MICRO-523: Optical Detectors

Week Five: Photodiodes

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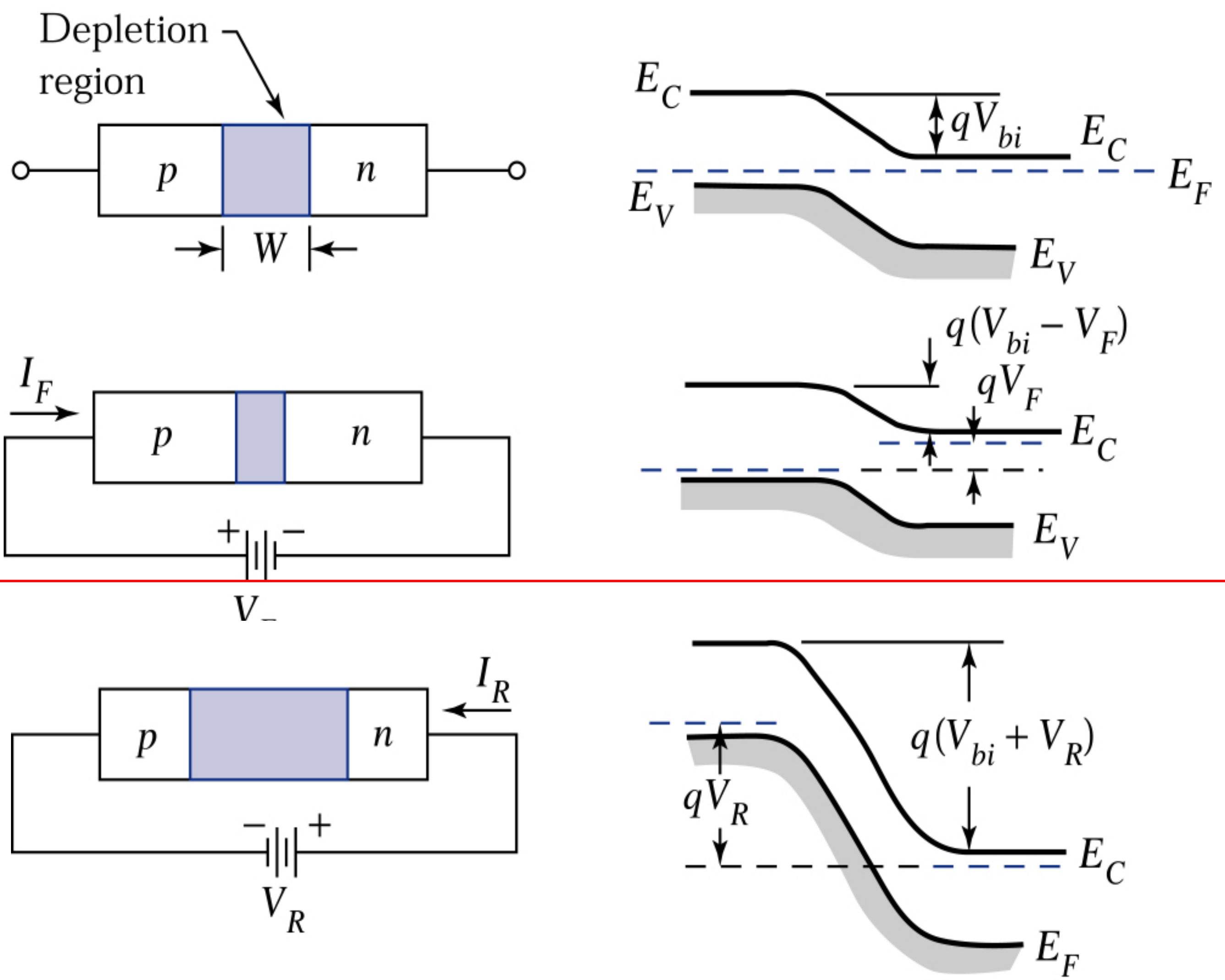
Based on MICRO-523, P.-A. Besse, 2023

TAs: Samuele Bisi, Yazan Lampert

EPFL

Outline

5.1 Non-Equilibrium p-n Junctions: Bands



Thermal equilibrium

Forward-bias (V>0)

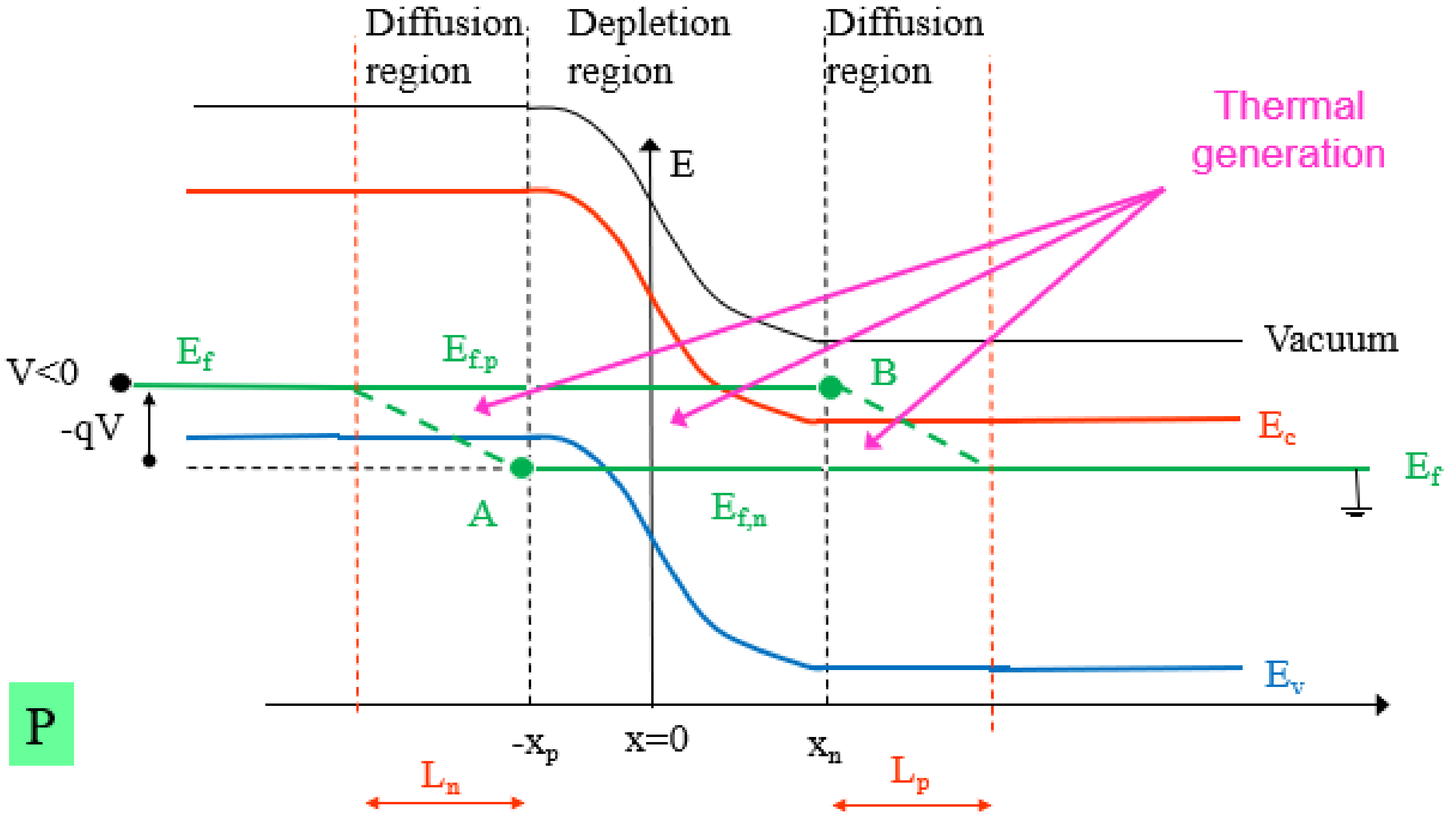
Reverse-Bias (V<0)

Related:

- Built-in potential
- Polarity
- I-V curve

S.M. Sze, « Semiconductor devices, physics and technology »

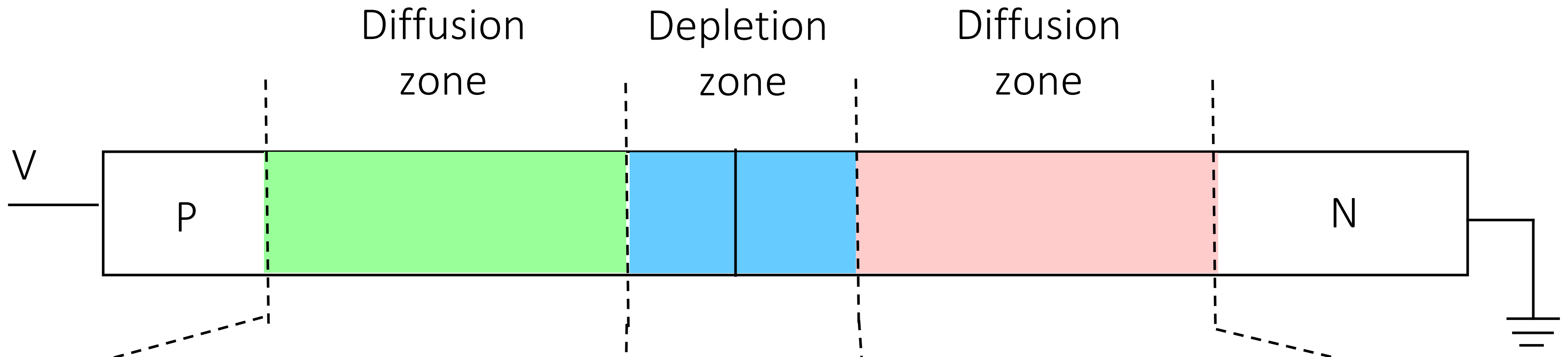
5.1 Quasi Fermi Levels: Reverse-Blocking Mode



$L_{n,p}$: diffusion lengths

Related:
- Dark current
- Recombination at edges
- Widths of regions
- p-i-n variant

5.1 Currents in p-n diodes



$$J_n \equiv q \cdot \int_{-\infty}^{-x_p} U_{th}(x)dx$$

$$J_{gr} \equiv q \cdot \int_{-x_p}^{x_n} U_{th}(x)dx$$

$$J_p \equiv q \cdot \int_{x_n}^{\infty} U_{th}(x)dx$$

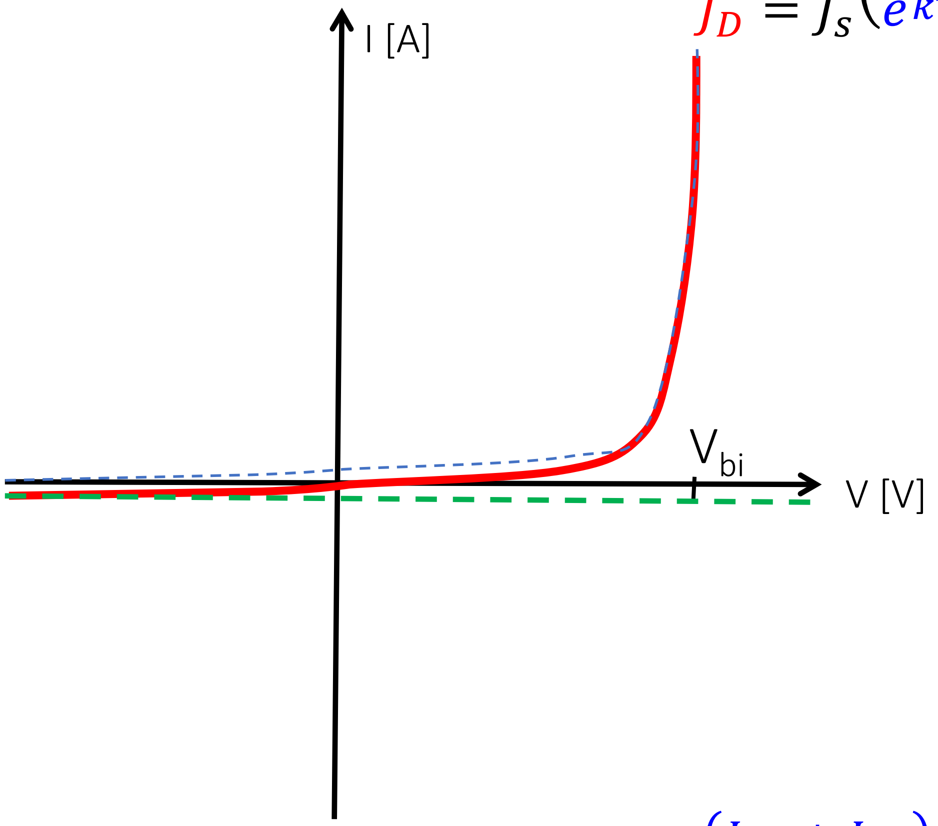
$U_{th} = R - G$ = net recombination rate (recombination – generation)

Dark current = sum of all generated carriers

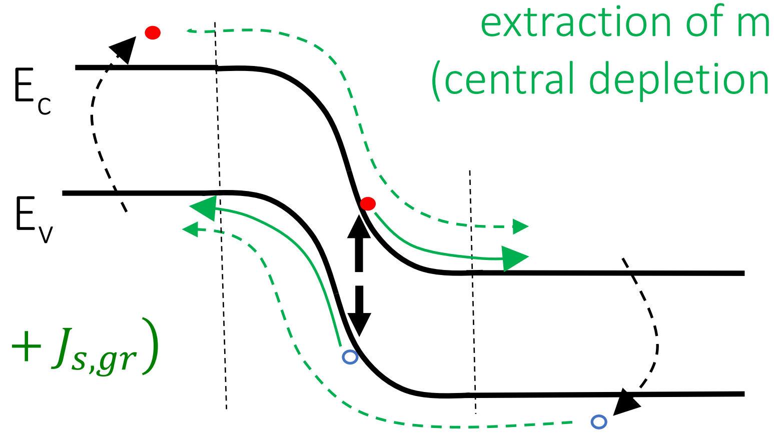
5.1 Currents in p-n diodes

Shockley equation
(ideal diode law)

$$J_D = J_s\left(e^{\frac{qV}{kT}} - 1\right)$$

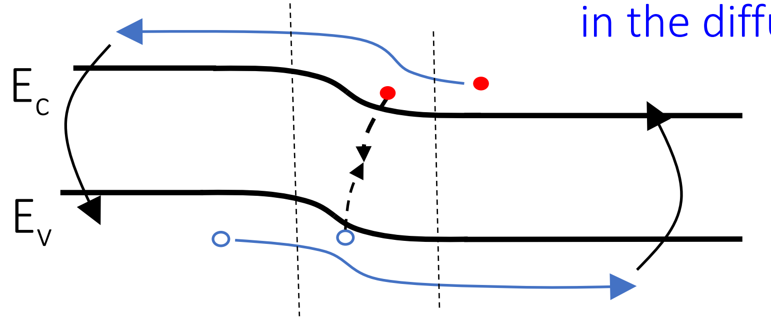


Generation and
extraction of minority carriers
(central depletion zone dominates)



$$-\left(\cancel{J_{s,n}} + \cancel{J_{s,p}} + J_{s,gr}\right)$$

Injection and recombination
of minority carriers
(recombinatin dominates
in the diffusion zones)



$$\left(J_{s,n} + J_{s,p}\right) \cdot e^{\frac{qV}{kT}} + \cancel{J_{s,gr} \cdot e^{\frac{1}{2}\cdot\frac{qV}{kT}}}$$

5.1 Initial Conclusions

1) The dark current in a p-n diode is the integral of thermal generation in:
   I) the diffusion region of the electrons in P
      (typical length L_n: a few tens of micrometers)
   II) the depletion region, with a strong electric field
      (typical length W: a few tenths of micrometers)
   III) the diffusion region of the holes in N
      (typical length L_p: a few tens of micrometers)

2) The dark current of a silicon p-n diode at room temperature is dominated by generation in the depletion zone.
   It is not the current of an ideal diode!

Outline

5.2 Photodiode: PRINCIPLE

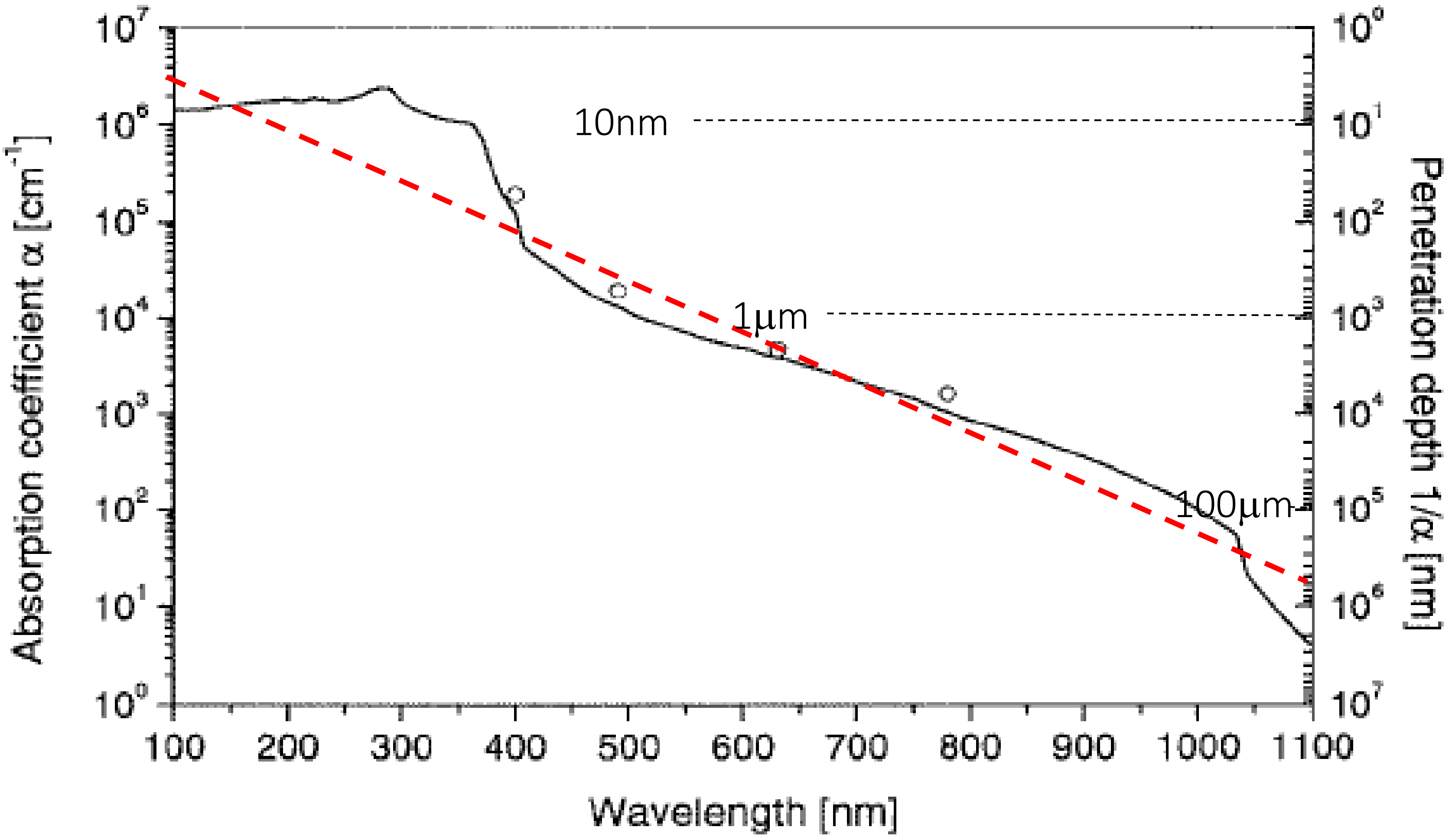
A two-step process to detect light:

1) Optical generation of electron-hole pairs
   = absorption of light by matter

2) Collection of charge carriers
   = extraction of carriers by the electrical field of the junction or by diffusion (main process in solar cells)

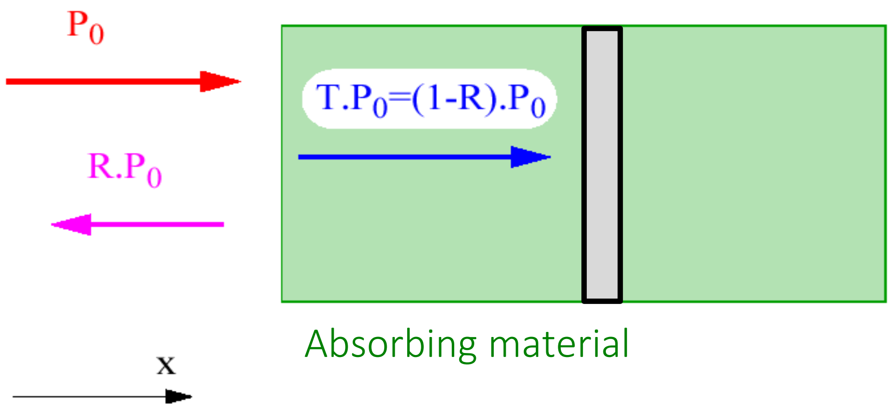
5.2 Absorption

For silicon



Approximation: $\alpha_{cm^{-1}}(\lambda_{\mu m}) \cong 10^{7.2-5.5\lambda}$

5.2 Absorption and Generation Rate



R= Reflection coefficient

T= Transmission coefficient

S= Surface of the detector

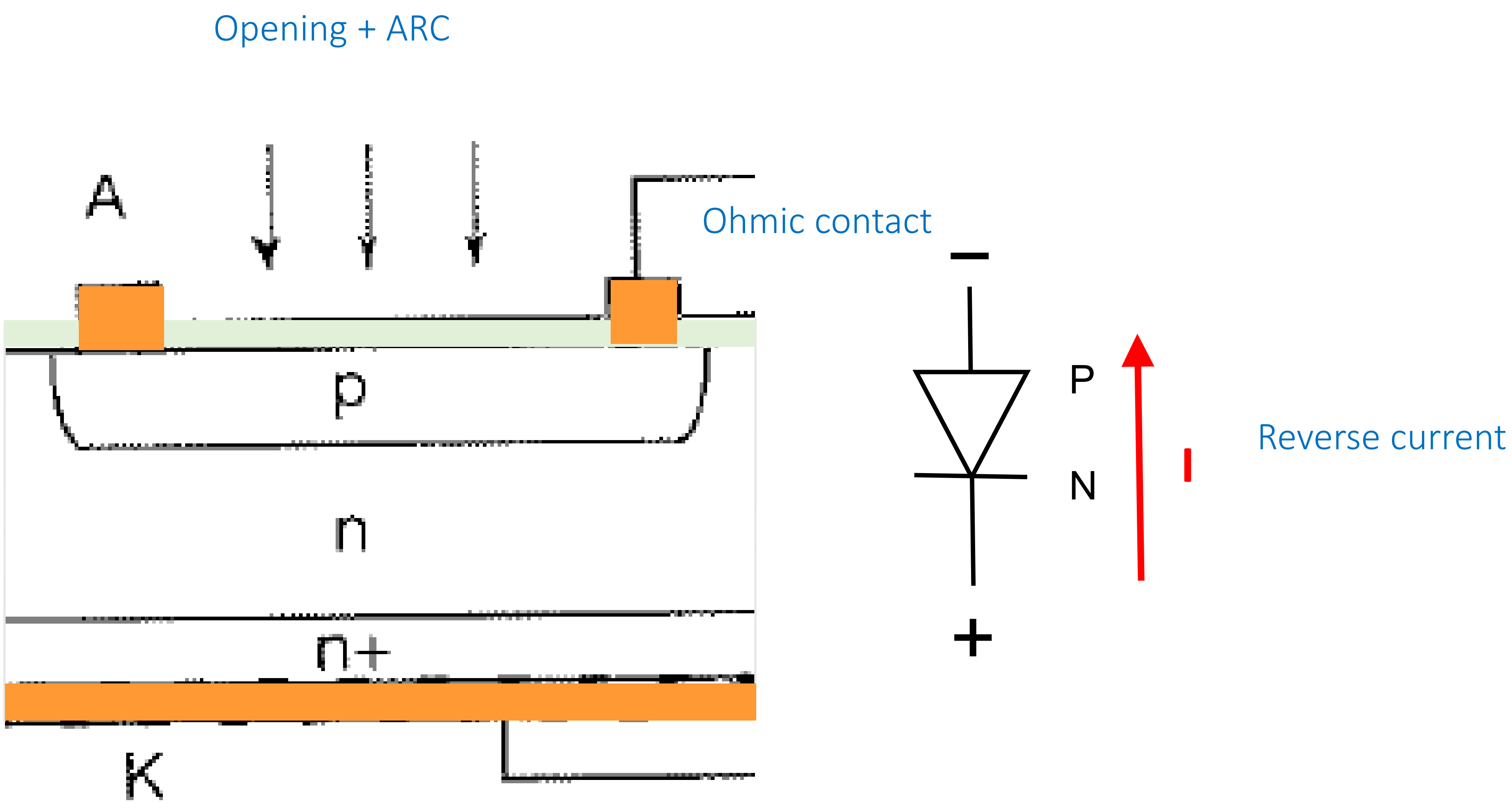
Same discussion as in 1.49 and Exercise 2.2

g(x)= generation rate of carriers

G_{opt}= generation rate per surface

$$G_{opt}(x) = \frac{P(x)/S}{h\nu} \cdot \alpha = \frac{P_0}{S}\frac{1}{h\nu} \cdot (1-R) \cdot e^{-\alpha x} \cdot \alpha \quad \left[\frac{1}{cm^3 \cdot s}\right]$$

5.2 Typical Structure of a p-n Photodiode

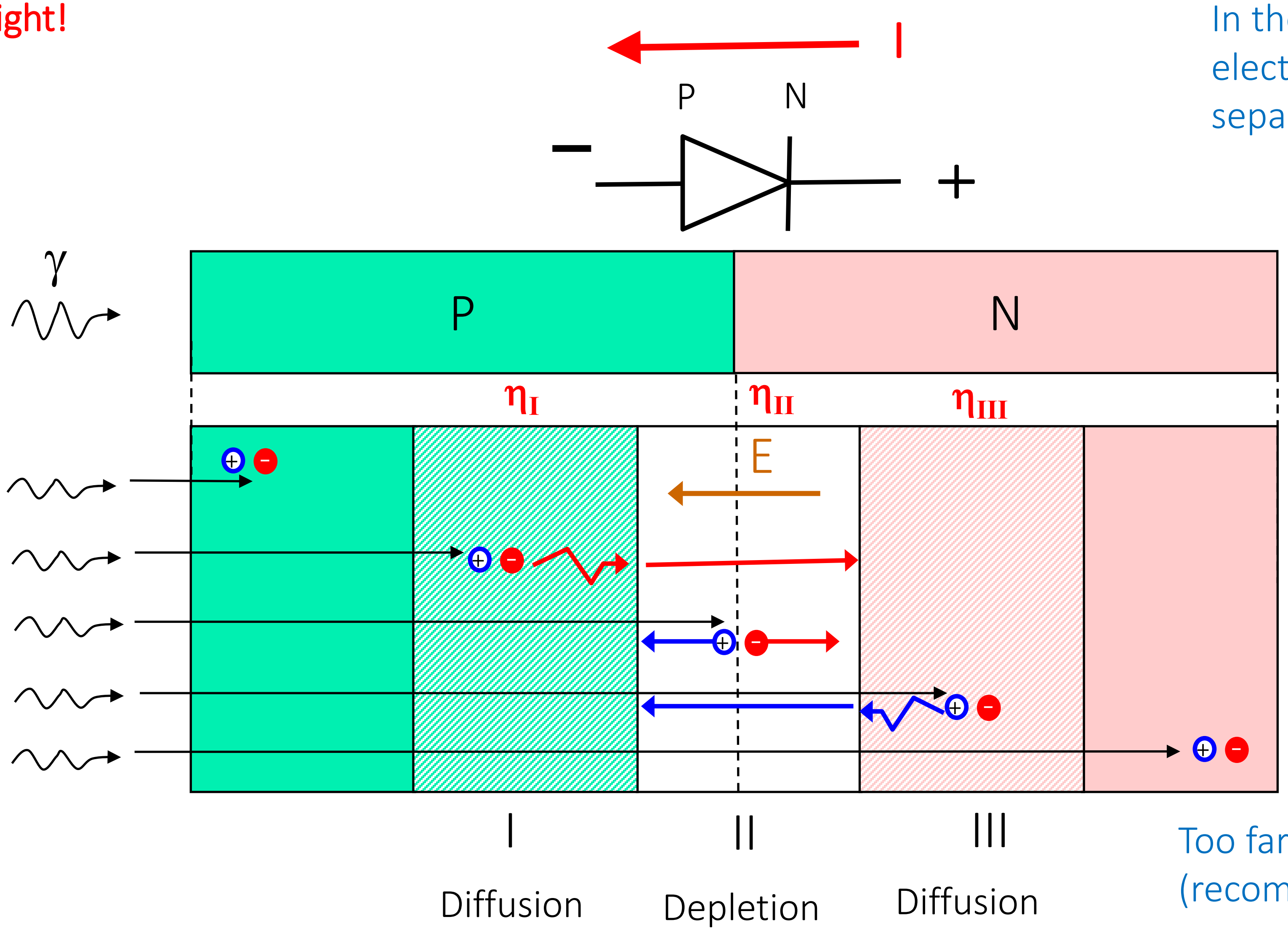


S. Donati, « Photodetectors, devices, circuits and applications »

5.2 Schematic Diagram of the Working Principle

Now subject to light!

