

# MICRO-523: Optical Detectors

## Week Five: Photodiodes (Solutions Ex5)

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Based on MICRO-523, P.-A. Besse, 2023

TAs: Samuele Bisi, Yazan Lampert

**EPFL**

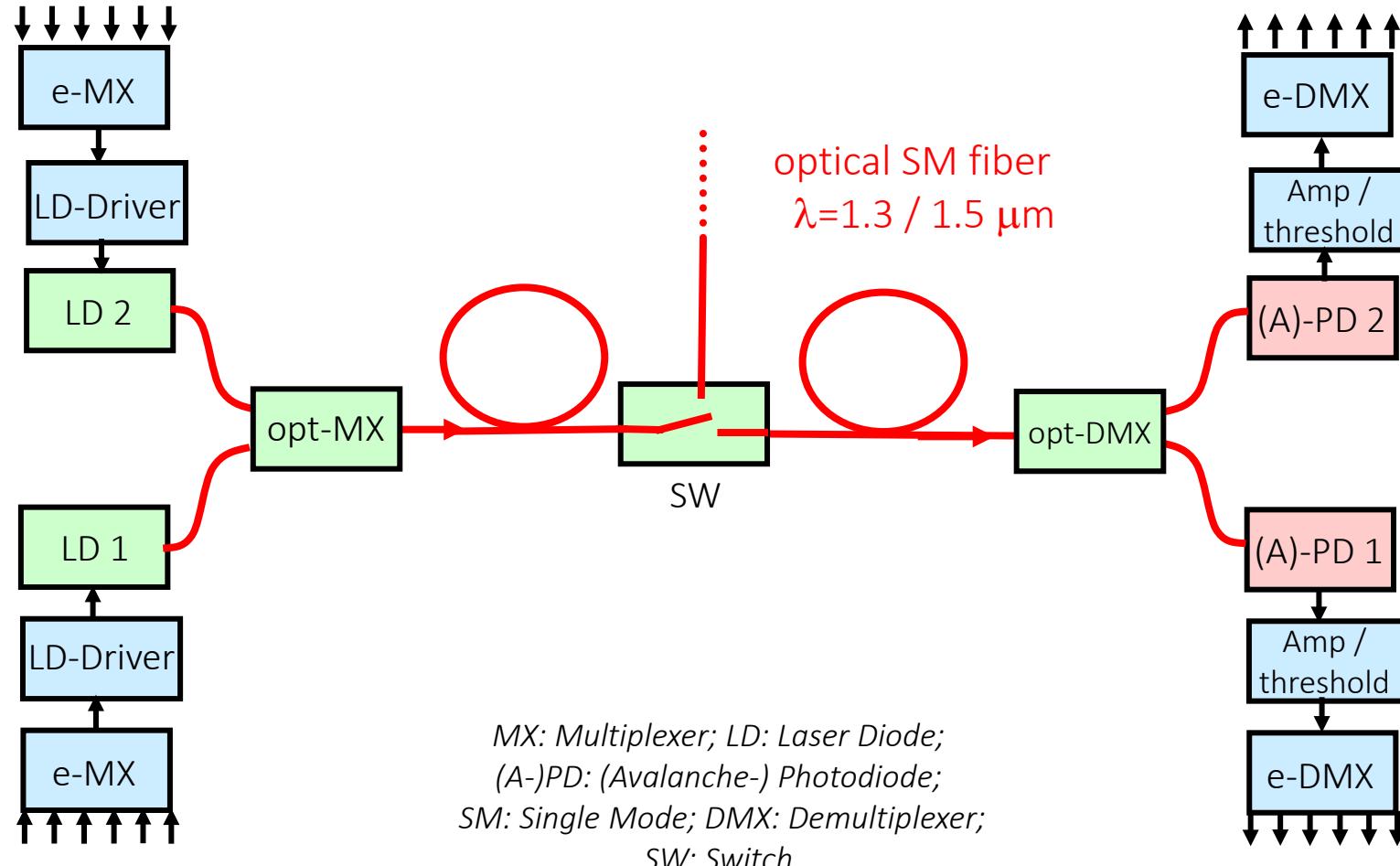
# Outline

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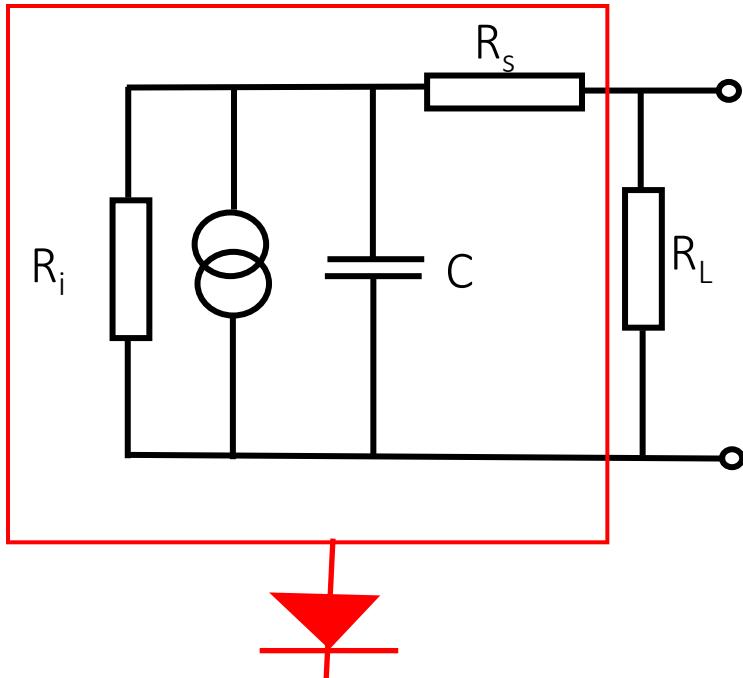
5.1 Fast photodiodes

5.2 PIN Heterostructure photodiodes

# Exercise 5.1: Schematic Diagram of a Digital Optical Communication System



# Exercise 5.1: Bandwidth Limits of PDs



1. RC-constant:

$$\tau_{RC} = 2(R_s + R_L)C$$

$(R_i \gg R_L)$

$$\text{but } C_j = S \cdot \frac{\epsilon_0 \epsilon}{W} = \pi \left(\frac{d}{2}\right)^2 \cdot \frac{\epsilon_0 \epsilon}{W}$$

2. Drift of carriers in the depletion layer:

$$\tau_{\text{drift}} = W/v_c$$

$W$ : width of depletion layer

$v_c$ : carrier velocity ( $10^7 \text{ cm/sec}$ )

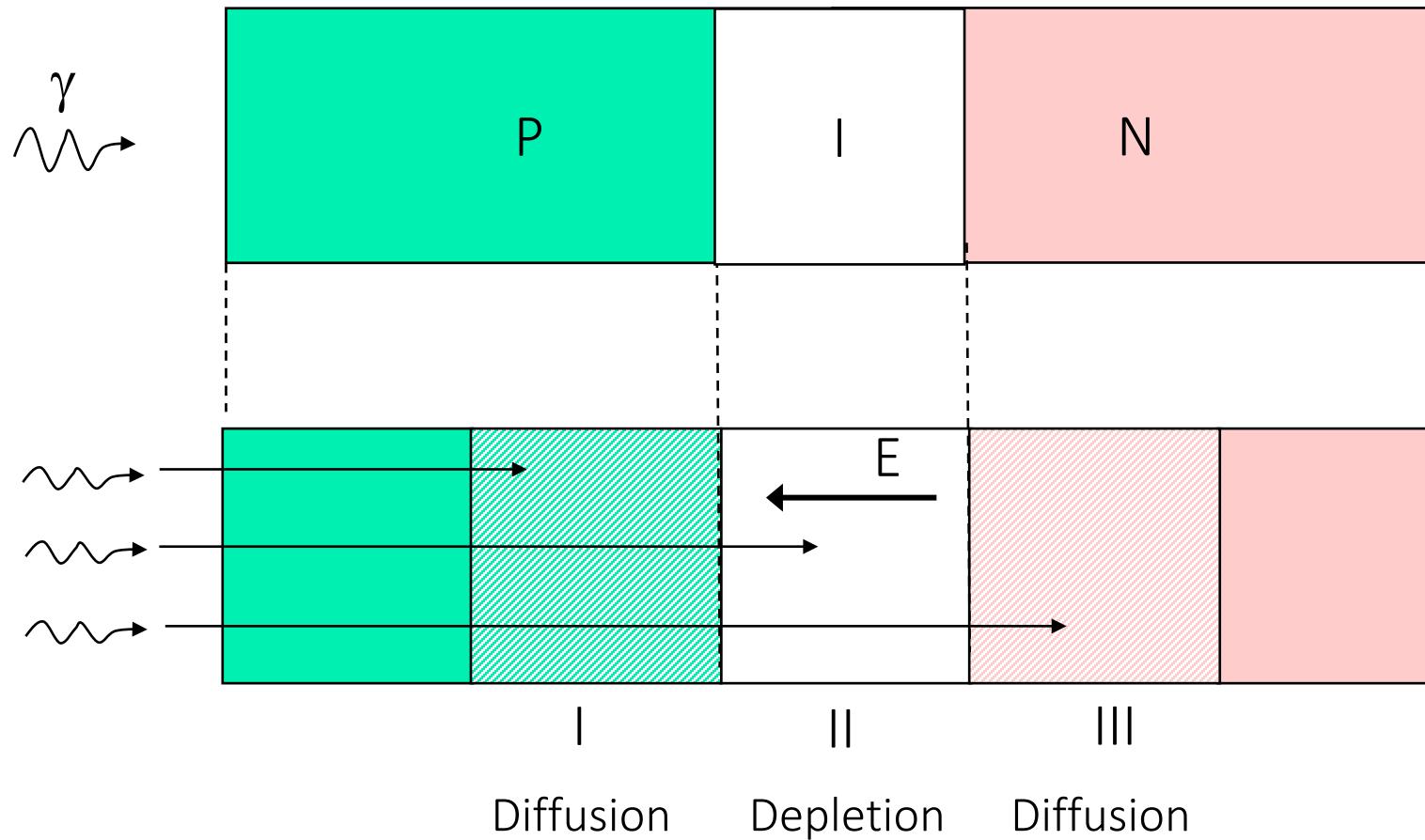
3. Diffusion of carriers to the p-n junction:

