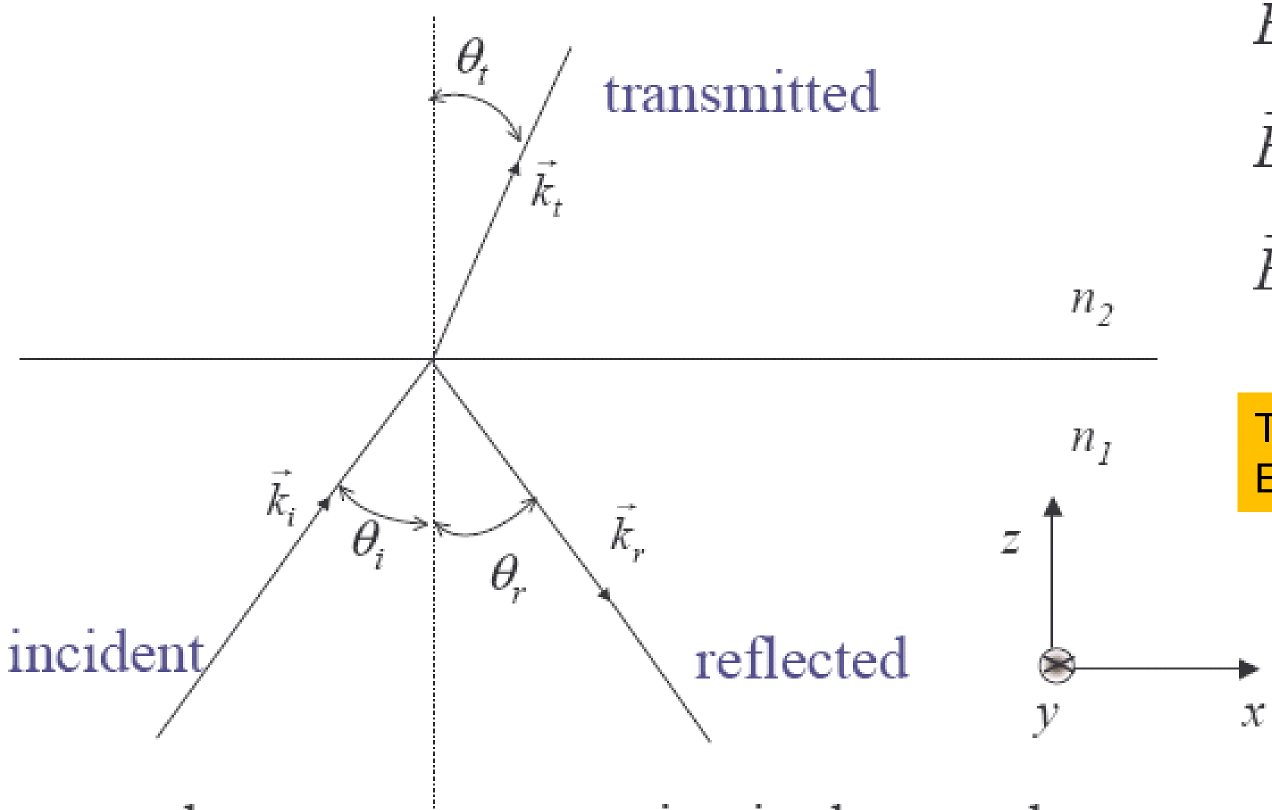
Total Internal Reflection (TIR)



$$\vec{E}_i = \vec{E}_{i0} e^{i(\vec{k}_i \cdot \vec{r} - \omega t)}$$

$$\vec{E}_r = \vec{E}_{r0} e^{i(\vec{k}_r \cdot \vec{r} - \omega t)}$$

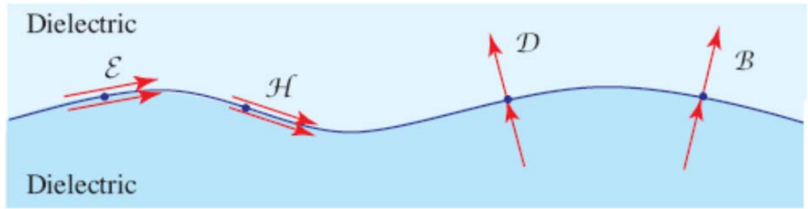
$$\vec{E}_t = \vec{E}_{t0} e^{i(\vec{k}_t \cdot \vec{r} - \omega t)}$$