In the depletion region: strong electric field → carriers quickly separated



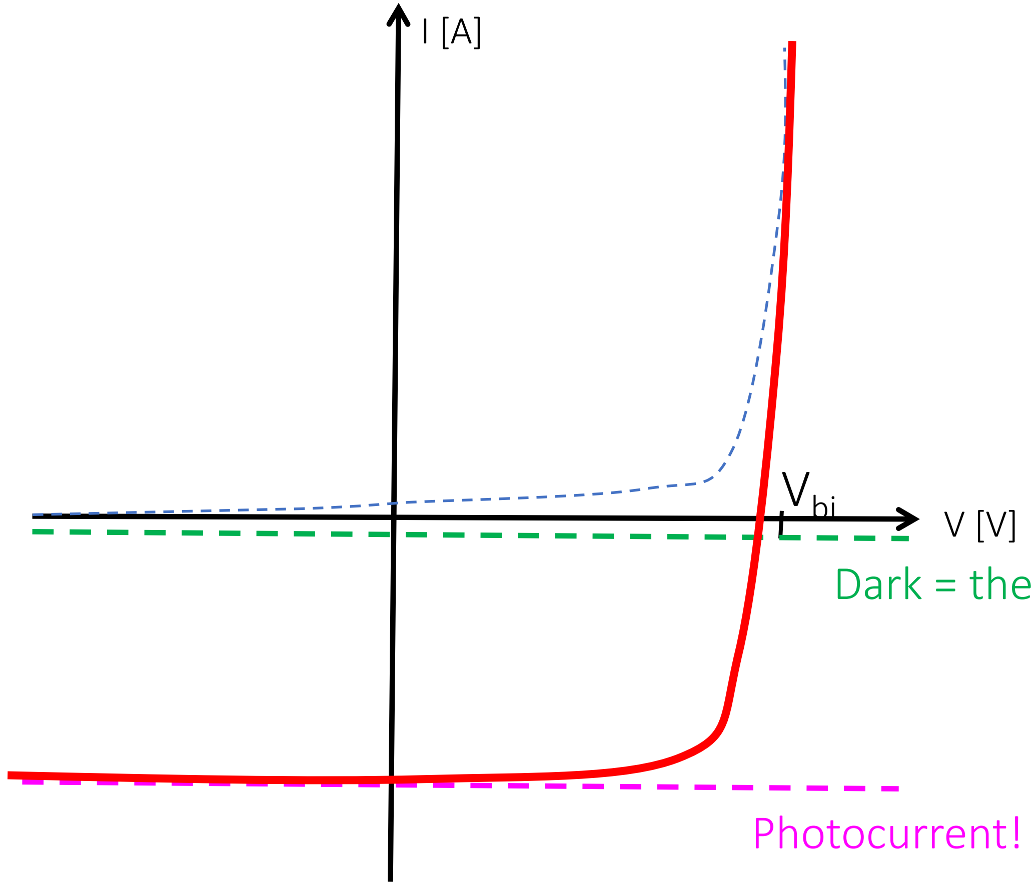
Diffusion = «slow» process (Brownian motion)

Too far from elecric field → «lost» (recombination)

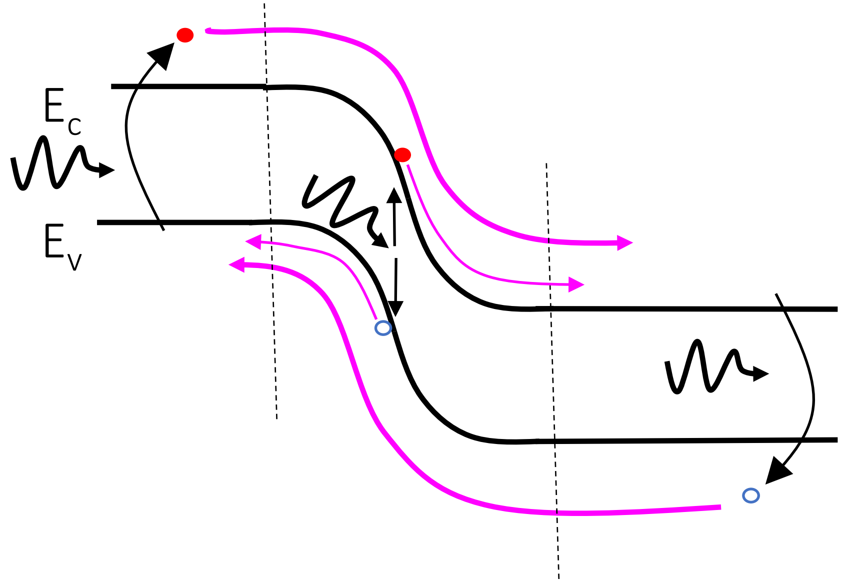
5.2 Current in photodiodes

Same as discussion §5.1 but
now under illumination

$$J = J_D - J_{ph}$$



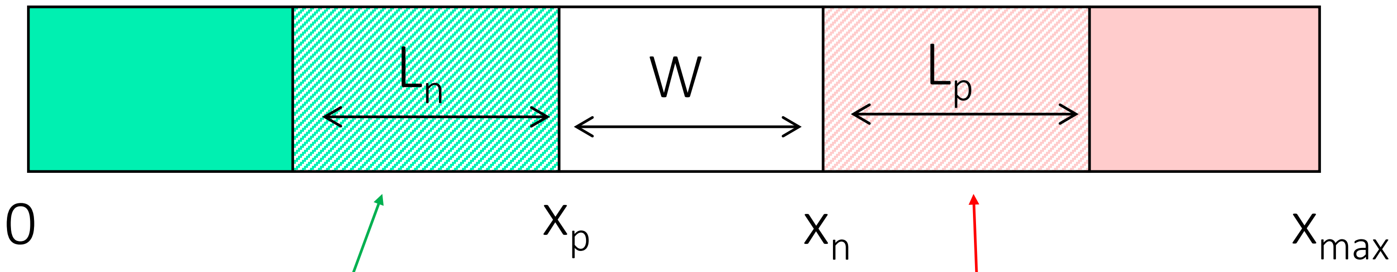
Generation and extraction of photocarriers



$$-J_{ph} = -\left(\frac{q}{h\nu}\right)(\eta_I + \eta_{II} + \eta_{III}) \cdot P_{opt}$$

5.2 Continuity Equations of the Minority Carriers

$$I_{II} = -S \cdot q \cdot \int_{x_p}^{x_n} G_{opt}(x)dx$$



$x \leq x_p$

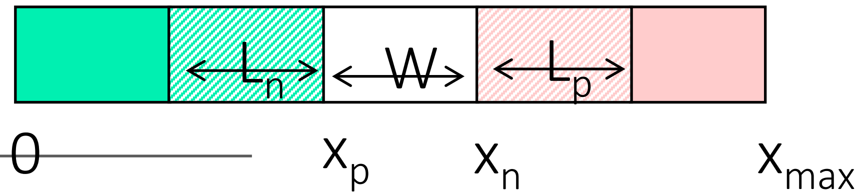
$$D_n \frac{\partial^2 n_p}{\partial x^2} - \frac{n_p - n_{p0}}{\tau_n} + G_{opt} = 0$$

$$D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{n0}}{\tau_p} + G_{opt} = 0$$

$x \geq x_n$

Diffusion equations

5.2 Solutions for the Photocurrent



General case:

$$I = I_{th} + I_{ph} = -I_d - \frac{q}{h\nu}(\eta_I + \eta_{II} + \eta_{III}) \cdot (1 - R) \cdot P_0$$

Specific case: Shallow junction $x_p \ll L_n$ and Deep substrate $x_{max} - x_n \gg L_p$

Main results:

$$\eta_{II} = e^{-\alpha x_p} \cdot (1 - e^{-\alpha W})$$

$$\eta_{III} = \frac{\alpha L_p}{1 + \alpha L_p} \cdot e^{-\alpha x_n}$$

See Exercise 2.2!

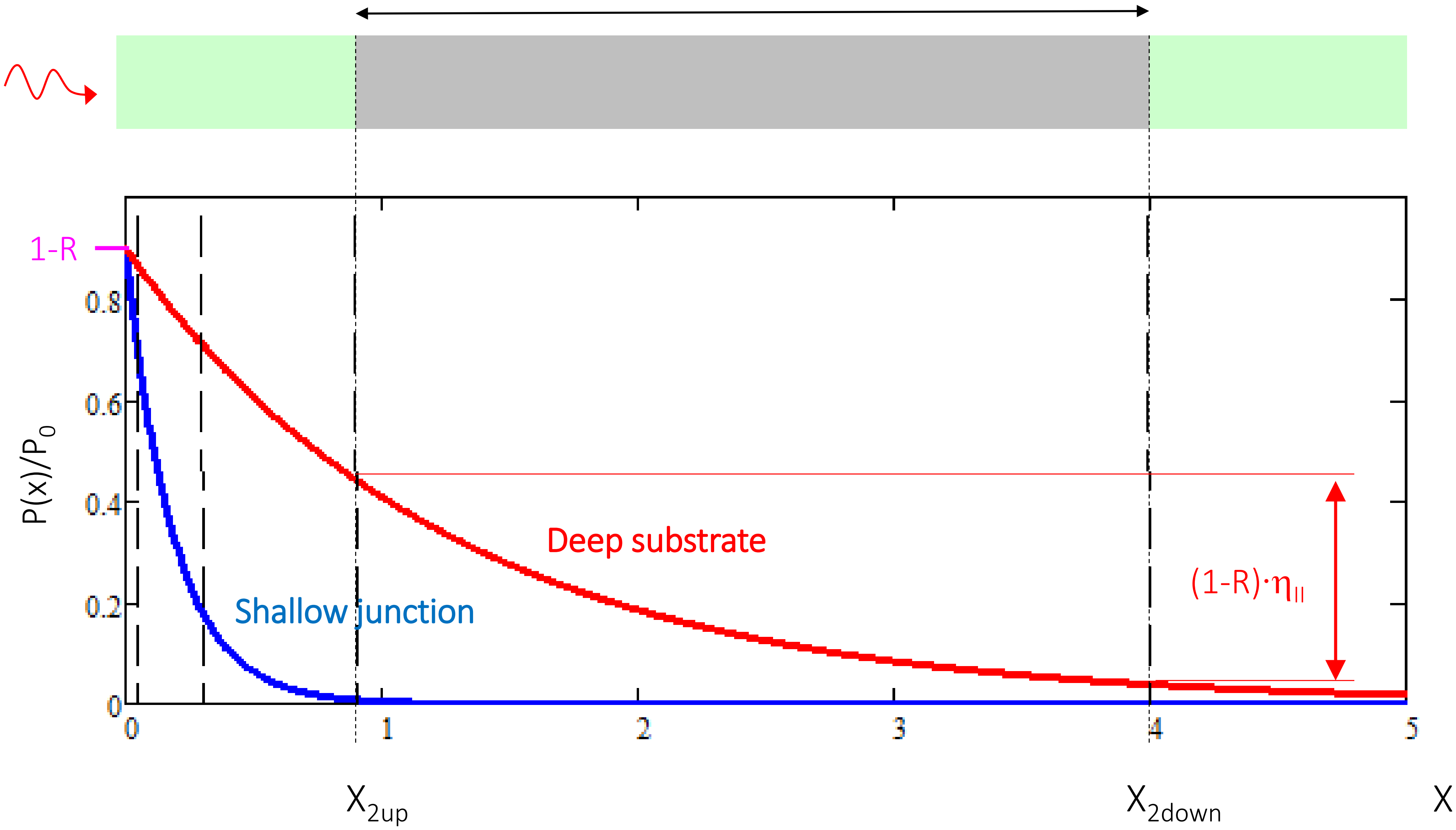
With recombination at the surface:

$$\eta_I = \left(\frac{1 - e^{-\alpha x_p}}{\alpha x_p} \right) - e^{-\alpha x_p}$$

Without recombination at the surface:

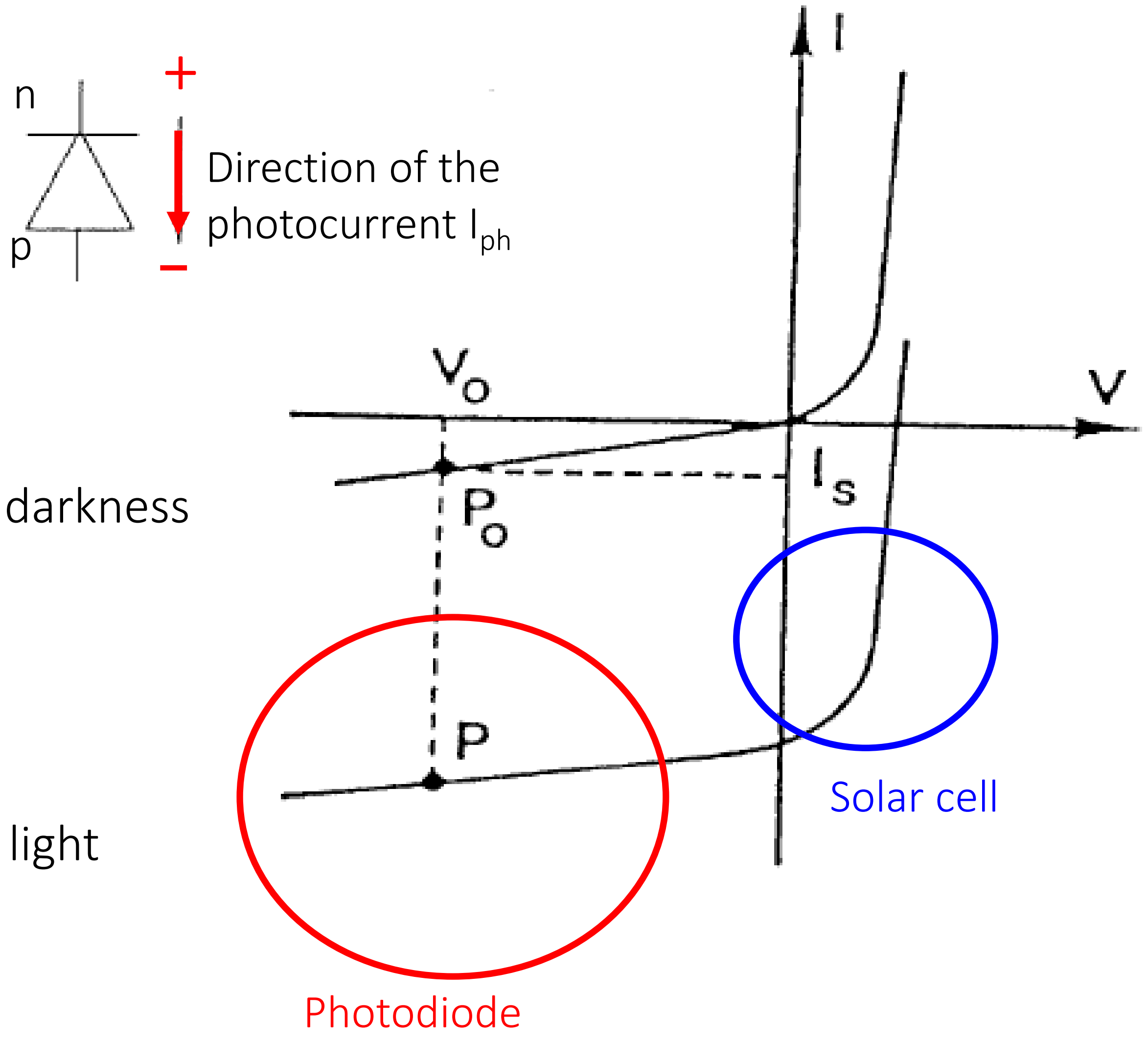
$$\eta_I = 1 - e^{-\alpha x_p}$$

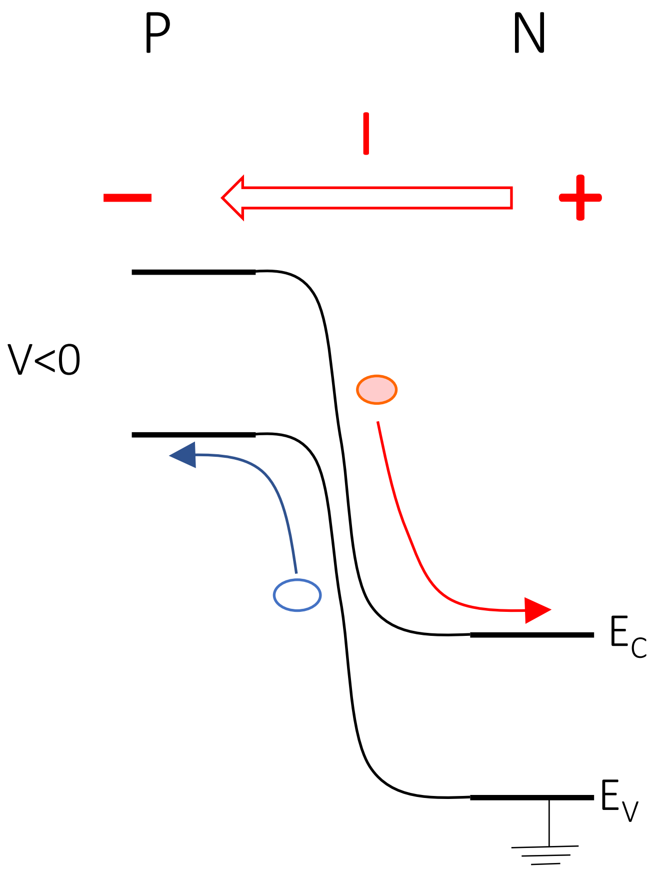
5.2 Interpretation: Quantum Efficiency η_{ll}



See Exercise 2.2

5.2 I(V) Plot


n
p
+
−
Direction of the photocurrent I_{ph}
I
V
V_o
I_s
P_o
P
darkness
light
Photodiode
Solar cell


P
N
I
−
+
V<0
E_C
E_V

Take-Home Messages/W5-1

5.1 p-n junction:

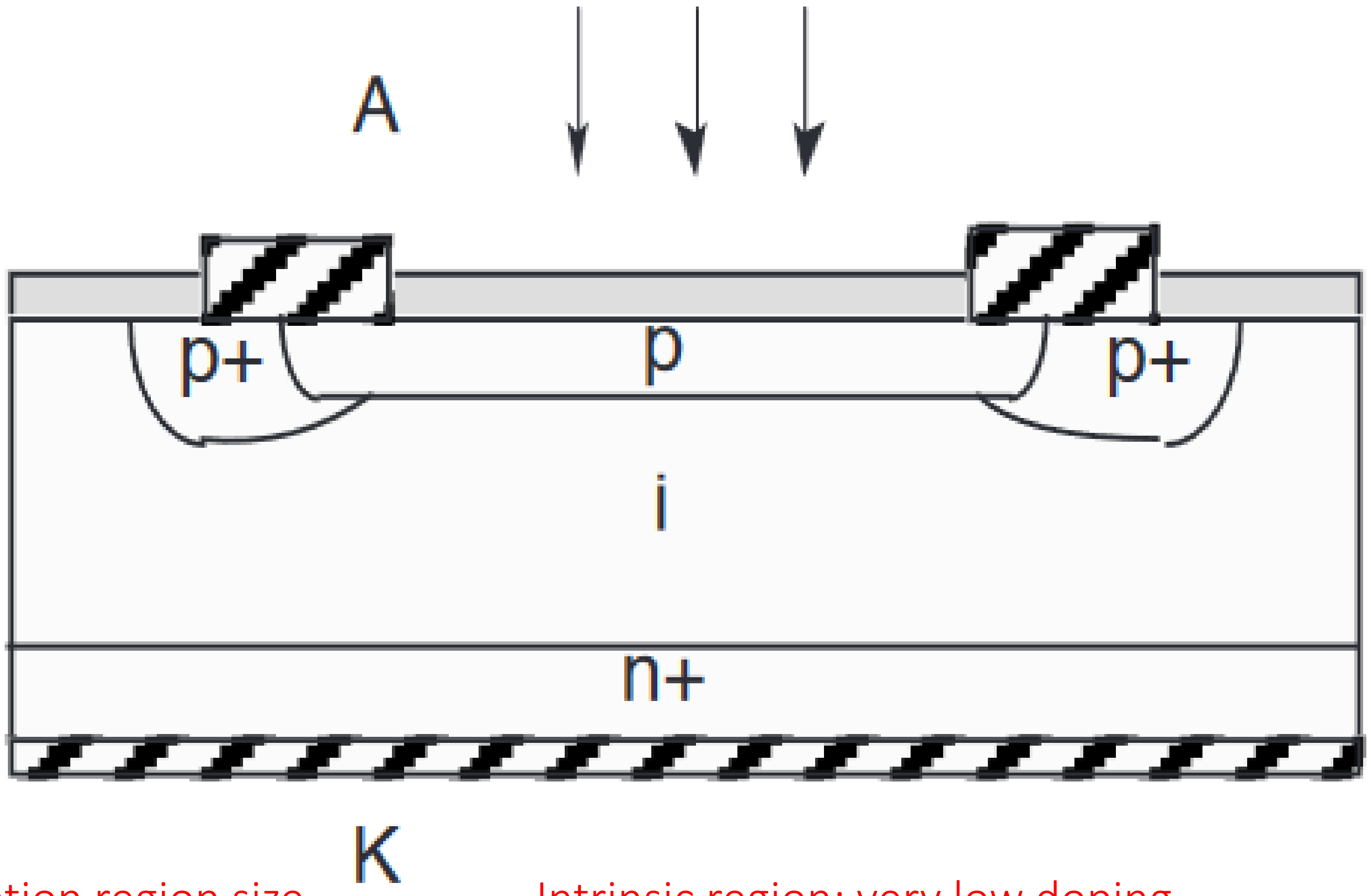
- Explain the working principle of a p-n junction?

5.2 p-n Photodiodes:

- Explain the working principle and structure of a p-n photodiode.
- Where should the light be absorbed to be correctly detected ?

Outline

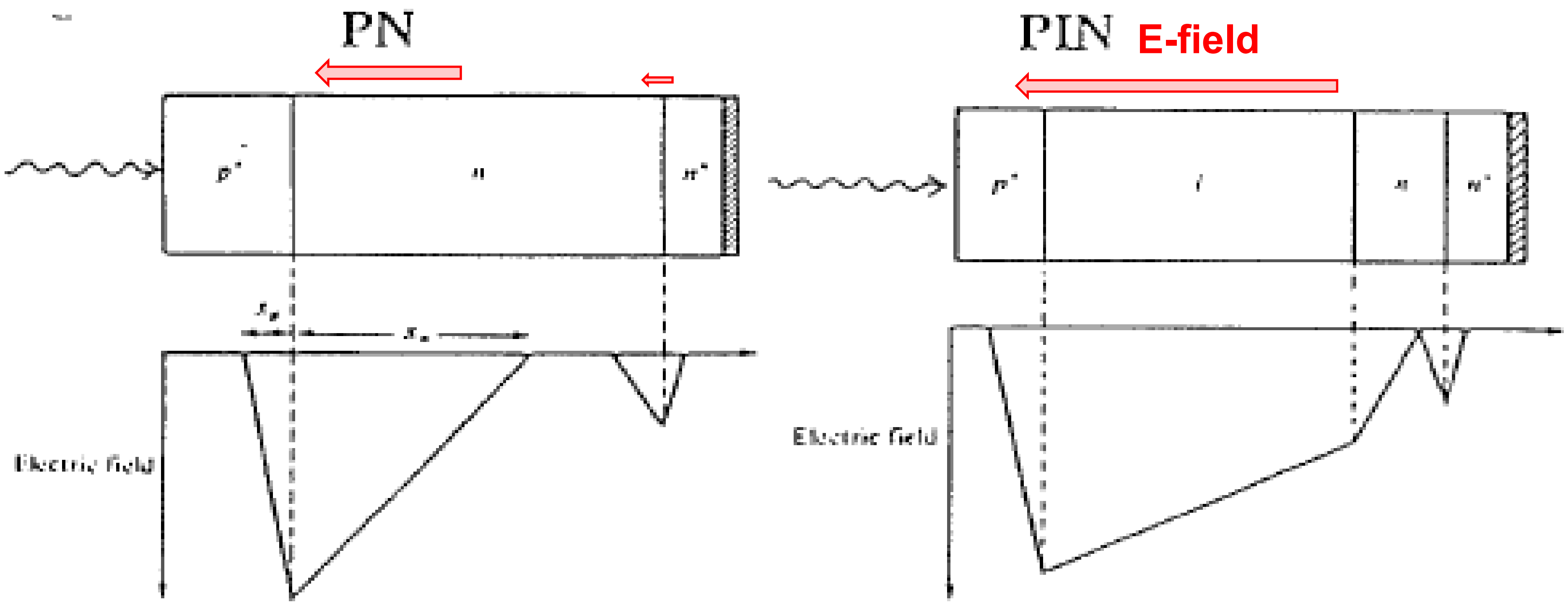
5.3 PIN Photodiode



Q: How can the depletion region size and η_{II} be increased?