$$\tau_{\text{diff}} = d^2/D$$

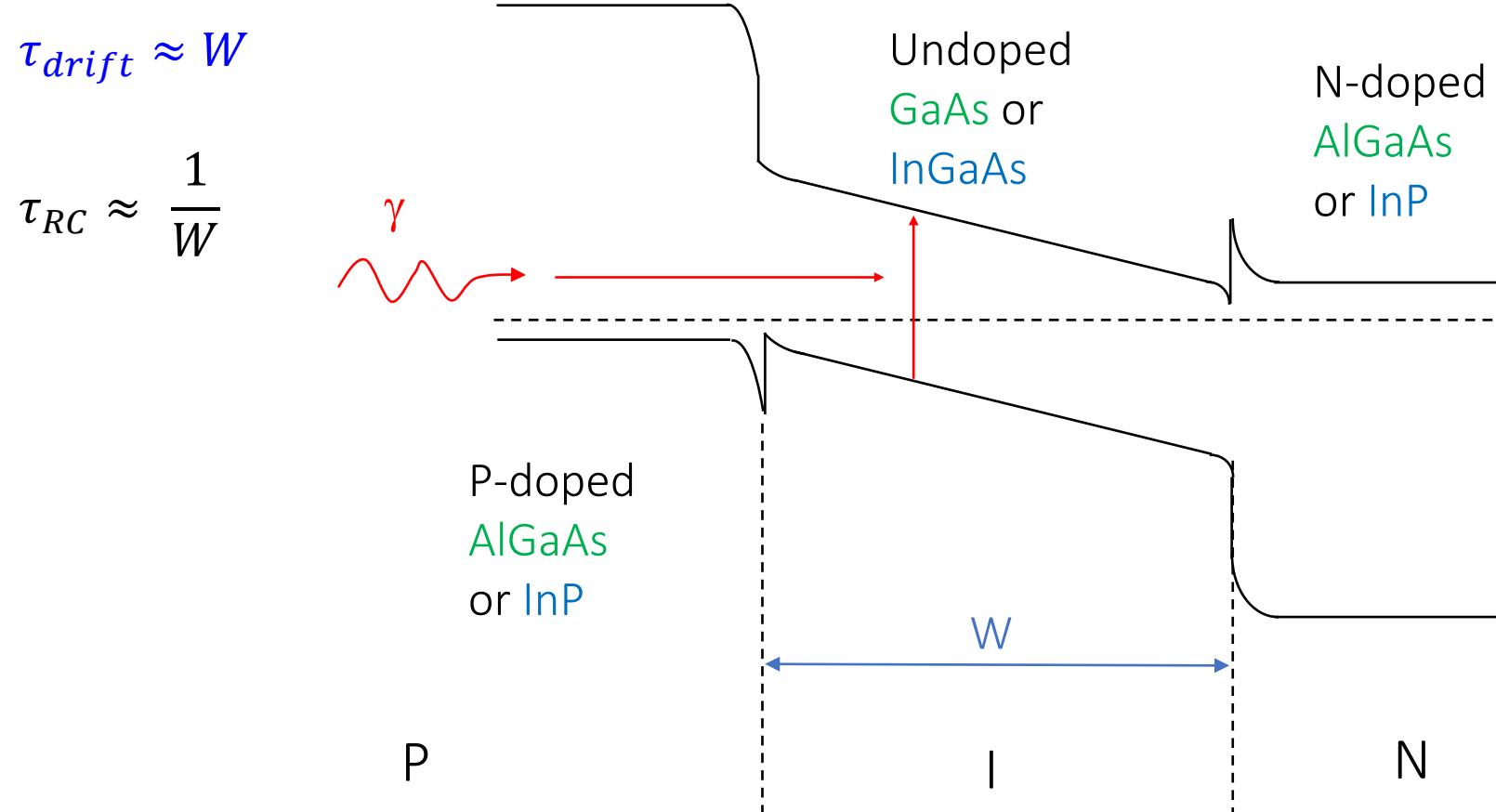
$d$ : distance to depletion layer

$D$ : diffusion constant

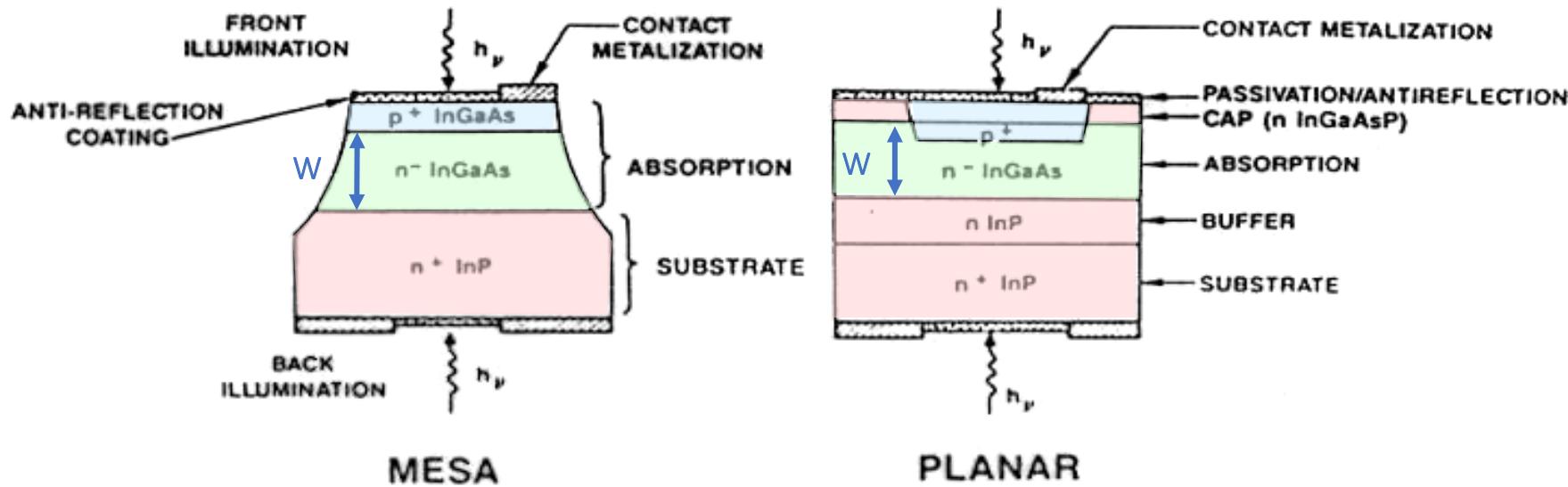
# Exercise 5.1: PIN: Diagram of the Working Principle



# Exercise 5.1: PIN Heterojunction



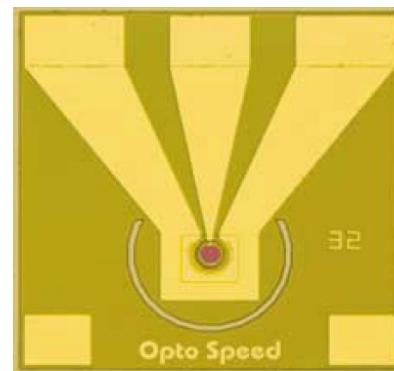
# Exercise 5.1: InGaAs PIN Heterojunction Photodiode