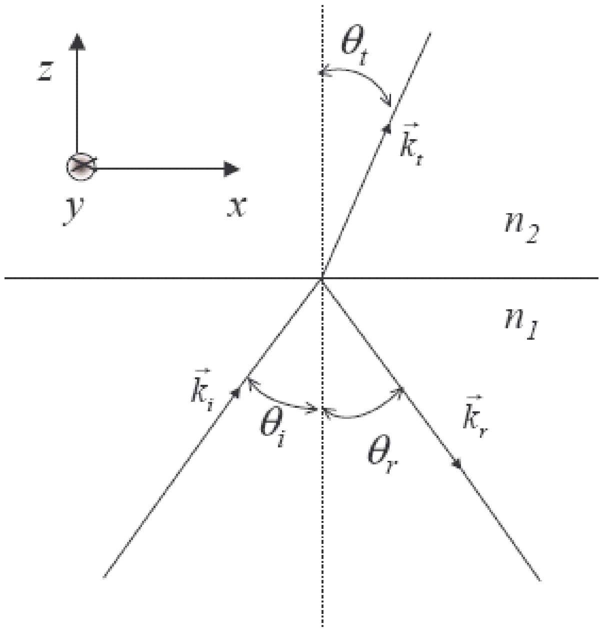
TE wave:
E is in y-direction

- plane wave propagating in the x-z plane
- incident onto a boundary between two media of different refractive index

Boundary Conditions

Dielectric Boundary:
tangential components of E & H
Normal components of D & B





Boundary conditions at z=0, at any instant of time *t*, for the tangential component of electric field:

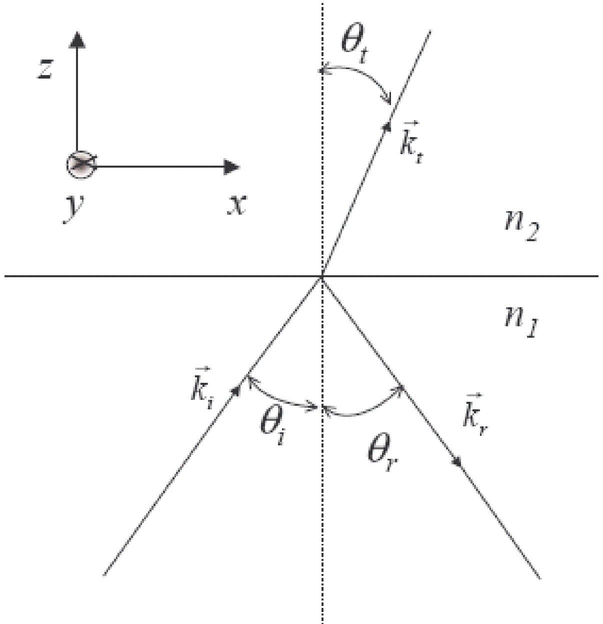
$$E_{i0,x} e^{i(\vec{k}_i\cdot\vec{r}-\omega t)}\Big|_{z=0} + E_{r0,x} e^{i(\vec{k}_r\cdot\vec{r}-\omega t)}\Big|_{z=0} = E_{t0,x} e^{i(\vec{k}_t\cdot\vec{r}-\omega t)}\Big|_{z=0}$$

This is possible only if: $\forall t$,@z=0

$$\vec{k}_i\cdot\vec{r}-\omega t = \vec{k}_r\cdot\vec{r}-\omega t = \vec{k}_t\cdot\vec{r}-\omega t$$

k_y=0 =>

Law of Reflection and Refraction



Boundary conditions at z=0, at any instant of time *t*, for the tangential component of electric field:

$$E_{i0,x} e^{i(\vec{k}_i \cdot \vec{r} - \omega t)}\Big|_{z=0} + E_{r0,x} e^{i(\vec{k}_r \cdot \vec{r} - \omega t)}\Big|_{z=0} = E_{t0,x} e^{i(\vec{k}_t \cdot \vec{r} - \omega t)}\Big|_{z=0}$$

This is possible only if: $\forall t$,@z=0

$$\vec{k}_i \cdot \vec{r} - \omega t = \vec{k}_r \cdot \vec{r} - \omega t = \vec{k}_t \cdot \vec{r} - \omega t$$

$$k_y = 0 \Rightarrow k_i^x x = k_r^x x = k_t^x x$$

$$\Rightarrow \quad k_i^x = k_r^x = k_t^x$$

The in-plane component of the wave-vector is preserved during reflection and refraction

$$\Rightarrow \frac{2\pi}{\lambda / n_1} \sin\theta_i = \frac{2\pi}{\lambda / n_1} \sin\theta_r = \frac{2\pi}{\lambda / n_2} \sin\theta_t$$