Intrinsic region: very low doping

5.3 PIN: Broad and Homogeneous Electric Field

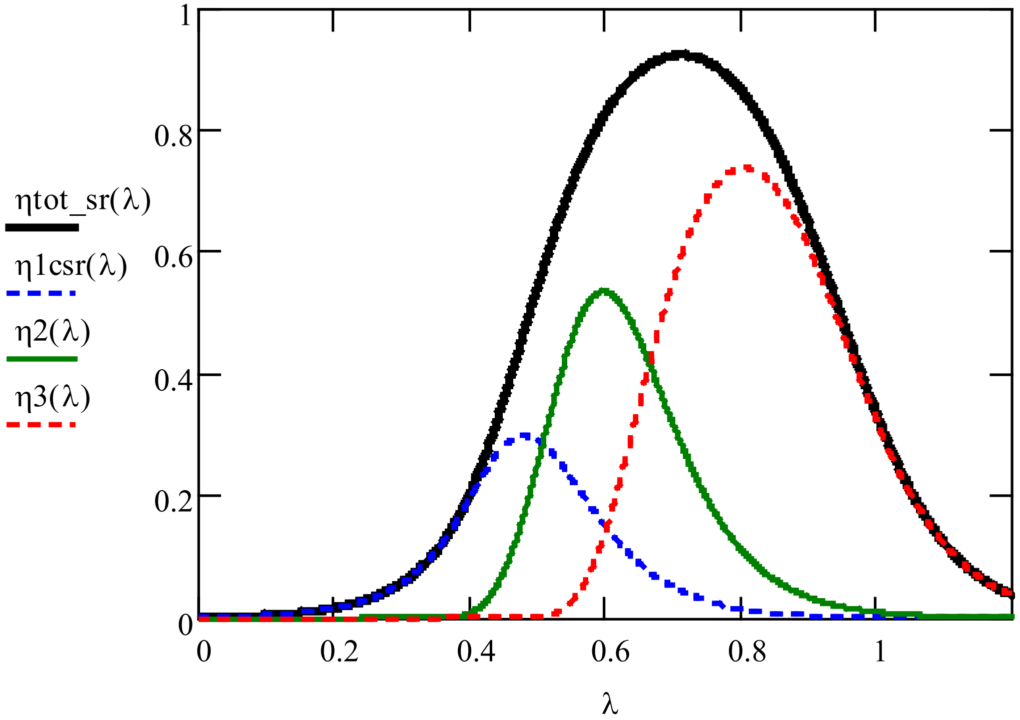


Quantum efficiency increased especially in the red and NIR

J. Wilson, J.F.B. Hawkes, « Optoelectronics, an introduction »

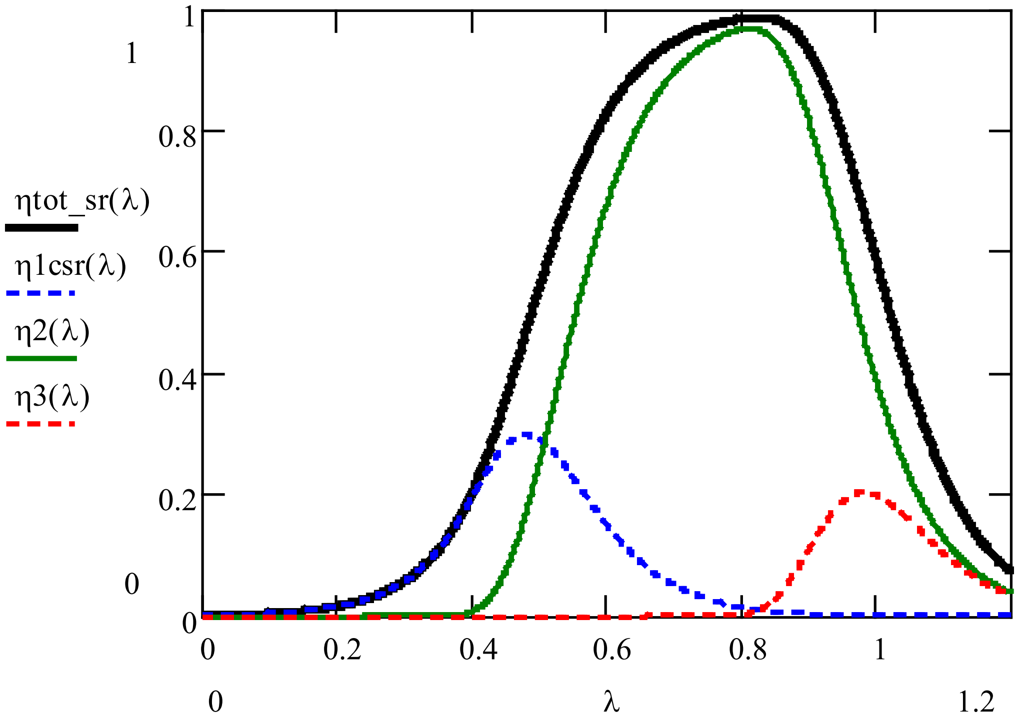
5.3 Examples of Quantum Efficiency

p-n



x_p=0.5 μm
W=2 μm

PIN



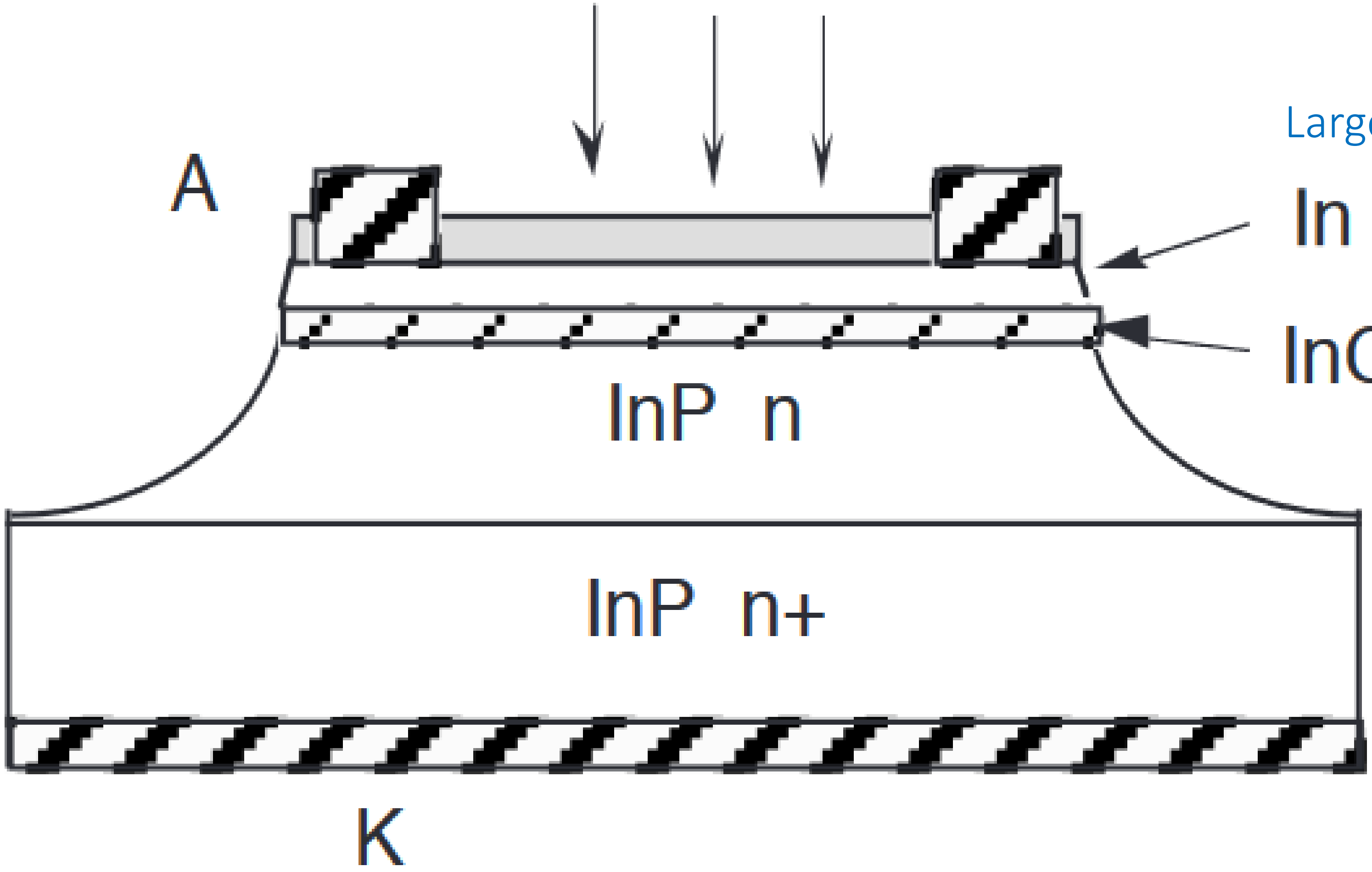
x_p=0.5 μm
W=100 μm

x_p: beginning of depletion region

Large QE increase in the intrinsic region

Drift dominates there → much faster response times/higher bandwidth

5.3 Heterojunctions for Photodiodes

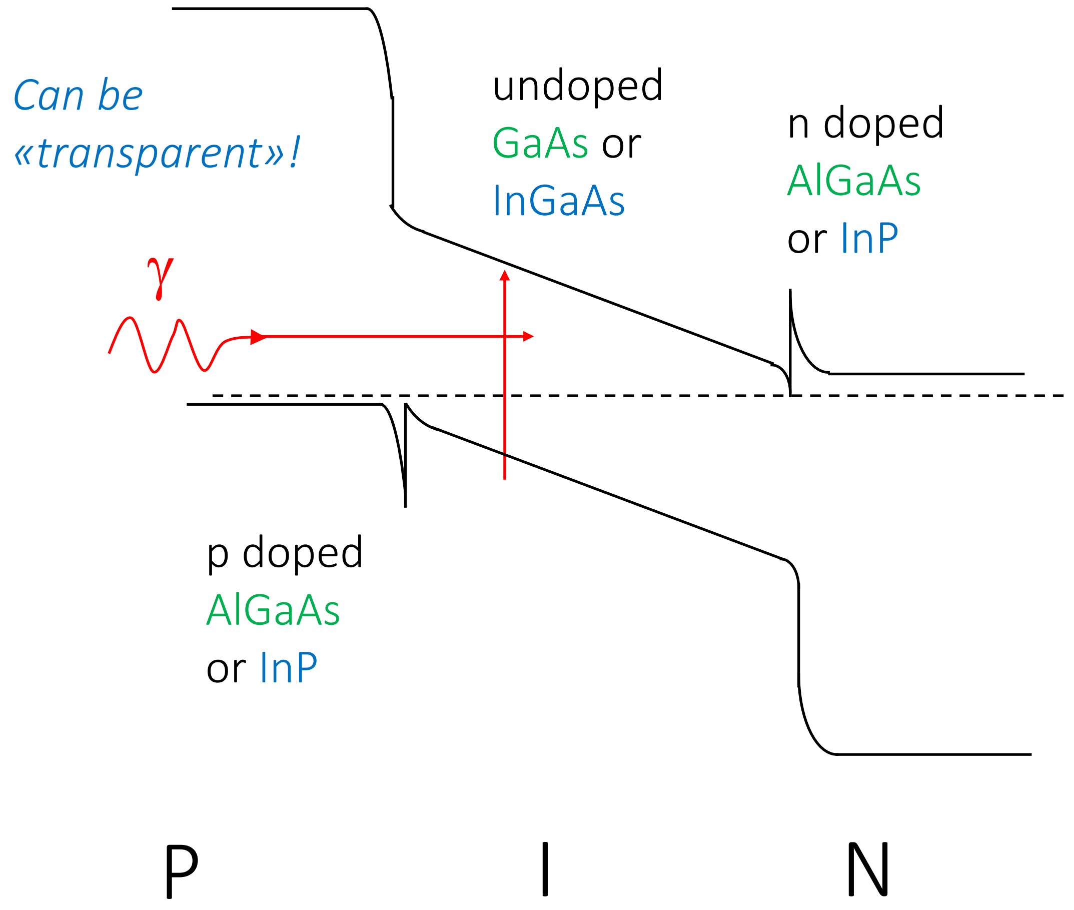


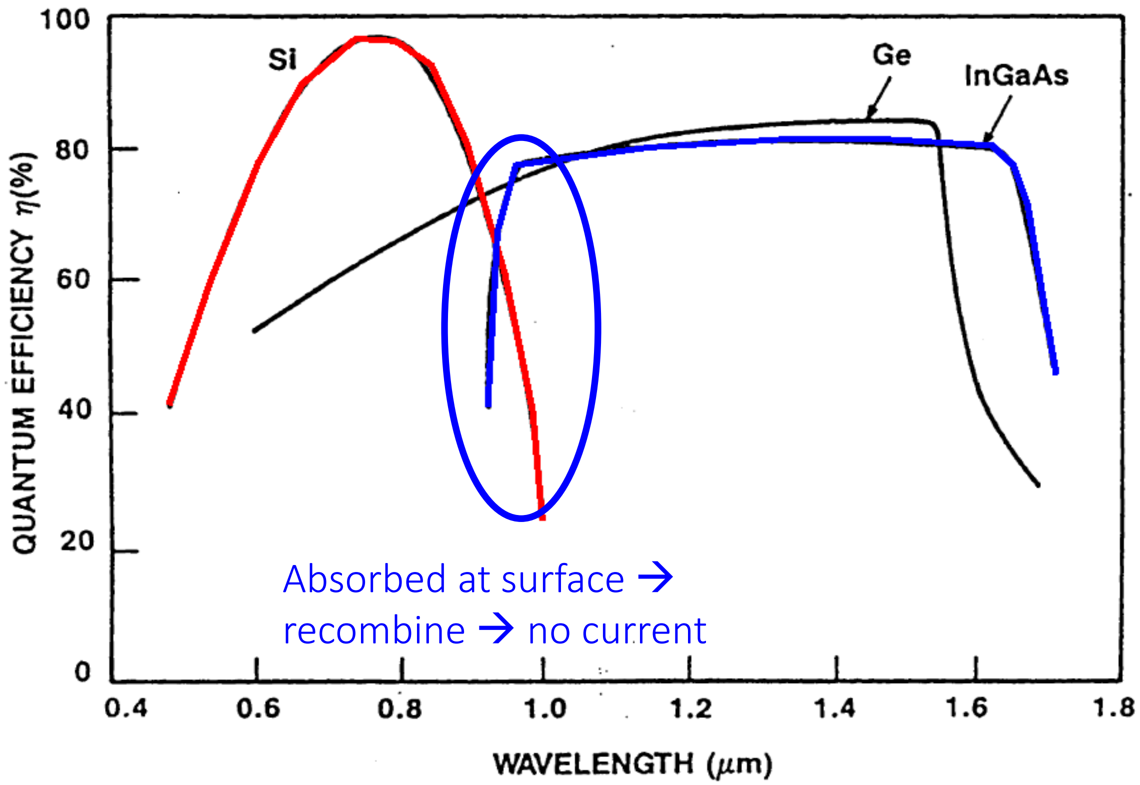
Larger bandgap

(Small) Direct bandgap material
→ Thin layer sufficient (1-5 μm)

S. Donati, « Photodetectors, devices, circuits and applications »

5.3 Heterojunctions for Photodiodes



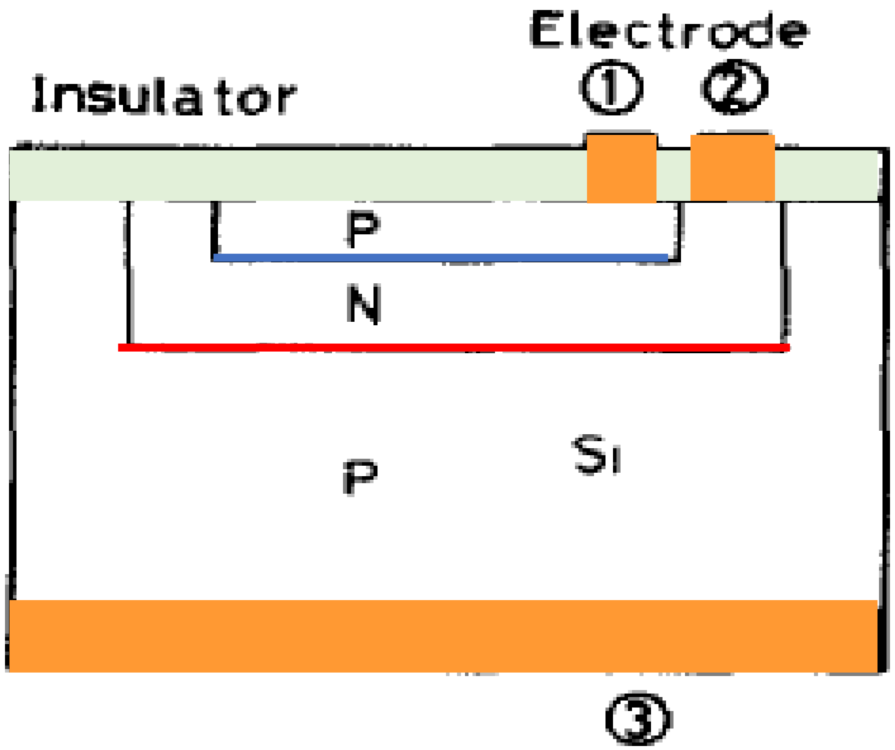


«Small» vs large bandgap (see also §1.5 Bandgaps: InP: bandgap 1.35 ev, cut-off 0.92 μm)

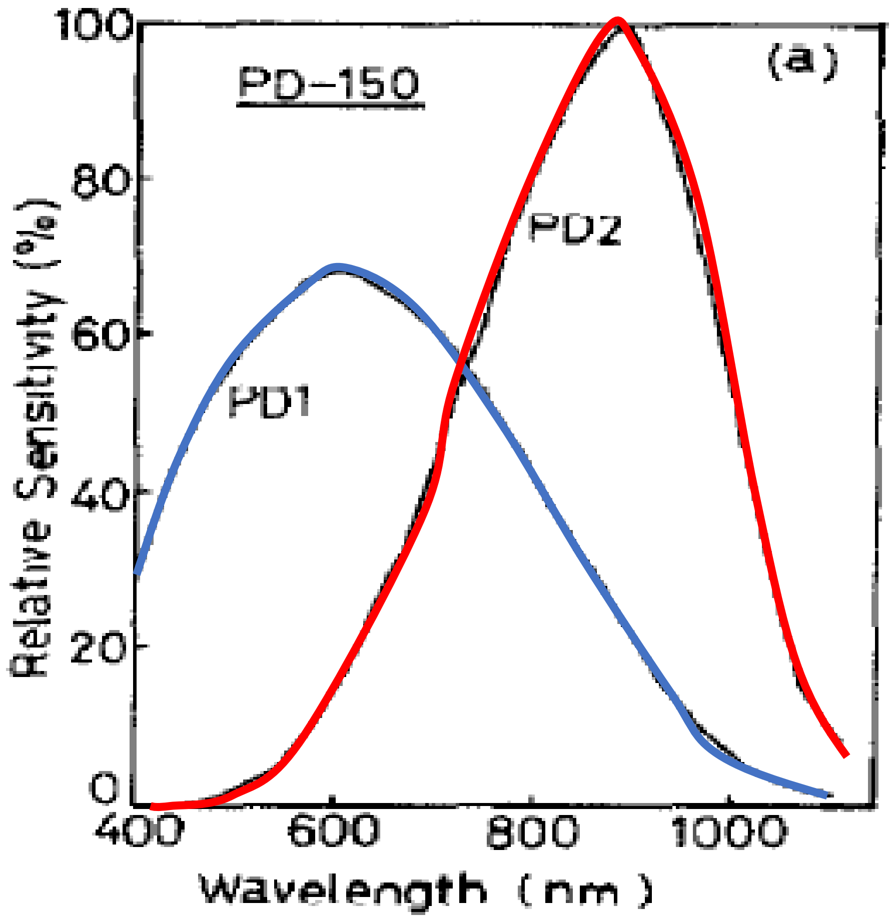
Outline

5.4 Case Study: Color Sensors

Double photodiode



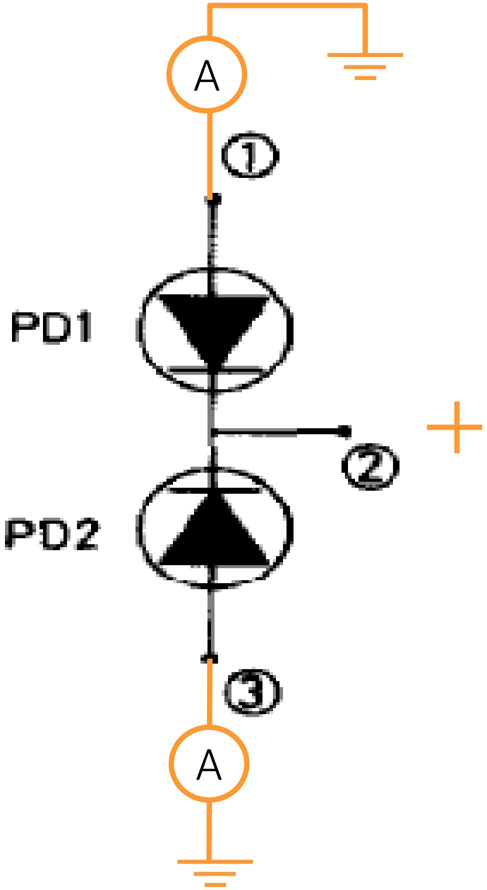
N KAKO, *Sensors and Actuators, 4 (1983) 655 - 660*



PNP transistor, two junctions in reverse one on top of the other → 1st diode: blue (surface), 2nd diode: red (depth)

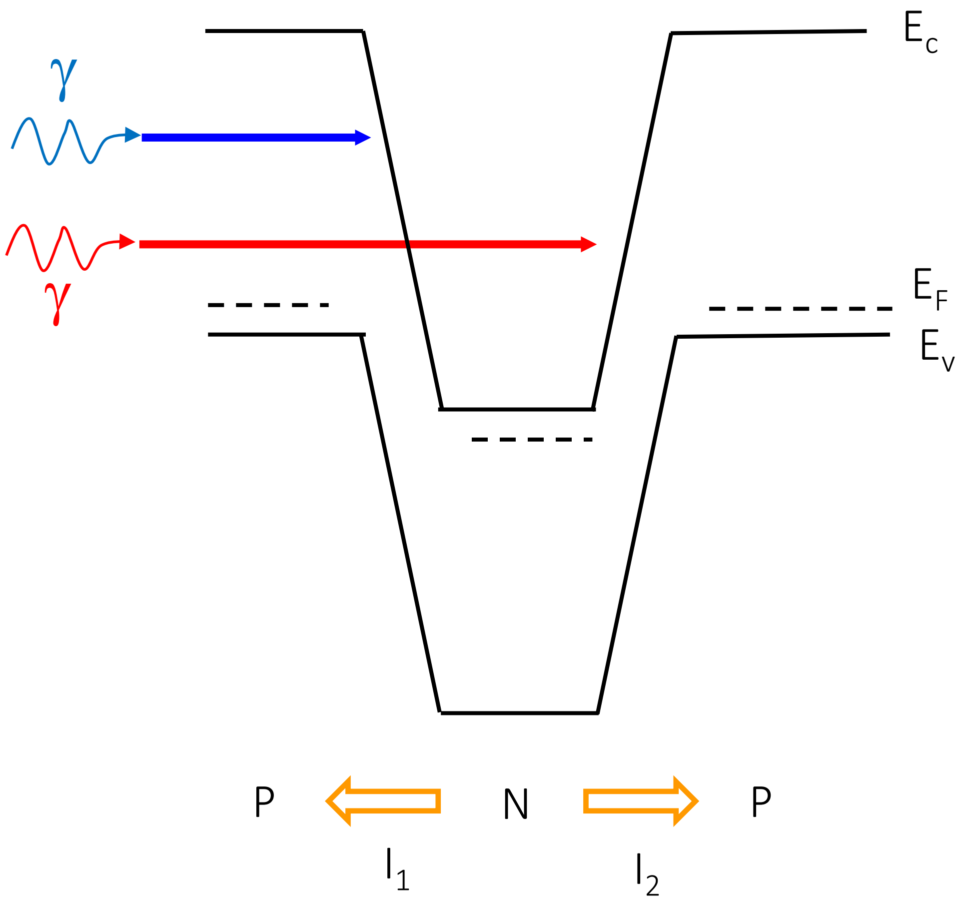
Example of application: flame colour detection (blue = ok; yellow = too little oxygen)

5.4 Color Sensors



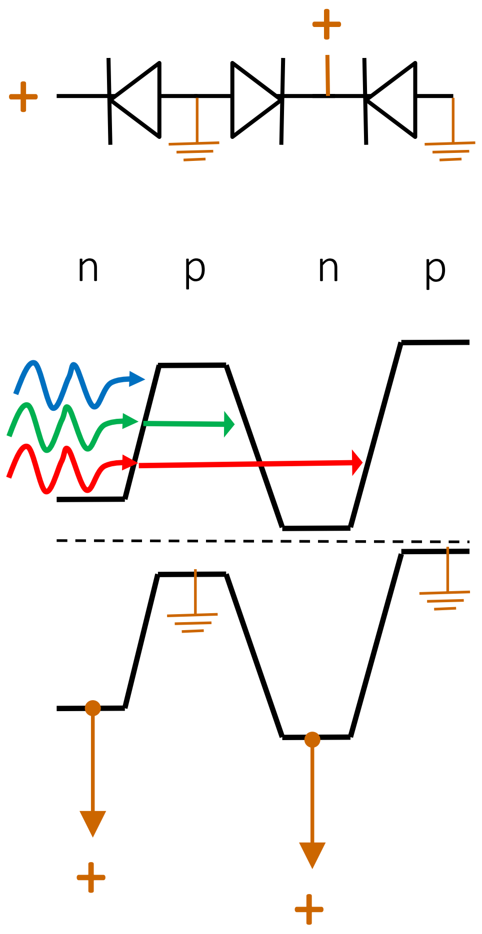
Both junctions in reverse

Bipolar PNP structure in reverse-bias (transistor in «cut-off mode»)