$$\tau_{drift} \approx W$$

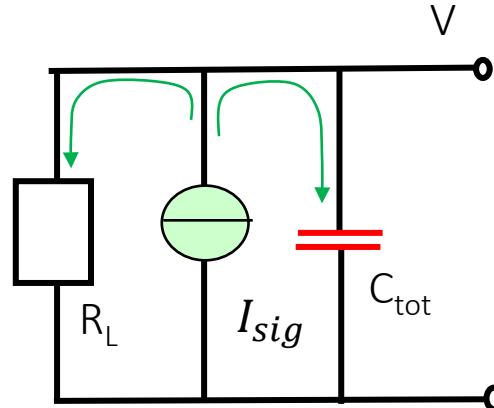
→ Decreased  $W$

→ Decreased absorption



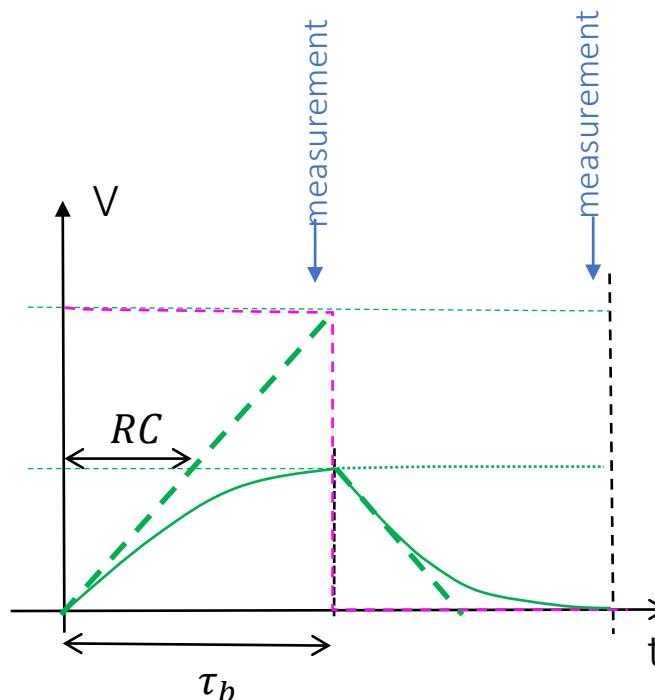
InGaAs/InP photodiode  
Opto Speed

# Exercise 5.1: Continuous Detection: High Speed Digital Transmission



## 1) At the beginning of the bit:

The current loads the capacitor



## 2) Once the capacitor is charged:

The current flows through the resistor

Number of electrons stored in the capacitor:

$$m_{c,sig} \cong \frac{RC}{\tau_b} m M \cong \frac{1}{2} m M$$

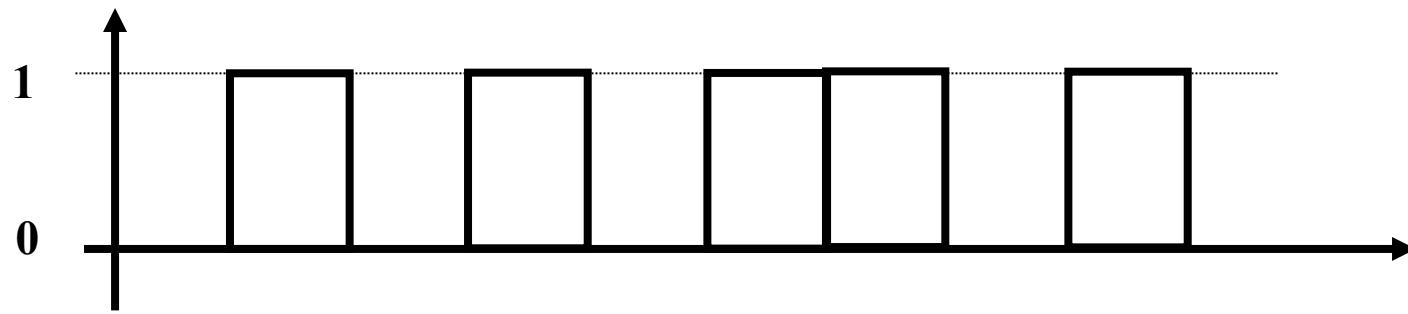
$m$ : # of generated electrons per bit

$M$ : amplification before the electronics (e.g. avalanche or optical amplification)

## Exercise 5.1: Bit Error Rate (BER)

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- On-off keying system: bits “0” and “1”

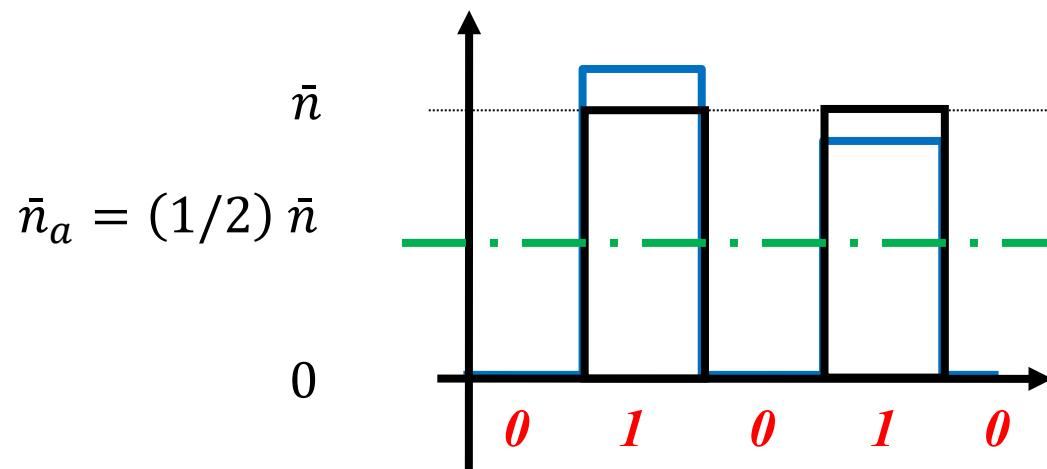


- BER: probability of error per bit
- If  $p_0$  = probability of mistaking a “0” for a “1”  
&  $p_1$  = probability of mistaking a “1” for a “0”, then

$$\text{BER} = p_0/2 + p_1/2 \quad (\text{BER definition})$$

## Exercise 5.1: Ideal BER (1)

Ideal = limited by the optical signal shot noise



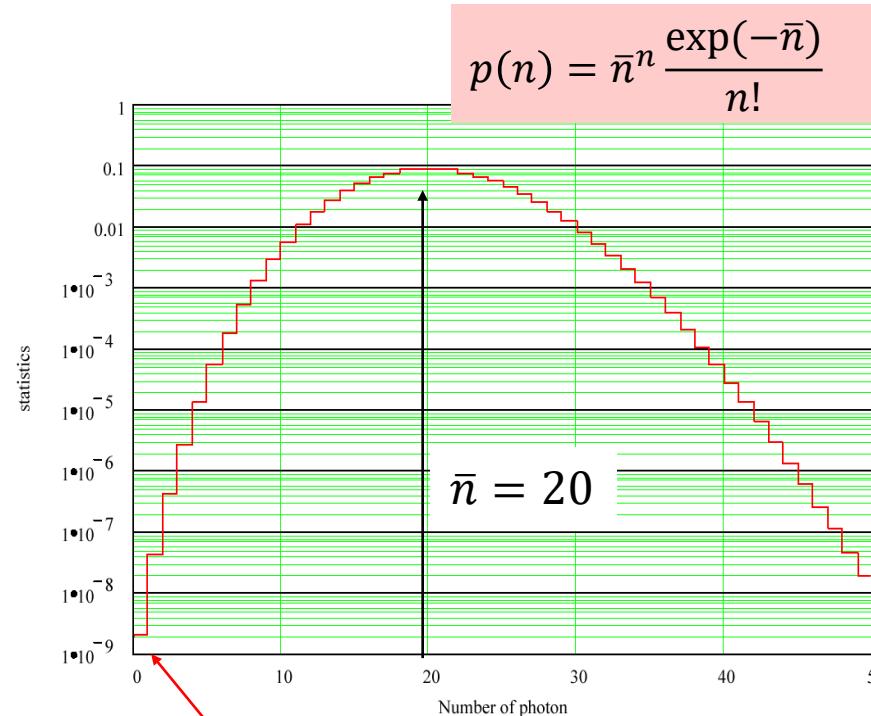
- If an average of  $\bar{n}$  photons is transmitted by a laser diode, the probability of detecting  $n$  photons is given by:

$$p(n) = \bar{n}^n \frac{\exp(-\bar{n})}{n!}$$

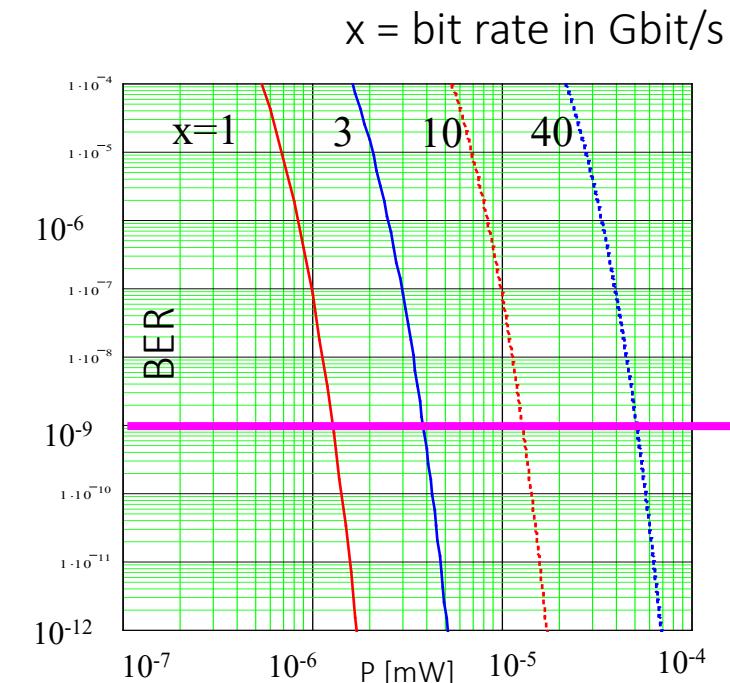
→ How many photons per “1” bit are needed to guarantee a BER of  $10^{-9}$ ?