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

The law of refraction (Snell's law)

$$\theta_i = \theta_r$$

The law of reflection

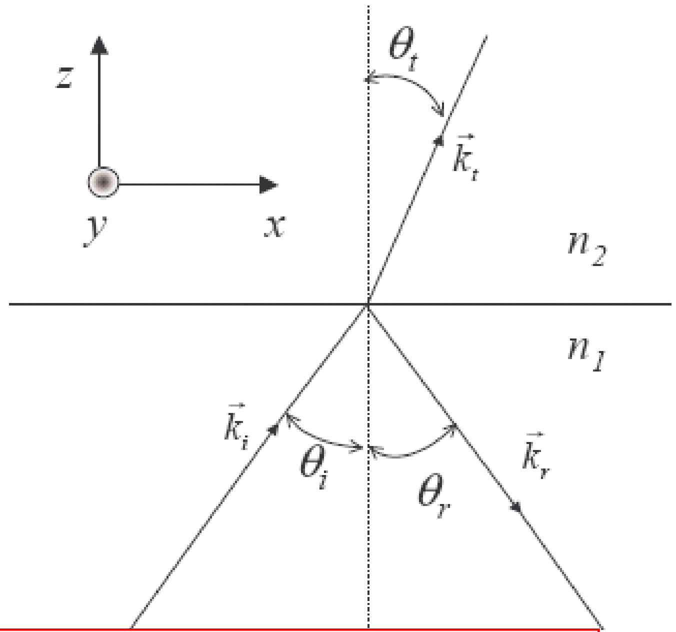
Total Internal Reflection

Snell's law: $\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i$

The maximum value of the incident angle for which the transmission of light into the second medium occurs:

$$\sin\theta_{t,\max} = \sin 90^o = 1 = \frac{n_1}{n_2}\sin\theta_{i,\max}$$

$$\theta_{i,\max} = \arcsin\frac{n_2}{n_1}$$



Total reflection (or total internal reflection): $\theta_i > \theta_{i,\max}$

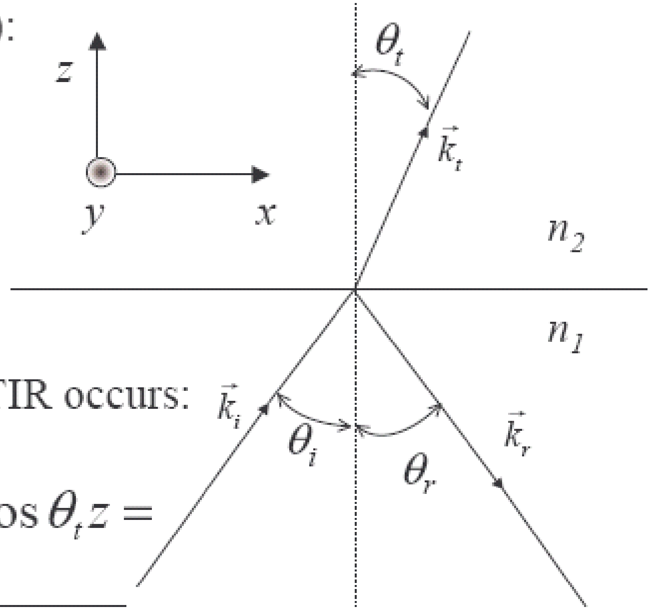
- θ_t does not have a real value
- all the incident light is reflected back into the first medium; no light enters (meaning no power flow) in the second medium
- EM field in the second medium is *evanescent*:

Total Internal Reflection

Total reflection (or total internal reflection - TIR):

$$\theta_i > \theta_{i,\max}$$

$$\theta_{i,\max} = \arcsin\frac{n_2}{n_1}$$



The field is evanescent in the 2nd medium when TIR occurs:

$$\vec{k}_t \cdot \vec{r} = k_t^x x + k_t^z z = \frac{2\pi}{\lambda / n_2}\sin\theta_t x + \frac{2\pi}{\lambda / n_2}\cos\theta_t z =$$

$$= \frac{2\pi n_1}{\lambda} x \sin\theta_i \pm i\frac{2\pi n_2}{\lambda} z\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1}$$

$$\vec{E}_t = \vec{E}_{t0} e^{i\left(\vec{k}_t\cdot\vec{r} - \omega t\right)} = \vec{E}_{t0} e^{i\left(\frac{2\pi n_1}{\lambda} x \sin\theta_i - \omega t\right)} e^{-\frac{2\pi n_2}{\lambda} z\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1}}$$

Real when TIR occurs

- Surface wave in the x-direction, and exponential decay in z

Total Internal Reflection

The field in second medium decays exponentially as a function of increasing distance from the boundary:

$$\vec{E}_t = \vec{E}_{t0} e^{i\left(\frac{2\pi n_1}{\lambda} x \sin\theta_i - \omega t\right)} e^{-\frac{2\pi n_2}{\lambda} z \sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1}} \propto e^{-z/z_0}$$

The penetration depth of the wave into the 2nd medium is proportional to:

$$z_0 = \frac{\lambda}{2\pi n_2 \sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1}},$$

$$I = I_0 \cdot e^{-\frac{z}{d}} \qquad d = \frac{\lambda}{4\pi \cdot n_2 \cdot \sqrt{\left(\frac{n_1}{n_2}\right)^2 \cdot (\sin\theta_i)^2 - 1}}$$

Larger incidence angle θ_i => smaller penetration depth into the 2nd medium.