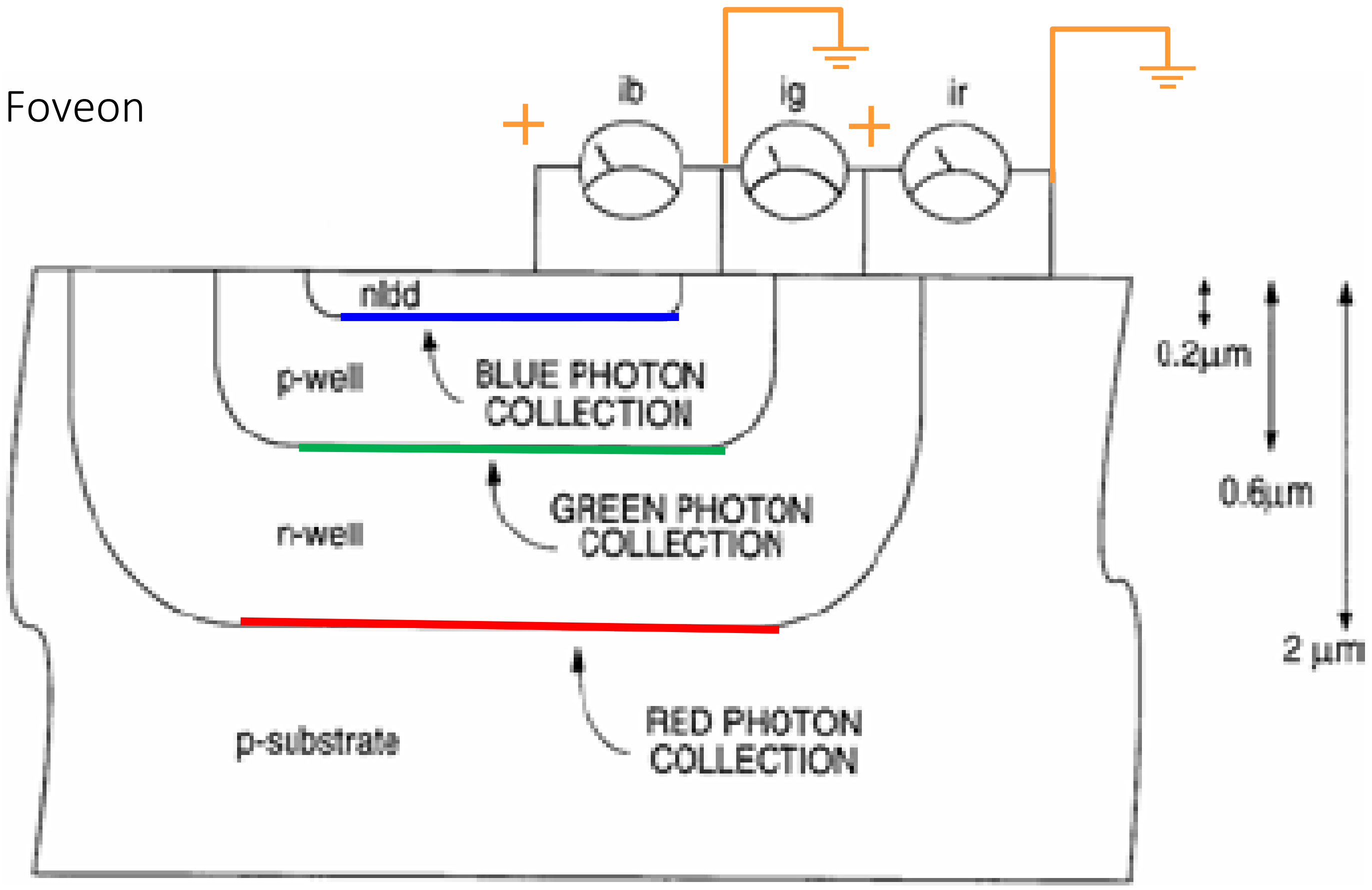


N KAKO, Sensors and Actuators, 4 (1983) 655 - 660

5.4 Foveon X3: Pixel Design on Silicon



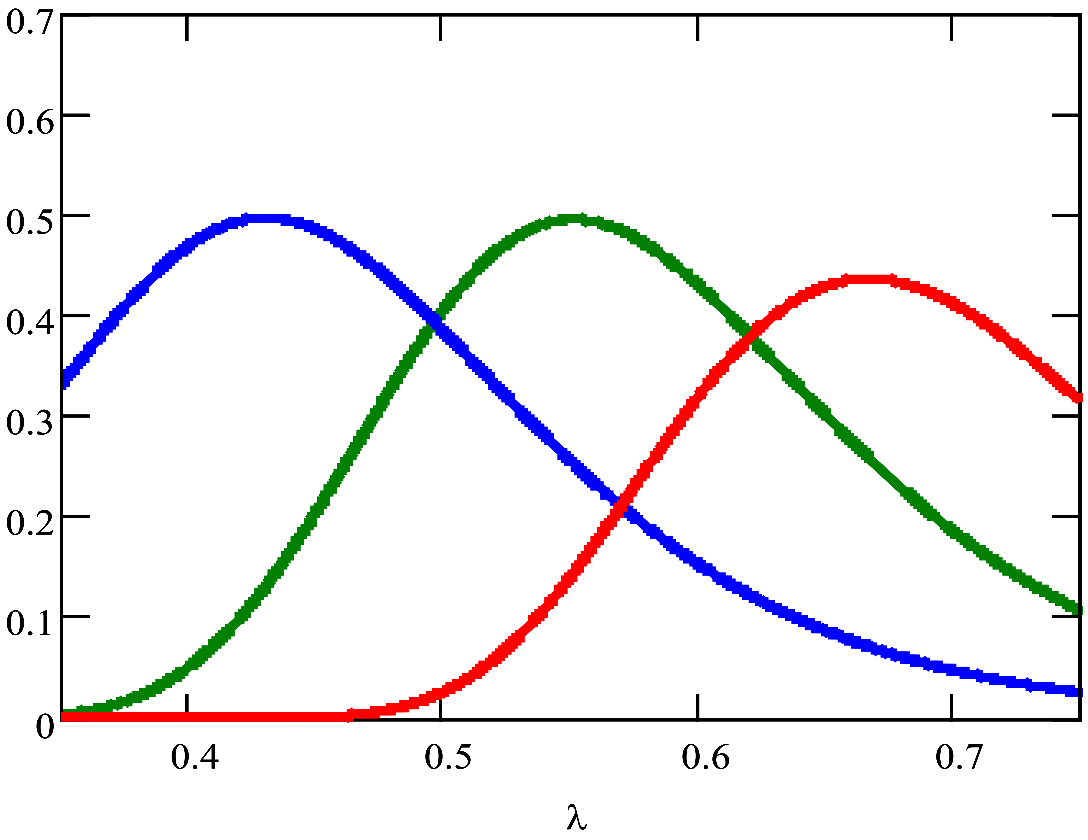
Foveon



Three junctions in reverse!
No colour filters.

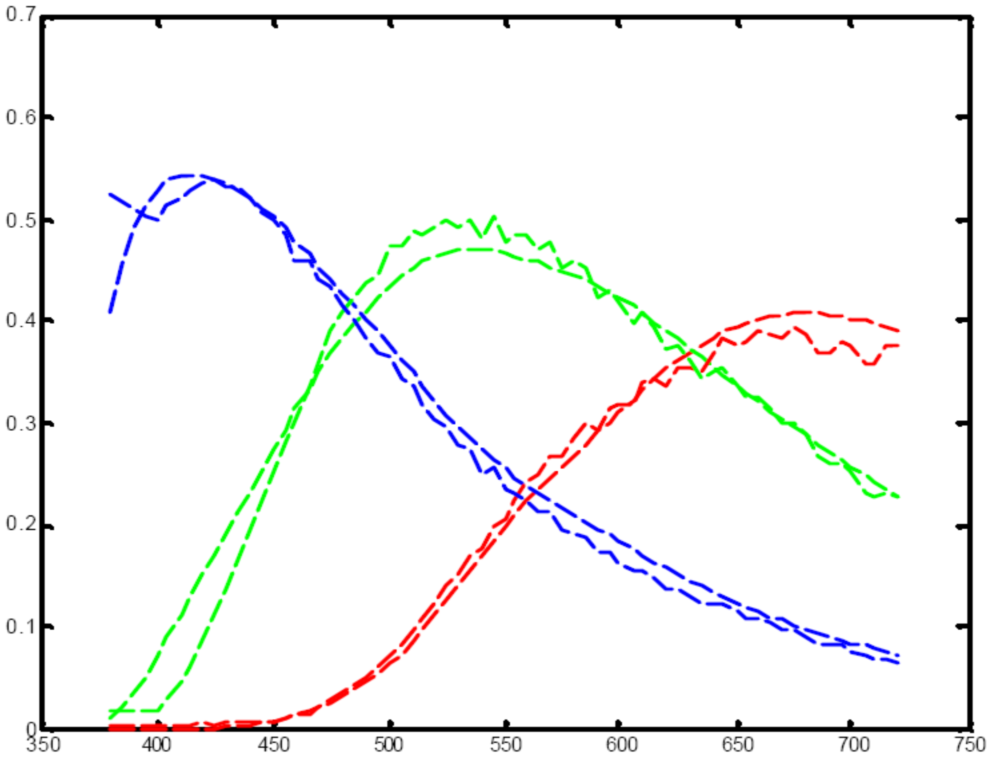
R.F. Lyon and P. M. Hubel: « Eyeing the camera: into the next century », 10th Color Imaging Conference, 2002

5.4 Spectral Response



Our simple model

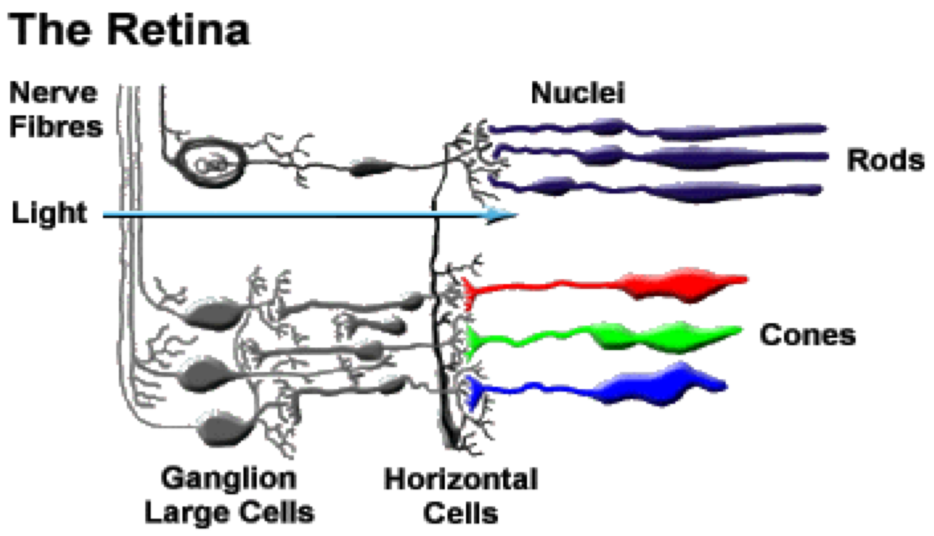
Any other sensor behaving like this?

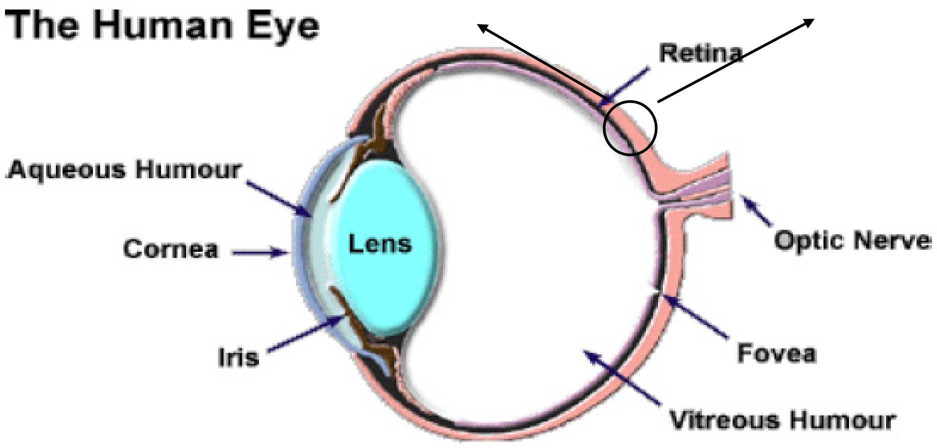


Foveon publication:
Measurements and model

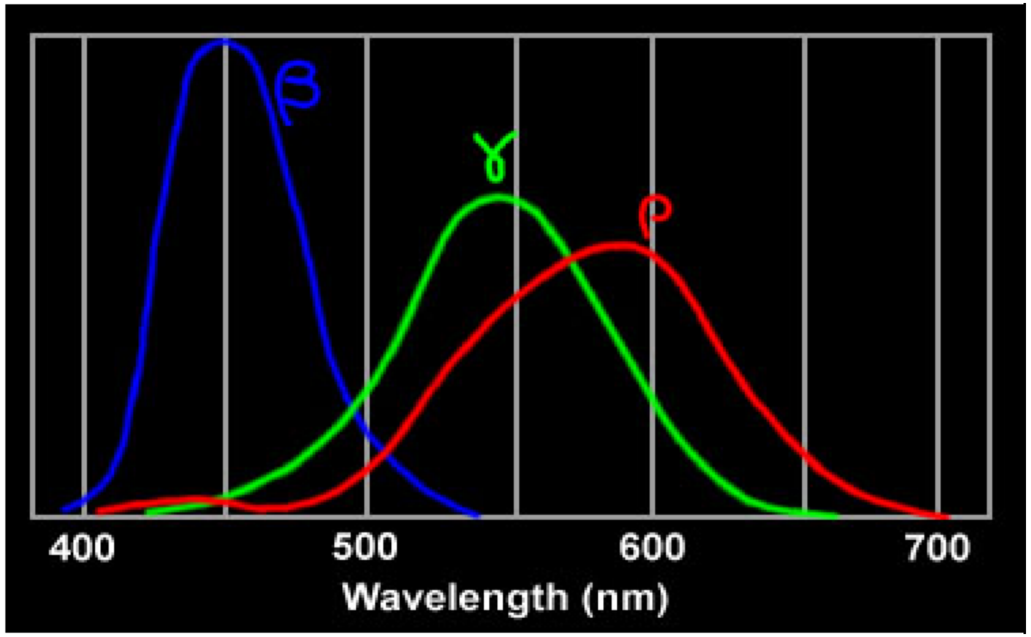
P. Hubel: «Foveon technology and the changing landscape of digital cameras », 13th IS&T Color Imaging Conference pp. 314-317, 2005

5.4 Human Vision



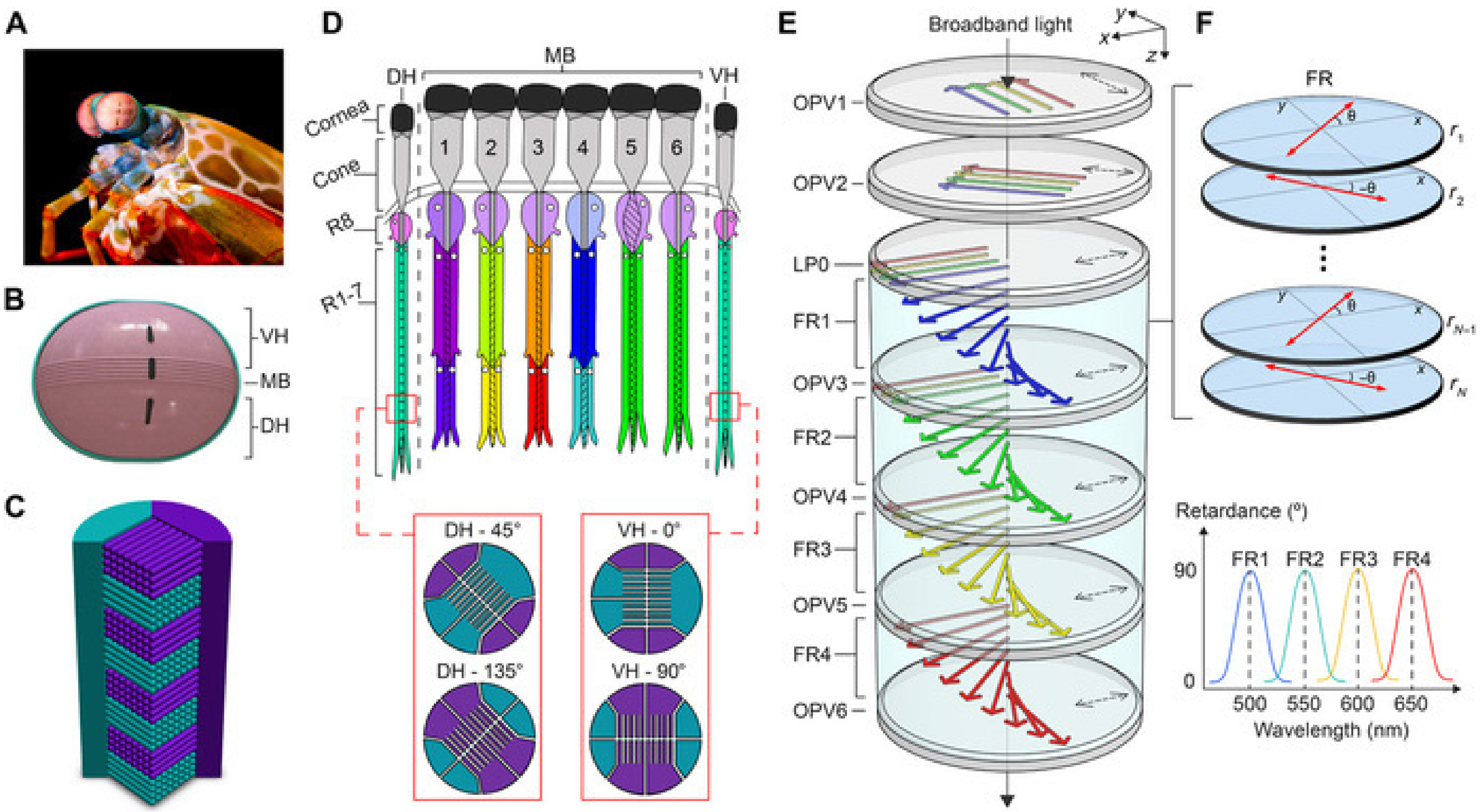


Spectral response of the cones



http://www.photo.net/photo/edscott/vis00010.htm

5.4 An example from nature: the Mantis shrimp

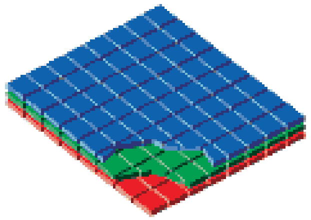


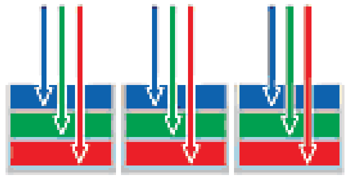
Mantis shrimp–inspired organic photodetector for simultaneous hyperspectral and polarimetric imaging, Volume: 7, Issue: 10, DOI: (10.1126/sciadv.abe3196)

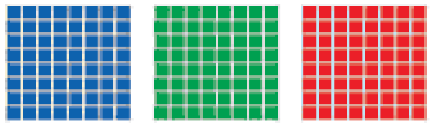
5.4 Vertical or Mosaic Structure

Foveon X3® Capture

Foveon







A Foveon X3 direct image sensor features three separate layers of pixel sensors embedded in silicon.

Since silicon absorbs different wavelengths of light at different depths, each layer records a different color. Because the layers are stacked together, all three colors are captured.

As a result, only Foveon X3 direct image sensors capture red, green, and blue light at every pixel location.

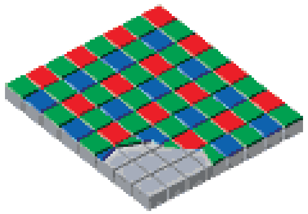
http://www.foveon.com/article.php?a=113

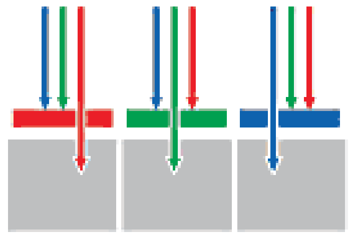
Polaroid x530

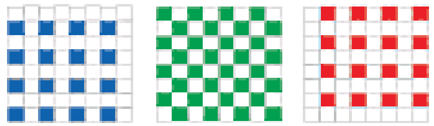
4.5 Megapixels → 5.5 Megapoints

Mosaic Capture

Bayer filter







In conventional systems, color filters are applied to a single layer of pixel sensors in a tiled mosaic pattern.

The filters let only one wavelength of light–red, green, or blue–pass through to any given pixel location, allowing it to record only one color.

As a result, mosaic sensors capture only 25% of the red and blue light, and just 50% of the green.

http://www.foveon.com/article.php?a=69

Sigma SD14

14.1 Megapixels → 4.7 Megapoints

http://www.sigma-photo.com/cameras/

«RAW» mode vs 3x fitted/interpolated colour pixels (vs monochrome!)

Take-Home Messages/W5-2

5.3 pin photodiodes:

- What are the advantages of PIN photodiodes for the photon detection?

5.4 Heterojunctions:

- Why are heterojunctions used to fabricate high-performance photodiodes?

5.5 Colour sensors:

- Describe the photon absorption in a semiconductor as function of the wavelength of the light.
- How can a color sensor be fabricated with photodiodes?

Outline

5.1 p-n junctions (review)

5.2 p-n photodiodes

5.3 PIN photodiodes

5.4 Case study: color sensors

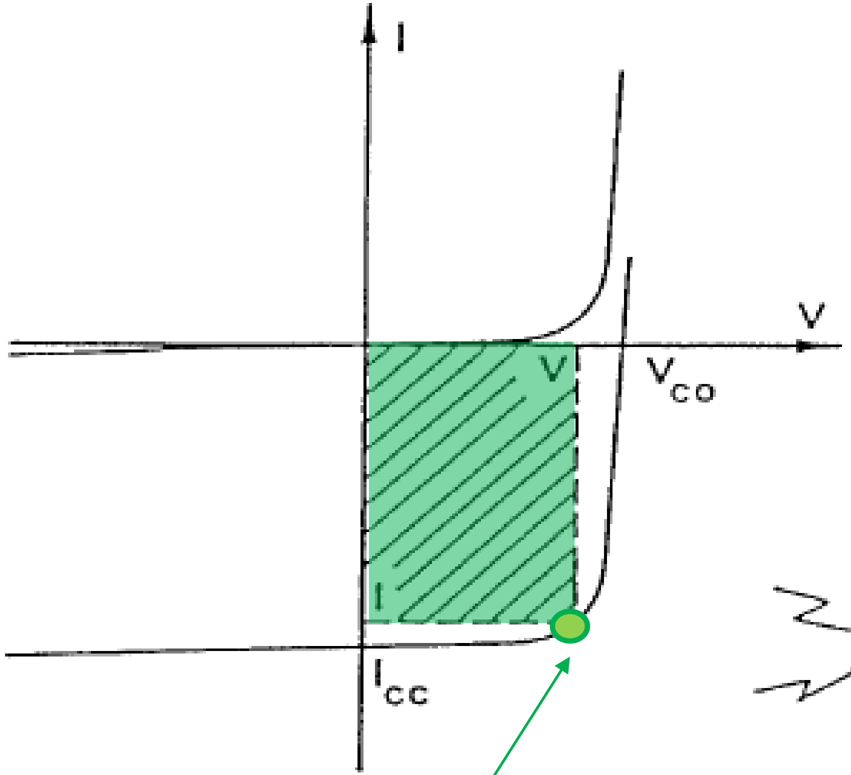
5.5 Solar cells

5.6 Electronic circuits, cut-off frequencies, noise

5.7 Avalanche photodiodes

5.5 Solar Cells

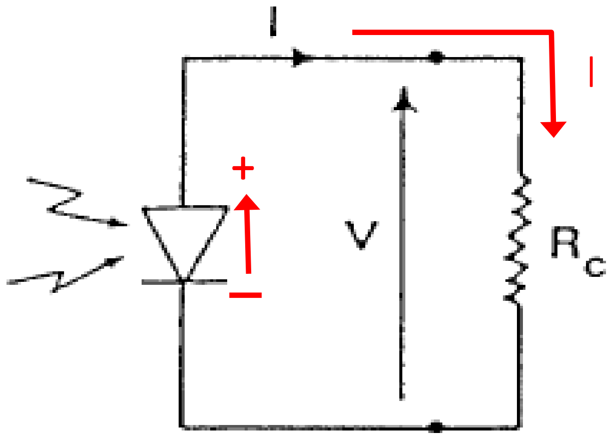
$$P_{el} = U \cdot I$$



Power = shaded area (rectangle) – direct power generation

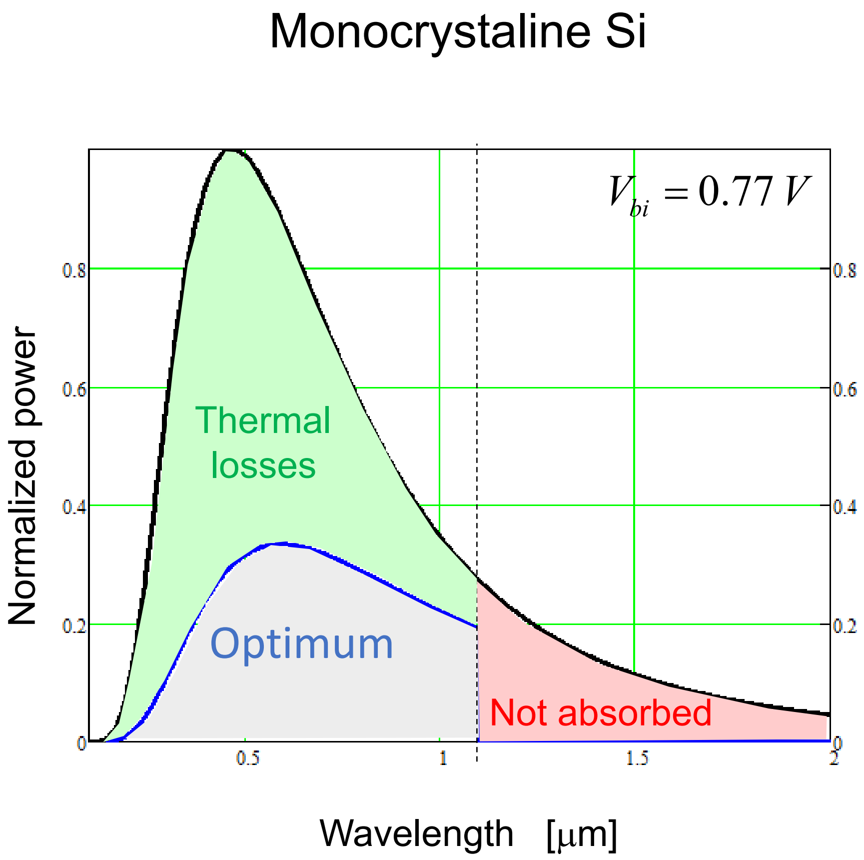
Optimal load resistance R_c:

$$R_c = \frac{V_{\max}}{I_{\max}}$$

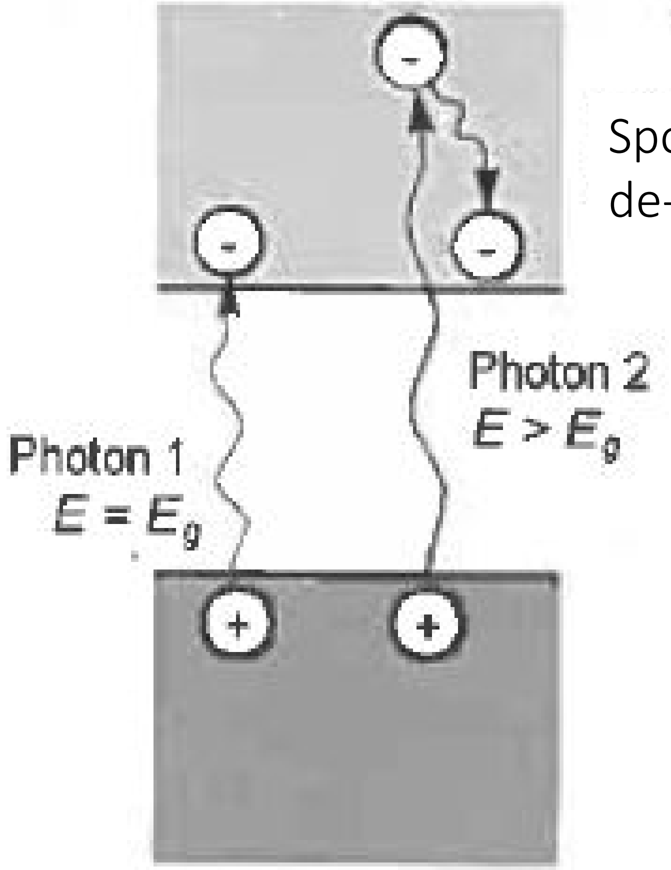


$$P_{el} \cong V_{bi} \cdot I_{ph} = V_{bi} \cdot R_I P_{opt}$$

5.5 Maximum Efficiency of Solar Cells



η_{tot} = 31 % (E_g=1.1 eV)