# Exercise 5.1: Ideal BER: Example

Because of the shot noise on each “1” bit, it takes 20 photons per “1” to guarantee a BER of  $10^{-9}$ .



$$p(0) = \exp(-\bar{n}) = 2 \cdot 10^{-9}$$



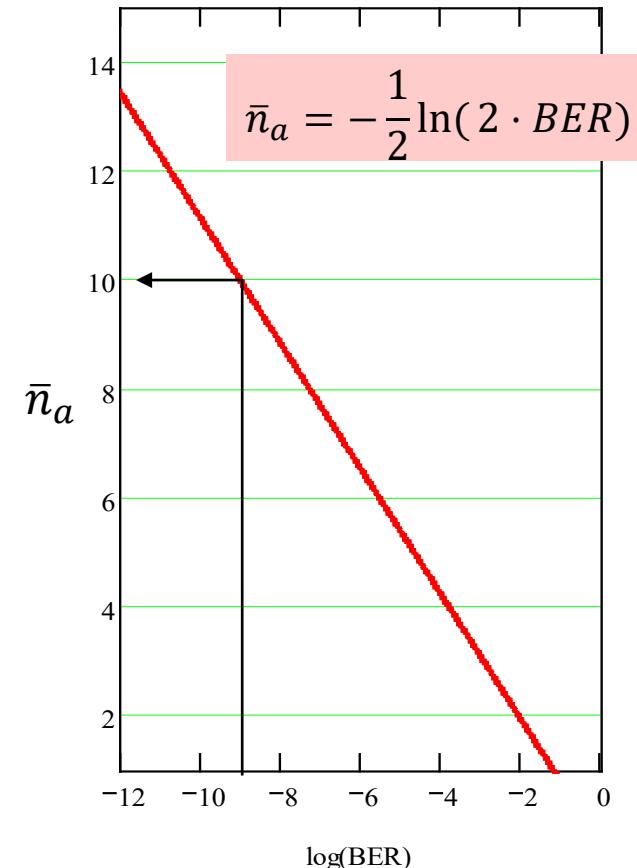
$$\bar{n}_a = \frac{P}{h\nu} T$$

$$BER = \frac{1}{2} e^{-2\left(\frac{P}{h\nu}\right) \cdot \left(\frac{1}{x \cdot 10^9}\right)}$$

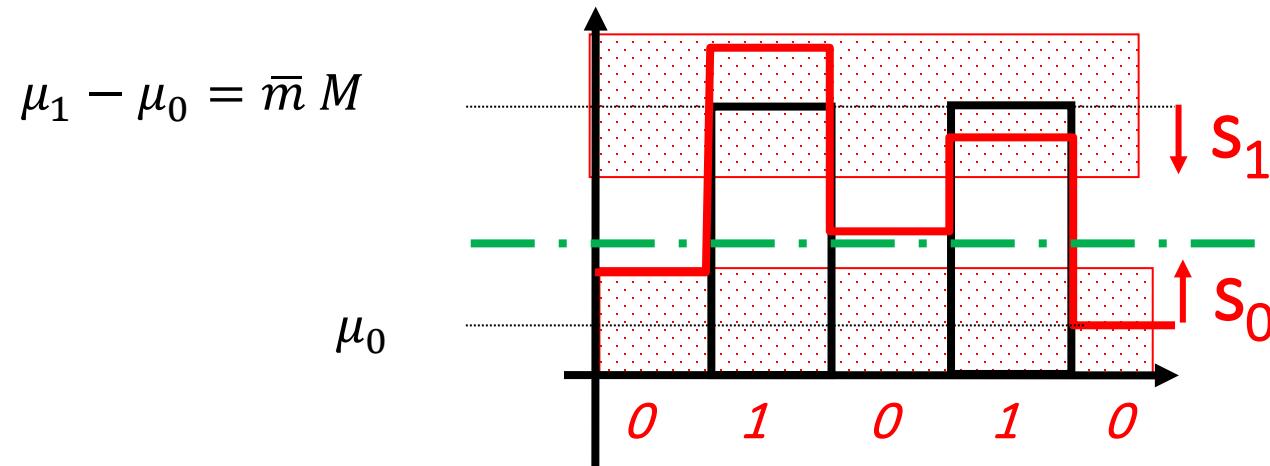
## Exercise 5.1: Receiver Sensitivity of Ideal Digital Systems

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- The receiver sensitivity of digital systems is defined as the average number of photons per bit required to achieve a certain BER (usually  $10^{-9}$ ).
- For an ideal receiver, 10 photons per bit are required to achieve a BER of  $10^{-9}$ . (20 photons for “1” bits).

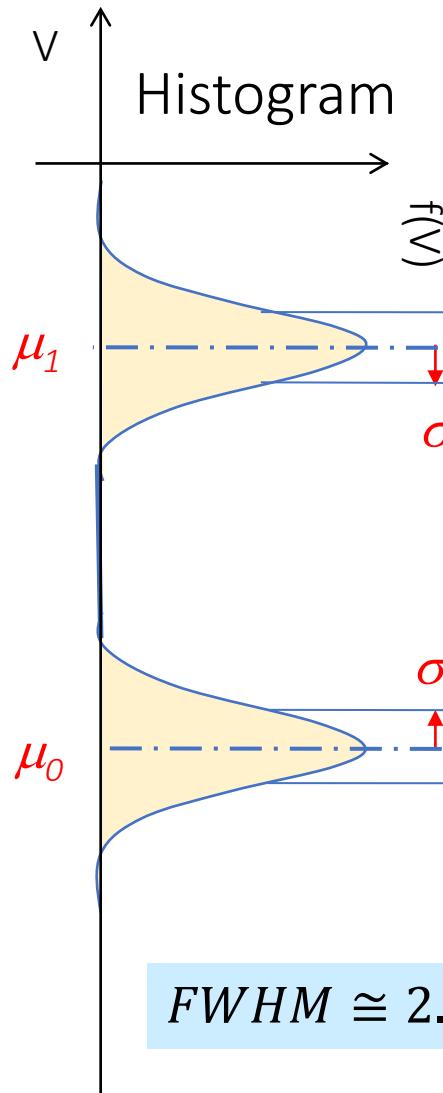


## Exercise 5.1: Real Receiver: Sensitivity of a Receiver with Gaussian Noise



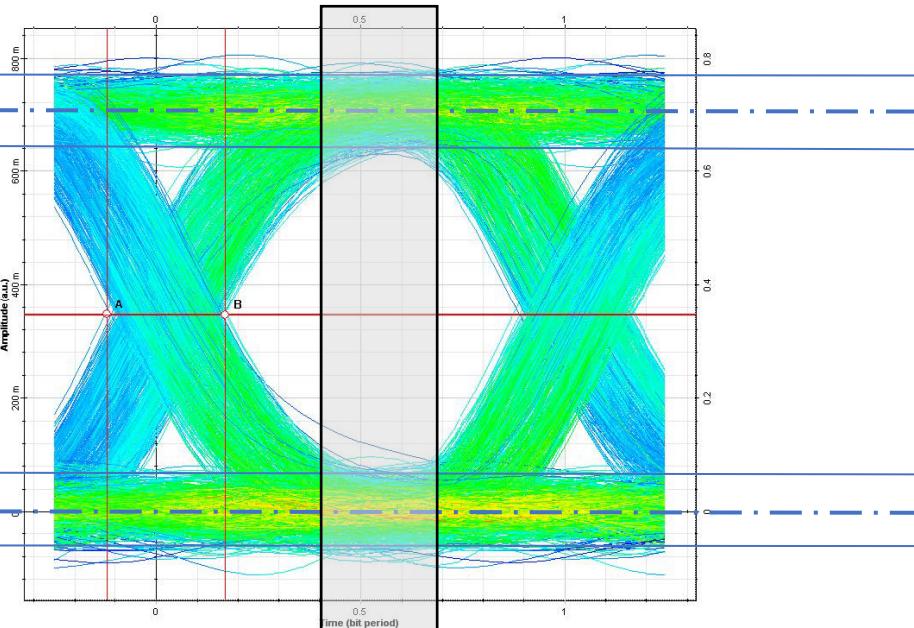
- Average number of photo-electrons:  
( $\eta_i$ : quantum efficiency) 
$$\bar{m} = \eta_i \bar{n}$$
- Gaussian noise of a photo-detector circuit characterized by a zero mean and an rms value of  $\sigma_q$  (noise parameter) within a bandwidth  $\Delta f$ .

