in classical ray optics):

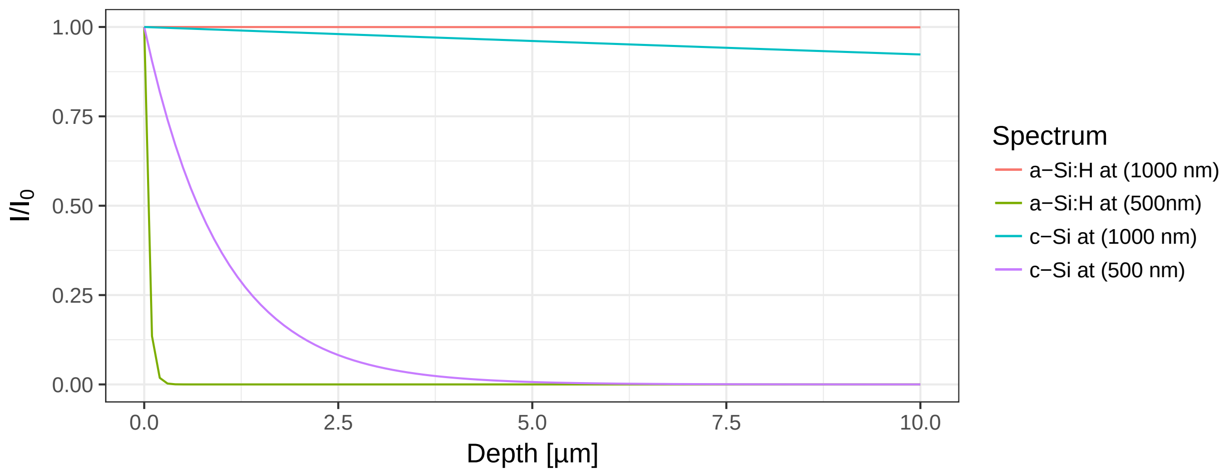

Figure 1: Light intensity decay in a-Si and c-Si for different wavelengths.

(a) reduce front reflection losses
use an antireflection coating
texture the front surface to create multiple bounces

(b) use a back "mirror", e.g., silver electrode to at least double the optical path
texture also the BACK surface to lengthen the optical path beyond a factor of 2, e.g., in crystalline silicon path length extension ca. 4x

Exercise 2: Antireflection layer

Consider a multilayer stack with refractive indices n_1 ,n_2, and n_3. The Fresnel equation of reflection at normal incidence for two layers i and j is $r_{ij} = ((n_i - n_j)/(n_i + n_j))^2$. To achieve minimal reflection the second reflection r_{23}, should nullify r_{21}. Derive a condition linking n_1 , n_2, and n_3.

Solution: At the first interface (n_1, n_2), the reflection coefficient can be given as:

$r_{21} = r_{12} = ((n_1 - n_2)/(n_1 + n_2))^2$

At the second interface (n_2,n_3), the reflection coefficient can be given as:

$r_{23} = ((n_2 - n_3)/(n_2 + n_3))^2$

In order to achieve minimum reflection, we want these two reflection coefficients to have the same magnitude and opposite sign, so the reflected waves destructively interfere:

$|r_{12}| = |r_{23}|$

Substitute the expressions for r_{12} and r_{23} and take the square root:

$|(n_1 - n_2)/(n_1 + n_2)| = |(n_2 - n_3)/(n_2 + n_3)|$

Now, cross-multiply:

$(n_1 - n_2)(n_2 + n_3) = (n_1 + n_2)(n_2 - n_3)$

Expanding both sides:

$n_1 n_2 - n_1 n_3 - n_2^2 + n_2 n_3 = n_1 n_2 + n_1 n_3 - n_2^2 - n_2 n_3$

Now, rearrange the equation to group the terms:

$2n_1 n_3 = 2n_2^2$

Divide both sides by 2 and take the square root:

$\sqrt{n_1 n_3} = n_2$, called geometric mean.

Under this condition the magnitude of the reflection coefficients at the two interfaces is equal and their signs are opposite; destructive interference between the reflected waves occurs and a minimum in reflection under normal incidence is achieved.