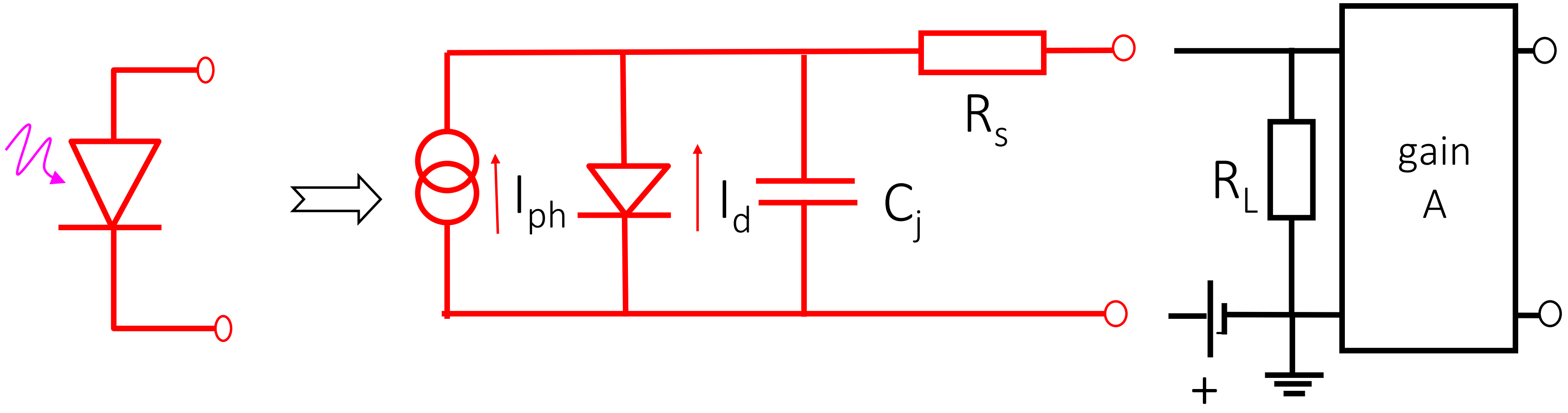
Spontaneous de-excitation

Thermal losses: we generate «only» the bandgap energy

A. Labouret, M. Villoz, « installations photovoltaiques », Dunod, 2012

5.5 Equivalent Circuit of a Photodiode

R_d = differential (shunt) resistance = $1/\frac{\delta I_d}{\delta V}$



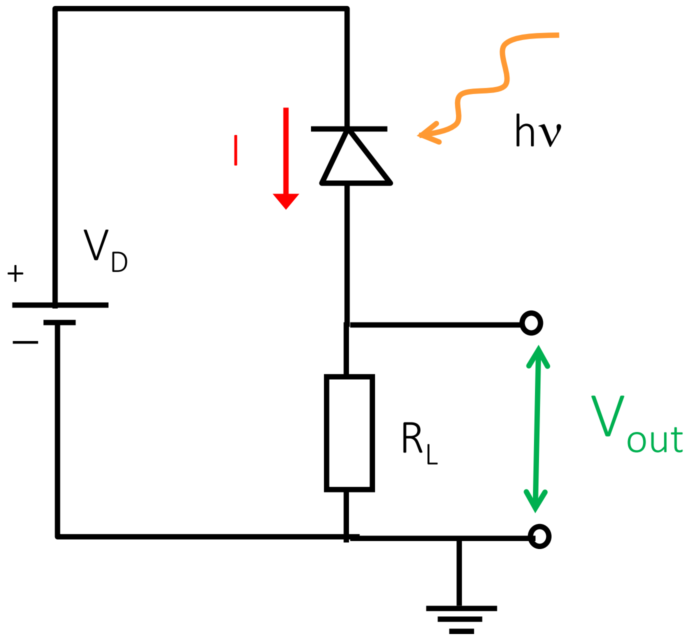
C_j = junction capacitance
R_L = load resistance
R_S = series contact resistance

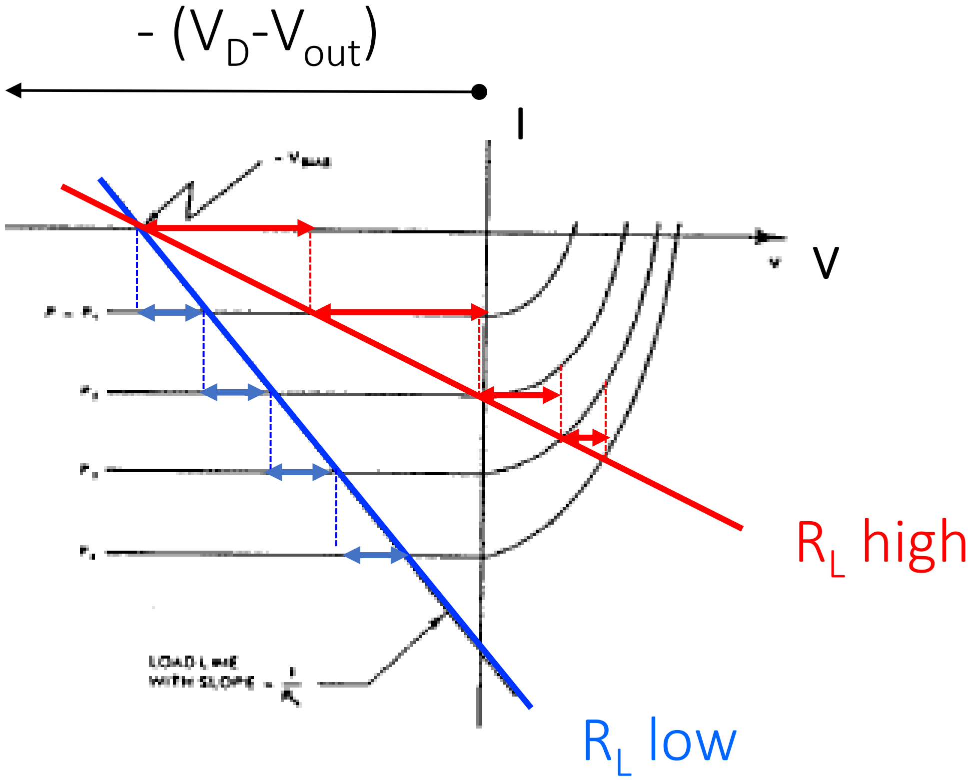
- Dark current: I_d (thermal generation)

- Photocurrent: $I_{ph} = I_{sig\text{-}AC} + I_{sig\text{-}DC} + I_{bg}$

Outline

5.6 Measuring Circuit (1): Load Resistance



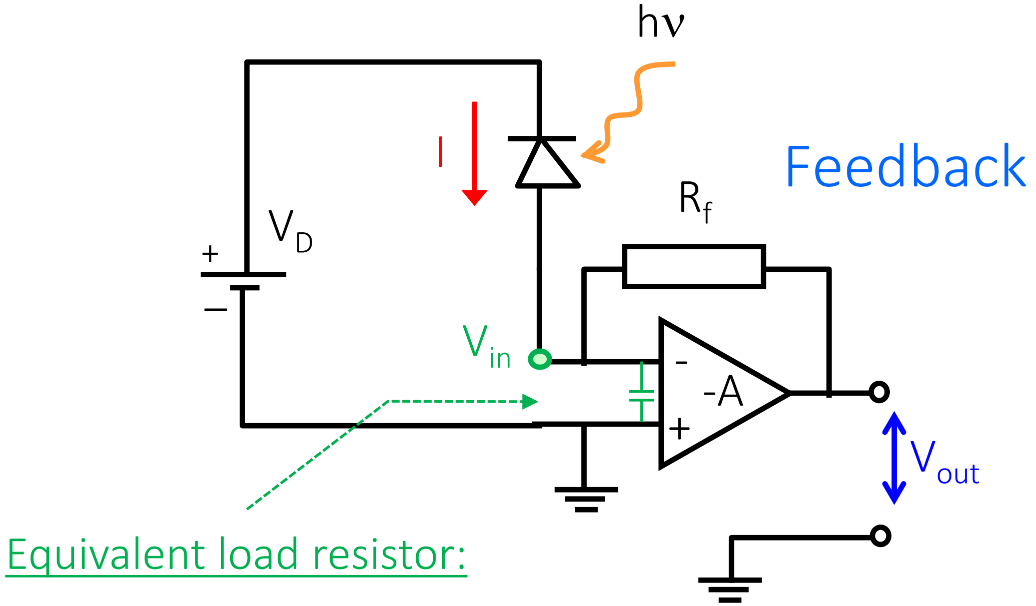


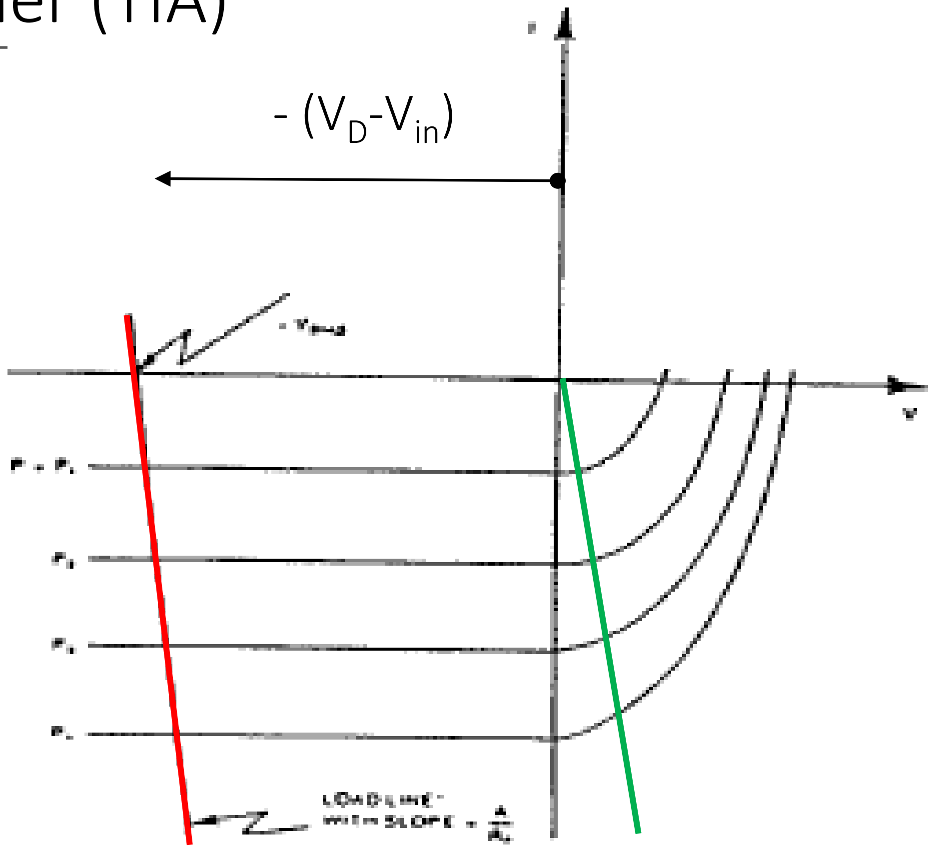
$$V_{out} = R_L \cdot I$$

Only rough measurements possible, small dynamic range

Load line (slope): $1/R_L$

5.6 Measuring Circuit (2): Transimpedance amplifier (TIA)





$$R_f I = V_{in} - V_{out} = (1 + A) \cdot V_{in} \qquad -A \cdot V_{in} = V_{out} \cong -R_f \cdot I$$

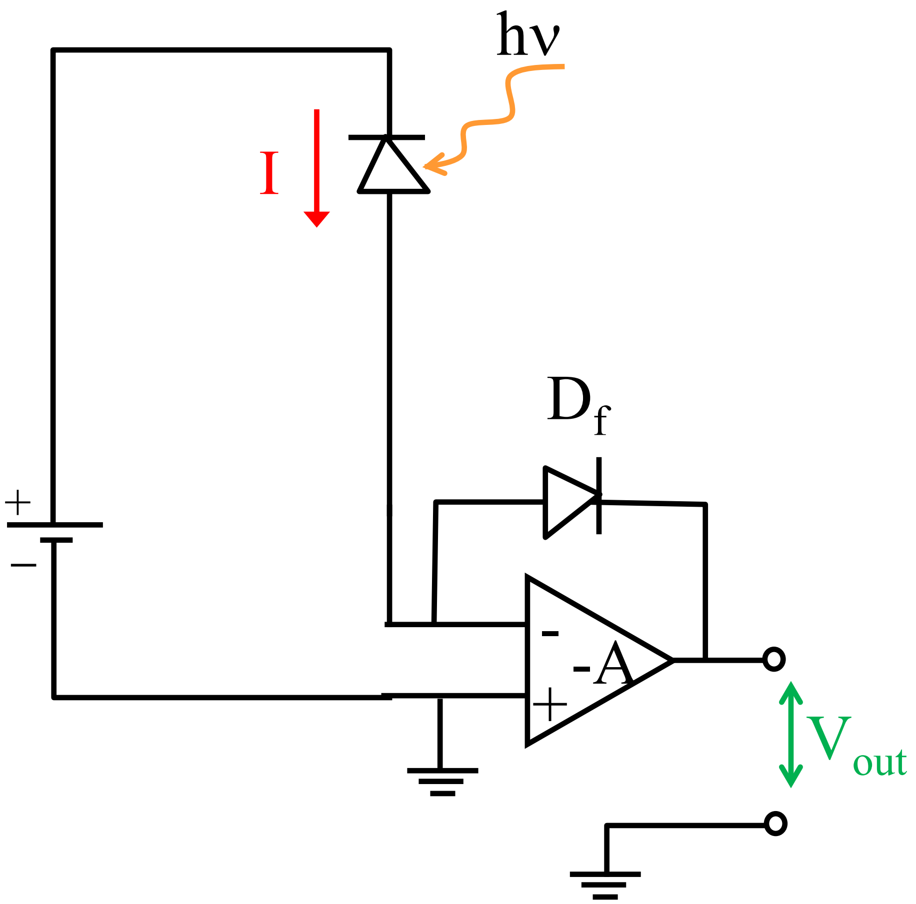
$$V_{in} = \frac{R_f}{A+1} \cdot I \equiv R_L \cdot I \quad \Rightarrow \quad R_L \equiv \frac{R_f}{A+1}$$

Load line: $1/R_L$

Improved linearity (R_L small)

with high gain (R_f large)

5.6 Measuring Circuit (3): Transimpedance and Logarithmic Output



Diode current: $I \cong I_0 \cdot e^{\frac{-qV_{out}}{kT}}$

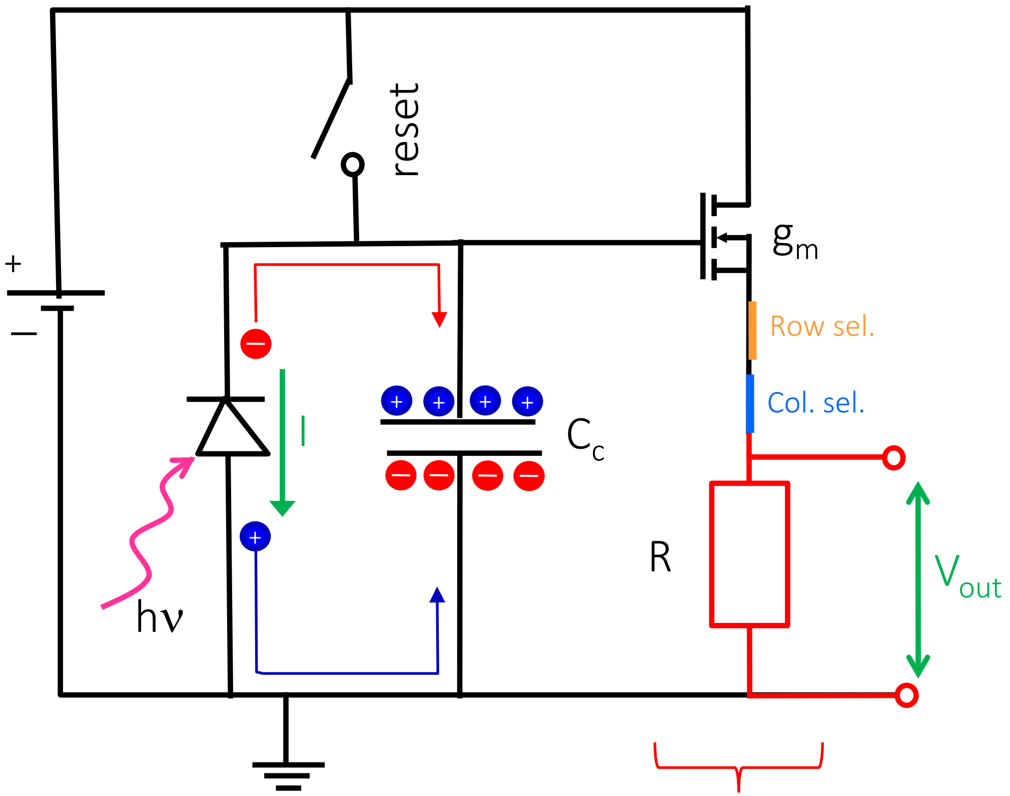
$$\Downarrow$$

$$V_{out} \cong V_0 - \frac{kT}{q}\ln(I) \quad < 0$$

$$V_0 \cong \frac{kT}{q}\ln(I_0)$$

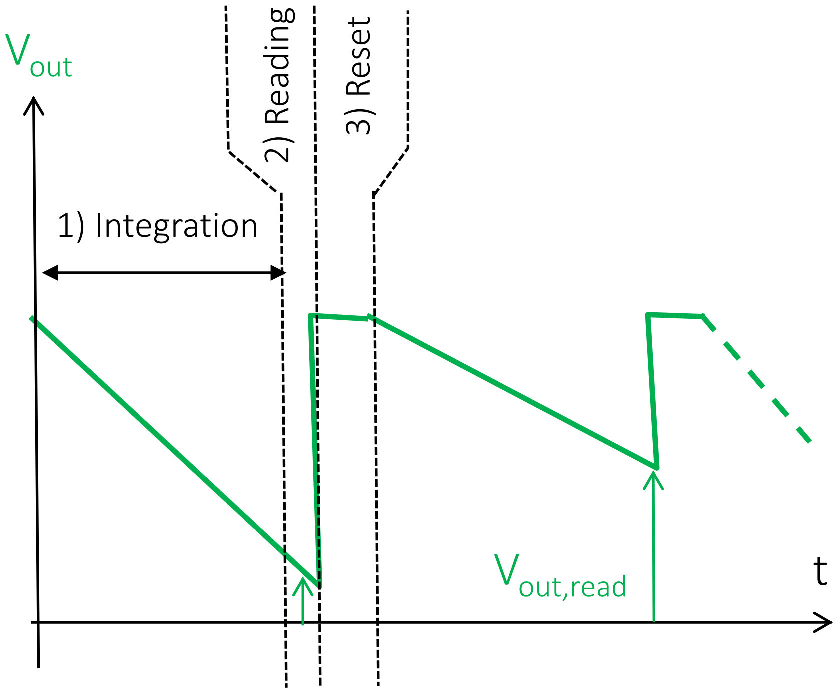
5.6 Measuring Circuit (4): Integration and Current-to-Voltage Conversion

3T APS – basic idea of CMOS camera pixel (one single pixel shown here)



Source Follower

NMOS transistors



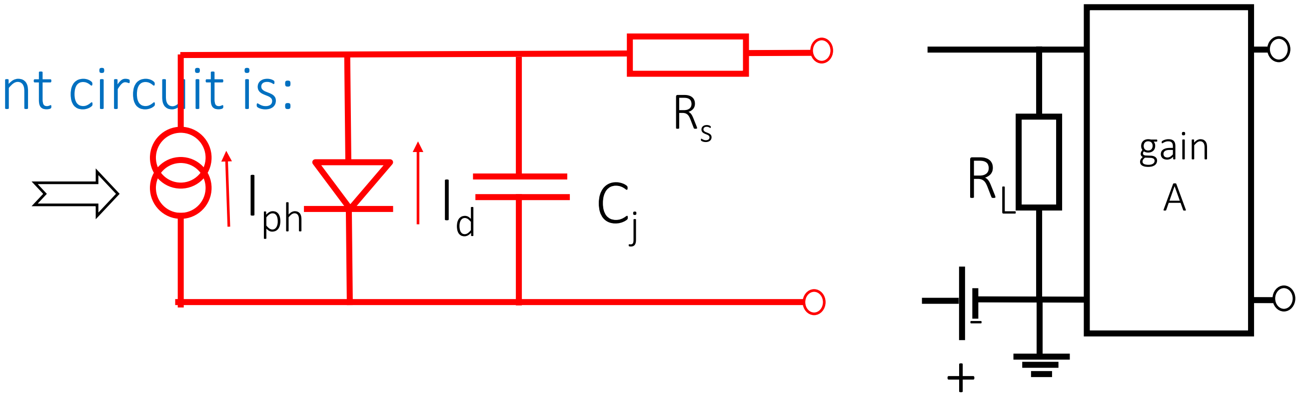
$$V_{out,read} = V_0 - \frac{Q}{C_c} = V_0 - \frac{1}{C_c} \cdot \int_0^{\tau} I \cdot dt$$

Charge integration

5.6 Cut-off Frequencies

1) The cutoff frequency of the equivalent circuit is:

$$f_2 = \frac{1}{4(R_s + R_L)C_j}$$



2) The drift time in the depletion region induces a cutoff frequency:

$$\tau_d \cong W / v_{sat} \quad \Rightarrow \quad f_{2d} \cong \frac{1}{2 \cdot \tau_d} = \frac{v_{sat}}{2 \cdot W}$$

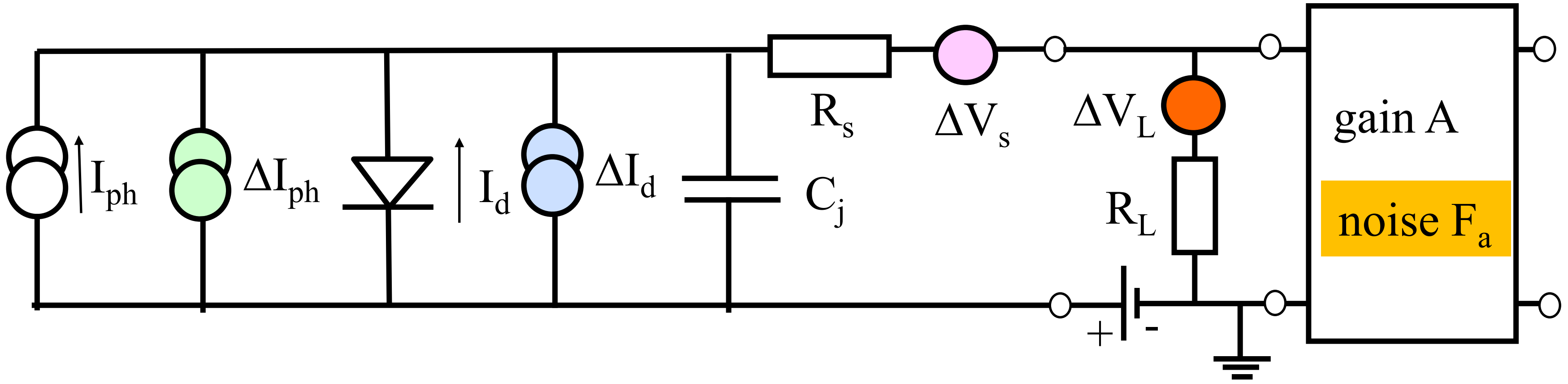
Typically ~ τ_d=10 ps

3) The diffusion time in diffusion regions is very long:

$$\tau_D \cong \frac{L^2}{D} \quad \Rightarrow \quad f_{2D} \cong \frac{1}{2\tau_D} = \frac{D}{2L^2}$$