# Exercise 5.1: Eye Diagram



«Signal degradation—Jitter», Optical System Tutorials  
<http://optiwave.com/resources/applications-resources/optical-system-signal-degradation-jitter/>

Eye diagram, NRZ (Non-Return to Zero)

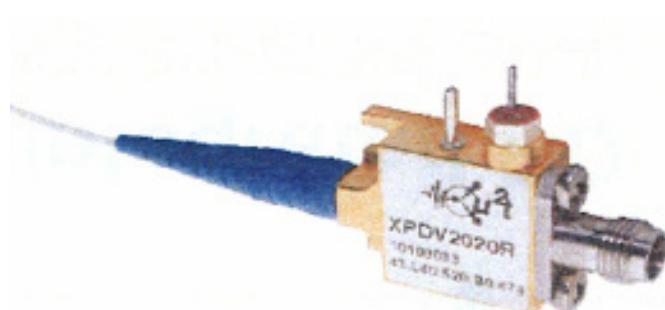
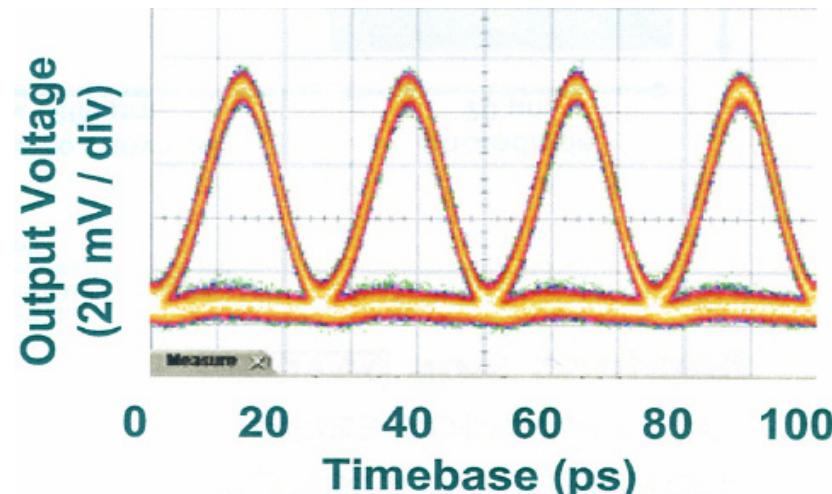
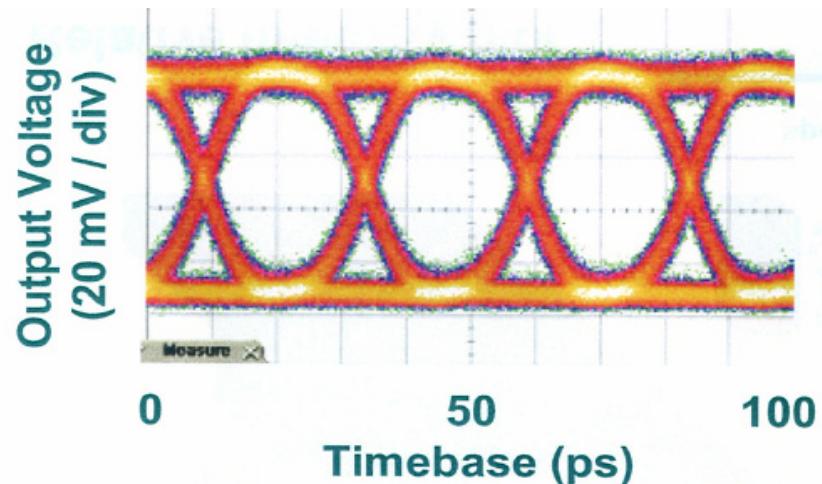


$$FWHM \cong 2.3 \cdot \sigma$$

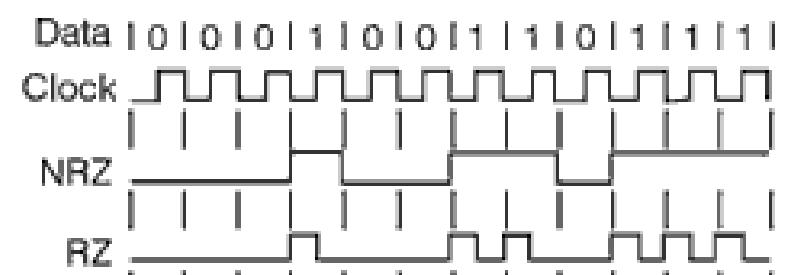
$$\rightarrow Q = (\mu_1 - \mu_0) / (\sigma_0 + \sigma_1) \rightarrow BER$$

# Exercise 5.1: High-Speed Detectors (1)

$u^2t$ : 40 GBits/s



Photoreceivers for DPSK, A.Umbach, May 2005. ©  $u^2t$  AG

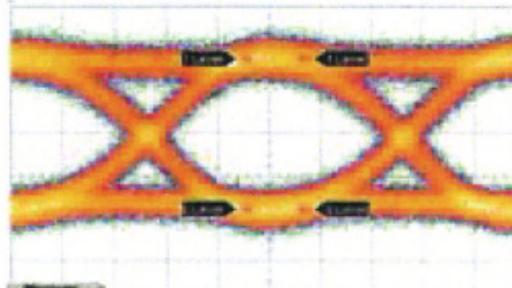


# Exercise 5.1: High-Speed Detectors (2)

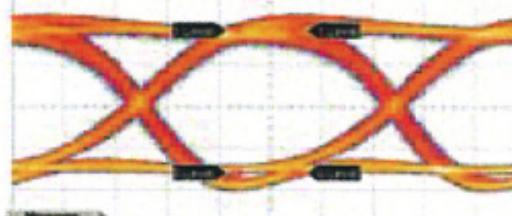
## 40 Gbit/s Eye Diagrams

Photoreceivers for DPSK, A.Umbach, May 2005. © u<sup>2</sup>t AG

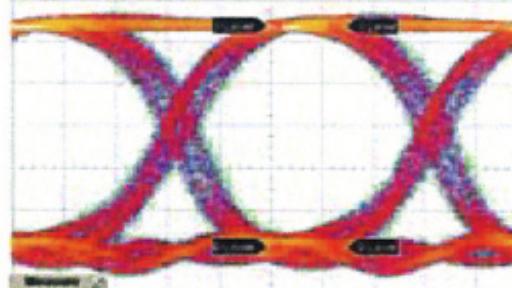
**-10 dBm, 0.1mW**  
**35 mV eye ampl.**  
**SNR = 7, RMS jitter = 1.1 ps**



**0 dBm, 1mW**  
**340 mV eye ampl.**  
**SNR = 10, RMS jitter = 0.7 ps**



**+5 dBm, 3mW**  
**520 mV eye ampl.**  
**SNR = 16, RMS jitter = 1.1 ps**

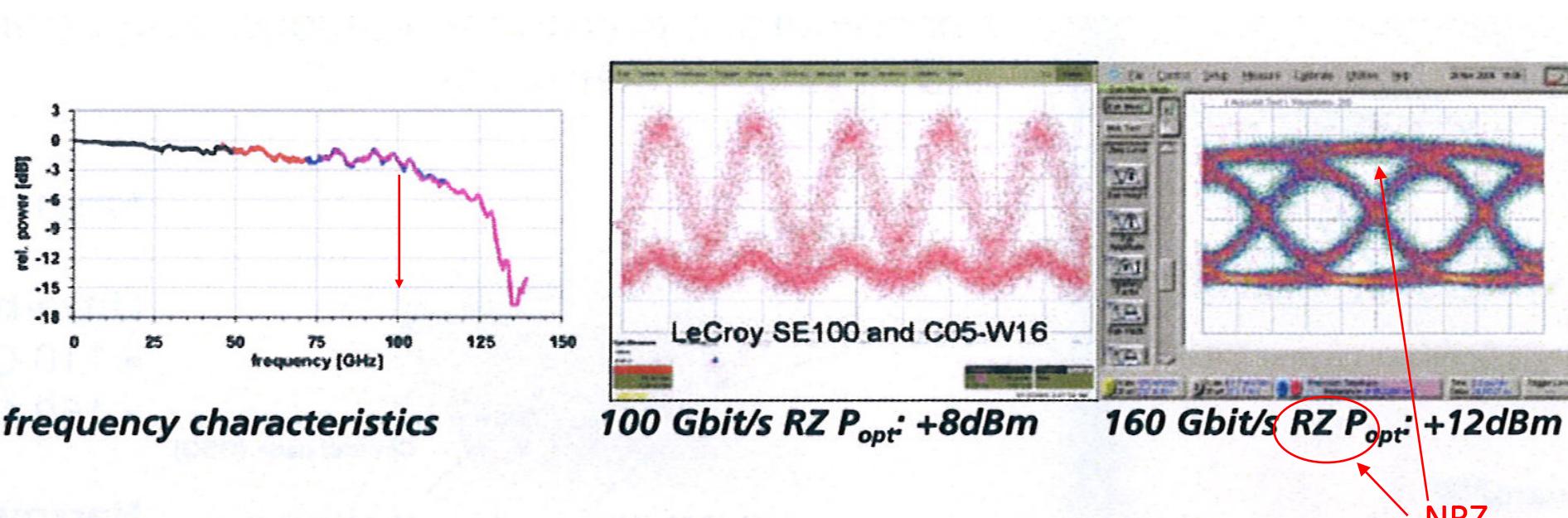


u<sup>2</sup>t: 40 GBits/s



Increasing input power  
→  
Better SNR  
“open eyes”

## Exercise 5.1: High Speed PIN Detectors (3)



- DC responsivity: 0.73 A/W
- pulse response (FWHM): 7.4 ps (@250 mV<sub>p</sub> output voltage (internally 50 Ω)).
- PDL: 0.41 dB at 1.55 μm