Typically ~ τ_D=100 ns

5.6 Equivalent Circuit with Noise Sources



$$F_a = \frac{(S/N)_{in}^2}{(S/N)_{out}^2}$$

Photonic (shot) noise:

$$\Delta I_{ph}^2 = 2q\left(|I_0| + \left|I_{background}\right|\right)\Delta f$$

Thermal noise on the resistances:

$$\Delta V_s^2 = 4kTR_s\Delta f$$

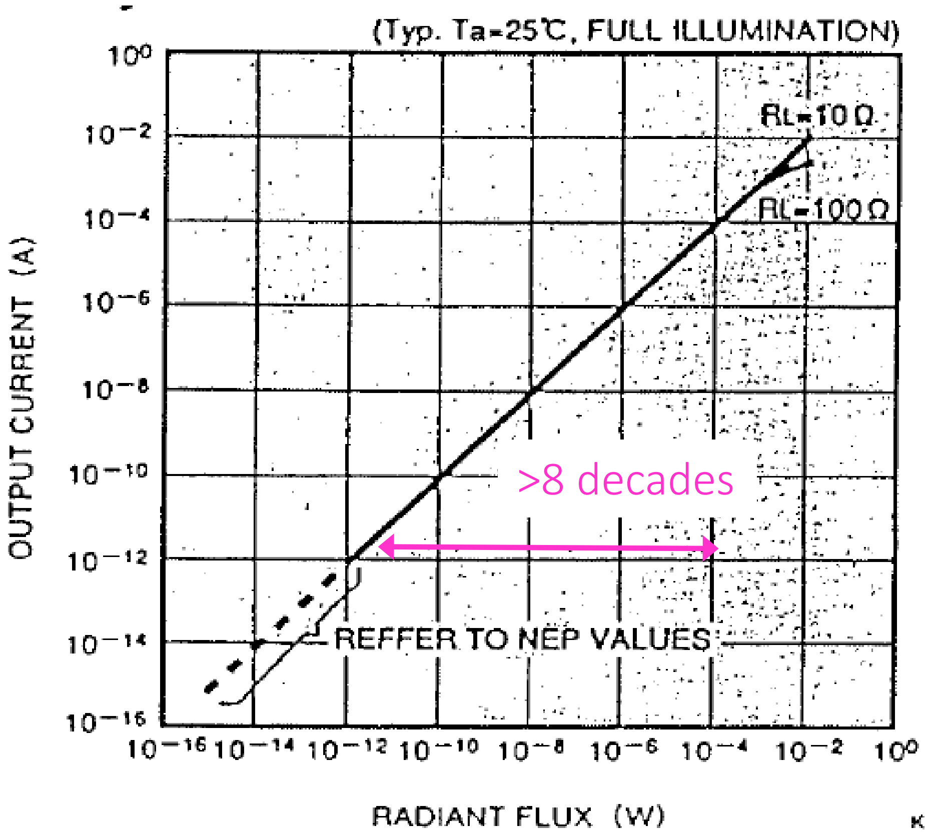
$$\Delta V_L^2 = 4kTR_L\Delta f$$

Dark current noise:

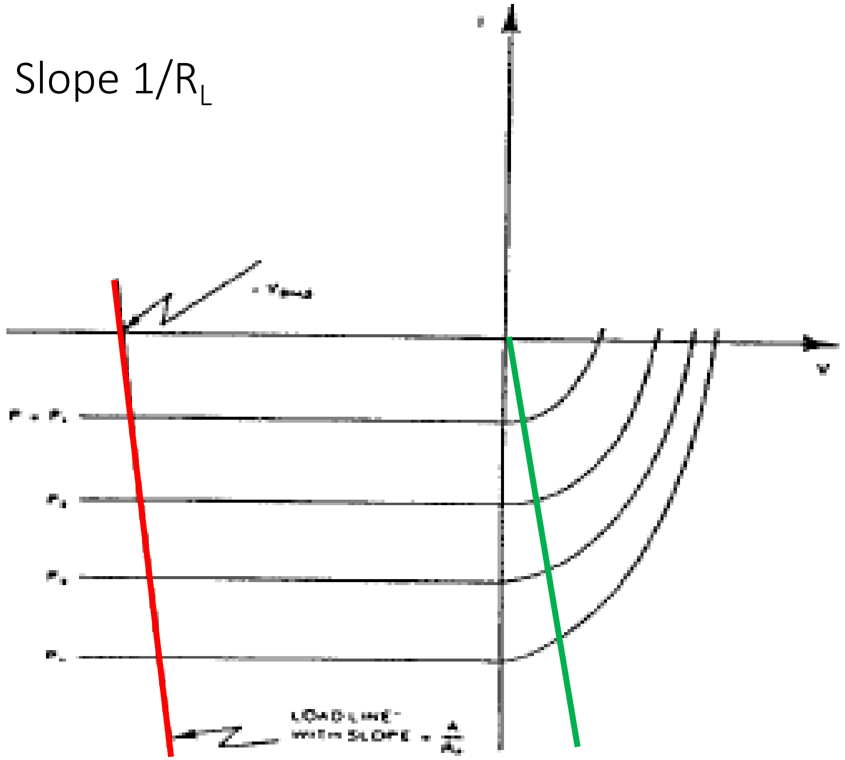
$$|\Delta I_{dark}|^2 = 2\frac{q}{\Gamma}|I_{dark}|\left(e^{qV/\Gamma kT} + 1\right)\Delta f$$

$$NB: for\ a\ practical\ diode:\ I = I_0\left(e^{qV/\Gamma kT} - 1\right)\ with\ \Gamma = ideality\ factor\ [1-2]$$

5.6 Dynamic Range



Slope $1/R_L$



$$NEP = \frac{\sqrt{F_a} \cdot \sqrt{\Delta I_{ph}^2 + \Delta I_{dark}^2 + \frac{\Delta V_s^2 + \Delta V_L^2}{R_L^2}}}{\eta q / h\nu}$$

Small R_L → less saturation effect

5.6 Photodiodes: Summary

Quantum Efficiency:	almost optimal	> 90%
Gain	no	1
Noise	electronic	Load resistance
Voltage	zero or small	0-10 V reverse
Spectrum	adaptable	UV - VIS - IR

Compatible with CMOS → cameras (see the respective chapters)

Outline

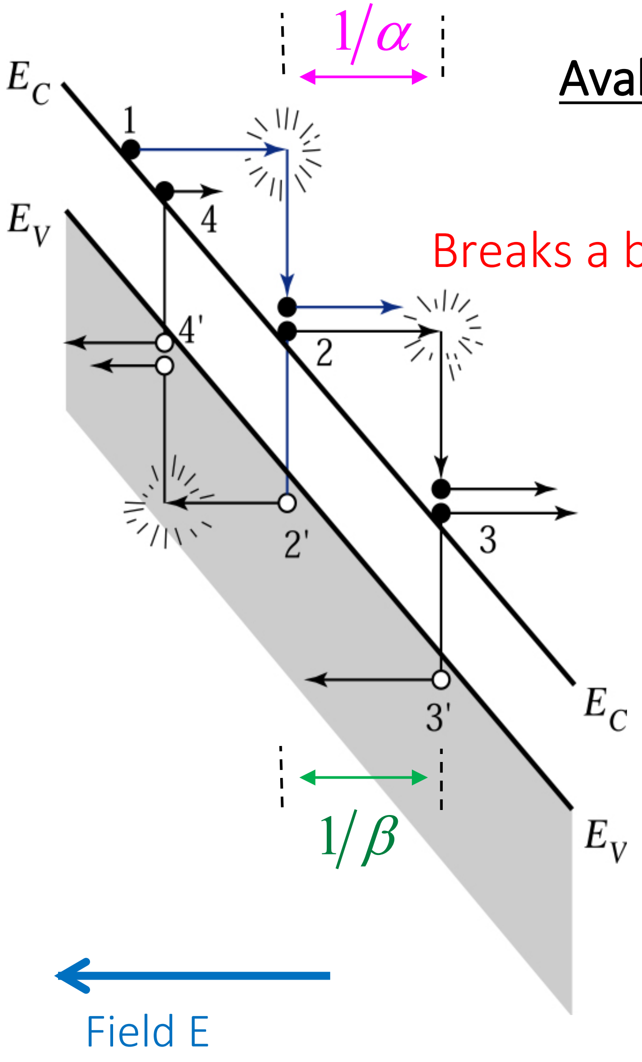
5.7 Avalanche Effect

α = ionisation coefficient of electrons [1/cm] (much more efficient than holes)

Silicon:

$$k \equiv \frac{\beta}{\alpha} \cong 0.1$$

β = ionisation coefficient of holes [1/cm]



Avalanche

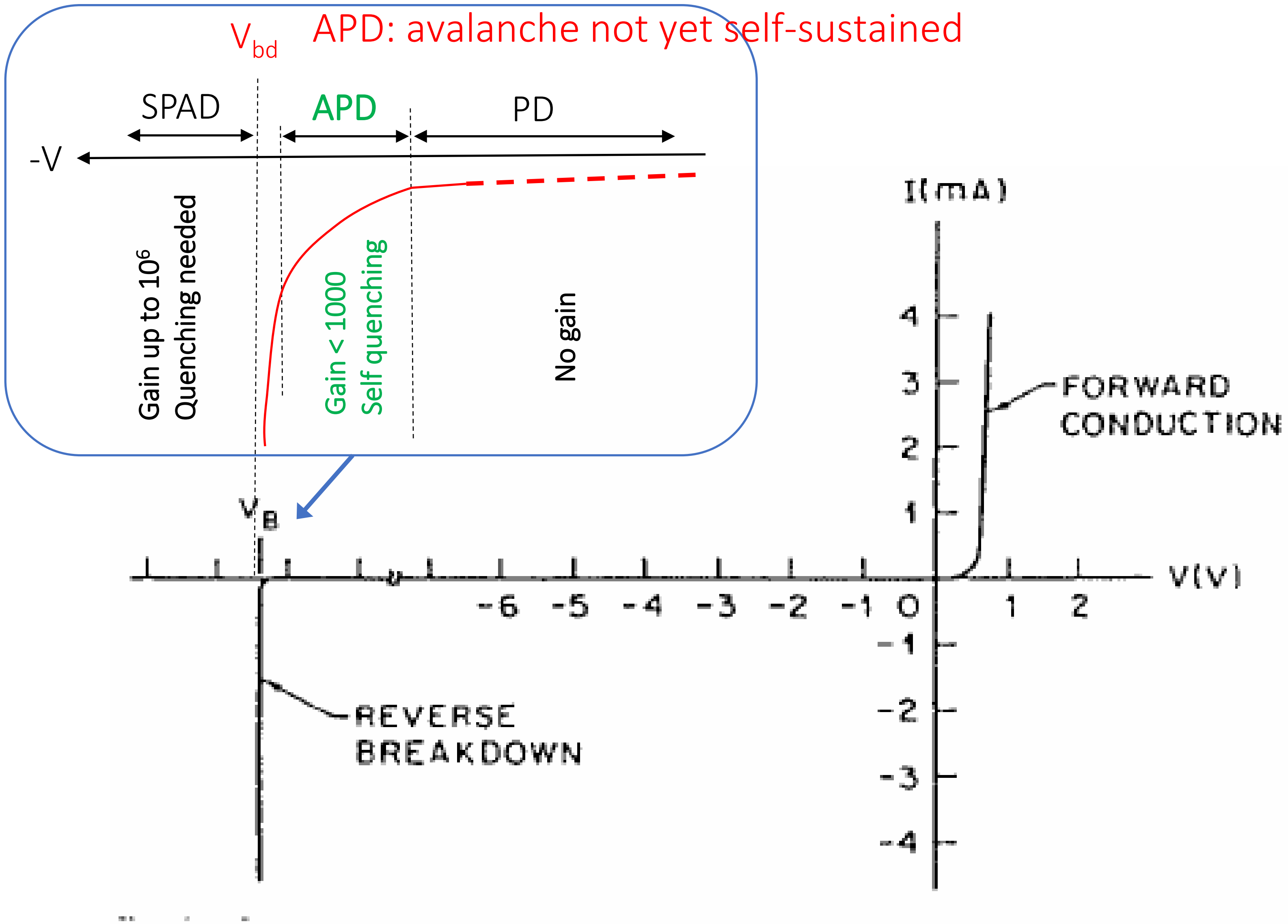
Breaks a bond → e/h pair creation

Bi-carriers !

Self-sustained above V_{bd}!

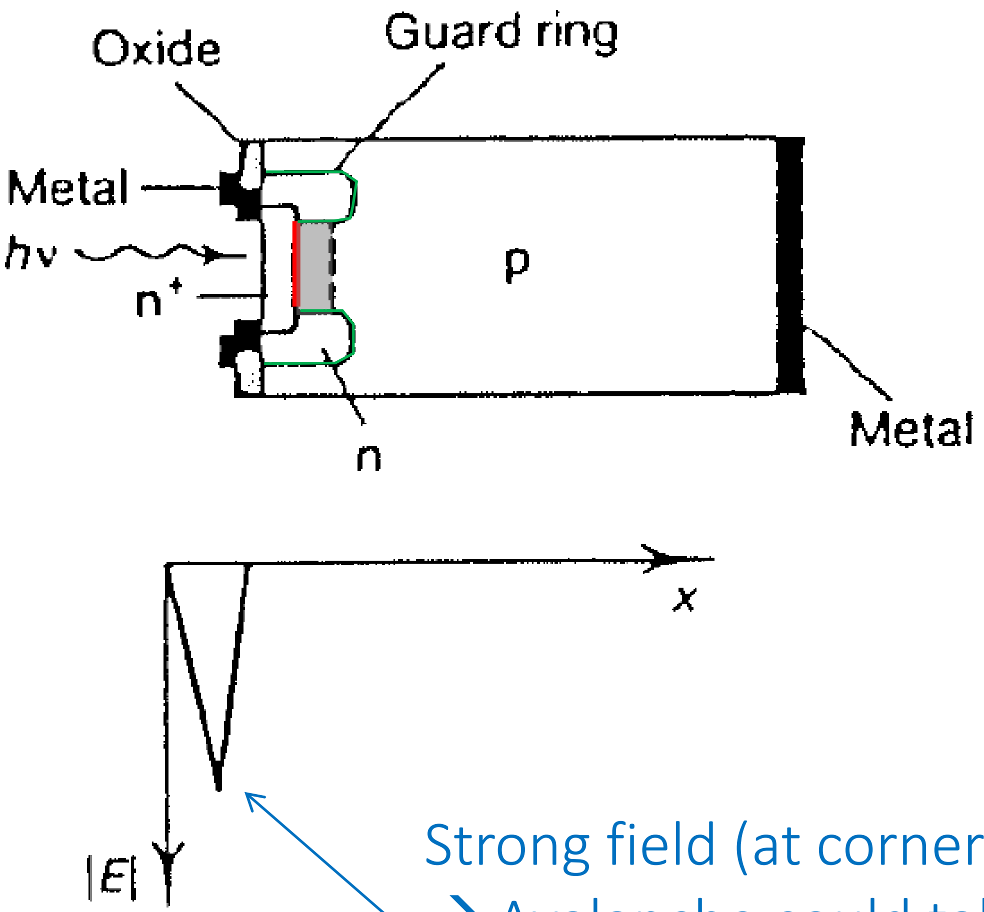
S.M. Sze, « Semiconductor devices, physics and technology »

5.7 I(V) Plot

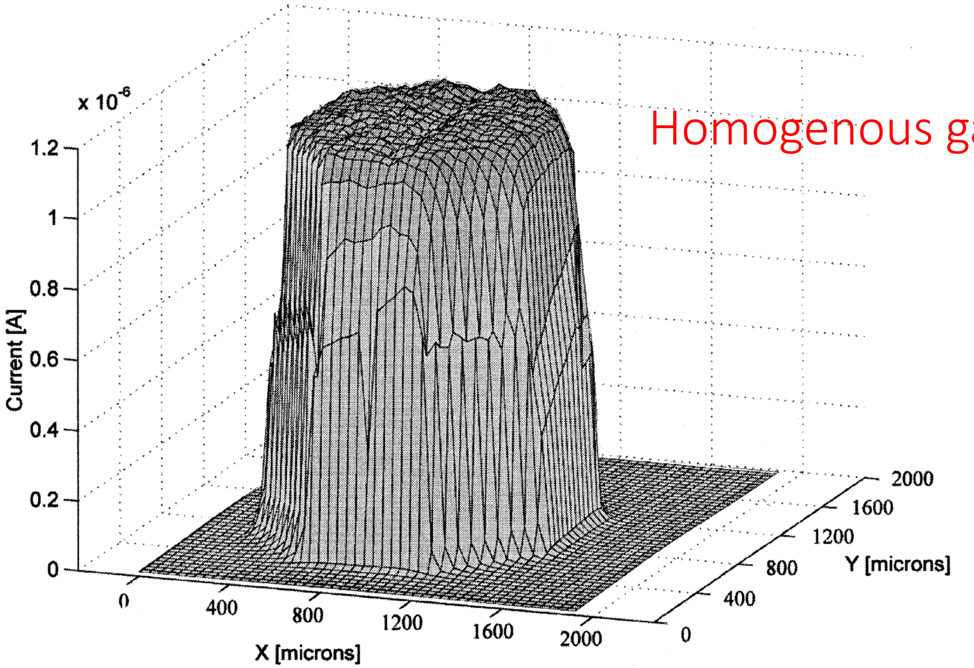


V_{bd}
APD: avalanche not yet self-sustained
SPAD
APD
PD
-V
Gain up to 10^6
Quenching needed
Gain < 1000
Self quenching
No gain
I(mA)
4
3
2
1
-1
-2
-3
-4
V_B
-6 -5 -4 -3 -2 -1 0 1 2
V(V)
FORWARD CONDUCTION
REVERSE BREAKDOWN

5.7 Structure with a Guard Ring



Avoids latch-up at the boundaries (lightning-rod effect)



Homogenous gain

Strong field (at corners)

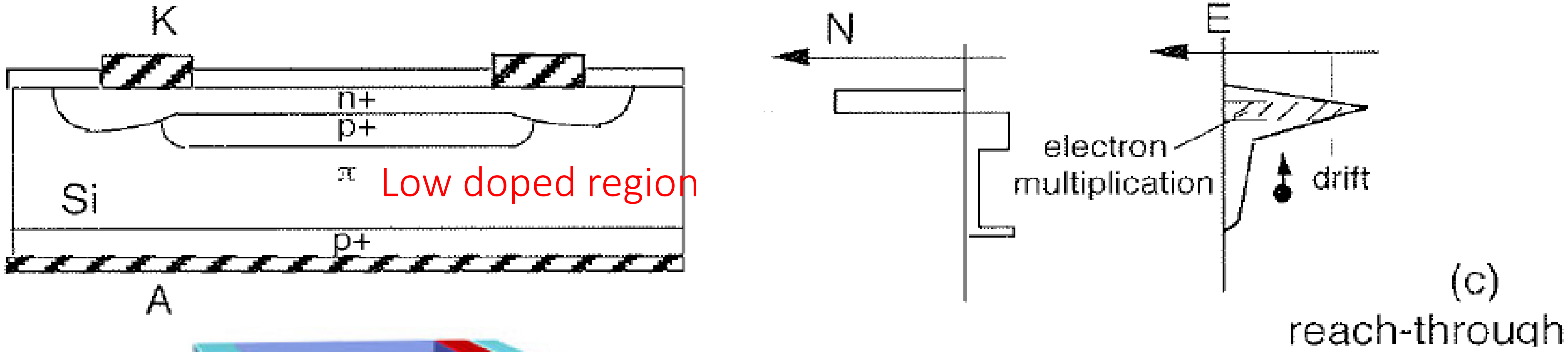
→ Avalanche could take place there

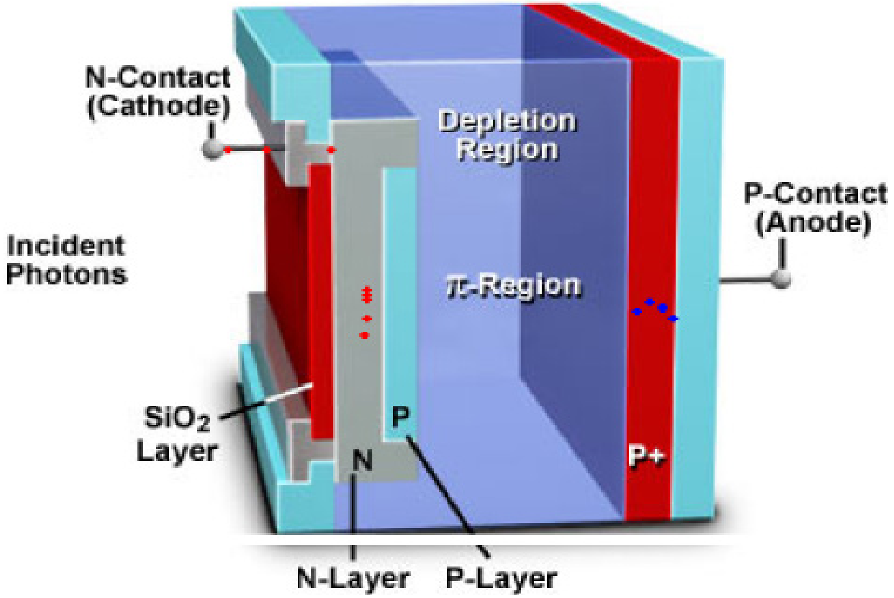
→ introduce a (weakly doped) guard ring to reduce it

A. Pauchard et al., Sensors and Actuators 82 (2000) 128–134

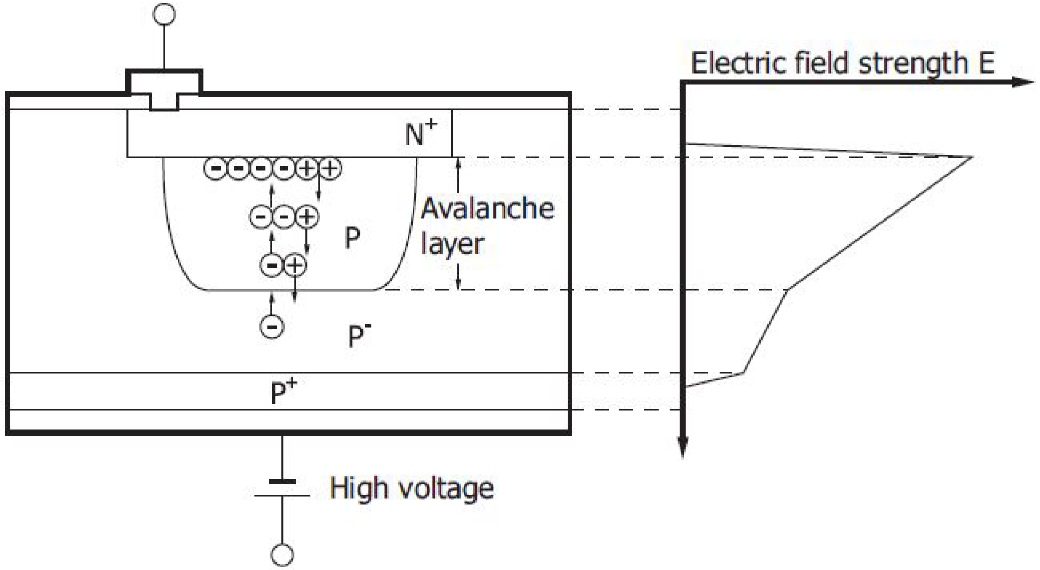
5.7 APD Structure: Reach-Through

S. Donati, « Photodetectors, devices, circuits and applications »





http://micro.magnet.fsu.edu/primer/java/digitalimaging/avalanche/index.html

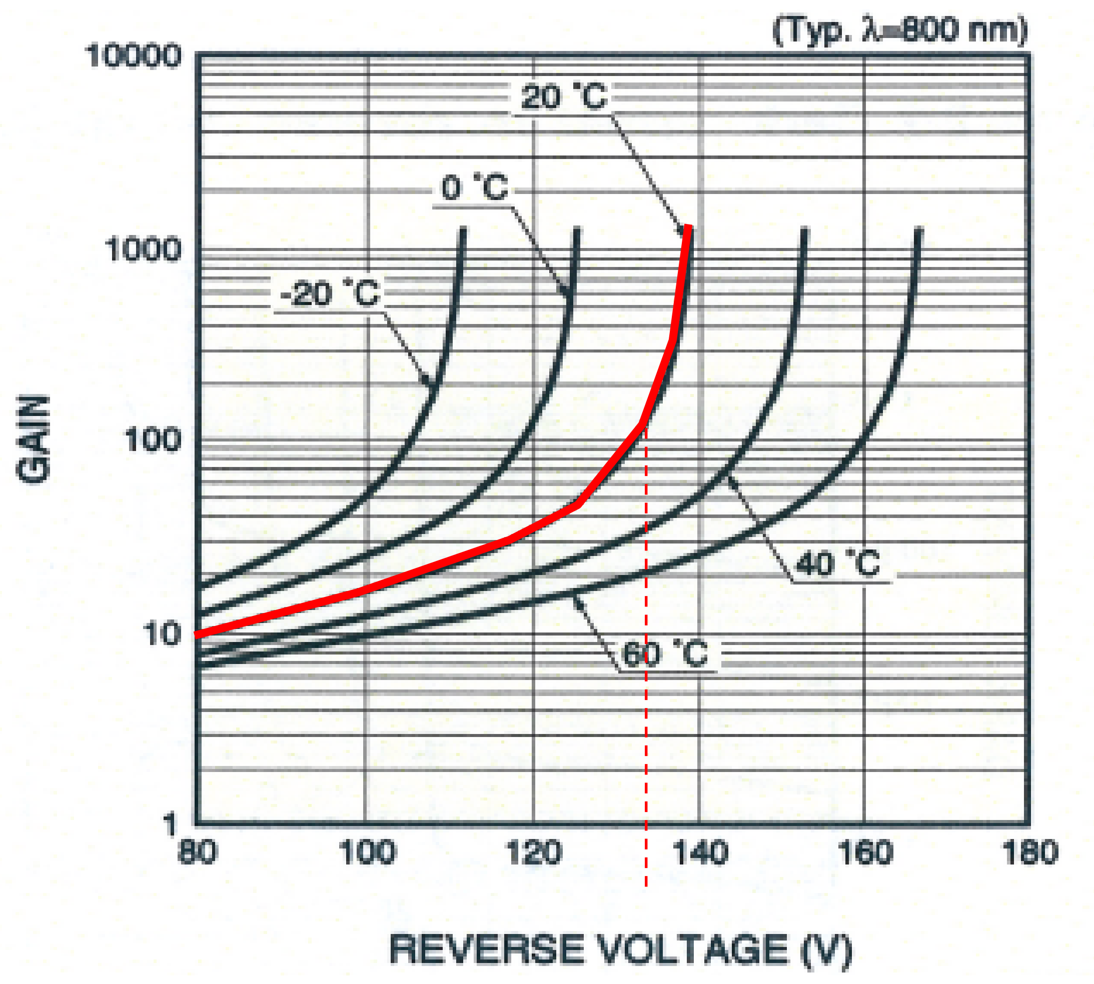


Near IR structure, Hamamatsu, Handbook 2014, chap.3

5.7 APD: Gain and Dark Current

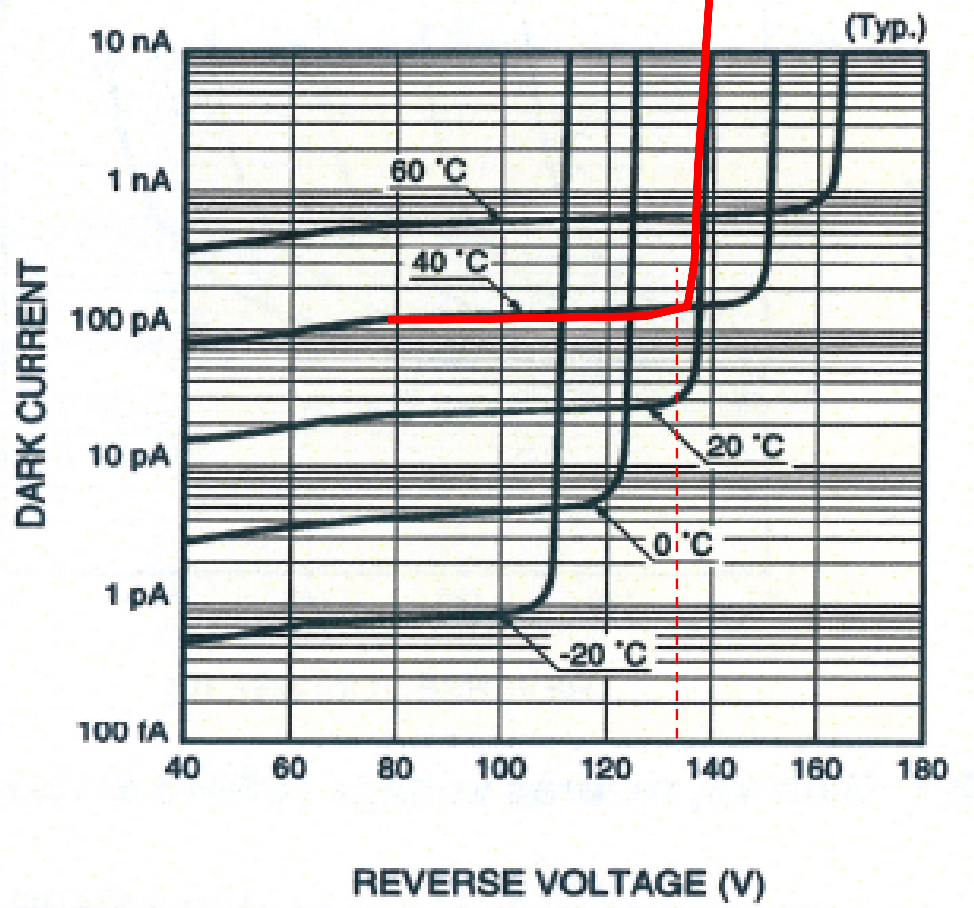
Gain

V_{BD} at room temp (stay below)



Typically: 100

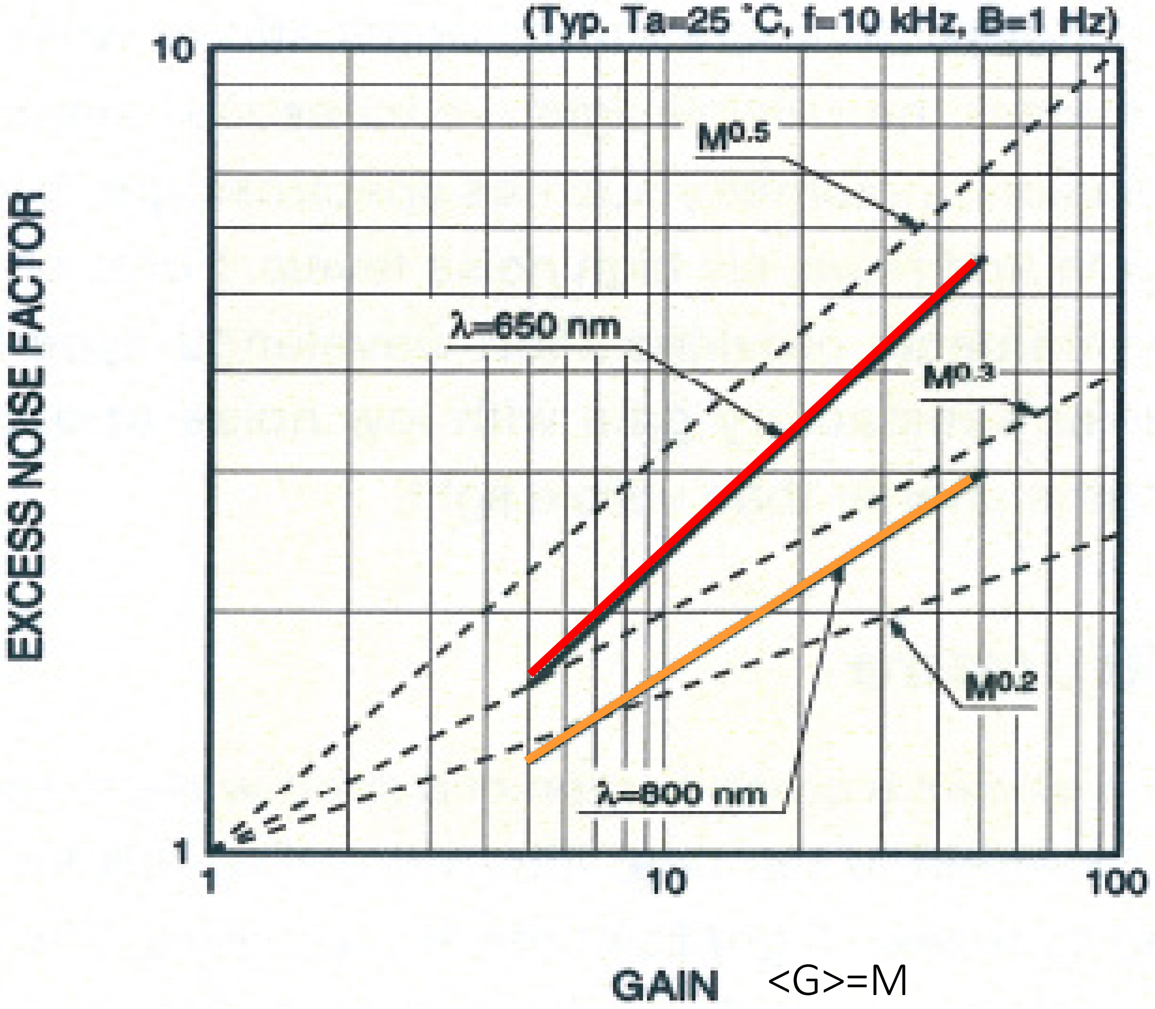
Dark Current



Typically: 10-100 pA

Hamamatsu
SI APD series S238x

5.7 APD: Noise and Excess Noise Factor F



Hamamatsu SI APD series S238x

$$F = \frac{(S/N)_{in}^2}{(S/N)_{out}^2}$$

$$F = \frac{\langle G^2 \rangle}{\langle G \rangle^2} \cong \langle G \rangle^x$$

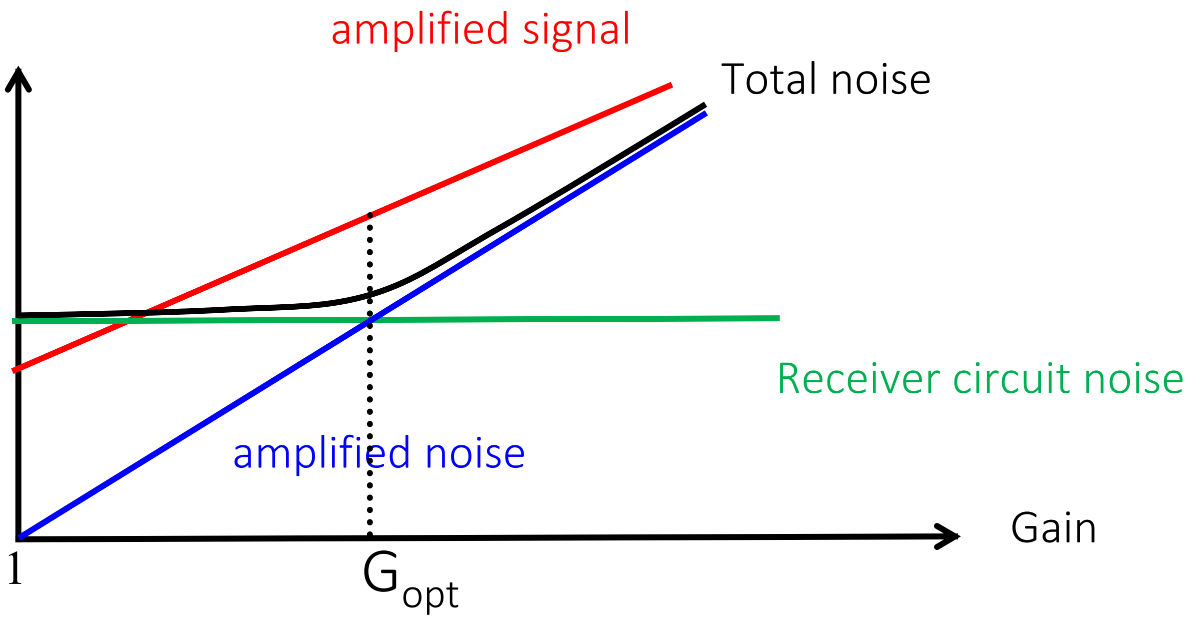
Typically:

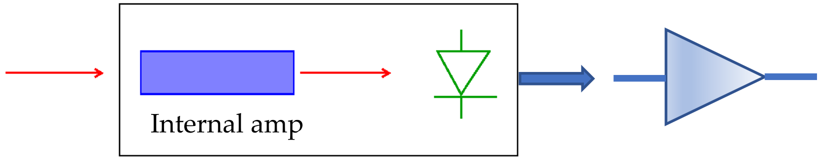
$x = 0.3$

$F = 3 \ldots 5$

5.7 Reminder: Optimal Amplification

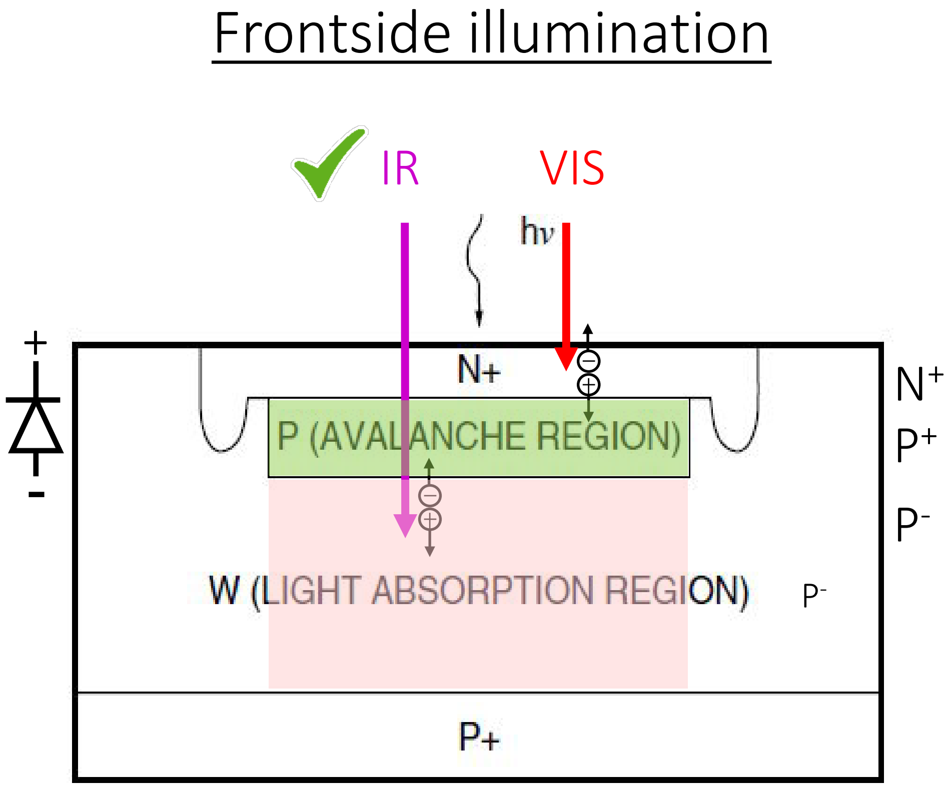
If noise is amplified more than the signal,
would it not be better not to amplify at all?

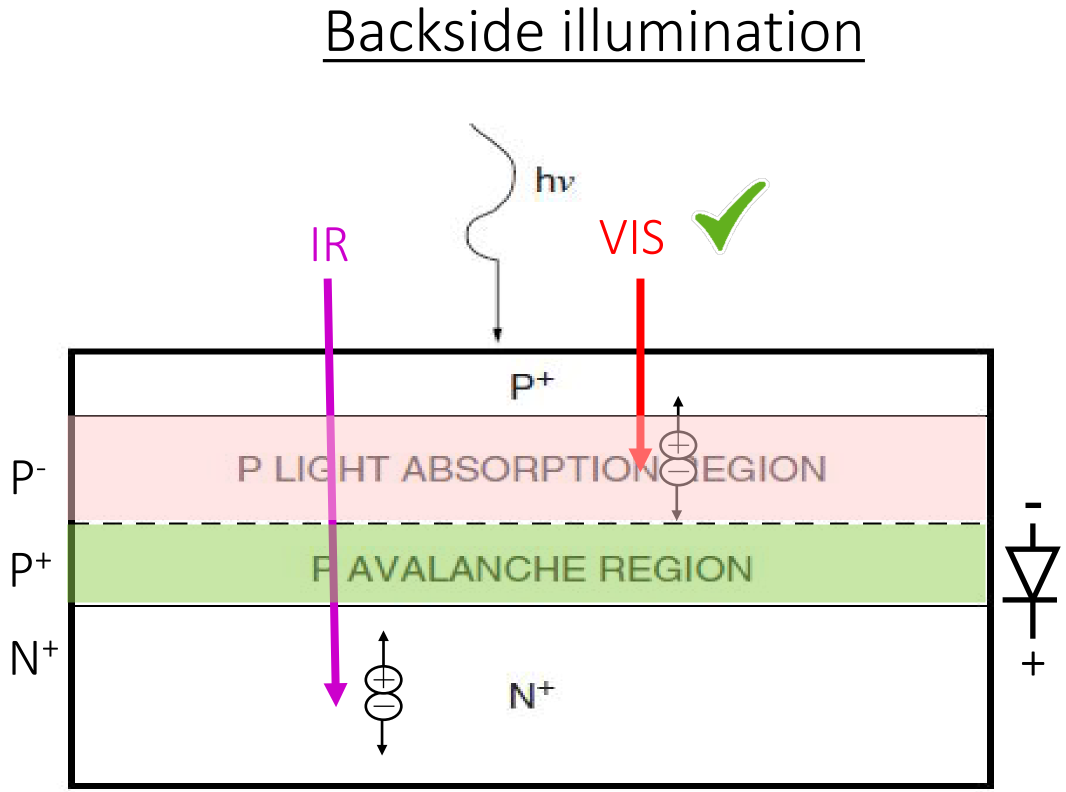




There is an optimal internal gain, at which the internal gain noise exceeds the receiver circuit noise.

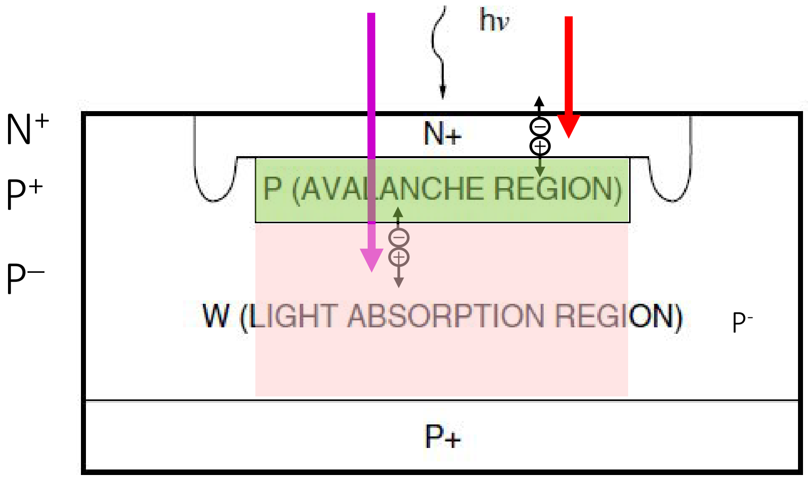
5.7 Front or Backside Illumination?



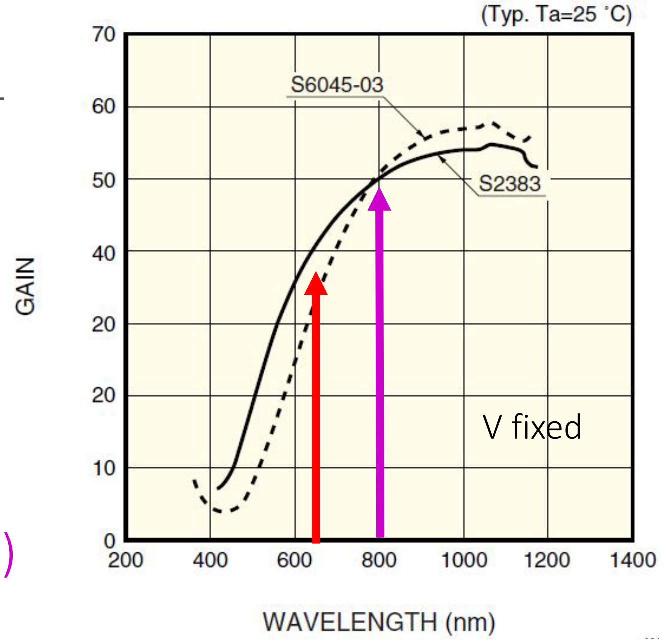


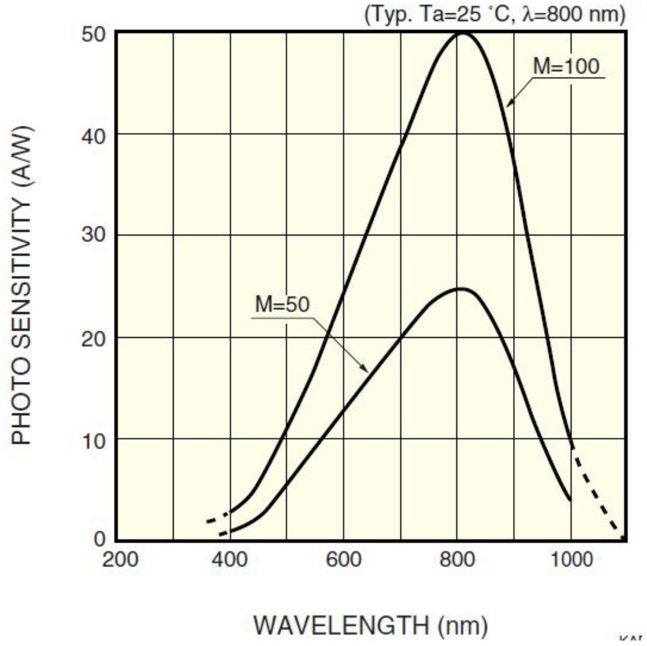
Electron injection is required for high gain and low (excess) noise

5.7 APD-structure in Silicon for the Near IR



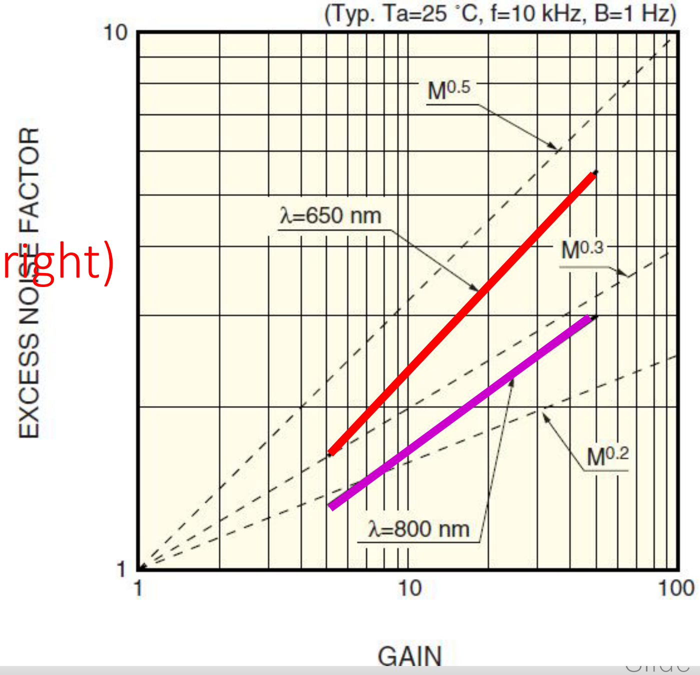
IR: electron injection
large gain (top right)
low noise (bottom right)



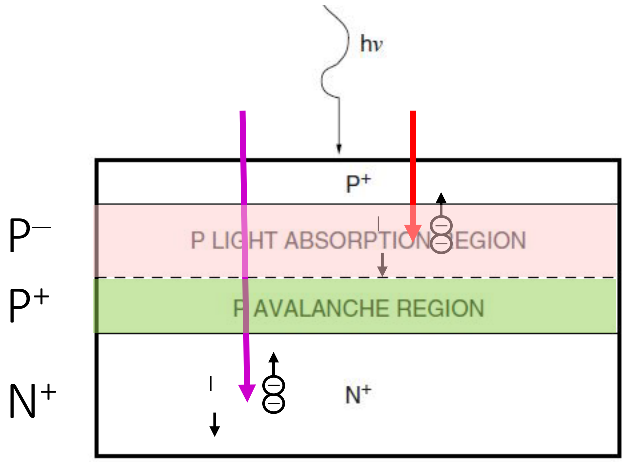


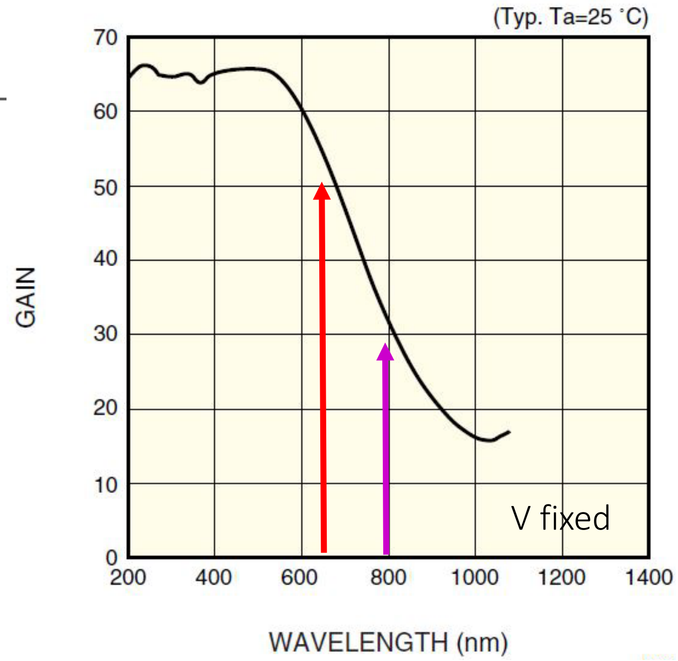
VIS: hole injection
lower gain (top right)
higher noise (bottom right)

Hamamatsu Solid State division, «Characteristics and use of Si APD», Technical information SD-28

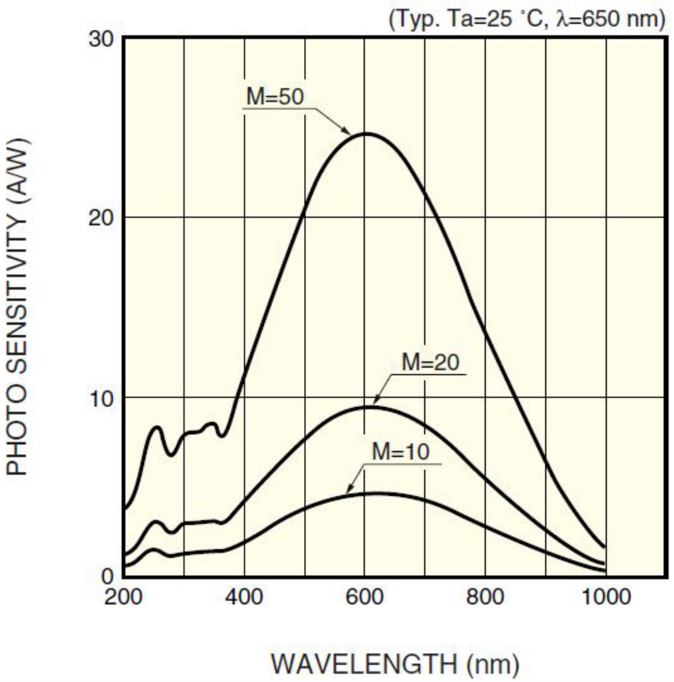


5.7 Structure APD in Silicium for the Visible



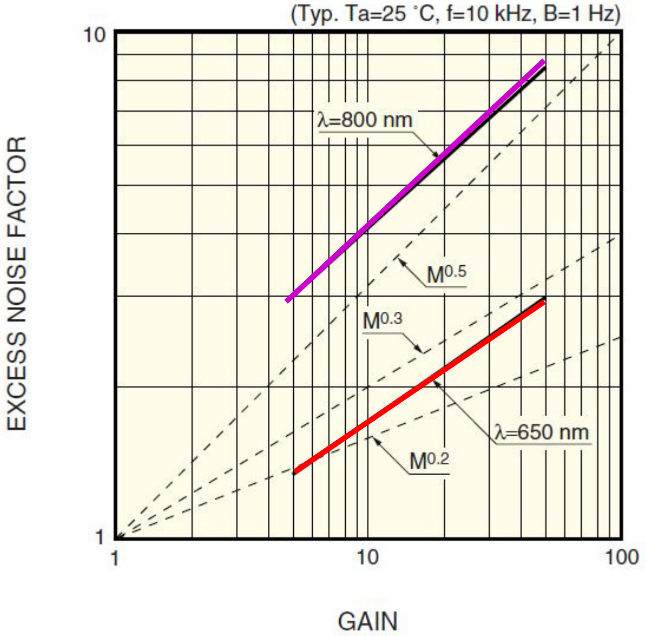


IR: hole injection
lower gain
higher noise



VIS: electron injection
larger gain
lower noise

Hamamatsu Solid State division, «Characteristics and use of Si APD», Technical information SD-28



5.7 Avalanche Photodiode: Summary

Quantum efficiency:	near-optimal	> 90%
Gain	average	50 - 200
Noise	good	dark current or signal shot noise
limited		
Excess noise	high	F: 3 - 5
Voltage	high	10-200 V
Spectrum	adaptable	UV - VIS - IR
Voltage stabilization		Critical

Photon counting possible

Geiger mode and SPAD for digital detection

Technologically compatible with microelectronics

Take-Home Messages/W5-3

5.6 Photodiode bandwidth & electronic circuits:

- Which phenomena limit the speed of a photodiode?
- Which electronic circuits are used with photodiodes? Describe their respective advantages.
- Describe the physical effects limiting the dynamics of a photodiode

5.7 Solar cells:

- How does a solar cell work?