Heinrich Hertz Institut (HHI): up to 160 GBit/s

# Outline

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5.1 Fast photodiodes

5.2 PIN Heterostructure photodiodes

# Exercise 5.2: PIN Heterostructure Photodiode

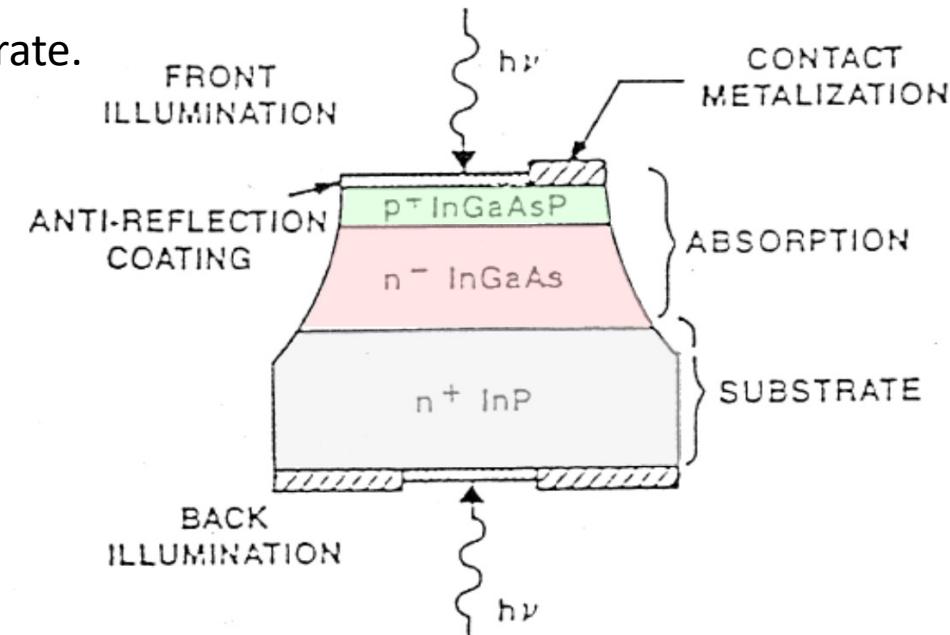
## $\text{In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$ system on an InP substrate

a) To grow a monocrystal, we have to preserve the lattice of the InP substrate.  
This involves satisfying the following relationship:

$$x = \frac{0.4562 \cdot y}{1 - 0.031 \cdot y}$$

b) In this case (“lattice matched to InP”) the gap can be changed according to:

$$E_g(y) = 1.35 - 0.72 \cdot y + 0.12 \cdot y^2 \quad [\text{eV}]$$



## Exercise 5.2: PIN Heterostructure Photodiodes

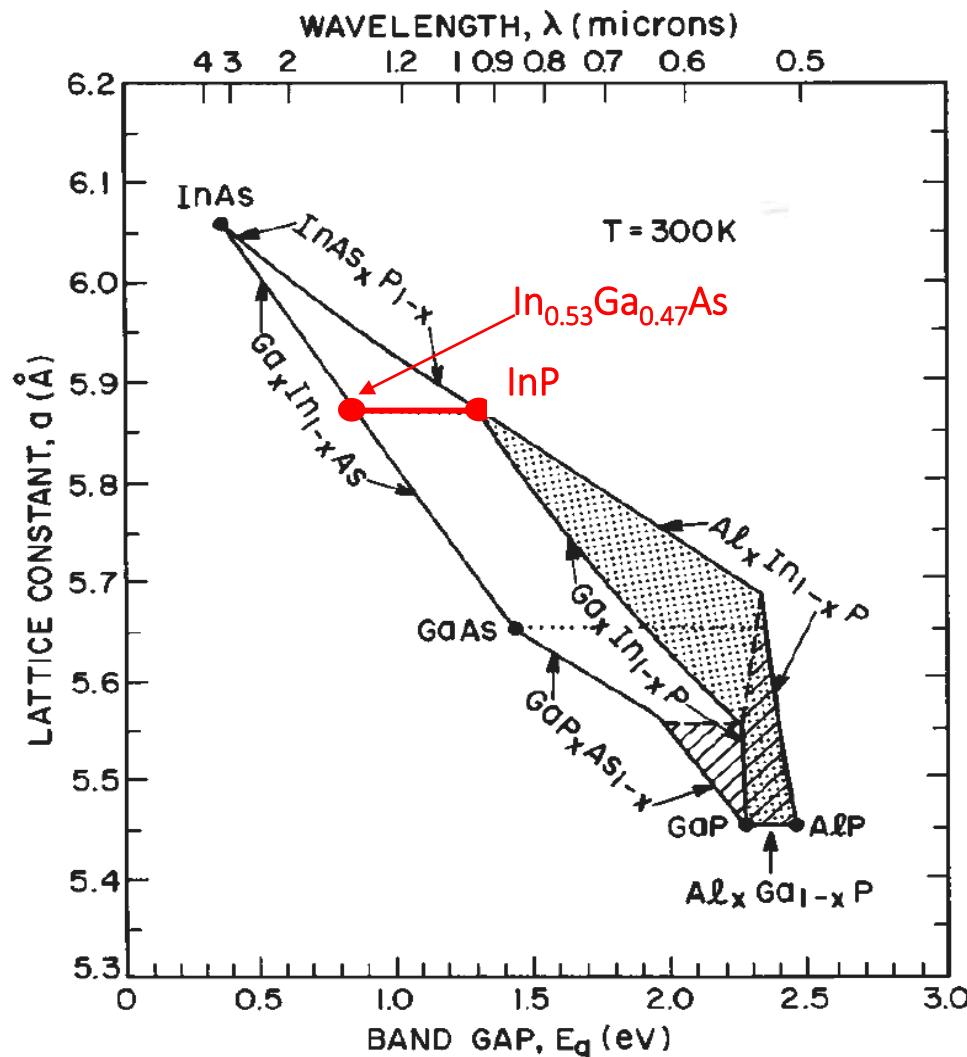
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This type of photodiode was designed for optical telecommunications and has to work at wavelengths between  $1.50 \mu\text{m}$  and  $1.60 \mu\text{m}$ . Its diameter is  $10 \mu\text{m}$ , corresponding to that of a single mode fiber optic cable.

A) Considering a superficial layer with the following composition:  $\text{In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$  with  $y=0.84$ . Sketch the quantum efficiency for front and back illumination. What is the main difference?

B) Estimate the width  $W$  of the intrinsic InGaAs region to optimize the bandwidth using a load resistance of  $R_L = 50 \Omega$ .  
(use  $\varepsilon = 12$  and  $v_{\text{sat}} = 10^5 \text{ m/s}$ )  
Does the diode have to be polarized, and if so, why?

## Exercise 5.2: Complement: $\text{In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$ Heterostructures



InGaAsP  
matched to InP

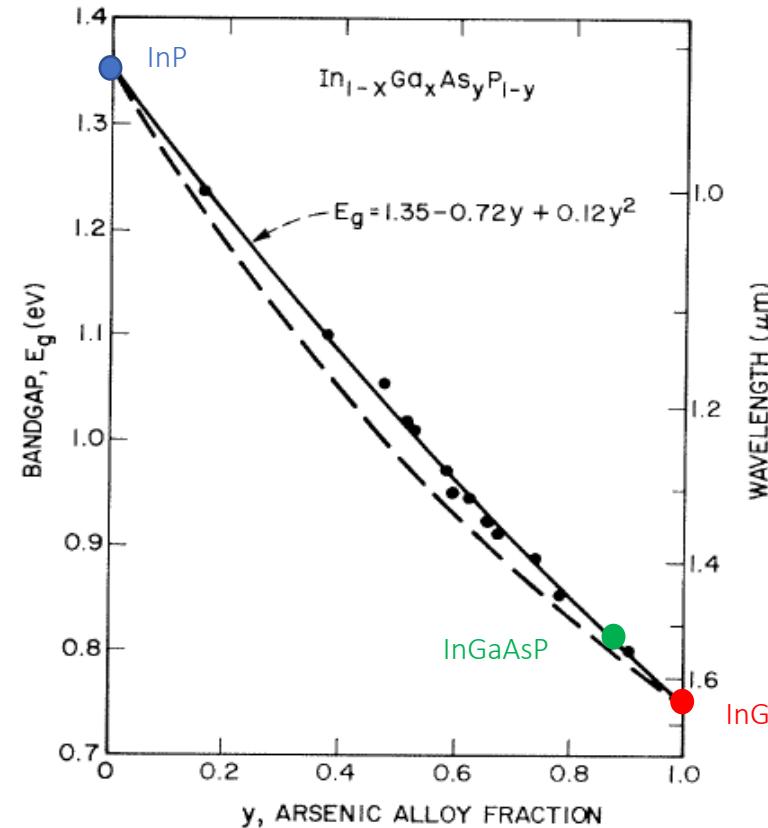
$$x = \frac{0.4562 \cdot y}{1 - 0.031 \cdot y} \cong 0.45 \cdot y$$

Agrawal, « Long-wavelength semiconductor lasers », VNR

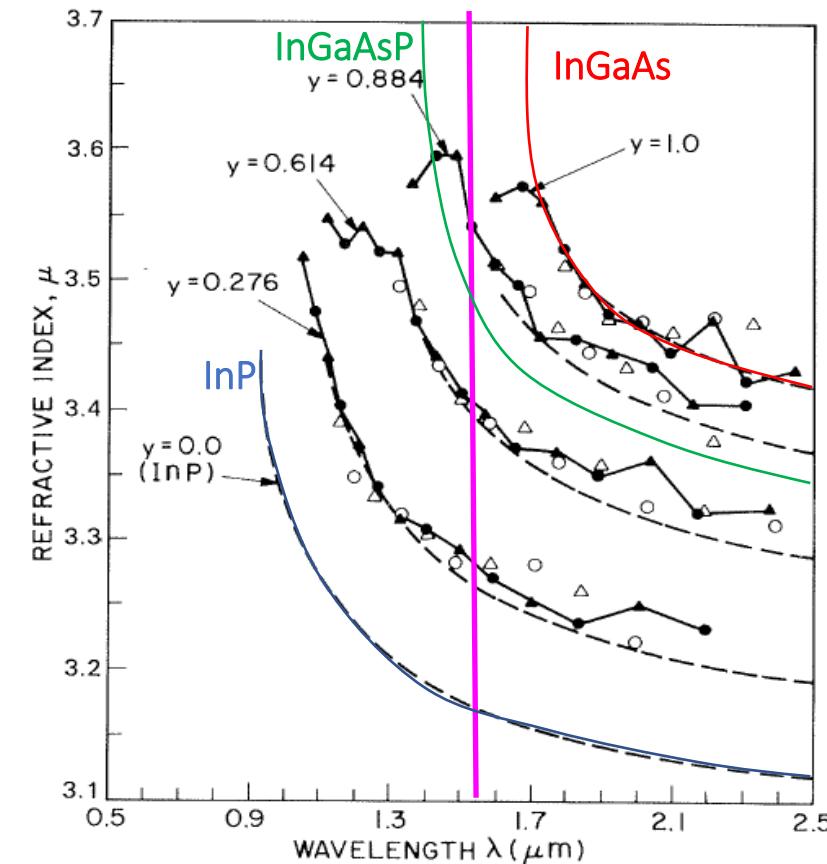
# Exercise 5.2: $\text{In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$ Matched to InP

Bandgap

$$E_g(y) = 1.35 - 0.72 \cdot y + 0.12 \cdot y^2 \quad [\text{eV}]$$

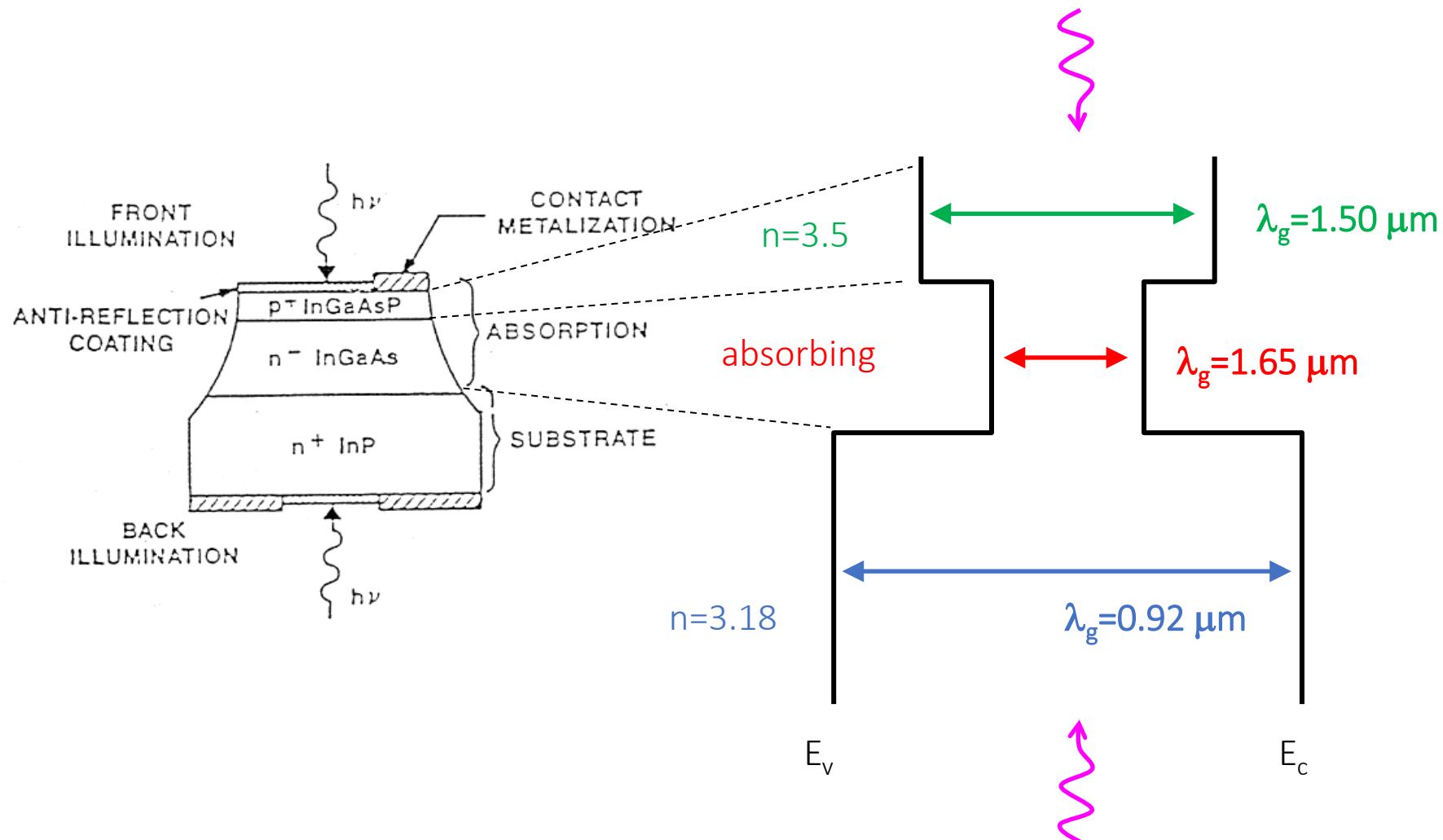


Refractive index



Agrawal, « Long-wavelength semiconductor lasers », VNR

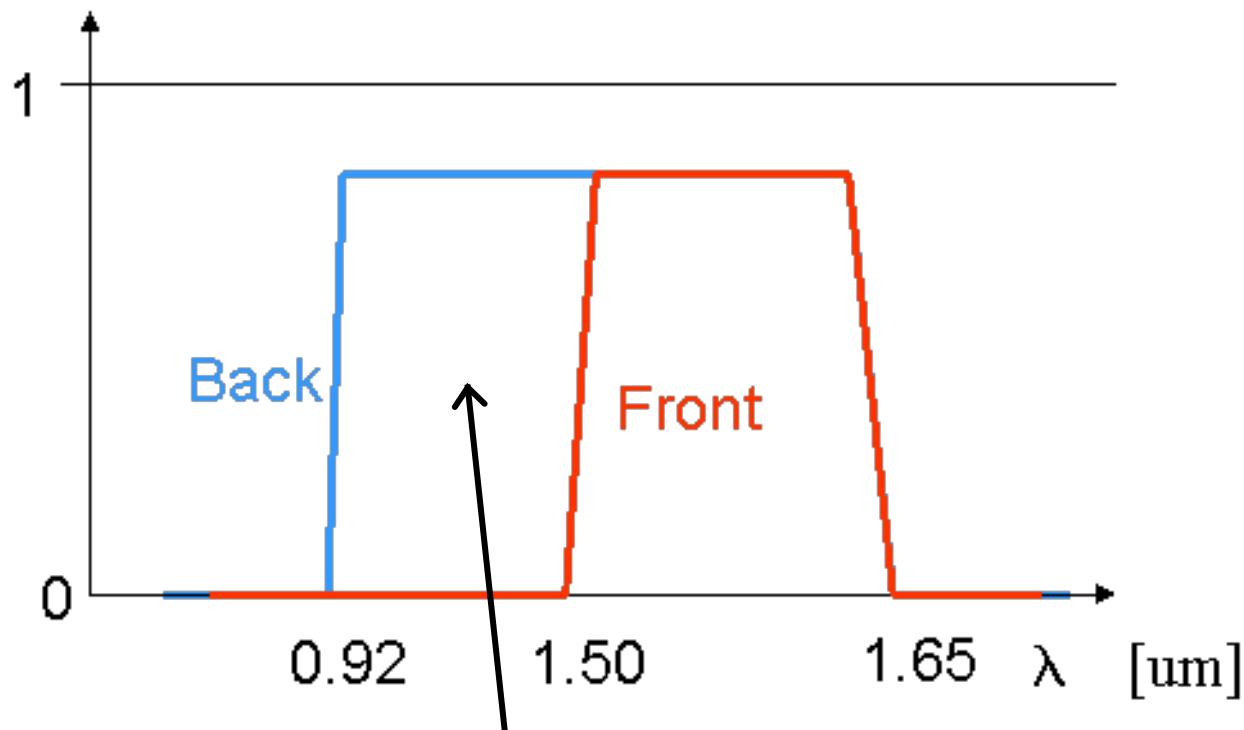
## Exercise 5.2: PIN Structure and Bands