5.8 Avalanche photodiode (APD):

- Describe an avalanche photodiode. Why is a « guard ring » required ?
- What is the « excess noise » in an avalanche photodiode? Explain the optimization of the internal gain as function of the electronic noise

Acknowledgements

Previous course version: P.-A. Besse, 2023

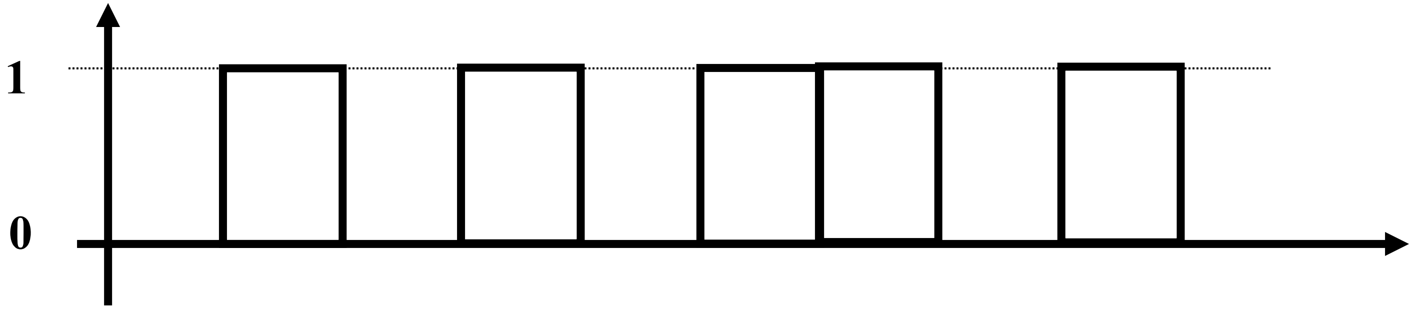
Slide preparation and first revision:

- Edwin Bertschy, EPFL MA
- Victoria Chalain, EPFL MA

Exercises – Week 5

Exercise 5.1: High-speed photodiodes and BER

- On-off keying system: bits “0“ and “1“

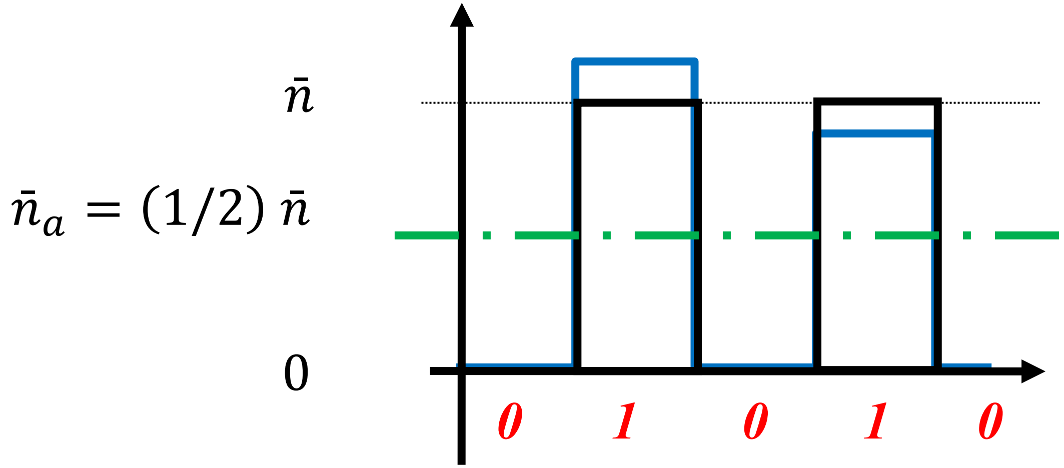


- BER: probability of error per bit
- If p_0 =probability of mistaking a “ 0“ for a “ 1“
  & p_1 =probability of mistaking a “1” for a “0”, then

$$BER = p_0 /2 + p_1/2$$ *(BER definition)*

Exercise 5.1: High-speed photodiodes and BER

Ideal = limited by the optical signal shot noise



- If an average of $\bar{n}$ photons is transmitted by a laser diode, the probability of detecting n photons is given by:

$$p(n) = \bar{n}^n \frac{\exp(-\bar{n})}{n!}$$

→ How many photons per "1" bit are needed to guarantee a BER of 10^{-9}?

Exercise 5.2: PIN Heterostructure Photodiode

$In_{1-x}Ga_xAs_yP_{1-y}$ system on an InP substrate

a) To grow a monocrystal, we have to preserve the lattice of the InP substrate.
This involves satisfying the following relationship:

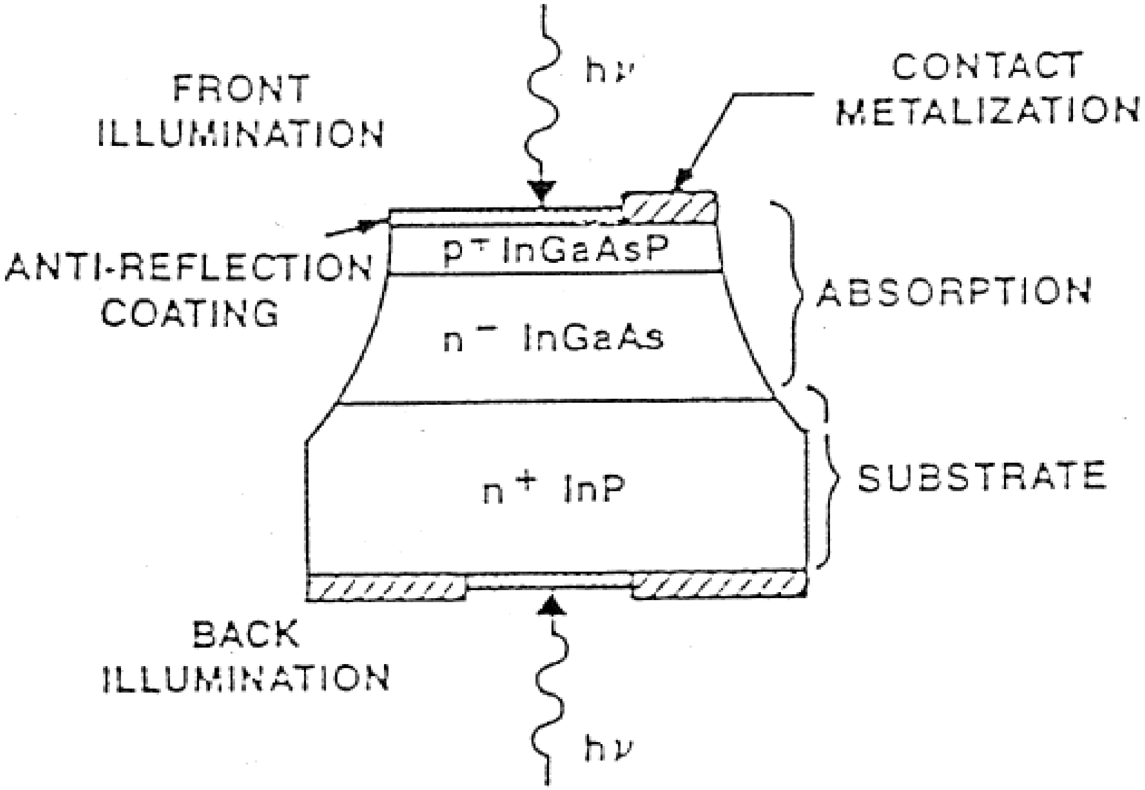
$$x = \frac{0.4562 \cdot y}{1 - 0.031 \cdot y}$$

b) In this case ("lattice matched to InP") the gap can be changed according to:

$$E_g(y) = 1.35 - 0.72 \cdot y + 0.12 \cdot y^2 \quad [eV]$$

Exercise 5.2: PIN Heterostructure Photodiode

Consider the PIN photodiode depicted below:



Exercise 5.2: PIN Heterostructure Photodiode

This type of photodiode was designed for optical telecommunications and has to work at wavelengths between 1.50 μm and 1.60 μm. Its diameter is 10 μm, corresponding to that of a single mode fiber optic cable.

A) Considering a superficial layer with the following composition: $In_{1-x}Ga_xAs_yP_{1-y}$, with y=0.84.
Sketch the quantum efficiency for front and back illumination. What is the main difference?

B) Estimate the width W of the intrinsic InGaAs region to optimize the bandwidth using a load resistance of $R_L = 50\ \Omega$.
(use $\varepsilon = 12$ and $v_{sat} = 10^5$ m/s)
Does the diode have to be polarized, and if so, why?

Appendix 5.1: Band diagrams and pn junction

A5.1 Fundamental equations (1)

1) Potential energy : $E_{pot} = E_{vac} = (-q) \cdot \varphi$ The potential energy is the vacuum level

2) Maxwell : $$div(\vec{E}) = \frac{\rho}{\varepsilon_0 \varepsilon} = \frac{q\ (p + N_d^+ - n - N_a^-)}{\varepsilon_0 \varepsilon}$$

1D ⇨ $E_x \propto + \int \rho \cdot dx$ The electric field is the integral of the net charges

∝ = proportional

3) Maxwell : $$rot(\vec{E}) = -\frac{\partial \vec{B}}{\partial t} = 0 \quad \Rightarrow \quad \vec{E} = -\overrightarrow{grad}(\varphi) = \frac{1}{q} \overrightarrow{grad}(E_{vac})$$

1D ⇨ $E_x \propto + \frac{\partial E_{vac}}{\partial x}$ The electric field is the slope of the vacuum level

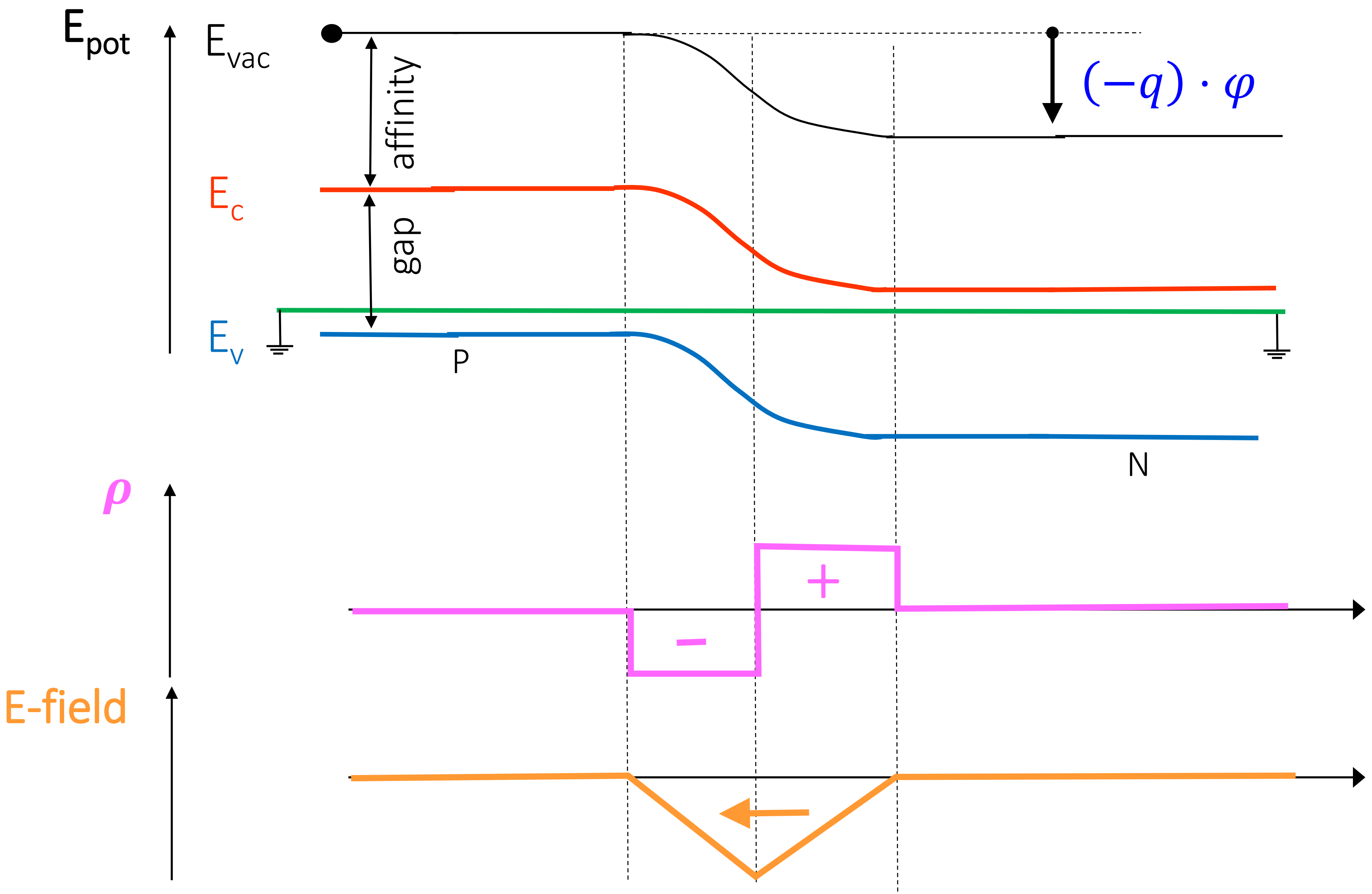
A5.1 Fundamental equations (2)

4) Poisson equation :
from 2) and 3)

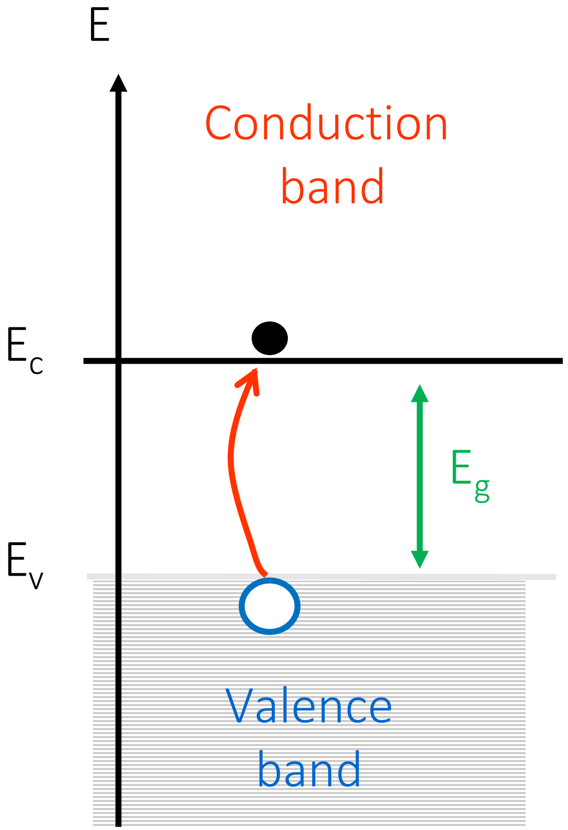
$$\Delta\varphi = -\frac{\rho}{\varepsilon_0 \varepsilon} = -\frac{q\ (p + N_d^+ - n - N_a^-)}{\varepsilon_0 \varepsilon} = -\frac{1}{q}\Delta E_{vac}$$

1D ⇒ $\rho \propto +\frac{\partial^2 E_{vac}}{\partial x^2}$ The net charges are the curvature of the vacuum level

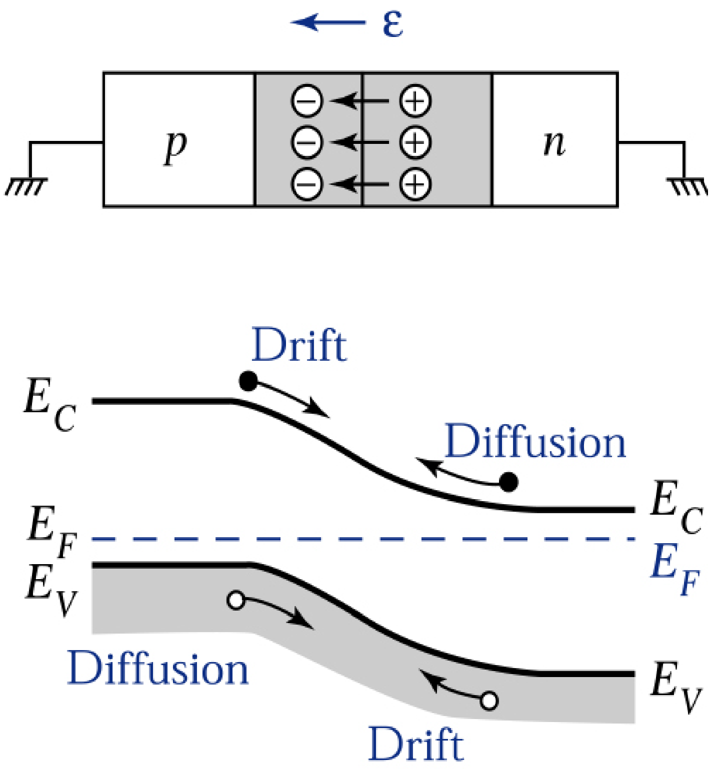
A5.1 Band diagram



A5.1 Band structure

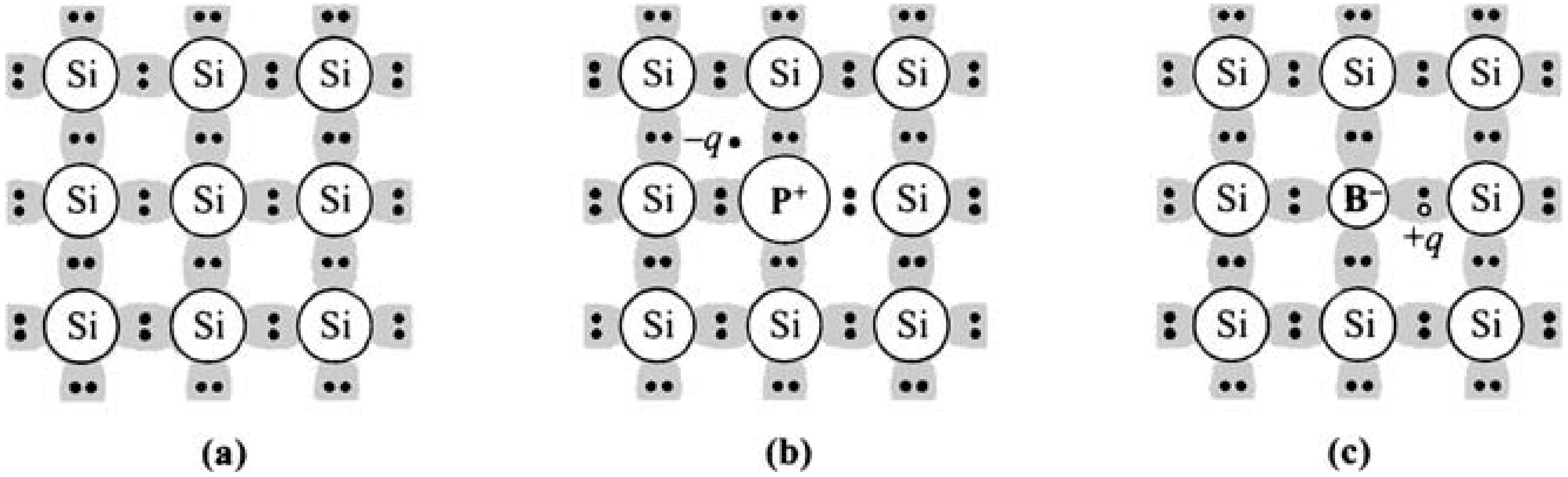


Jonction p/n



S.M. Sze, « Semiconductor devices, physics and technology »

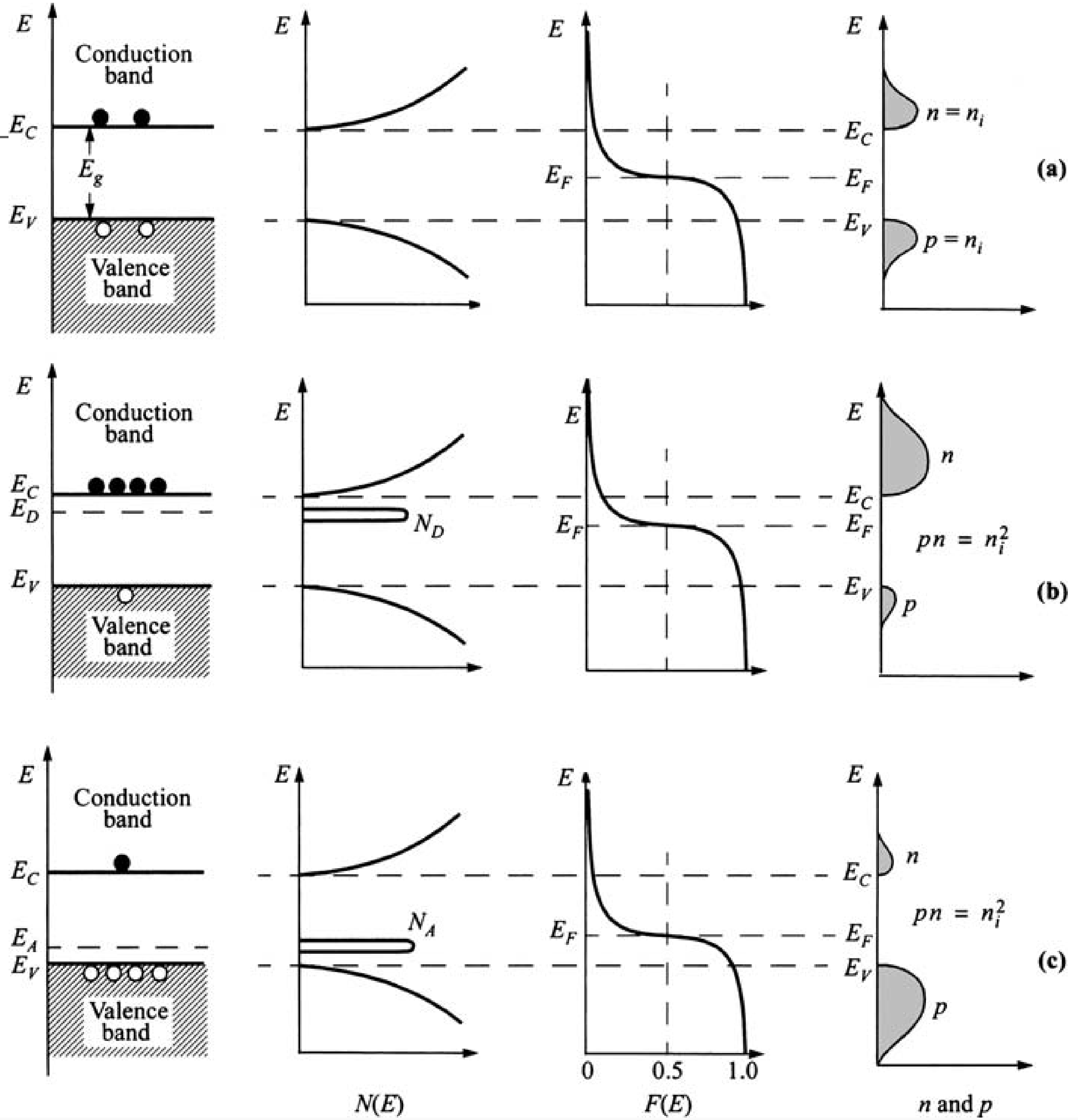
A5.1 Semiconductor doping



Three basic bond pictures of a semiconductor. (a) Intrinsic Si with no impurity. (b) n-type Si with donor (phosphorus). (c)p-type Si with acceptor (boron).

S.M. Sze, K.K. Ng, « Physics of Semiconductor devices », 3rd edition

A5.1 Mass-action law



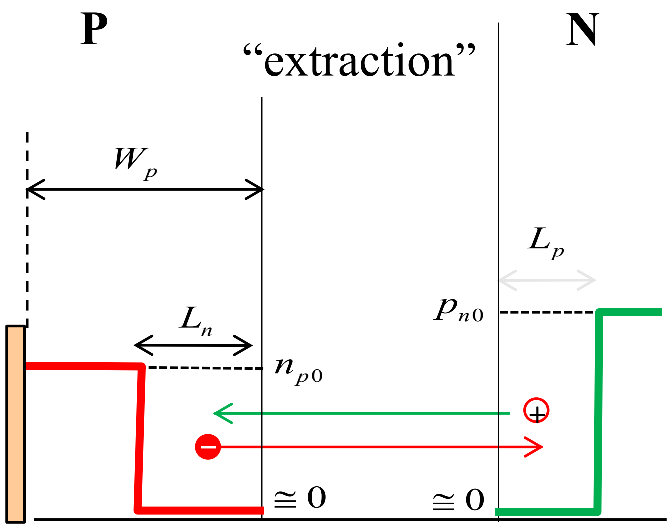
Nondegenerate semiconductors:

$$p \cdot n = N_C N_V \exp\left(-\frac{E_g}{kT}\right) = n_i^2$$

S.M. Sze, K.K. Ng, « Physics of Semiconductor devices », 3rd edition

A5.1 Approximation and Interpretation: Blocking Junction

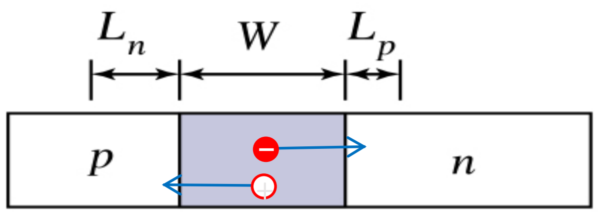
Ideal diode: minority carrier profile



$$J_n = q \cdot \int U_{th}\, dx = q \cdot L_n \cdot \frac{n_{p0}}{\tau_n} \equiv I_{sn}$$

$$J_p = q \cdot \int U_{th}\, dx = q \cdot L_p \cdot \frac{p_{n0}}{\tau_p} \equiv I_{sp}$$

Depletion region: Current due to generation



$$J_{gen} = q \cdot \int_{-x_p}^{x_n} U_{th}(x) dx \cong q \cdot U_{th}(0) \cdot W$$

$$U_{th} \cong \frac{1}{\tau} \cdot \left(\frac{np - n_i^2}{n + p + 2n_i} \right) \cong -\frac{n_i}{2\tau}$$

$$J_{gen} \cong -\frac{qn_i}{2\tau} \cdot W \approx \frac{n_i}{\tau} \cdot \sqrt{V_{bi} - V}$$