## Exercise 5.2: PIN Heterostructure Photodiodes

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### A) Quantum Efficiency



With front illumination, the diode is sensitive between 1.50 and 1.65  $\mu\text{m}$

With back illumination, the diode is sensitive over a wider region between 0.92 and 1.65  $\mu\text{m}$

## Exercise 5.2: PIN Heterostructure Photodiodes

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a) The capacity of the depletion zone:

$$C_j = S \cdot \frac{\epsilon_0 \epsilon}{W} = \pi \left( \frac{d}{2} \right)^2 \cdot \frac{\epsilon_0 \epsilon}{W}$$

b) The cutoff frequency due to RC time:

$$f_2 = \frac{1}{4R_L C_j} = \frac{W}{R_L \pi d^2 \epsilon_0 \epsilon}$$

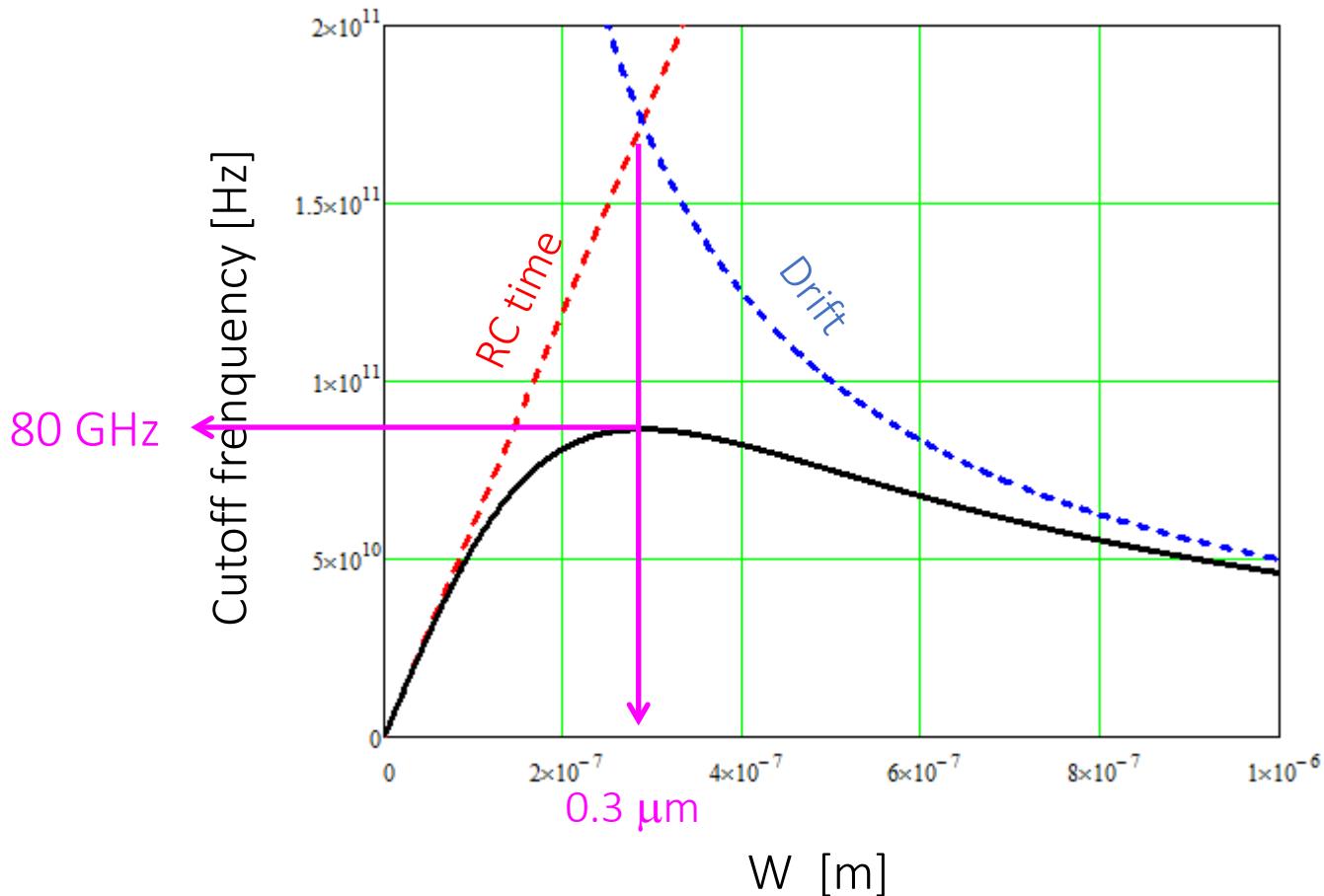
c) The cutoff frequency due to the drift time in the intrinsic zone:

$$f_{2d} = \frac{1}{2\tau_{drift}} \approx \frac{v_{sat}}{2W}$$

Optimal:

$$f_2 = f_{2d} \quad \rightarrow \quad W_{opt} = d \cdot \sqrt{\frac{\pi}{2} v_{sat} R_L \epsilon_0 \epsilon} \cong 0.3 \quad [\mu m]$$

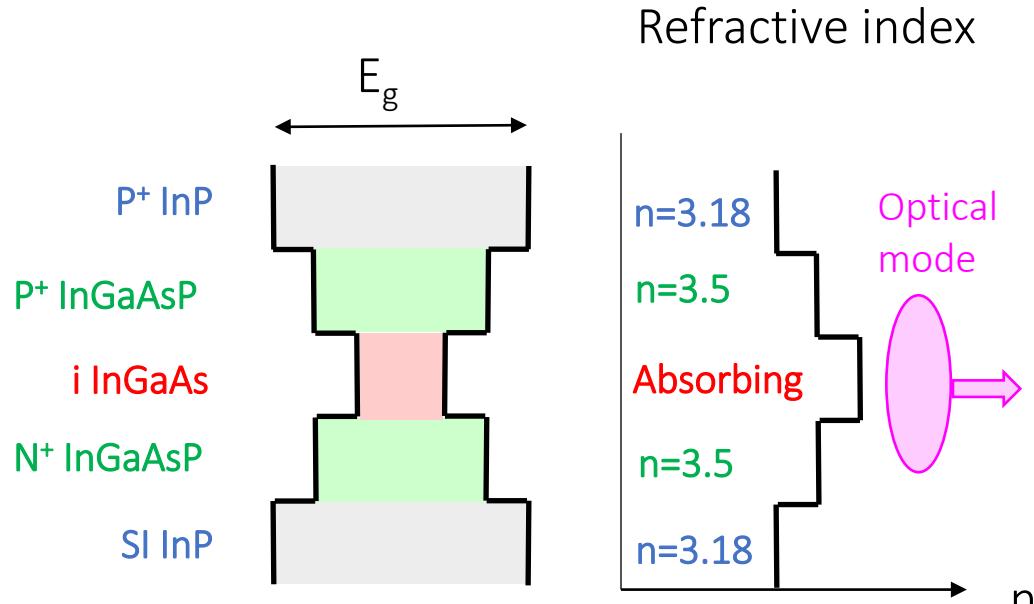
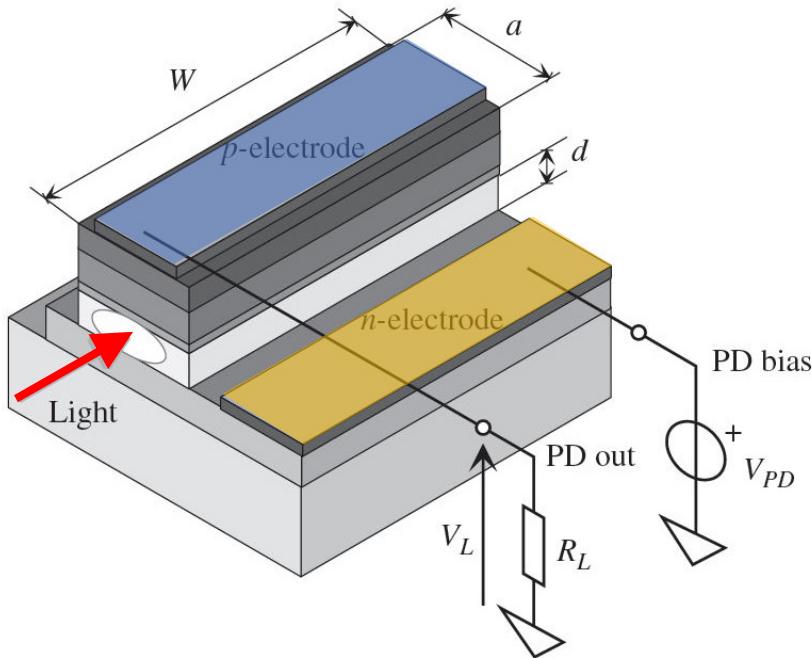
## Exercise 5.2: Example



Thin absorbing layer !!

$$\frac{1}{f_{tot}} = \frac{1}{f_2} + \frac{1}{f_{2d}}$$

# Exercise 5.2: Waveguide Photodiode (WG-PD)



G. Ghione, «Semiconductor devices for high-speed optoelectronics», Chap. 4, Cambridge

Large gap  $\rightarrow$  lower refractive index  $\rightarrow$  waveguide